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Reliability Maps for Probabilistic Guarantees of Task Motion for Robotic Manipulators

Hamid Abdi[a]*, Anthony A. Maciejewski[b] and Saeid Nahavandi[a]

[a]Centre for Intelligent Systems Research, Deakin University, Waurn Ponds Campus, Victoria 3217, Australia; [b]Electrical and Computer Engineering Department, Colorado State University, Fort Collins, CO 80523, USA

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Abstract

There are many applications for which reliable and safe robots are desired. For example, assistant robots for disabled or elderly people and surgical robots are required to be safe and reliable to prevent human injury and task failure. However, different levels of safety and reliability are required for different tasks so that understanding the reliability of robots is paramount. Currently, it is possible to guarantee the completion of a task when the robot is fault tolerant and the task remains in the fault-tolerant workspace (FTW). The traditional definition of FTW does not consider different reliabilities for the robotic manipulator's different joints. The aim of this paper is to extend the concept of a FTW to address the reliability of different joints. Such an extension can offer a wider FTW while maintaining the required level of reliability. This is achieved by associating a probability with every part of the workspace to extend the FTW. As a result, reliable fault-tolerant workspaces (RFTWs) are introduced by using the novel concept of conditional reliability maps. Such a RFTW can be used to improve the performance of assistant robots while providing the confidence that the robot remains reliable for completion of its assigned tasks.

Keywords

fault tolerant robots, workspace analysis, reliability, assistive robots, redundant robots

1. Introduction

Robots are increasingly applied for more advanced and complex tasks in various applications. This requires further study to improve their performance. For example, assistant robots have been designed for helping disabled or elderly people [1,2]. These robots are required to be safe and reliable to prevent human injury and task failure. In some tasks, if the robot is unable to accomplish the tasks, then there will be the possibility of serious human injury. Furthermore, safety

*To whom correspondence should be addressed. E-mail: hamid.abdi@deakin.edu.au

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and reliability of surgical robots are critical to prevent patient injury because of robot failure [3,4]. This is true for most applications of fault-tolerant robots from space exploration to hazardous material disposal where the robot failure can result in a catastrophic outcome [5–9]. Fault tolerance is important because it increases the dependability of the robots. However, robots are being applied to more advanced tasks where different levels of safety and reliability are required. Therefore, understanding the reliability of the robots is paramount. Currently, it is possible to guarantee the completion of a task when the robot is fault tolerant and the task remains in the fault-tolerant workspace (FTW).

Research on fault-tolerant manipulators has focused on either the control of the manipulators (e.g. fault analysis and fault-tolerant motion planning or fault-tolerant controllers) or the design of the manipulators (e.g. fault-tolerant serial manipulator or parallel manipulators [10–14]). Within the literature of the fault-tolerant control of manipulators, various subjects such as failure analysis, fault detection, fault isolation, fault identification, and fault accommodation of the manipulators are studied [15–17]. In the category of manipulator, fault tolerance for serial link manipulators can be achieved by adding extra kinematic redundancy as well as other types of redundancies, such as redundant actuators. A manipulator with extra kinematic redundancy is called serial link redundant manipulator. The added kinematic redundancy not only improves the fault tolerance of the manipulators [13,18–21], but also can improve other static and/or dynamic properties of the manipulator. There have been a number of studies on FTW of redundant manipulators [6,22–25]. There have also been studies on the reliability analysis of manipulators [16,26–28]. However, the connection between FTW and reliability has not been explicitly studied. Currently, the traditional definition of the FTW does not consider the probability of failure for different joints. Extending the definition to do so has major benefits:

(1) It accommodates the concept of reliability into the definition of a FTW.
(2) It is more consistent with the nature of failure, because failures have a probabilistic behavior that should be reflected in the definition of a FTW to be more realistic.

These benefits are especially useful when robots are applied to a wide range of applications requiring different guarantees of reliability. Such a reliable FTW can be used to improve the performance of assistant robots while providing the confidence that the robot remains reliable for the completion of its assigned tasks.

The paper is organized as follows: in Section 2, the effect of a locked joint failure on the workspace of a manipulator and the traditional definition of a FTW are presented. Section 3 discusses the reliability for different regions in the manipulator workspace and proposes two novel maps to define reliable FTWs. Then in Section 4, the properties of different regions of the workspace and maps are presented. A case study is demonstrated in Section 5 and the reliability map is used to obtain a reliable fault-tolerant workspace (RFTW). Then in Section 6, additional extensions of the proposed method are discussed. Finally, the concluding remarks are presented.

2. Fault-Tolerant Workspace

2.1. Workspace

The forward kinematics of a serial manipulator is introduced by

$$\mathbf{x} = \mathbf{f}(\mathbf{q}) \tag{1}$$

where $\mathbf{f} : R^n \rightarrow R^m$ is the forward kinematic function, $\mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^{\mathrm{T}}$ is the joint variable and $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_m]^{\mathrm{T}}$ is the end-effector pose and orientation variables.

The joint variables define the n-dimensional configuration space (C-space) and positional variables define the m-dimensional workspace of the manipulator. The workspace of a manipulator is a set of possible pose and orientations using all possible values of joint angles within their range of joint limits. In this paper, we assume an assistive robot arm is providing a motion task from a start point $\mathbf{x}_{\text{start}}$ to an end point $\mathbf{x}_{\text{end}}$. For example, the robot may be assisting elderly or disabled people to do a critical task. For surgical robots, the task could be cutting a specific location of the body or doing a surgical procedure.

2.2. Reduced Workspaces

Any locked joint failure eliminates a part of the workspace. The eliminated part depends on the geometry of the manipulator and the configuration of the manipulator at the time of failure. If we assume that W is the workspace of the healthy manipulator, then a fault of the kth joint reduces W to ${}^kW$ (where ${}^kW \subset W$), which is called kth reduced workspace. The reduced workspace represented by ${}^kW$ defines the range of points that the manipulator can reach after the failure in the kth joint. Therefore, the motion tasks in this region can be accomplished despite the failure of the kth joint. For single joint failures, there are n reduced workspaces corresponding to failures of joint 1 to n and are denoted by ${}^iW$ for $i = 1, 2, \ldots, n$. Reduced workspaces for multiple failures can be obtained by performing a workspace analysis while considering the faulty joints and will be denoted with a list of the failed joints in the preceding superscript, e.g. for failures in the first and second joints the reduced workspace is denoted by ${}^{1,2}W$.

2.3. Traditional Definition of FTW

Traditionally, the intersection between ${}^iW$ for all $i = 1, 2, \ldots, n$ is called the FTW. This region is fault tolerant for any single joint failure and the points in the region are reachable regardless of which joint fails. Assume that the FTW is depicted by $\tilde{W}$ then

$$\tilde{W} = \bigcap_i {}^iW, \quad i = 1, 2, \ldots, n \tag{2}$$

It is important to note that the FTW depends on the configuration of the manipulator at the time of failure. Here, we focus on single joint failures because the single joint failures are much more probable than multiple joint failures. For instance, if a single joint failure probability is 0.001, then the failure probability of the manipulator for single joint failures is $0.001n$ and for two joint failures is $n(n-1)5 \times 10^{-7}$.

The FTW for two locked joint failures is

$$\tilde{W} = \bigcap_{i,j} {}^{i,j}W, \quad 1 \leq i<j \leq n \tag{3}$$

3. Reliability Analysis

3.1. Reliability and Failure Possibility

Reliability of manipulators is defined by the possibility that manipulators work without any interruption over a specific time period [28]. If the failure probability density function for the kth joint of the robot is given by $p_k(t)$, then the cumulative distribution function (CDF) is obtained using

$$P_k(t) = \int_{\tau=0}^{\tau=t} p_k(t)d\tau \tag{4}$$

where $P_k(t)$ is the probability of the joint failure until time t.

The possibility that the joint of the manipulator remains healthy at time t_f is one minus the value of CDF or $1 - P_k(t_f)$. This probability is known as the reliability of the joint and is denoted by $r_k(t_f) = 1 - P_k(t)$. For the sake of simplicity, we use the convention where r_k represents $r_k(t_f)$. In the remainder of this paper, the reliability values of r_k for $k = 1, 2, \ldots, n$ are assumed to be given.

The manipulator is a serial system so that the total reliability of the manipulator is obtained by

$$r = \prod_k r_k(t_f) \tag{5}$$

3.2. Connection Between Reliability and Workspace

Different joints have different failure possibilities and consequently a different level of reliability. The postfailure workspace of the manipulator is inherently related to the joint reliability. For example, if the reliability of a joint is 100% then the joint will never fail and knowing that affects the FTW. Therefore, when combining the reduced workspaces, one can consider the effect of different reliabilities for the different joints.

The intersection rule for the FTW in Equation (2) does not consider the reliability of different joints. Every reduced workspace is accounted for in the same way regardless of whether the associated joint has a low reliability or a high reliability. The lack of probability information motivated us to define a new measure for combining the reduced workspaces. The present paper extends the FTW in order to address the different reliabilities of different joints. This extension results in a more refined definition of the FTW that allows one to quantify the level of reliability. This extended FTW will be referred to as the RFTW.

In order to do this, we suggest using the information on the reliability of the joints when intersecting the reduced workspaces. This can be achieved by assigning a reliability to each point of the workspace. Therefore, the workspace is no longer divided into fault-tolerant and fault intolerant workspaces. To achieve this goal we first introduce the concept of a reliability map in the following.

3.3. *Regions of Workspace with Uniform Reliability*

To introduce the reliability maps, we use the conceptual diagram in Fig. 1. For the conceptual case, assume that the workspace of the manipulator is W and the workspace includes three reduced workspaces depicted by ${}^{1}W$, ${}^{2}W$, and ${}^{3}W$. This case can correspond to a 3DoF planar manipulator. We define different regions in W that are not overlapping and have constant reliability. We denote the regions by S with a preceding superscript. These regions in Fig. 1 are ${}^{0}S$, ${}^{1}S$, ${}^{2}S$, ${}^{3}S$, ${}^{12}S$, ${}^{13}S$, ${}^{23}S$ and ${}^{123}S$. These regions are mathematically defined by

$$ {}^{0}S = W - \left(\bigcup_{i} {}^{i}W \right), \quad i = 1, 2, 3 \tag{6} $$

$$ {}^{i}S = {}^{i}W - \left(\bigcup_{k \neq i} {}^{k}W \right), \quad i, k = 1, 2, 3 \tag{7} $$

$$ {}^{ij}S = ({}^{i}W \cap {}^{j}W) - {}^{k}W, \quad i, j, k = 1, 2, 3, \quad i \neq j \neq k \tag{8} $$

$$ {}^{123}S = \bigcap_{i} {}^{i}W, \quad i = 1, 2, 3 \tag{9} $$

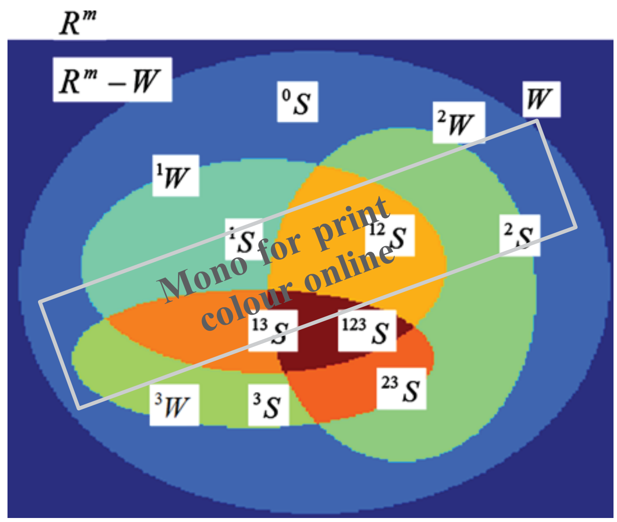

Figure 1. Regions of the workspace include ^0S (light blue color region), ^1S (the green part of ^1W), ^2S (the green part of ^2W), ^3S (the green part of ^3W), ^{12}S (is the between ^1W and ^2W), ^{13}S (is between ^1W and ^3W), ^{23}S (is between ^2W and ^3W), and ^{123}S (is between the three reduced workspaces).

The above equations can be explained as: ^0S is the set of points in the workspace with no intersection with any reduced workspaces, ^iS is the subset of ^iW with no intersection with other reduced workspaces, ^{ij}S is a subset of $(^iW \cap {}^jW)$ with no intersection with other reduced workspaces, and ^{123}S is the intersection of all reduced workspaces.

3.4. *Properties of the Different Regions of Workspace*

The following properties of these sets can be shown and are illustrated via the conceptual case in Fig. 1.

Property 1: The intersection between regions is null. For example $^{12}S \cap {}^1S = \{\}$.
Property 2: The union of the regions is equal to the workspace. For the conceptual case, $^0S \cup {}^1S \cup {}^2S \cup {}^3S \cup {}^{12}S \cup {}^{13}S \cup {}^{23}S \cup {}^{123}S = W$.
Property 3: Each region has a constant reliability related to the reliability of the associated joints. This property is discussed in Section 3.5.
Property 4: Note that the whole space is R^m and the dark blue part of the space is $R^m - W$ and $^1W = {}^1S \cup {}^{12}S \cup {}^{13}S \cup {}^{123}S$.

Similar regions can be defined for manipulators with more DoF. The extension is discussed later half of this paper. Two types of reliability values can be defined for the regions of the workspace. The first is a reliability value and the second is a conditional reliability value. The conditional reliability value is subjected to single joint failures. These two values will be introduced in the next two subsections.

3.5. *Reliability Values*

In this section, we discuss the reliability for the different regions of Fig. 1. However, it is possible to use the method for other manipulators and the general case will be discussed later in the paper. For the 3DoF manipulator of the conceptual case, the reliability of the three joints are assumed to be given by r_1, r_2, and r_3. If a point belongs to ${}^0S = W - ({}^1W \cup {}^2W \cup {}^3W)$ then this part of the workspace is reachable if all joints are functional. The reliability that the point in ${}^0S$ is reachable is $r({}^0S) = r_1r_2r_3$. If a point belongs to ${}^1S = {}^1W - ({}^2W \cup {}^3W)$ then this point is reachable when joints 2 and 3 are healthy regardless of the status of joint 1. Therefore, the reliability that the point is reachable is $r({}^1S) = r_2r_3$. Similarly, $r({}^2S) = r_1r_3$ and $P({}^3S) = r_1r_2$. If the point belongs to ${}^{12}S = ({}^1W \cap {}^2W) - {}^3W$ then the point is reachable with the reliability of $r({}^{12}S) = r_2r_3 + r_1r_3 - r_1r_2r_3$. Similarly, $r({}^{13}S) = r_1r_2 + r_2r_3 - r_1r_2r_3$ and $r({}^{23}S) = r_1r_2 + r_1r_3 - r_1r_2r_3$ and finally, if the point belongs to ${}^{123}S = ({}^1W \cap {}^2W \cap {}^2W)$ then it is reachable when all the joints are healthy or any single joint failure case. Therefore, the reliability is $r({}^{123}S) = r_1r_2r_3 + (1 - r_1)r_2r_3 + r_1(1 - r_2)r_3 + r_1r_2(1 - r_3) = r_1r_2 + r_1r_3 + r_2r_3 - 2r_1r_2r_3$. Finally, for the point in $R^m - W$, we assign a zero reliability value. This region in Fig. 1 is the exterior region of the largest ellipse shown with dark blue color.

3.6. *Conditional Reliability Values*

Assume that J indicates a healthy joint and $\bar{J}$ indicates a faulty joint. If a single joint failure occurs then the postfailure workspace is $\hat{W}$, and the probability of single joint failure is

$$\begin{aligned} p(\hat{W}) &= P(\bar{J}_1J_2J_3) + P(J_1\bar{J}_2J_3) + P(J_1J_2\bar{J}_3) \\ &= (1 - r_1)r_2r_3 + r_1(1 - r_2)r_3 + r_1r_2(1 - r_3) \\ &= r_1r_2 + r_1r_3 + r_2r_3 - 3r_1r_2r_3 \end{aligned} \tag{10}$$

where p indicates the probability.

Obviously, the conditional reliability of ${}^0S$ is zero because ${}^0S$ is not reachable after any single joint failure, therefore

$$cr\left(\frac{{}^0S}{\hat{W}}\right) = 0 \tag{11}$$

where $cr(\,)$ indicates the conditional reliability value of the region.

The conditional reliability for the case of a single joint failure in the regions of ^{1}S, ^{2}S, and ^{3}S is

$$cr({}^{i}S) = p\left(\frac{{}^{i}S}{\hat{W}}\right) = \frac{p\left({}^{i}S \cap \hat{W}\right)}{p\left(\hat{W}\right)} = \frac{p({}^{i}S \cap (\bar{J}_1 J_2 J_3 \cup J_1 \bar{J}_2 J_3 \cup J_1 J_2 \bar{J}_3))}{p\left(\hat{W}\right)}$$
$$= \frac{(1 - r_i) r_j r_k}{r_1 r_2 + r_1 r_3 + r_2 r_3 - 3 r_1 r_2 r_3} \tag{12}$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$.

The conditional reliability value for ^{12}S, ^{13}S, and ^{23}S are obtained using

$$cr\left({}^{12}S\right) = p\left(\frac{{}^{12}S}{\hat{W}}\right) = \frac{(1 - r_1) r_2 r_3 + r_1 (1 - r_2) r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3 - 3 r_1 r_2 r_3} \tag{13}$$

where $cr(\,)$ indicated the conditional reliability.

In closed form, the conditional reliability values for ^{12}S, ^{13}S, and ^{23}S are obtained using

$$cr({}^{ij}S) = cr({}^{i}S) + cr({}^{i}S), \quad i, j = 1, 2, 3 \quad \text{and} \quad i < j \tag{14}$$

Finally, the conditional probability of ^{123}S is obtained as 1. This conditional reliability is physically meaningful because the region ^{123}S is 100% fault tolerant

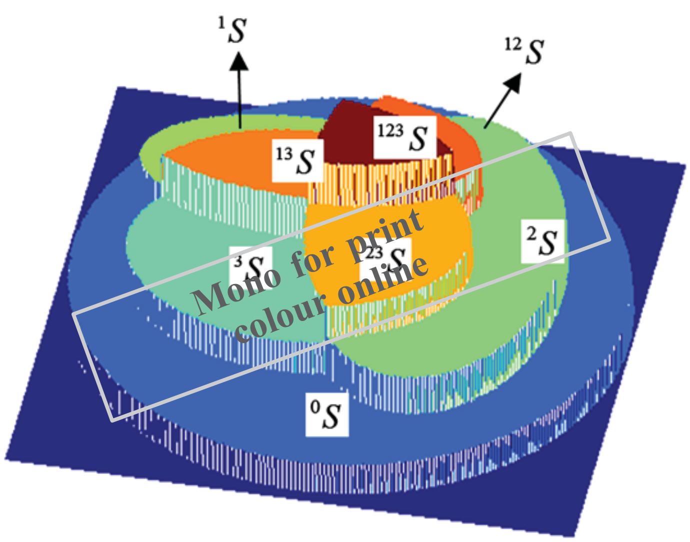

Figure 2. Illustrative reliability map for the conceptual case where the height of each region indicates the reliability value. The conditional reliability map is similar to the reliability map and the conditional reliability values are related to the reliability values via Equation (15).

for a single joint failure. For the point in $R^m - W$, we assign a zero conditional reliability value.

3.7. Reliability and Conditional Reliability Maps

From the reliability and conditional reliability values, it is possible to define a map of reliability and a map of conditional reliability for the workspace. The maps are easily illustrated for planar manipulators. The availability of the maps for 3D positional manipulators and spatial manipulators is discussed later.

If the reliability and conditional reliability values are calculated for the regions of Fig. 1, then two maps for reliability and conditional reliability are obtained. Figure 2 demonstrates the regions with different reliability values in a 3D scheme.

4. Reliable Fault-Tolerant Workspace

The reliability map and the conditional reliability map specify a reliability value for different regions of the workspace. The following properties can be observed for the reliability and the conditional reliability maps.

4.1. Properties of the Reliability and Conditional Reliability Maps

Property 1: The points in $R^m - W$ have the reliability and conditional reliability values of zero. This is because these points are not reachable under any condition.

Property 2: The points in ${}^0S = W - \bigcup_i {}^iW$ have the reliability value of $r_1r_2r_3$ and conditional reliability value of zero. Therefore, they are reachable only when there is no failure.

Property 3: The points in $\tilde{W} = \bigcap_i {}^iW$ have the reliability value of $r_1r_2 + r_1r_3 + r_2r_3 - 2r_1r_2r_3$ and the conditional reliability value of 1. This region is 100% fault tolerant for any single joint failures.

Property 4: The regions that are the intersection of two or more reduced workspaces have higher reliability than the regions of the reduced workspaces with no intersection with the other reduced workspaces.

Property 5: The reduced workspace and the different regions of the workspace for a given manipulator entirely depend on the initial configuration of the manipulator. If the manipulator has an unknown single locked joint failure at the initial time, then the regions intersecting two or more reduced workspaces can be reachable with the specified reliability.

Property 6: The values of the conditional reliability are related to reliability by

$$cr(.) = \frac{r(.) - r_1 r_2 r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3 - 3 r_1 r_2 r_3} \tag{15}$$

where $r(.)$ is the reliability value and $cr(.)$ is the conditional reliability value of any region.

4.2. Reliable Workspace

Robotic technology is going to introduce robots that are suitable for a wide range of tasks. Each task requires a specific level of reliability for fault tolerance. From the task reliability, one can set a threshold for the desired reliability of the fault tolerance and choose the regions in the reliability map that have higher reliability value than the threshold. These regions are suitable for operation while they are reliable enough for fault tolerance. The threshold can be specified for the reliability or conditional reliability maps. In general, the reliability threshold helps to decide whether a region is reliable to work in or not, but if the main concern is the fault tolerance, then the conditional reliability threshold is more suitable. This is because the FTW has the conditional reliability of 1 and the fault intolerant workspace has the conditional reliability of 0. After applying the threshold, the union of the regions that have higher reliability or conditional reliability of the given threshold is used to determine the reliable workspace. We call this region a “RFTW”. Knowing the reliable workspace is useful, because if the entire path of the motion tasks is in the reliable workspace, then the task can be accomplished with the desired level of reliability. In contrast, when a part or the entire path lies outside of the reliable workspace, then the task can be aborted due to low reliability. The information from the reliability maps can also be used to perform motion planning for reliable operation.

4.3. Selecting the Threshold Value

The threshold can be determined from the desired tasks. For example, if the manipulator is being applied to a noncritical task such as a regular pick and place task, then a low reliability threshold can be selected. This selected value extends the reliable workspace considerably. In contrast, if the manipulator is used in a surgical task on the human body, then a high threshold is selected. This brings higher reliability in a smaller FTW. If the reliability threshold is set to 100%, then the RFTW proposed in the present paper results in the common definition of FTW in (2), which is the intersection between all the reduced workspaces. Therefore, the “RFTW” is a consistent extension of FTW of the manipulator.

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5. Case Study

5.1. Case Study Parameters

In this case study, a planar manipulator with three DOF is modeled using Matlab robotic toolbox [29]. The link lengths are 50, 35, and 20 cm for the first, second, and the third link. Let us assume the first joint has the reliability of $r_1 = 0.99$, while the other two have the reliability of $r_2 = 0.9$ and $r_3 = 0.8$. The joint angles' limits are selected as $\pm 100^\circ$, $\pm 130^\circ$, and $\pm 160^\circ$, respectively for the joints 1–3. In Fig. 3, the workspace of the healthy manipulator and in Figs. 4–6 the reduced workspaces due to single locked joint failures are illustrated.

The configuration of the manipulator is shown at the time of failure where the joint angles at failure time are 70°, -96°, and -50° for joints 1–3, respectively. The two cuts in the top side of the reduced workspaces in Figs. 5 and 6 are because of the joint limits. The workspace and the reduced workspaces are obtained by sweeping the joint angle values between the minimum and the maximum joint angle values and calculating the forward kinematics of the manipulator.

5.2. Fault-Tolerant Workspace

The FTW of the manipulator at the given configuration is obtained by using the intersection rule in Equation (2). The procedure to find the FTW is shown in Fig. 7 by overlapping the reduced workspaces, with the results of overlapping

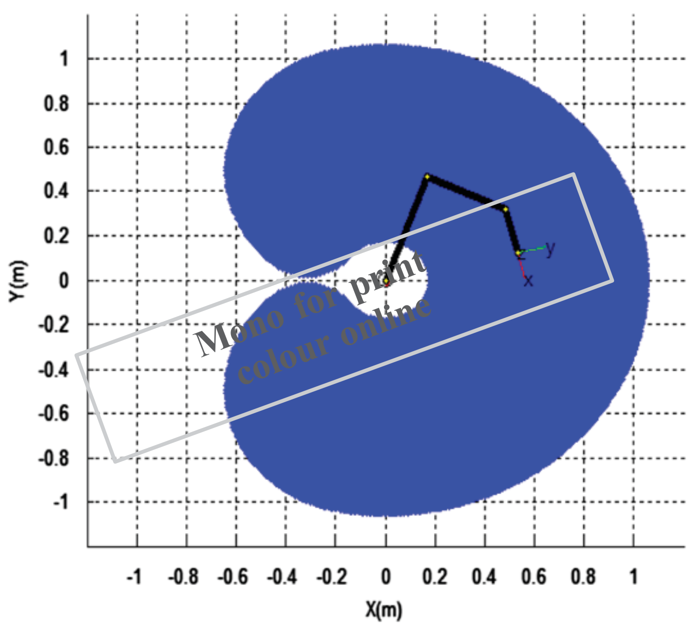

Figure 3. Workspace of a 3DoF planar manipulator.

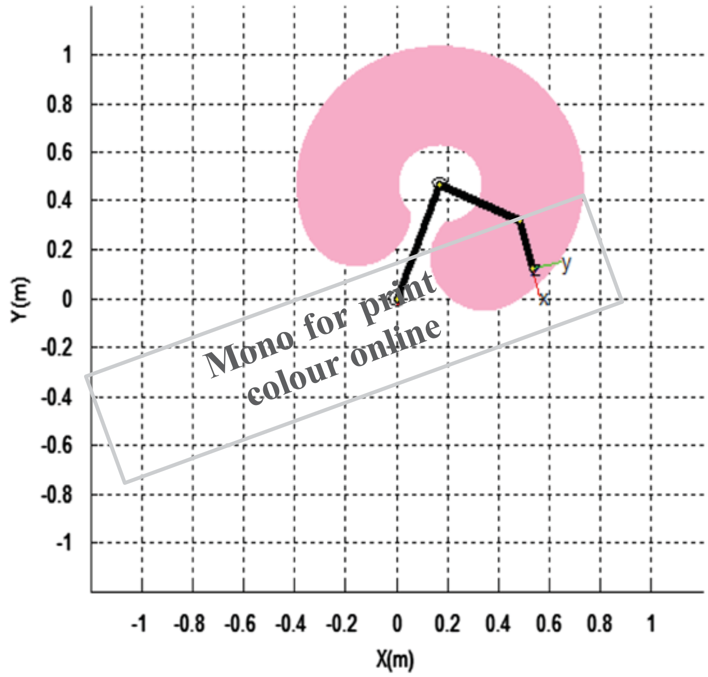

Figure 4. Reduced workspace due to failure of the first joint.

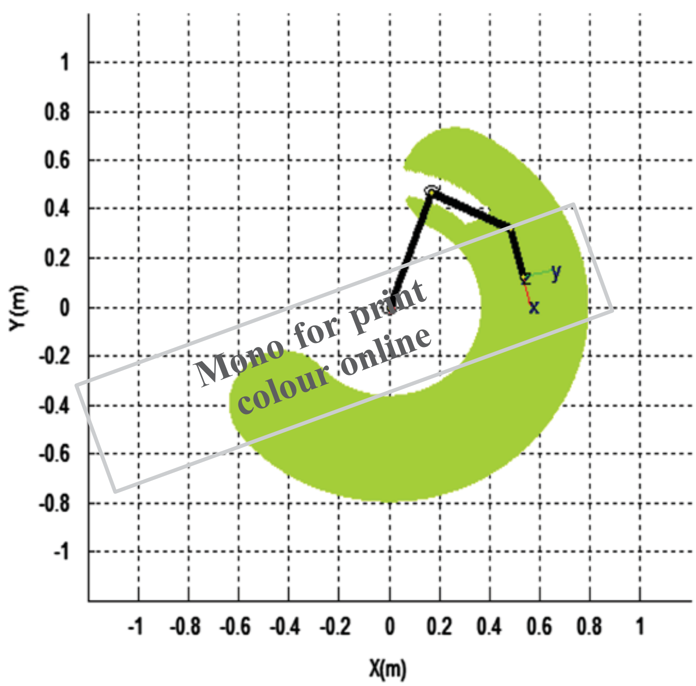

Figure 5. Reduced workspace due to failure of the second joint.

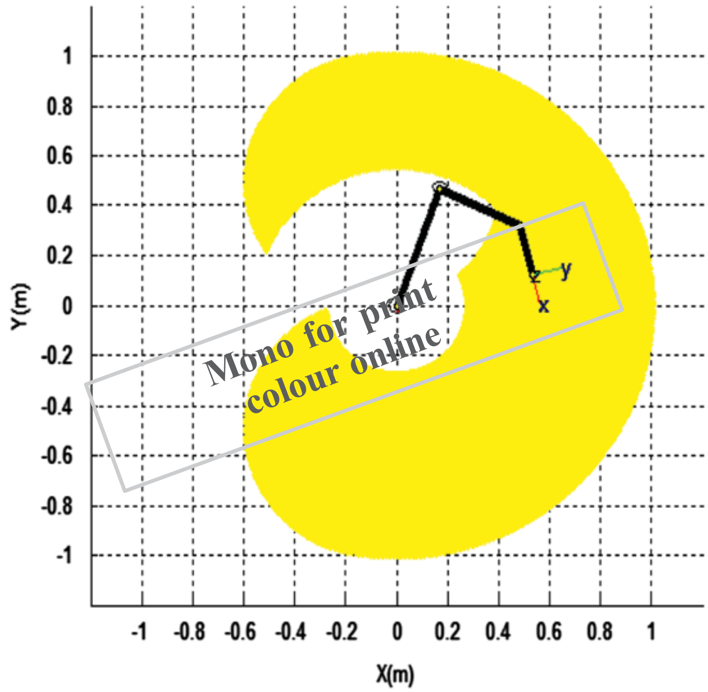

Figure 6. Reduced workspace due to failure of the third joint.

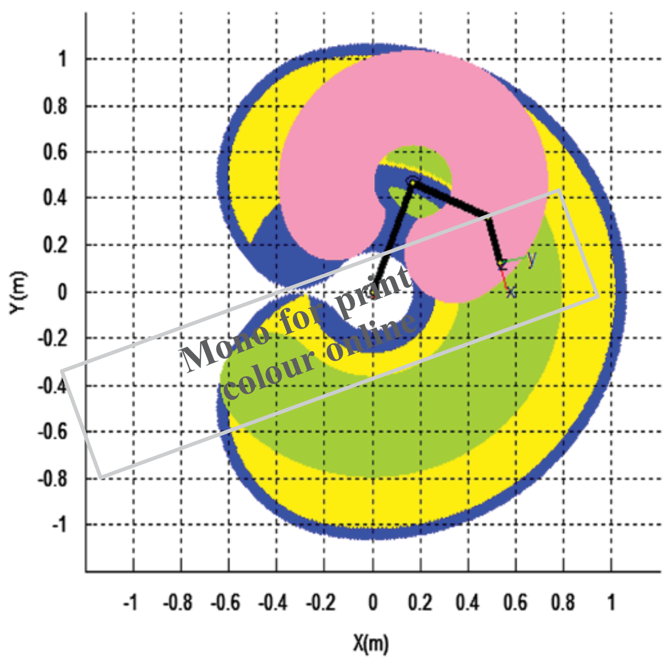

Figure 7. Overlapping the reduced workspaces.

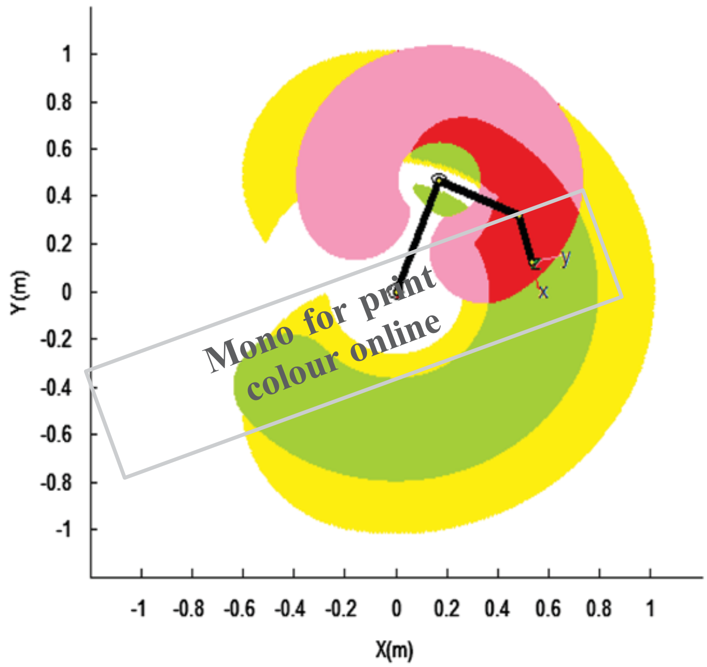

Figure 8. FTW using the conventional approach (the red region).

indicated in Fig. 8, where each region is shown with a different color. From Fig. 8, it is clearly seen that the region in red is the FTW and it is approximately 10 times smaller than the original workspace in Fig. 3. If the manipulator starts the task from a point in this region and the ending point is in this region, then even with any single joint failure the motion tasks can be accomplished with 100% confidence. Therefore, if one wants to plan a task to be fully fault tolerant, it has to be in this region.

5.3. *Reliability and Conditional Reliability Map*

Different regions of the workspace for the manipulator in this case study are shown in Fig. 9. The legend in the figure indicates the color of the different regions of the workspace. The reliability map of the workspace for the manipulator is calculated based on the values of $r_1 = 0.99$, $r_2 = 0.9$, and $r_3 = 0.8$. The reliability and the conditional reliability values for the regions are presented in Table 1.

The reason that ${}^1S$ has a low conditional reliability is because it requires that joints 2 and 3 be healthy. If we compare the reliability of joint 1 ($r_1 = 0.99$) with the reliability of joints 2 and 3 ($r_2 = 0.9$, $r_3 = 0.8$), then their much lower reliabilities explain the value of 0.027 for the conditional reliability of ${}^1S$. The results shown in the table indicate that the regions that require the third joint to be healthy have a lower conditional reliability. This is because the reliability of the third joint is so low.

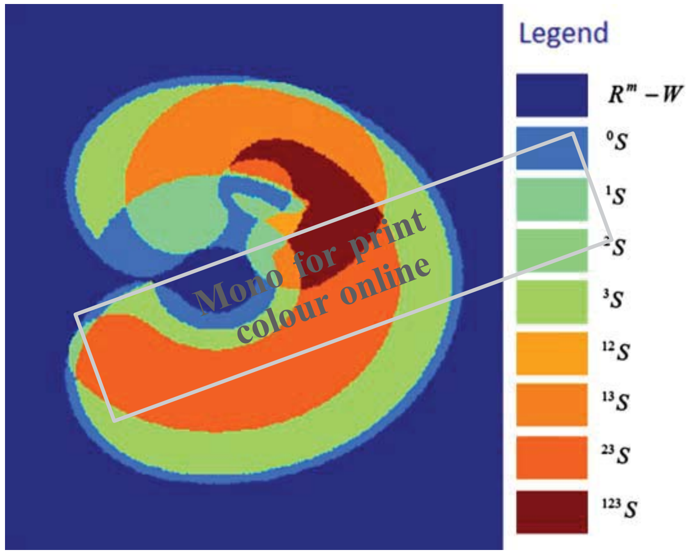

Figure 9. Regions of the workspace with constant reliability values. The reliability and conditional reliability values for each region are shown in Table 1.

Table 1.
Reliability and conditional reliability values for the 3DoF planar manipulator

Region	Reliability value equations	Reliability value	Conditional reliability value
$R^m - W$	0	0.000	0.000
${}^{0}S$	$r_1r_2r_3$	0.713	0.000
${}^{1}S$	r_2r_3	0.720	0.027
${}^{2}S$	r_1r_3	0.792	0.299
${}^{3}S$	r_1r_2	0.891	0.674
${}^{12}S$	$r_2r_3 + r_1r_3 - r_1r_2r_3$	0.799	0.327
${}^{13}S$	$r_1r_2 + r_2r_3 - r_1r_2r_3$	0.898	0.701
${}^{23}S$	$r_1r_2 + r_1r_3 - r_1r_2r_3$	0.970	0.973
${}^{123}S$	$r_1r_2 + r_1r_3 + r_2r_3 - 2r_1r_2r_3$	0.977	1.000

5.4. Reliable Fault-Tolerant Workspace

In the reliability map and conditional reliability map, each region has a value between 0 and 1. The highest reliability and conditional reliability is for the intersection of the three reduced workspaces, i.e. 0.977 for the reliability and 1.000 for the conditional reliability. The reliability is not one because of the possibility of multiple joint failures. From Table 1, one can see that some regions have higher reliability and higher conditional reliability values than others. From

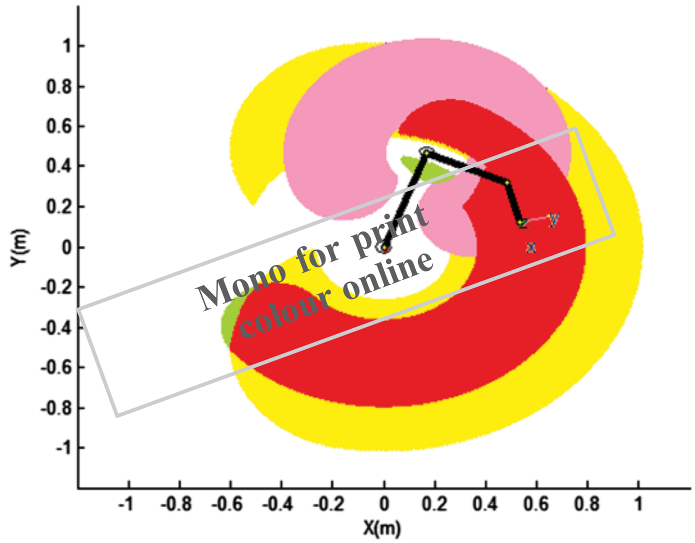

Figure 10. RFTW (the red region) with the reliability of 0.977 and conditional reliability of 0.973.

the maps, one can see that region ${}^{123}S$ with a reliability of 0.977 and conditional reliability of 1 is 100% fault tolerant for a single joint failure. After ${}^{123}S$, the region ${}^{23}S$ has the reliability value 0.970 and the conditional reliability value of 0.973, i.e. in terms of reliability there is only 0.3% loss and in terms of conditional reliability there is 3% loss. If the conditional reliability of 97% is acceptable then one can assume that region ${}^{23}S$ is sufficiently reliable for fault tolerance of single joint failures and add it to ${}^{123}S$. Therefore, the RFTW will be ${}^{123}S \cup {}^{23}S$. This is shown in Fig. 10.

By comparing Fig. 10 with Fig. 8, it is easy to see that the RFTW is nearly four times bigger than the FTW, while the conditional reliability of the RFTW is only 3% lower than the conditional reliability of the FTW. Therefore, if the RFTW is used instead of FTW, there will be a 300% increase in the workspace and only 3% decrease in conditional reliability.

6. Extensions

The method to obtain the RFTW was only illustrated for planar and serial manipulators. Further research is required to develop a method for parallel manipulators. This is because the calculation of the reliability values for parallel manipulator is completely different from serial manipulators.

6.1. *Extension for Planar Manipulators with Higher DoF*

The approach can be extendable for serial manipulators with higher degrees of freedom. For example, the reduced workspaces of the 4DoF planar manipulator

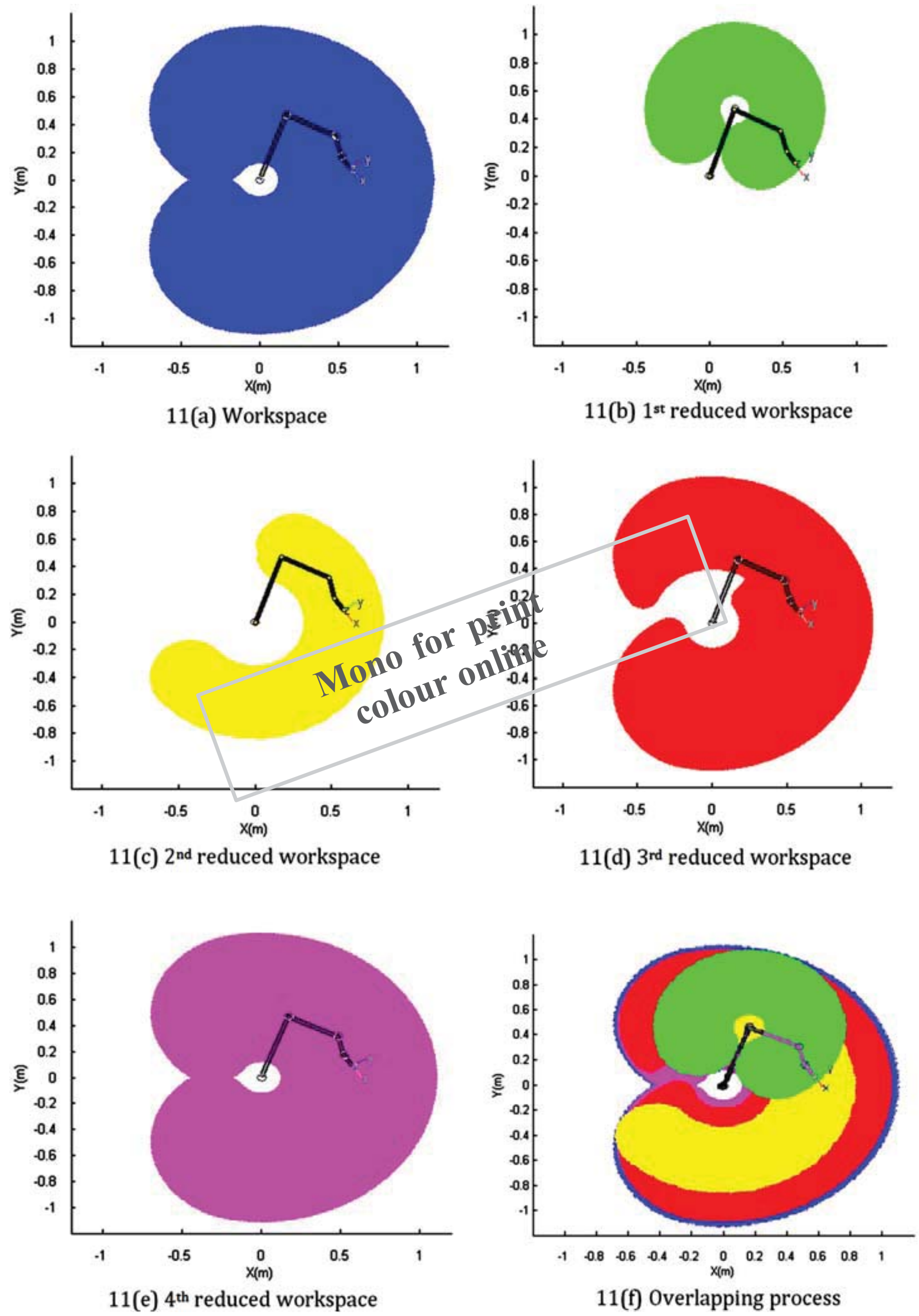

Figure 11. Different stages of finding RFTW for 4DoF planar manipulators.

in Fig. 11(a) are shown in Fig. 11(a)–(e). Finding the regions for this manipulator can be easily performed by overlapping the reduced workspaces and identify-

Table 2.
Reliability values for the 4DoF planar manipulator

Region	Reliability value
$R^m - W$	0
${}^0S$	$r_1r_2r_3r_4$
${}^1S$	$r_2r_3r_4$
${}^2S$	$r_1r_3r_4$
${}^3S$	$r_1r_2r_4$
${}^4S$	$r_1r_2r_3$
${}^{12}S$	$r_1r_3r_4 + r_2r_3r_4 - r_1r_2r_3r_4$
${}^{13}S$	$r_1r_2r_4 + r_2r_3r_4 - r_1r_2r_3r_4$
${}^{14}S$	$r_1r_2r_3 + r_1r_2r_4 - r_1r_2r_3r_4$
${}^{23}S$	$r_1r_2r_3 + r_1r_2r_4 - r_1r_2r_3r_4$
${}^{24}S$	$r_1r_2r_3 + r_1r_3r_4 - r_1r_2r_3r_4$
${}^{34}S$	$r_1r_2r_3 + r_1r_2r_4 - r_1r_2r_3r_4$
${}^{123}S$	$r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4 - 2r_1r_2r_3r_4$
${}^{124}S$	$r_1r_2r_3 + r_1r_3r_4 + r_2r_3r_4 - 2r_1r_2r_3r_4$
${}^{134}S$	$r_1r_2r_3 + r_1r_2r_4 + r_2r_3r_4 - 2r_1r_2r_3r_4$
${}^{234}S$	$r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 - 2r_1r_2r_3r_4$
${}^{1234}S$	$r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4 - 3r_1r_2r_3r_4$

ing the regions. For this manipulator, the link lengths are 50, 35, 15, and 10 cm for the first, second, third, and fourth link, respectively. The joint angles' limits are selected as $\pm 100^\circ$, $\pm 130^\circ$, $\pm 160^\circ$, and $\pm 170^\circ$ for joints 1–4, respectively. In Fig. 11(a), the workspace of the healthy manipulator; and in Fig. 11(b)–(e), the reduced workspaces due to single locked joint failures are illustrated. The configuration of the manipulator is shown at the time of failure where the joint angles at failure time are 70°, -96°, -50°, and 20° for joints 1–4, respectively. This robot is very similar to the previous robot except with the added fourth link.

The regions in this case are ${}^0S$, ${}^1S$, ${}^2S$, ${}^3S$, ${}^4S$, ${}^{12}S$, ${}^{13}S$, ${}^{14}S$, ${}^{23}S$, ${}^{24}S$, ${}^{34}S$, ${}^{123}S$, ${}^{134}S$, ${}^{124}S$, ${}^{234}S$ and ${}^{1234}S$ with the reliability values shown in Table 2.

The similarities between the equations given for the reliability values in Tables 1 and 2 can be used for generalization purpose. However, at this stage, we do not have a mathematical proof of the reliability values for any regions of manipulator with higher DoF.

6.2. *Extension for 3D Positional Manipulators with Higher DoF*

The workspace and the reduced workspaces of 3D positional redundant manipulators consist of 3D volumes. For these manipulators, the different regions of the workspace obtained by identifying the part of the volume of the reduced workspaces that has no intersection with other reduced workspaces, the part of the volume that has intersection with another reduced workspace, and the part of the volume that has intersection with the n-reduced workspaces. However, illustrating the results for these manipulators will be challenging.

6.3. Extension for Spatial Manipulators with Higher DoF

The workspace for spatial manipulators has six dimensions including three for position and three for orientation. Thus, the workspace and the reduced workspaces for spatial redundant manipulators consist of 6D volumes. The method in this paper can be applied for these manipulators, however, further research is required and the illustration of the results will be extremely hard.

6.4. Extension for Prismatic Joint Manipulators

The techniques in this paper were discussed for serial manipulators with revolute joints; however, it is applicable for serial manipulators that include prismatic joints as well. The workspace and reduced workspaces for a planar revolute joint manipulator were shown in Figs. 3–10. A similar concept can be used for 2D planar prismatic joint manipulators where the reduced workspaces are rectangular shape. For the case of 3D positional prismatic joint manipulators, the workspace and reduced workspaces of manipulators consist of a number of cubes. Identifying the different regions of the workspace can be performed using a similar method that was discussed in Section 3.3.

6.5. Extension for Multiple Joint Failures

The method can be also extendable for multiple joint failures of serial manipulators. For obtaining reduced workspaces for the case of multiple joint failures, it is necessary to consider different combinations of the locked joints and finding the corresponding reduced workspaces. This was shown for two faulty joints in Equation (3).

6.6. Extension for Other Types of Failures

Locked joint failures are the most common type of failures and it occurs because of an actuator failure or other mechanical failure at the joint. The method in this paper is not applicable for other types of failures such as free swing joints or sensor failures. The researchers in the fault-tolerant community commonly use active braking to convert other types of failures to locked joint failures.

7. Conclusion

Understanding reliability of fault-tolerant robots is critical to prevent task failure. In this paper, we connected the concept of reliability with the concept of

FTW for robotic manipulators. This connection extended the FTW to the novel concept of “RFTW.” The extension was a compromise between a smaller value of reliability to obtain a considerable amount of additional RFTW. In order to define the RFTW, reliability and conditional reliability maps were proposed. This provided a framework that specifies a reliability and conditional reliability value for different regions of the workspace. The union of the regions that have higher reliability or conditional reliability value than the specific threshold can be considered as the RFTW. The decision on whether to consider a region as a reliable fault-tolerant region depends on specifying a reliability threshold. By adjusting the reliability threshold, one can extend or retract the RFTW to suit the application.

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About the Authors

Hamid Abdi received his BSc and MSc in control from Sharif University of Technology, Tehran–Iran, and received his PhD degree from Deakin University, Australia. He has worked in department of electrical engineering, Iran University of science and technology, Arak–Iran, 2000–2008. He has published several journal and conference papers, and received several awards including Iran's minister of petroleum award and minister of science and technology certificate, the UNESCO entrepreneurship certificate, and a golden medal of Sheikh Bahaee. He owns three patents for his innovative robot for traffic control and road safety. He has been selected in the top 10 researchers among all academic and research staffs, in Markazi-Iran. His research interest includes advanced industrial control, fault-tolerant systems, robotics, and energy.

Anthony A. Maciejewski, (M'87–SM'00–F'05) received the BSEE, MS, and PhD degrees from Ohio State University, Columbus, in 1982, 1984, and 1987, respectively. From 1988 to 2001, he was a professor of electrical and computer engineering at Purdue University, West Lafayette, IN. He is currently the department head of Electrical and Computer Engineering at Colorado State University, Fort Collins.

Saeid Nahavandi, (SM'07) received the BSc (hons.), MSc, and PhD degrees in automation and control from Durham University, Durham, UK. He is the Alfred Deakin professor, chair of engineering, and the leader for the Intelligent Systems Research Centre, Deakin University, Geelong, VIC, Australia. He has published over 300 peer-reviewed papers in various international journals and conferences. He designed the world's first 3D interactive surface/motion controller. His research interests include modeling of complex systems, simulation-based optimization, robotics, haptics and augmented reality. He was a recipient of the Young Engineer of the Year Title in 1996 and six international awards in Engineering. He is the associate editor of the *IEEE Systems Journal*, an editorial consultant board member for the *International Journal of Advanced Robotic Systems*, and an editor (South Pacific Region) of the *International Journal of Intelligent Automation and Soft Computing*. He is a fellow of Engineers Australia (FIEAust) and IET (FIET).