Fundamental Limitations on Designing Optimally Fault-Tolerant Redundant Manipulators

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***Abstract*—In this paper, the authors examine the problem of designing nominal manipulator Jacobians that are optimally fault tolerant to one or more joint failures. Optimality is defined here in terms of the worst-case relative manipulability index. While this approach is applicable to both serial and parallel mechanisms, it is especially applicable to parallel mechanisms with a limited workspace. It is shown that a previously derived inequality for the worst-case relative manipulability index is generally not achieved for fully spatial manipulators and that the concept of optimal fault tolerance to multiple failures is more subtle than previously indicated. Lastly, the authors identify the class of 8-DOF Gough–Stewart platforms that are optimally fault tolerant for up to two joint failures. Examples of optimally fault-tolerant 7- and 8-DOF mechanisms are presented.**

***Index Terms*—Fault tolerance, kinematic redundancy, manipulability, parallel manipulators.**

I. Introduction

FAULT-TOLERANT design of serial or parallel manipulators is critical for tasks requiring robots to operate in remote and hazardous environments where repair and maintenance tasks are extremely difficult [1]–[10]. In such cases, operational reliability is of prime importance. By adding kinematic redundancy to the robotic system, the robot may still be able to perform its task even if one or more joint actuators fail [11]. However, simply adding kinematic redundancy to the system does not guarantee fault tolerance [12]. One must strategically plan how the kinematic redundancy should be added to the system to ensure that fault tolerance is optimized [13].

A number of studies have been dedicated to the assessment [5], [14] and analysis [15]–[17] of robot safety and reliability, including robots designed primarily for this purpose [18]. Early work on the kinematic evaluation of fault tolerance includes [11]. Other studies related to enhancing a robot's tolerance to failure include work on failure detection [19], low-level failure avoidance and recoverability [20], layered failure tolerance control [21], [22], failure tolerance by trajectory planning [23], and kinematic failure recovery [24]. Work that considers obstacles in the workspace include [25] and [26].

One approach to the problem of designing fault-tolerant robots is to optimize some measure of fault tolerance. This measure can be either global, i.e., over a specified region of the workspace, or local, i.e., at a specific configuration. Global measures, such as those in [25] and [27], are more appropriate for tasks that require large motions throughout the workspace, whereas local measures [11], [28] are more appropriate for dexterous operations in a relatively small location, e.g., laser pointing [6] and manipulation of nuclear material [9]. In this paper, we focus on a local measure called the relative manipulability index, which was first introduced in [12] to quantify the fault tolerance of kinematically redundant serial manipulators.

For a serial manipulator, the relative manipulability index is defined in terms of the manipulator Jacobian J, which relates the manipulator's joint velocity $\dot{\theta}$ to its end-effector velocity $\mathbf{v}$ by the equation

$$\mathbf{v} = J\dot{\theta}. \tag{1}$$

In this paper, we will assume that the manipulator is not operating at a kinematically singularity so that J has full rank. When a locked joint failure occurs, say in joint i, that component of the joint velocity is zero. Consequently, the end-effector motion is characterized by ${}^{i}J$, i.e., the Jacobian J with its ith column removed. Multiple locked joint failures are handled in the same way, i.e., the corresponding columns of the Jacobian are removed. The *relative manipulability index* corresponding to locked joint failures in joints $i_1, \ldots, i_f$ is defined to be

$$\rho_{i_1,\ldots,i_f} = \frac{w({}^{i_1,\ldots,i_f}J)}{w(J)} \tag{2}$$

where ${}^{i_1,\ldots,i_f}J$ denotes the manipulator Jacobian after the columns $i_1, \ldots, i_f$ corresponding to the failed joints are removed and where $w(J) = \sqrt{\det(JJ^T)}$ is the manipulability index for J [29]. This quantity is a local measure of the amount of dexterity that is retained when a manipulator suffers from one or more locked joint failures. The value of a relative manipulability index ranges from zero to one. A zero value would indicate a loss of full end-effector motion at that configuration after the failed joints are locked. In other words, a zero relative manipulability index means that the reduced manipulator Jacobian ${}^{i_1\cdots i_f}J$ does not have full rank. A relative manipulability index of one would indicate that no dexterity is lost at that configuration. In this case, the joints in question do not contribute to end-effector motion at the operating configuration prior to their failure, i.e., those joints only produce self-motion [12], [30].

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Relative manipulability indices have also been used to study the fault tolerance of redundant Gough–Stewart platforms (GSPs) [31]. A GSP is a parallel mechanism consisting of a base, a moving platform, and struts. For a GSP, the inverse Jacobian M maps the generalized velocity of the payload to the corresponding joint velocities of the individual struts. The matrix M has the same form as the transpose of a manipulator Jacobian J. In other words, the first three components of each row forms a unit vector that is orthogonal to the vector given by the last three components of that row. If $M^T M$ is a diagonal matrix, then one says that the mechanism is an orthogonal GSP (OGSP) [32]–[34]. OGSPs are a special class of GSPs that are particularly well suited to various precision applications due to the local kinematic and dynamic decoupling of the Cartesian directions they provide [35], [36]. The concept of an OGSP is related to the concept of isotropy [37], [38].

The types of failures for the GSP considered here are called hard failures and torque failures [10]. A *hard failure* is caused by mechanical fatigue or blown-off struts. When this occurs, the system acts as if the failed struts are totally lost. In this case, not only is actuation lost, but so are the mechanical constraints implemented by the strut. A *torque failure*, also known as a free swing failure [3], refers to a hardware or a software fault in a robotic manipulator that causes the loss of torque (or force) on a joint. Examples include a ruptured seal on a hydraulic actuator, the loss of electric power and brakes on an electric actuator, and a mechanical failure in a drive system. A joint with torque failure can move passively. Like a hard failure, a torque failure can be tolerated by designing the original system to be kinematically redundant. Similar to the case of a serial manipulator, the kinematic equations for a GSP following a hard or torque failure are obtained by removing the corresponding row of M. A class of OGSPs was identified in [31] that possesses optimal fault-tolerant manipulability for single joint failures based on maximizing the minimum relative manipulability index about an operating point.

In this paper, the authors determine a family of manipulators that are optimally fault tolerant to multiple failures at their nominal operating configuration. In the next section, the relationship between relative manipulability indices and the null space of the manipulator Jacobian is established using the principal minors of the null space projection operator. Based on this formulation of fault tolerance, it is easy to establish identities and inequalities for relative manipulability indices. Motivated by the observation that the relative manipulability indices are completely determined by the null space of the manipulator Jacobian, we then discuss some of the theoretical limitations of designing manipulator Jacobians with a prescribed null space. An optimally fault-tolerant 7-DOF manipulator is then determined in Section III. In Section IV, the authors consider the concept of equally fault-tolerant configurations, i.e., configurations for which any combination of a specified number of joint failures results in the same local manipulability. It is shown through a series of results that such configurations are truly rare. In Section V, the authors identify the class of 8-DOF fully spatial manipulators that have the property that in their nominal operating configuration, the manipulators are optimally fault tolerant for up to two joint failures. Section VI contains a discussion on some of the fundamental limitations of designing for fault tolerance to multiple joint failures and explores some other measures of fault tolerance. Lastly, conclusions appear in Section VII.

II. Fault Tolerance and the Null Space of the Manipulator Jacobian

It turns out that the amount of fault tolerance that a manipulator possesses is closely related to the null space of the manipulator Jacobian. This important fact motivates the problem of designing operating configurations for robotic mechanisms based on choosing the manipulator Jacobian to have a prescribed null space. After characterizing the relative manipulability indices in terms of the null space of the manipulator Jacobian, we will discuss the amount of freedom that a designer has in choosing the null space of a nominal manipulator Jacobian.

A. Relative Manipulability Indices and the Null Space of the Manipulator Jacobian

We begin by demonstrating that the subdeterminants of the null space projection operator of the manipulator Jacobian completely characterize the relative manipulability indices. Our analysis is applicable to serial and parallel mechanisms, so throughout this paper, we will use M and J^T interchangeably. Let J be a full rank $m \times n$ matrix with $m < n$ and let $r = n - m$. For a manipulator, m denotes the dimension of the workspace, n denotes the number of joints, and r denotes the degree of redundancy. We will call an $n \times r$ matrix N a *null space matrix* of J if the columns of N form an orthonormal basis for the null space of J. Although the null space matrix N is not unique for a given J, any two null space matrices N and N' of J are related by an orthogonal matrix Q in the following way: $N' = NQ$. We will see later that we can use Q to place N into a canonical form that can help us to properly view the null space and its relationship to fault tolerance.

It was shown in [12] that the relative manipulability index is related to the null space matrix by the relationship

$$\rho_{i_1,\ldots,i_f} = w(N_{i_1,\ldots,i_f}) = \sqrt{|N_{i_1,\ldots,i_f} N^T_{i_1,\ldots,i_f}|} \qquad (3)$$

where $N_{i_1,\ldots,i_f}$ is the $f \times r$ matrix consisting of rows $i_1, \ldots, i_f$ of the matrix N. We thus have the interesting observation that the relative manipulability indices are strictly a function of the null space of J. We will build on this result to address the issue of designing manipulators that are optimally fault tolerant to one or more joint failures.

The relative manipulability index squared, $\rho^2_{i_1,\ldots,i_f} = |N_{i_1,\ldots,i_f} N^T_{i_1,\ldots,i_f}|$, is perhaps best viewed as a principal minor of the null space projection operator $P_N = I - J^+ J$, where J^+ denotes the pseudoinverse of J. The $n \times n$ matrix P_N represents the orthogonal projection of the joint space onto the null space of J. Unlike a null space matrix, P_N is unique for a given J; however, given a corresponding null space matrix N, we have that $P_N = NN^T$. It then follows from (3) that the relative manipulability index squared is equal to the determinant of the submatrix consisting of the $i_1, \ldots, i_f$ rows and columns of P_N.

Recall that a $k \times k$ *minor* of an $n \times n$ matrix $A = [a_{ij}]$ with $k < n$ is a subdeterminant of the form

$$A\begin{pmatrix} i_1 & \cdots & i_k \\ j_1 & \cdots & j_k \end{pmatrix} \triangleq \begin{vmatrix} a_{i_1 j_1} & a_{i_1 j_2} & \cdots & a_{i_1 j_k} \\ a_{i_2 j_1} & a_{i_2 j_2} & \cdots & a_{i_2 j_k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i_k j_1} & a_{i_k j_2} & \cdots & a_{i_k j_k} \end{vmatrix} \quad (4)$$

where $1 \le i_1 < \cdots < i_k \le n$ and $1 \le j_1 < \cdots < j_k \le n$. If $(j_1, \ldots, j_k) = (i_1, \ldots, i_k)$, then this quantity is called a *principal minor* of A. Hence, we have that $\rho^2_{i_1,\ldots,i_f}$ is the $(i_1, \ldots, i_f)$ principal minor of $P_N = NN^T$:

$$\rho^2_{i_1,\ldots,i_f} = P_N\begin{pmatrix} i_1 & \cdots & i_f \\ i_1 & \cdots & i_f \end{pmatrix}. \quad (5)$$

It is well known that the coefficients of the characteristic polynomial $p_A(\lambda) = |\lambda I - A|$ of A are given in terms of the sums of the principal minors of A. To be more specific, for $p_A(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0$, we have

$$a_{n-k} = (-1)^k \sum_{1 \le i_1 < \cdots < i_k \le n} A\begin{pmatrix} i_1 \cdots i_k \\ i_1 \cdots i_k \end{pmatrix}. \quad (6)$$

Since P_N is a projection, it is idempotent, i.e., $P_N^2 = P_N$, so its only possible distinct eigenvalues are 0 and 1. Furthermore, because $\mathrm{rank}(P_N) = r < n$, where $r = n - m$, it follows that the characteristic polynomial of P_N is

$$p(\lambda) = \lambda^m(\lambda - 1)^r = \sum_{k=0}^{r} \binom{r}{k} (-1)^k \lambda^{n-k}. \quad (7)$$

Equations (5)–(7) then imply that

$$\sum_{1 \le i_1 < \cdots < i_f \le n} \rho^2_{i_1,\ldots,i_f} = \binom{r}{f}. \quad (8)$$

This result, written as a slightly different but equivalent expression, was also proven in [31]; however, the proof provided there was based on repeated application of the Binet–Cauchy theorem and was less direct than applying principal minors. It is important to note, however, that the approach just given is not merely a different proof of the result in [31]. More importantly, it provides us with an approach that will be used in Section IV to address multiple joint failures.

As noted in [31], (8) can be used to obtain an upper bound for the worst-case relative manipulability index by noting that the minimum value of any set of numbers must be less than or equal to the average so that

$$\min_{1 \le i_1 < \cdots < i_f \le n} \rho_{i_1,\ldots,i_f} \le \sqrt{\frac{\binom{r}{f}}{\binom{n}{f}}}. \quad (9)$$

Similarly

$$\max_{1 \le i_1 < \cdots < i_f \le n} \rho_{i_1,\ldots,i_f} \ge \sqrt{\frac{\binom{r}{f}}{\binom{n}{f}}}. \quad (10)$$

B. Designing Nominal Fully Spatial Manipulator Jacobians With a Prescribed Null Space

Based on the inequality in (9), Ukidve *et al.* [31] convincingly argue the importance of designing for fault tolerance. This is especially true when there may be multiple faults. One approach to ensuring local fault tolerance is to design the manipulator based on null space properties. This is particularly applicable when the required workspace is very small, as is the case in [31]. However, there are limitations to how much redundancy can be used when designing nominal manipulator Jacobians with a prescribed null space.

These limitations follow from the fact that the manipulator Jacobian for a fully spatial manipulator must satisfy certain constraints on its columns. In particular, the vector given by the first three components of a column must have unit length and must be orthogonal to the vector given by the last three components of that column. For a manipulator with n joints, this results in $2n$ constraints. If the manipulator Jacobian is required to have a prescribed null space matrix, then each of its six rows must be orthogonal to the r rows of N^T, where $r = n - 6$ is the number of degrees of redundancy of the manipulator. Consequently, the manipulator Jacobian must satisfy $6r$ null space constraints. Since the manipulator Jacobian has $6n$ parameters, it follows that one has $6n - 2n - 6r = 4(6 + r) - 6r = 24 - 2r$ degrees of freedom to satisfy the design constraints. Hence, one cannot expect to arbitrarily find a manipulator with $r > 12$ degrees of redundancy that has a configuration where the manipulator Jacobian has a prescribed null space matrix.

If the mechanism is required to be an OGSP, then there is a further reduction in the degrees of freedom that one has in choosing a manipulator Jacobian with a prescribed null space. If JJ^T is required to be a diagonal matrix, there would be 15 additional constraints, decreasing the degrees of freedom to $9 - 2r$. In this case, one should not expect to be able to arbitrarily specify the null space of a manipulator with $r > 4$ degrees of redundancy. Of course, there are cases where this is possible for the right choice of the null space. Furthermore, there could be cases where there is no real solution to the problem even though r is sufficiently small. The dimension arguments presented here do, however, provide the designer with a tool to assess the likely feasibility of designing a mechanism with prescribed null space properties.

III. Designing Manipulator Jacobians That Are Optimally Fault Tolerant to Single Locked-Joint Failures

According to (9), the maximum worst-case relative manipulability index for a 7-DOF manipulator is $1/\sqrt{7}$. This optimal value is achieved if and only if the null vector of the manipulator Jacobian has components of equal magnitude, i.e., $|\hat{n}_i| = 1/\sqrt{7}$, where $\hat{n}_i$ is the ith component of the unit-length null vector $\hat{\mathbf{n}}_J$. Hence, we can specify the null vector to obtain an optimally fault-tolerant manipulator configuration, provided, of course, that we can simultaneously satisfy the manipulator Jacobian constraints described in the previous section. Based on the dimension arguments in Section II-B, we have 22 degrees

TABLE I
PARAMETERS FOR THE OPTIMALLY FAULT TOLERANT 6×7 MANIPULATOR JACOBIAN GIVEN IN (11)

i	$\mathbf{n}_i^T$			$\mathbf{r}_i^T$		
1	0.000	0.000	1.000	−1.065	0.113	0.000
2	−0.172	−0.827	−0.536	0.272	−0.520	0.715
3	0.877	0.418	−0.239	0.302	−0.785	−0.263
4	−0.408	−0.004	−0.913	0.530	0.761	−0.240
5	0.473	−0.802	0.364	0.460	−0.098	−0.814
6	0.065	0.983	−0.174	−1.007	0.062	−0.031
7	−0.836	0.233	0.497	0.507	0.467	0.633

of freedom in choosing a 7-DOF manipulator Jacobian with a prescribed null vector. If we further require that JJ^T be diagonal, the number of degrees of freedom in choosing J with a prescribed null vector reduces to 7. An example of a nominal manipulator Jacobian that is optimally fault tolerant to a single failure is given by (11), shown at the bottom of the page.

This manipulator Jacobian corresponds to a 7-DOF manipulator, and its null vector components are all equal. Consequently, all seven relative manipulability indices corresponding to (11) are equal to $1/\sqrt{7}$. In this case, JJ^T is diagonal, so (11) corresponds to an OGSP. The parallel mechanism parameters for the corresponding manipulator Jacobian are given in Table I. For a parallel manipulator, the unit vector $\mathbf{n}_i$ in the table indicates the direction of the ith strut, while $\mathbf{r}_i$ represents the point on the axis of the ith strut that is closest to the origin.

There are a number of different possible manipulator realizations that can be generated from the Jacobian in (11). Clearly, the desired failure tolerance properties are not affected by multiplying one or more of the columns of J by -1. Two different parallel manipulators generated from this Jacobian are shown in Figs. 1 and 2. A fault-tolerant configuration for a serial manipulator can also be generated from an optimal Jacobian. One such realization is depicted in Fig. 3 with its Denavit–Hartenberg parameters given in Table II. There is additional flexibility when designing serial manipulator configurations from a prescribed Jacobian because one can permute the columns of the Jacobian to alter the manipulator geometry without affecting its fault-tolerant properties.

If one is concerned with symmetry, it is easy to see that the following null space represents an 8-DOF manipulator that is fault tolerant to a single joint failure:

$$N = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (12)$$

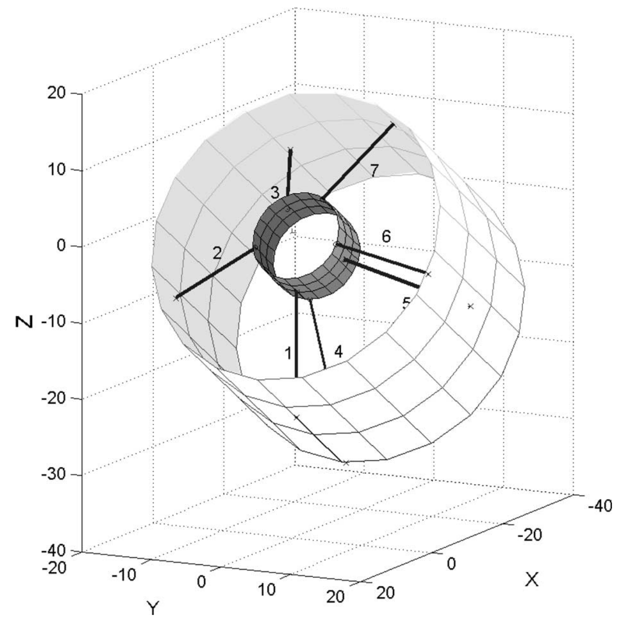

Fig. 1. Example of a cylindrical geometry for an OGSP corresponding to a realization of the optimally fault-tolerant 6×7 manipulator Jacobian given in (11). The labels on the struts correspond to the respective columns of J (rows of M). Similar parallel mechanisms have been proposed for mounting in aerospace vehicles [32]. The $\mathbf{n}_i$'s for struts 2, 4, 6, and 7 have been reversed, i.e., the corresponding rows of (11) have been multiplied by -1.

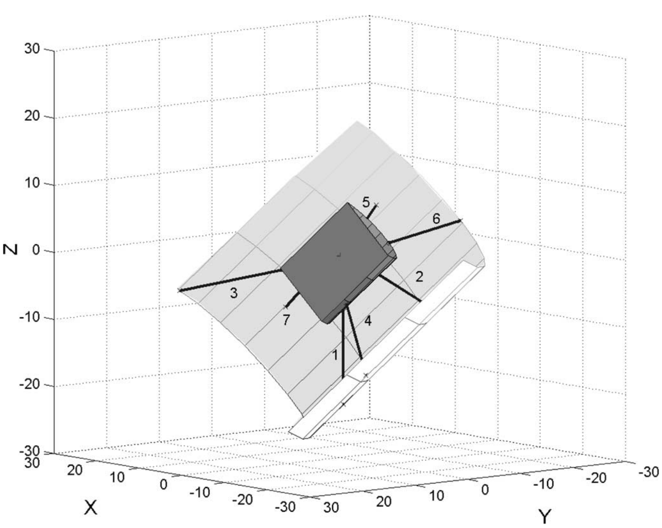

Fig. 2. Another variation of a parallel mechanism related to the optimally fault-tolerant 6×7 Jacobian given in (11). In this case, the $\mathbf{n}_i$'s for struts 2–5 have been reversed, i.e., the corresponding rows of (11) have been multiplied by -1.

$$J^T = \begin{bmatrix} 0.000 & 0.000 & 1.000 & 0.113 & 1.065 & 0.000 \\ -0.175 & -0.827 & -0.536 & 0.870 & 0.023 & -0.314 \\ 0.877 & 0.418 & -0.239 & 0.297 & -0.159 & 0.814 \\ -0.408 & -0.004 & -0.913 & -0.696 & 0.581 & 0.308 \\ 0.473 & -0.802 & 0.364 & -0.689 & -0.553 & -0.323 \\ 0.065 & 0.983 & -0.174 & 0.020 & -0.177 & -0.993 \\ -0.836 & 0.233 & 0.497 & 0.085 & -0.781 & 0.508 \end{bmatrix}. \qquad (11)$$

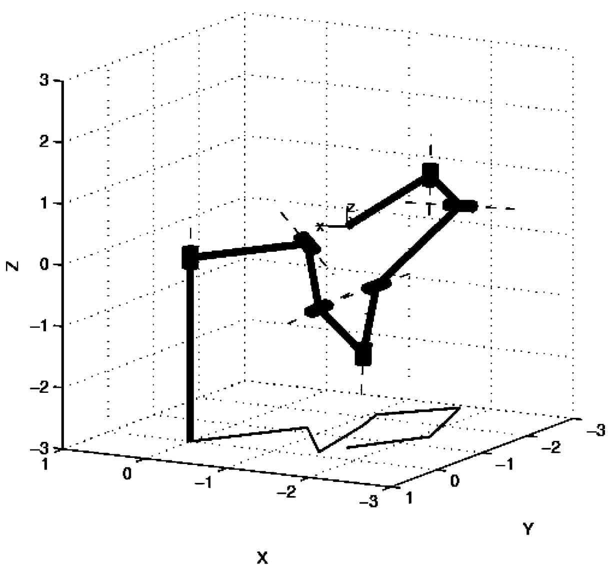

Fig. 3. Serial mechanism corresponding to the Denavit–Hartenberg parameters given in Table II. This manipulator is in a configuration with an optimally fault-tolerant 6 × 7 manipulator Jacobian. (This image was generated using the Robotics Toolbox described in [39].

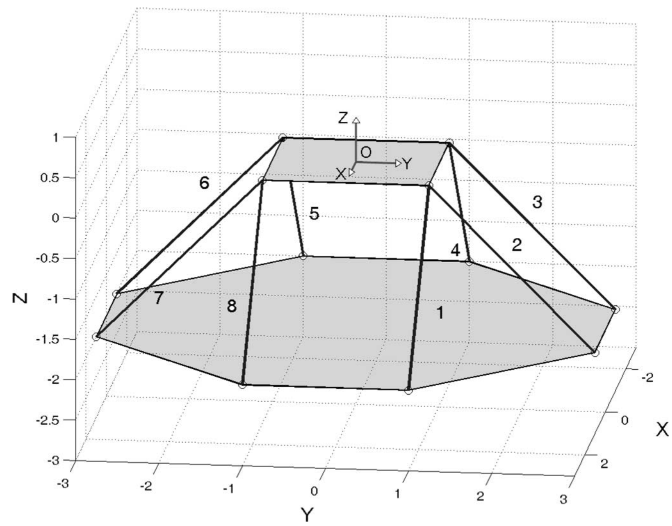

Fig. 4. Symmetric 8-DOF GSP. The geometric symmetry ensures enough algebraic symmetry in the manipulator Jacobian and the null space matrix that optimal fault tolerance is guaranteed for single joint failures.However, in spite of having two degrees of redundancy, the manipulator is fault intolerant to certain combinations of failures in two joints.

TABLE II
DENAVIT–HARTENBERG TABLE FOR AN OPTIMALLY FAULT-TOLERANT 6 × 7 MANIPULATOR JACOBIAN

i	α_i	a_i	θ_i	d_i
1	2.137	1.438	-0.205	0.000
2	1.947	-0.528	0.982	1.329
3	1.712	1.319	-2.159	1.325
4	2.120	0.124	-1.475	1.084
5	2.533	1.300	-0.906	-0.817
6	1.483	1.407	0.955	-1.119
7	0.000	0.875	-2.720	-0.880

with a corresponding symmetric parallel manipulator shown in Fig. 4. However, it is equally easy to see that this design is fault intolerant to two joint failures. From (12), it is clear that locking the joints corresponding to two even numbered struts or two odd numbered struts results in a reduced Jacobian that is singular. Thus, additional care must be given if one wants to address fault tolerance to multiple failures. This important issue is addressed in the next two sections.

IV. EQUALLY FAULT-TOLERANT CONFIGURATIONS

Equation (9) served as a motivation in [31] for defining a manipulator operating about a single point in the workspace to be optimally fault tolerant to $f \leq r$ failures if all of its relative manipulability indices $\rho_{i_1,\ldots,i_f}$ are equal, i.e.,

$$\rho_{i_1,\ldots,i_f} = \sqrt{\frac{\binom{r}{f}}{\binom{n}{f}}} \tag{13}$$

for $1 \leq i_1 < \cdots < i_f \leq n$. In this paper, we will prefer to say that a manipulator is *equally fault tolerant to* $f \leq r$ *failures* at an operating configuration if (13) holds for $1 \leq i_1 < \cdots < i_f \leq n$ at that configuration. Note that equal fault tolerance is a local property since it would apply to specific configurations and would be most applicable for manipulators operating in a small workspace. If a manipulator is equally fault tolerant to $f \leq r$ failures, then by (9), it is optimally fault tolerant in a worst-case relative manipulability index sense to $f \leq r$ failures. However, while it is clear that an optimal value exists, it is possible that a manipulator may not have a configuration that is equally fault tolerant to f failures. In this case, the optimal value is smaller than the bound given in (9). It is the goal of this section to show that this is typically the case.

Note that the definition of equal fault tolerance given in (13) is specifically for f failures and that (13) does not necessarily say anything about fault tolerance to a different number of failures. However, as we can see from the next result, (13) implies that the manipulator is in a configuration that is also equally fault tolerant to $k < f$ failures for $f \geq 2$.

Theorem 1: If a manipulator is equally fault tolerant to f failures where $1 < f \leq r$, then it is also equally fault tolerant to $f - 1$ failures. Furthermore, the manipulator is equally fault tolerant to k failures for $k = 1, 2, \ldots, f$.

Proof: The proof of this nontrivial result is given in the Appendix.

We note (see the proof of Theorem 1 in the Appendix for details) that the null space matrix N can be chosen to have a particularly simple form when $r = 2$

$$N^T = \begin{bmatrix} N_{11} & N_{12} & N_{13} & \cdots & N_{1n} \\ 0 & N_{22} & N_{23} & \cdots & N_{2n} \end{bmatrix}. \tag{14}$$

Equal fault tolerance to two failures would imply that $|N_{11}N_{2j}|^2 = 2/n(n-1)$ for $j = 2, \ldots, n$, implying that the quantities $|N_{2j}|$ are all equal for $j = 2, \ldots, n$. The fact that the rows of N^T have unit length further implies that $|N_{2j}| = 1/\sqrt{n-1}$ for $j = 2, \ldots, n$. Hence, $|N_{11}| = \sqrt{2/n}$. Indeed, by Theorem 1, each column of N^T has norm $\sqrt{2/n}$ so that $|N_{1j}| = \sqrt{(n-2)/[n(n-1)]}$ for $j = 2, \ldots, n$. Consequently, the last $n-1$ columns of N^T have the form $[\pm\alpha \quad \pm\beta]^T$, where $\alpha = \sqrt{(n-2)/[n(n-1)]}$ and $\beta = 1/\sqrt{n-1}$. By the pigeonhole principle, if $n > 3$, then at least two columns among the last $n-1$ columns of N^T will be ± 1 multiples of each other, contradicting our requirement of fault tolerance; hence, N^T has at most three columns. We thus conclude that $n = 3$ and $m = 1$. In other words, the workspace can only be one-dimensional.

The earlier observations prove the following result.

Theorem 2: Suppose that J is a nominal Jacobian for a manipulator with $r = 2$ degrees of redundancy. Then, J is equally fault tolerant to two simultaneous joint failures if and only if J is a 1×3 matrix of the form $J = [\pm\alpha \quad \pm\alpha \quad \pm\alpha]$ for some $\alpha > 0$.

As a consequence of Theorem 2, it follows that no 8-DOF spatial manipulator can be equally fault tolerant to two simultaneous joint failures. We will examine the issue of designing *optimally* fault tolerant 8-DOF manipulator Jacobians in the next section.

While the null space matrix (14) was sufficient to prove that no 8-DOF manipulator is equally fault tolerant to any two failures, we will need the principal minors of P_N to address equal fault tolerance for manipulators with higher degrees of redundancy. The reason that Theorem 1 will play such an important role in this regard is the fact that it forces P_N to have a particularly simple structure when the manipulator is equally fault tolerant to more than one failure. If J is equally fault tolerant to a single failure, then the diagonal elements of P_N are all equal to r/n. If J is equally fault tolerant to $f \geq 2$, then by Theorem 1, it is equally fault tolerant to single failures and to two failures. Hence, the (i, j) principal minor of the symmetric matrix P_N is

$$\begin{vmatrix} r/n & p_{ij} \\ p_{ji} & r/n \end{vmatrix} = \frac{r^2}{n^2} - p_{ij}^2 = \frac{r(r-1)}{n(n-1)} \tag{15}$$

where we have used the fact that $p_{ji} = p_{ij}$ and the last equality follows from the assumption of equal fault tolerance to two failures. Solving for p_{ij} gives $p_{ij} = (\pm 1/n)\sqrt{r(n-r)/(n-1)}$ for all $1 \leq i < j \leq n$. Hence, when J is equally fault tolerant to $f \geq 2$ failures, the diagonal elements of P_N are all equal and the off-diagonal elements of P_N all have the same magnitude, i.e., P_N has the form

$$P_N = \begin{bmatrix} a & \pm b & \pm b & \cdots & \pm b \\ \pm b & a & \pm b & \cdots & \pm b \\ \pm b & \pm b & a & \cdots & \pm b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pm b & \pm b & \pm b & \cdots & a \end{bmatrix} \tag{16}$$

where $a = \frac{r}{n}$ and $b = \frac{-1}{n}\sqrt{r(n-r)/(n-1)}$.

Once again, consider a manipulator with two degrees of redundancy, and suppose that the manipulator is equally fault tolerant to two failures. Since the rank of P_N would then be 2, it follows that the 3×3 principal minors of P_N are zero; otherwise, the rank of P_N would be greater than or equal to 3. Any 3×3 principal minor of P_N necessarily has the form

$$\begin{vmatrix} a & \pm b & \pm b \\ \pm b & a & \pm b \\ \pm b & \pm b & a \end{vmatrix} = a^3 - 3ab^2 \pm 2b^3. \tag{17}$$

Since one of these two quantities is zero, so is their product so that

$$\begin{aligned} 0 &= (a^3 - 3ab^2 + 2b^3)(a^3 - 3ab^2 - 2b^3) \\ &= (a-b)^2(a+2b)(a+b)^2(a-2b) \\ &= (a^2 - b^2)^2(a^2 - 4b^2). \end{aligned} \tag{18}$$

We thus conclude that $a^2 = b^2$ or $a^2 = 4b^2$. Substituting in the expressions for a and b yields that $n = 0$ or $n = 3$, respectively. As $n = 0$ does not make sense, we conclude that $n = 3$. Equivalently, the workspace has $m = n - r = 3 - 2 = 1$ degree of freedom so that the corresponding Jacobian is a 1×3 matrix. Equal fault tolerance then dictates that the Jacobian has the form $J = [\pm\alpha \quad \pm\alpha \quad \pm\alpha]$ for some $\alpha > 0$.

Note that this implies once again Theorem 2 and, in particular, that an 8-DOF GSP cannot be equally fault tolerant to two simultaneous joint failures regardless of its geometry. An 8-DOF optimally fault-tolerant manipulator Jacobian will be determined in the next section. Of course, the worst-case relative manipulability index to two failures will necessarily be smaller than $1/\sqrt{28}$, the upper bound given by (9).

We are now ready to consider the case when J is equally fault tolerant to $f \geq 3$ failures. Applying similar arguments as before, we obtain the following result.

Theorem 3: A Jacobian is equally fault tolerant to $f \geq 3$ simultaneous joint failures if and only if J is a $1 \times n$ matrix of the form $J = [\pm\alpha \quad \pm\alpha \quad \cdots \quad \pm\alpha]$, where $\alpha > 0$ and $n \geq f + 1$.

Proof: See the Appendix.

In particular, Theorem 3 implies that no fully spatial manipulator Jacobian can be equally fault tolerant to three or more failures regardless of how many joints it may have. The same is true for planar manipulators.

We now consider the case when a fully spatial manipulator is equally fault tolerant to two failures. We have already shown that this is impossible for $r = 2$. To simplify matters, note that multiplying any of the columns of J by -1 does not affect the fault tolerance properties of J. In doing so, the corresponding rows and columns of P_N are also multiplied by -1 so that we can assume without loss of generality that P_N has the form

$$P_N = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & \pm b & \cdots & \pm b \\ b & \pm b & a & \cdots & \pm b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & \pm b & \pm b & \cdots & a \end{bmatrix}. \tag{19}$$

We use the property that P_N is a projection to determine restrictions on the number of degrees of redundancy that a fully spatial manipulator can have for the equal fault tolerance property to hold. As a projection, $P_N^2 = P_N$ so that for $j > 1$

$$b = p_{1j} = (P_N)_{1j} = (P_N^2)_{1j} = 2ab + qb^2 \tag{20}$$

where q is the integer $q = n_1 - n_2 - 1$, where n_1 denotes the number of elements in the jth column of P_N that are equal to b and n_2 denotes the number of elements equal to $-b$. Clearly, $n_1 + n_2 = n - 1$ as $(P_N)_{jj} = a$ and $(P_N)_{ij} = \pm b$ for $i \neq j$.

Since $b \neq 0$, (20) yields

$$q = \frac{1-2a}{b}. \quad (21)$$

For a redundant fully spatial manipulator, $m = 6$ and $n = r + 6$. Substituting the expressions for a and b into (21) gives

$$q = \frac{1-(2r/n)}{(-1/n)\sqrt{r(n-r)/(n-1)}} = (r-6)\sqrt{\frac{r+5}{6r}}. \quad (22)$$

The requirement that q is an integer is a necessary condition for the existence of a manipulator having $r > 1$ degrees of redundancy with the property that it is equally fault tolerant to two failures.

Unfortunately, the requirement that q is an integer automatically eliminates most spatial manipulator designs since only specific values of r are feasible. To see this, we need to identify those positive integers r such that the resulting q is an integer. We will do this by first solving a simpler problem. If q is an integer, then so is $6q^2 = r^2 - 7r - 24 + (180/r)$. Since the first three terms in the expansion of $6q^2$ are integers, so is the last term, $(180/r)$, i.e., r divides $180 = 2^2 \times 3^2 \times 5$. There are exactly 18 positive integers that divide 180, each having the form $r = 2^i 3^j 5^k$ with $i, j = 0, 1, 2$ and $k = 0, 1$. Those positive integers r for which q is an integer are among these 18 candidates. Testing all 18, we find that q is an integer only for $r = 1$, 3, 6, and 10. We are not interested in the case $r = 1$ since the manipulator could only be fault tolerant to single joint failures (in this case, the manipulator is equally fault tolerant to two failures in the undesirable sense that the relative manipulability index is zero for any combination of two distinct failures). We can also eliminate the case where $r = 3$ using the following argument. We have already noted that $n_1 - n_2 = q + 1$ and $n_1 + n_2 = n - 1 = r + 5$ so that $2n_1 = q + r + 6$, or equivalently, $q + r = 2n_1 - 6$. Hence, $q + r$ is an even number so that q and r have the same parity, i.e., both are even or both are odd. However, for $r = 3$, we have $q = -2$, implying that $r = 3$ is not a feasible solution. Thus, if a redundant fully spatial manipulator with $r > 1$ degrees of redundancy is equally fault tolerant to two joint failures, then $r = 6$ or 10. These last two cases can be eliminated using arguments concerning the row structure of N^T. We can thus formally state the following result.

Theorem 4: Regardless of a manipulator's geometry or the amount of kinematic redundancy present in a manipulator, no fully spatial manipulator Jacobian can be equally fault tolerant to two joint failures.

Proof: See the Appendix.

V. Optimally Fault-Tolerant Configurations

In the last section, it was shown that no 8-DOF GSP could be designed to operate at a configuration that is equally fault tolerant to two simultaneous failures. The same is true for planar serial manipulators. However, one can still design for worst-case optimal fault tolerance for up to any two simultaneous failures at a nominal configuration in a manner that will be made precise later. An example of a planar 4R manipulator in a configuration that is optimally fault tolerant to one and two failures is given in [12]. The goal of the current section is to determine a class 8-DOF GSPs that are optimally fault tolerant for up to any two simultaneous joint failures.

We have already seen that, from a relative manipulability index perspective, the null space of the manipulator Jacobian completely characterizes fault tolerance. Basically, the fault tolerance of a manipulator Jacobian depends on how spread out the rows of the null space matrix N are. It is natural then to try to spread these rows out as far as possible. This intuitive notion of spreading out the rows of the null space matrix will serve as a guide to identifying a configuration that is optimally fault tolerant in terms of the worst-case relative manipulability index. This is particularly simple when there is only one degree of redundancy. In this case, the magnitudes of the components of the null vector indicate how the redundancy is distributed among the various joints. If the magnitude of a particular component is zero, that would indicate that no redundancy lies in that joint, and consequently, the joint is critical for full end-effector motion control near that operating configuration. On the other hand, if only one component of the null vector is nonzero, then all of the redundancy resides in that joint, i.e., all of the other joints are critical for full end-effector motion at that configuration, while the joint in question is only capable of producing self-motion there [12]. For greater fault tolerance at the nominal operating configuration, it is desirable that the nominal manipulator Jacobian have a null vector whose components have equal magnitude. Such a Jacobian was given in (11).

The case for manipulators with multiple failures is similar. In designing a fault-tolerant nominal manipulator Jacobian, we first want the manipulator to be optimally fault tolerant to a single failure, i.e., we require the null space matrix N to have rows of equal norm. Among all manipulator Jacobians meeting this requirement, we choose one that optimizes fault tolerance to a second joint failure. This process can be continued to $f \leq r$ failures. In this case, we will say that the manipulator configuration is *optimally fault tolerant in terms of the worst-case relative manipulability index for up to f failures*.

For a manipulator with two degrees of redundancy, it is convenient to consider the rows of the $n \times 2$ null space matrix N to be points or vectors in a plane with each vector originating from the origin. Optimal fault tolerance to a single failure dictates that the norm of the rows be equal, i.e., that the terminal points of the vectors lie on a circle centered at the origin with radius $\sqrt{2/n}$. Now the relative manipulability index ρ_{ij} is equal to the absolute value of the determinant of the matrix consisting of rows i and j of N. Since these rows have length $\sqrt{2/n}$, we have $\rho_{ij} = (2/n)|\sin\phi_{ij}|$, where ϕ_{ij} is the angle between the corresponding ith and jth vectors. Fault tolerance to two failures is then characterized by the $\binom{n}{2}$ angles between the n vectors in the plane.

Before proposing a candidate optimal solution, we note the invariance of the unordered set $\{\rho_{i_1,\cdots,i_f} \mid 1 \leq i_1 < \cdots < i_f \leq n\}$ to two simple operations on the columns of J. If J' is obtained from J by multiplying some of the columns of J by -1, by reordering some of the columns, or by a combination of these two operations, then J and J' have the same unordered set of relative manipulability indices. Now rearranging columns and/or multiplying some of them by -1 affects the rows of the null space matrix in exactly the same way. These observations help simplify our study of fault tolerance. In particular, when identifying an optimal null space matrix N for the $r = 2$ case, we can assume that when the rows of N are viewed as points in the plane, they appear in the upper half plane, for if a particular row does appear in the lower half plane, simply multiply that row by -1 and its point representation will appear in the upper half plane.

A natural candidate for an optimal N can now be obtained by spreading the rows of N out as points on the upper half circle of radius $\sqrt{2/n}$. In other words, we choose the n rows of N to appear as points at the ordered angles

$$\phi_k = \frac{\pi k}{n}, \qquad k = 0, 1, \ldots, n-1 \tag{23}$$

on the circle with radius $\sqrt{2/n}$ and centered at the origin. The fact that (23) determines an optimal null space matrix follows directly from the following result.

Theorem 5: Let N be the $n \times 2$ matrix whose ith row is given by

$$N_i = \left[\sqrt{\frac{2}{n}} \cos\left(\frac{\pi(i-1)}{n}\right) \quad \sqrt{\frac{2}{n}} \sin\left(\frac{\pi(i-1)}{n}\right) \right] \quad i = 1, 2, \ldots, n. \tag{24}$$

Then, N is the null space matrix for an $(n-2) \times n$ matrix J that is optimally fault tolerant for up to two failures.

Proof: See the Appendix.

The null space matrix given by (24) can be used to determine manipulator configurations that are optimally fault tolerant for up to two joint failures. An example of a planar 4R serial manipulator that is in a configuration that is optimally fault tolerant to two failures is given in [12].

Equation (23) is not the only optimal null space matrix. As mentioned earlier, postmultiplication of a null space matrix by an arbitrary $r \times r$ orthogonal matrix will result in a null space matrix corresponding to the same null space of J. Also note that permuting or multiplying rows of N by -1 will not change the set of values of the relative manipulability indices. These operations on the rows result in precisely the same operations on the columns of J. Note that while permuting the columns of J does not affect the overall geometry of a parallel mechanism, it has a significant impact on the geometry of a serial mechanism.

We will now identify an example of a fully spatial manipulator configuration that is optimally fault tolerant to two failures.

TABLE III
PARAMETERS FOR AN OPTIMALLY FAULT-TOLERANT 6×8 MANIPULATOR JACOBIAN CORRESPONDING TO (26)

i	$\mathbf{n}_i^T$	$\mathbf{r}_i^T$
1	[0.000 0.000 1.000]	[−0.765 −0.639 0.000]
2	[−0.835 0.503 −0.225]	[−0.402 −0.264 0.903]
3	[−0.017 0.148 −0.989]	[0.965 −0.245 −0.054]
4	[0.719 −0.688 −0.101]	[−0.519 −0.426 −0.787]
5	[0.714 0.533 0.454]	[−0.382 0.865 −0.415]
6	[−0.831 0.054 0.554]	[0.020 1.025 −0.069]
7	[−0.482 −0.875 −0.052]	[0.785 −0.399 −0.565]
8	[0.163 0.978 −0.131]	[0.294 0.082 0.986]

Using (23) directly gives a null space matrix

$$N = \begin{bmatrix} 0.500 & 0.000 \\ 0.462 & 0.191 \\ 0.354 & 0.354 \\ 0.191 & 0.462 \\ 0.000 & 0.500 \\ -0.191 & 0.462 \\ -0.354 & 0.354 \\ -0.462 & 0.191 \end{bmatrix} \tag{25}$$

with the property that any corresponding manipulator Jacobian is optimally fault tolerant for up to two joint failures. A specific manipulator Jacobian corresponding to (25) is (26), shown at the bottom of the page.

In this case, the rows of the manipulator Jacobian are mutually orthogonal, and (26) corresponds to an OGSP that is optimally fault tolerant for up to two joint failures with a worst-case relative manipulability index $\sqrt{2/8} = 0.5$ for single failures and $2\sin(\pi/8)/8 = 0.0957$ for two simultaneous failures. Note that the second quantity compares reasonably well to the upper bound $1/\sqrt{28} = 0.1890$ given in (9). The parallel mechanism parameters for the corresponding manipulator Jacobian are given in Table III. As before, for a parallel manipulator, the unit vector $\mathbf{n}_i$ in the table indicates the direction of the ith strut while $\mathbf{r}_i$ represents the point on the axis of the ith strut that is closest to the origin.

Once again, there are a number of alternative manipulator realizations that can be generated from the Jacobian in (26). Two different parallel manipulators generated from this Jacobian are shown in Figs. 5 and 6. One fault-tolerant configuration for a serial manipulator is depicted in Fig. 7 with its Denavit–Hartenberg parameters given in Table IV. Also, as the dimension arguments presented in Section II-B indicate, a designer has a significant amount of freedom in choosing a

$$J^T = \begin{bmatrix} 0.000 & 0.000 & 1.000 & -0.639 & 0.765 & 0.000 \\ -0.835 & 0.503 & -0.225 & -0.395 & -0.845 & -0.423 \\ -0.017 & 0.148 & -0.989 & 0.251 & 0.955 & 0.139 \\ 0.719 & -0.688 & -0.101 & -0.498 & -0.618 & 0.663 \\ 0.714 & 0.533 & 0.454 & 0.614 & -0.123 & -0.822 \\ -0.831 & 0.054 & 0.554 & 0.572 & 0.046 & 0.853 \\ -0.482 & -0.875 & -0.052 & -0.474 & 0.314 & -0.879 \\ 0.163 & 0.978 & -0.131 & -0.975 & 0.199 & 0.277 \end{bmatrix}. \tag{26}$$

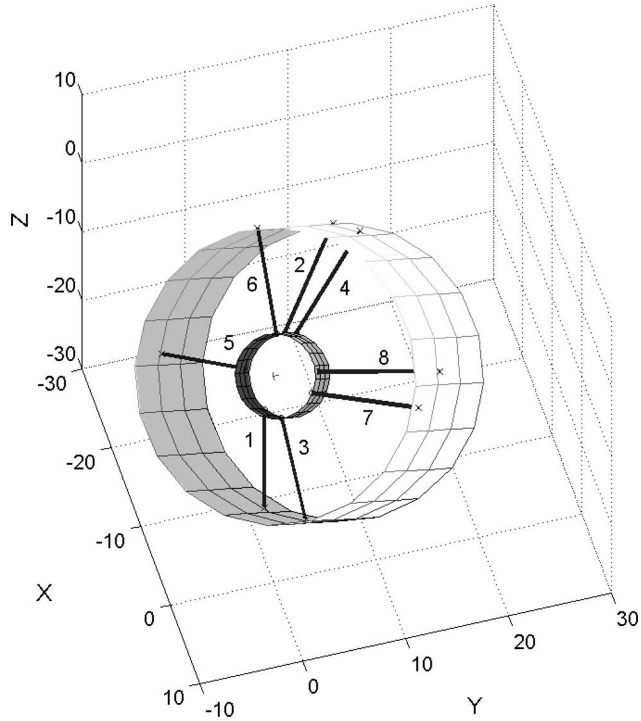

Fig. 5. Example of a cylindrical geometry for an OGSP corresponding to a realization of the optimally fault-tolerant 6×8 manipulator Jacobian given in (26). The $\mathbf{n}_i$'s for struts 2–5 have been reversed,i.e., the corresponding rows of (26) have been multiplied by -1.

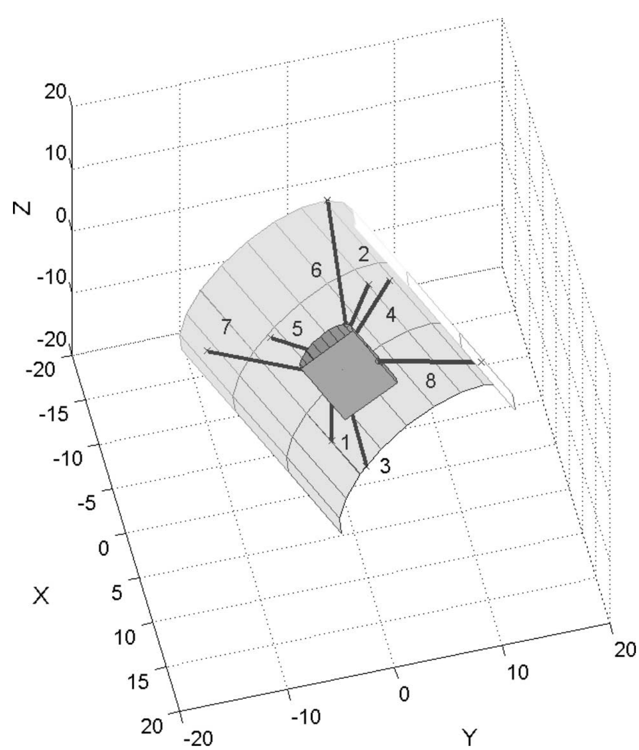

Fig. 6. Another variation of a parallel mechanism related to the optimally fault tolerant 6×8 Jacobian given in (26). In this case, the $\mathbf{n}_i$'s for struts 2, 3, 6, 7, and 8 have been reversed, i.e., the corresponding rows of (26) have been multiplied by -1.

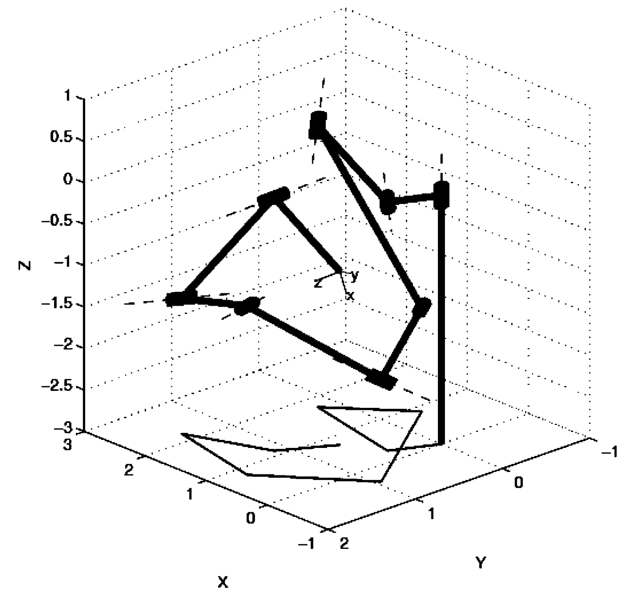

Fig. 7. Serial mechanism corresponding to the Denavit–Hartenberg parameters given in Table IV. This manipulator is in a configuration with an optimally fault-tolerant 6×8 manipulator Jacobian. (This image was generated using the Robotics Toolbox described in [39].

TABLE IV
DENAVIT–HARTENBERG TABLE FOR AN OPTIMALLY FAULT-TOLERANT 6×8 MANIPULATOR JACOBIAN

i	α_i	a_i	θ_i	d_i
1	1.798	-0.508	-2.113	0.000
2	1.254	-0.568	-0.124	-1.454
3	1.585	1.230	0.259	2.052
4	1.470	-0.226	-1.178	-1.791
5	1.889	0.160	0.799	1.588
6	1.241	0.462	0.617	-1.396
7	2.757	-1.668	-1.994	0.711
8	0.000	1.068	0.759	-0.419

manipulator Jacobian for a given optimal null space matrix. The Jacobians in (11) and (26) were chosen to minimize the condition number of the numerical Jacobian subject to the constraint that the manipulator is an orthogonal GSP. The designer could instead choose an objective function to address geometric issues such as increased symmetry.

VI. Discussion

The relative manipulability index is a measure for comparing the remaining manipulability of a redundant manipulator following one or more failures to the original manipulability before failure. The previous sections provide optimal values for the worst-case relative manipulability index for the cases of a 7- and an 8-DOF manipulator. For a 7-DOF manipulator, the optimal value is $1/\sqrt{7} = 0.3780$ for the case of a single joint failure, while the worst-case relative manipulability index for an 8-DOF manipulator is 0.5 and 0.0957 for one and two joint failures, respectively. Viewed as percentages (i.e., 37.8%, 50%, and 9.57%), it is tempting to conclude that designing for tolerance to multiple failures is impractical. Such numbers would steadily worsen for systems designed for a higher number of failures. Admittedly, these values appear unappealing at first glance; however, as we will explain, one reason that these numbers, especially 9.57%, initially appear to be so poor is due to the power law effect inherent in the manipulability index.

The fact that the relative manipulability indices are nonzero at these configurations means that full end-effector control is maintained in spite of the failures. Consequently, the manipulator is still capable of continuing its task, but the speed at which the task can be completed is, of course, significantly decreased. If we compare the speeds prior to and after the joint failures, we see that the system performance is not nearly as degraded as the earlier numbers seem to intuitively indicate. To see this, consider the following observations.

The singular values of the manipulator Jacobian provide insight into the velocity characteristics of the manipulator; however, as pointed out in the literature, one should exercise caution when utilizing the singular values of the manipulator Jacobian for this purpose as this approach assumes a prespecified weighting between linear and angular velocities [40]. The manipulability index is equal to the product of the six singular values so that the sixth root of the manipulability index is equal to the geometric mean of the singular values, which is one possible measure of how efficiently a serial manipulator transforms the joint motion into end-effector motion. Thus, the overall speed at which the manipulator can continue its task following failures in joints $i_1, i_2, \ldots, i_f$ is proportional to the product of $\rho_{i_1,i_2,\ldots,i_f}^{1/6}$ with the end-effector speed prior to failure. In terms of relative speed following the various joint failures, the previous three percentages now become 85.03%, 89.09%, and 67.63%, respectively. So, one can argue that a relative manipulability index of 0.0957 is actually fairly good, i.e., it retains approximately two-thirds of the velocity scaling. Still, the numbers indicate that there are fundamental limitations to what can be achieved in terms of designing for multiple joint failures.

With all of that said, it is important to remember that there are many considerations when designing a manipulator for a specific task. For certain applications, fault tolerance can be a critical consideration; however, fault tolerance is generally not the only desirable attribute for a manipulator. In most cases, merely achieving a certain level of fault tolerance is sufficient. The examples provided in this paper are optimal for fault tolerance and consequently serve as a guide to what can be achieved in this regard. This allows a designer to focus on other important aspects of manipulator design while being assured that fault tolerance has been adequately addressed given these limitations.

Before concluding, we briefly describe some other local measures of fault tolerance. The minimum singular value of ${}^iJ$ was used as a measure of fault tolerance in [27]. The minimum singular value provides a measure of how close the numerical Jacobian is to losing full rank. Unlike the relative manipulability index, it is generally not possible to obtain closed-form solutions for the minimum singular value of the reduced Jacobians ${}^iJ$, a notable exception being the case when the original Jacobian is in an isotropic configuration (i.e., a configuration where all the singular values are equal) [11], [41]. Consequently, fault-tolerance measures based on the minimum singular value do not enjoy as rich a theory as measures based on the relative manipulability index do. Furthermore, measures based on the singular values of the manipulator Jacobian are usually dependent upon the dimensions used. This also includes the manipulability index. However, while the relative manipulability index is defined in terms of the manipulability index, it is a ratio of the manipulability index of two Jacobians and is not dependent upon the units chosen. In fact, as a function of the null space of the manipulator Jacobian, the relative manipulability index is a geometric property of the manipulator and is independent of the coordinate frame and units chosen by the operator.

Another possible measure of the fault tolerance based on relative manipulability indices is the mean relative manipulability index:

$$\mu = \frac{1}{\binom{n}{f}} \sum_{1 \le i_1 < \cdots < i_f \le n} \rho_{i_1,\ldots,i_f}. \tag{27}$$

It turns out that this measure has many of the same characteristics as the minimum relative manipulability index. For example, one can show that (27) is also bounded by (9) and that this upper bound, like the minimum relative manipulability index, would be achieved precisely when all of the relative manipulability indices are equal. Furthermore, the optimal solution described in Section V is also optimal if the mean relative manipulability index is used in place of the minimum relative manipulability index. However, if one is really more concerned with the robot not being able to finish its job after the failures occur, it would be more natural to use the worst-case relative manipulability index. Note that even though the optimal solutions of these two measures are the same, away from the optimal solution, a relatively good mean relative manipulability does not generally imply a good worst-case relative manipulability index. If one were to incorporate a fault-tolerance measure as part of a combined objective function that took into account multiple criteria, the worst-case relative manipulability index would be preferable over the mean relative manipulability index.

VII. Conclusion

In this paper, the authors examined the issue of designing kinematically redundant manipulators that are optimally fault tolerant to multiple joint failures. The authors provided an alternative proof of the recent result that the sum of the squares of the relative manipulability indices corresponding to f failures is equal to $\binom{r}{f}$. This result provides an upper bound for the remaining amount of dexterity of a manipulator with one or more failed joints. Previously, this upper bound was used to characterize optimal fault tolerance to multiple failures. However, in this paper, it was shown that this upper bound for the worst-case relatively manipulability index is typically not achieved, and is therefore, not suitable for identifying fully spatial manipulators that are optimally fault tolerant to multiple failures. This clearly indicates the need for further consideration when designing robotic systems that are tolerant to multiple joint failures. By identifying the required properties of the null space of the manipulator Jacobian, the authors presented a general method for finding a family of 8-DOF GSPs with optimal worst-case fault tolerance for up to two failures.

APPENDIX
PROOFS OF THEOREMS 1, 3, AND 4

Theorem 1: If a manipulator is equally fault tolerant to f failures where $1 < f \leq r$, then it is also equally fault tolerant to $f-1$ failures. Furthermore, the manipulator is equally fault tolerant to k failures for $k = 1, 2, \ldots, f$.

Proof: We can prove the result by demonstrating that $\rho^2_{i_1,\ldots,i_{f-1}} = \binom{r}{f-1}/\binom{n}{f-1}$ for any $1 \leq i_1 < \cdots < i_{f-1} \leq n$. Rearranging the columns of N^T does not affect the overall fault tolerance analysis, so we can assume without loss of generality that $i_1 = 1, \ldots, i_{f-1} = f-1$. Likewise, premultiplying N^T by an $r \times r$ orthogonal matrix Q does not affect the fault tolerance analysis. Hence, by applying a QR factorization, we can further assume without loss of generality that N^T has the form given by (A1), shown at the bottom of the page.

Now

$$\rho^2_{1,\cdots,f} = |N_{11}N_{22}\cdots N_{f-1,f-1}N_{ff}|^2 = \frac{\binom{r}{f}}{\binom{n}{f}} \tag{A2}$$

where the first equality in (A2) follows by direct calculation from (A1) and the second equality follows from the assumption that the manipulator is equally fault tolerant to f failures. Since premultiplying N^T by an orthogonal matrix does not affect the values of the relative manipulability indices, we can easily determine $\rho^2_{1,\ldots,f-1,j}$ for $j = f, \ldots, n$ by first premultiplying (A1) by $\mathrm{diag}(I_{f-1}, U_j)$, where I_{f-1} is an $(f-1) \times (f-1)$ identity matrix and U_j is an $(r-f+1) \times (r-f+1)$ orthogonal matrix that zeros out the last $r-f$ elements of the jth column of N^T so that the (f,j) component of N^T becomes $\pm\alpha_j$, where $\alpha_j = \sqrt{N^2_{fj} + \cdots + N^2_{rj}}$. We then have for $j = f, \ldots, r$

$$\rho^2_{1,\ldots,f-1,j} = |N_{11}N_{22}\cdots N_{f-1,f-1}|^2 \alpha_j^2. \tag{A3}$$

Equating (A3) to the second term in (A2), we conclude that $\alpha_j = |N_{ff}|$ for $j = f, \ldots, n$ and zero otherwise so that $\sum_j \alpha_j^2 = (n-f+1)|N_{ff}|^2$. Now, the quantity $\sum_j \alpha_j^2$ is equal to the sum of the squares of the components of the last $r-f+1$ rows of N^T and since the rows of N^T have unit length, we have that $\sum_j \alpha_j^2 = r-f+1$ so that

$$|N_{ff}|^2 = \frac{r-f+1}{n-f+1}. \tag{A4}$$

It then follows that

$$\rho^2_{1,\ldots,f-1} = \frac{\rho^2_{1,\ldots,f}}{|N_{ff}|^2} = \frac{\binom{r}{f}}{\binom{n}{f}}\,\frac{n-f+1}{r-f+1} = \frac{\binom{r}{f-1}}{\binom{n}{f-1}}. \tag{A5}$$

Since the order of the columns does not matter, we conclude that the relative manipulability index for any $f-1$ failures is given by (A5). Repeated application of this result implies that the manipulator is equally fault tolerant to k failures for $k = 1, 2, \ldots, f$. □

Theorem 3: A Jacobian is equally fault tolerant to $f \geq 3$ simultaneous joint failures if and only if J is a $1 \times n$ matrix of the form $J = [\pm\alpha \quad \pm\alpha \quad \cdots \quad \pm\alpha]$, where $\alpha > 0$ and $n \geq f+1$.

Proof: By Theorem 1, we only need to prove the result for $f = 3$ failures. To simplify matters, note that multiplying any of the columns of J by -1 does not affect the fault tolerance properties of J. In doing so, the corresponding columns of N^T are multiplied by -1, in which case the corresponding rows and columns of P_N are multiplied by -1. Hence, without loss of generality, we can assume that the first row and column of P_N consists of a single a followed by $n-1$ b's. Thus, for $1 < i < j \leq n$

$$P_N \begin{pmatrix} 1 & i & j \\ 1 & i & j \end{pmatrix} = \begin{vmatrix} a & b & b \\ b & a & \pm b \\ b & \pm b & a \end{vmatrix}. \tag{A6}$$

One can then equate (A6) to $\binom{r}{3}/\binom{n}{3}$ and deduce through a series of algebraic simplifications that $r = 1$ or $n = r+1$. This process, however, is tedious and not very insightful. Instead, we will take a less direct but more insightful approach using the eigenvalues of P_N. With $+b$, (A6) becomes $a^3 - 3ab^2 + 2b^3$, and with $-b$, it becomes $a^3 - 3ab^2 - 2b^3$. These quantities are equal if and only if $b = 0$ and since $b = \frac{-1}{n}\sqrt{\frac{r(n-r)}{n-1}} \neq 0$, it follows that the various p_{ij}'s must all be equal for $1 < i < j \leq n$ for the equal fault-tolerance property to hold. If $p_{ij} = b$ for $1 < i < j \leq n$, then we can write

$$P_N = (a-b)I + b\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \tag{A7}$$

$$N^T = \begin{bmatrix}
N_{11} & N_{12} & N_{13} & \cdots & N_{1,f-1} & N_{1f} & N_{1,f+1} & \cdots & N_{1n} \\
0 & N_{22} & N_{23} & \cdots & N_{2,f-1} & N_{2f} & N_{2,f+1} & \cdots & N_{2n} \\
0 & 0 & N_{33} & \cdots & N_{3,f-1} & N_{3f} & N_{3,f+1} & \cdots & N_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & N_{f-1,f-1} & N_{f-1,f} & N_{f-1,f+1} & \cdots & N_{f-1,n} \\
0 & 0 & 0 & \cdots & 0 & N_{f,f} & N_{f,f+1} & \cdots & N_{f,n} \\
0 & 0 & 0 & \cdots & 0 & 0 & N_{f+1,f+1} & \cdots & N_{f+1,n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & N_{r,f+1} & \cdots & N_{r,n}
\end{bmatrix}. \tag{A1}$$

which has eigenvalues $\{a-b,\ldots,a-b,a+(n-1)b\}$. On the other hand, if $p_{ij}=-b$ for $1<i<j\leq n$, then we can write

$$P_N=(a+b)I-b\begin{bmatrix}-1\\1\\\vdots\\1\end{bmatrix}\left[-1\quad 1\quad\cdots\quad 1\right]. \qquad \text{(A8)}$$

In this case, the eigenvalues of P_N are $\{a+b,\ldots,a+b,a-(n-1)b\}$. Since P_N is a projection matrix, its set of eigenvalues consists of ones and zeros. There are $n-1$ eigenvalues of (A8) that are equal to $a+b=\frac{r}{n}-\frac{1}{n}\sqrt{\frac{r(n-r)}{n-1}}<a<1$. Since $1<r<n$, it follows that the quantity $a+b$ is greater than zero but less than one, implying that (A8) cannot correspond to a projection matrix. Consider now the eigenvalues of (A7). The $n-1$ eigenvalues that are equal to $a-b=\frac{r}{n}+\frac{1}{n}\sqrt{\frac{r(n-r)}{n-1}}$ are positive, so they must equal one if (A7) is a projection operator. Setting this quantity equal to one yields the result that $r=n-1$, which upon substitution into $a+(n-1)b$ yields zero. We thus conclude that P_N has rank $r=n-1$ and that the workspace dimension is $m=1$. Hence, any J that is equally fault tolerant to three or more joint failures necessarily has the form $J=[\pm\alpha\quad\cdots\quad\pm\alpha]$ for some $\alpha>0$, where $n\geq f+1$. □

Theorem 4: Regardless of a manipulator's geometry or the amount of kinematic redundancy present in a manipulator, no fully spatial manipulator Jacobian can be equally fault tolerant to two joint failures.

Proof: We have already eliminated all possible cases except $r=6$ and $r=10$. We now eliminate the case $r=6$. We do this by systematically studying the row structure of N^T.

We can assume that N^T has the form given in (A1). Equal fault tolerance to single failures implies that each column of N^T has norm $1/\sqrt{2}$. Furthermore, equal fault tolerance to two failures implies that $N_{22}=\pm\sqrt{5/11}$, and since the second column of N^T has norm $1/\sqrt{2}$, it follows that $N_{12}=\pm 1/\sqrt{22}$. Equal fault tolerance to two failures also implies that the absolute values of the dot products of distinct columns of N^T are equal. Examining those dot products that include the first column, we conclude that $N_{1k}=\pm 1/\sqrt{22}$ for $k=3,4,\ldots,12$. Since multiplying any column by -1 does not change the fault tolerance properties of N^T, we can assume without loss of generality that $N_{11}=1/\sqrt{2}$ and $N_{1k}=1/\sqrt{22}$ for $k=2,3,\ldots,12$. This determines the first row of N^T.

To determine the second row, we begin by noting that multiplying any row by -1 does not affect the fault tolerance, so we can assume that $N_{22}=\sqrt{5/11}$. Equating the absolute values of the dot product of the second column of N^T with the kth column for $k>2$ and the dot product of the first and second columns of N^T gives

$$\frac{1}{22}+\sqrt{\frac{5}{11}}N_{2k}=\frac{\pm 1}{2\sqrt{11}} \qquad \text{(A9)}$$

so that N_{2k} is equal to $a=(\sqrt{11}-1)/\sqrt{220}$ or $b=(-\sqrt{11}-1)/\sqrt{220}$. Let n_1 denote the number of N_{2k}'s equal to a and n_2 denote the number of N_{2k}'s equal to b. Then, n_1 and n_2 are nonnegative integers that sum to 10. It is not difficult to show that the orthogonality of the first two rows implies that $n_1=n_2=5$. We can, without affecting the fault tolerance properties of N^T, arrange the order of columns so that $N_{2k}=a$ for $k=3,4,\ldots,7$ and $N_{2k}=b$ for $k=8,9,\ldots,12$.

Continuing in a similar fashion, we can set the third row so that $N_{33}=\sqrt{\frac{2}{5}+\frac{1}{10\sqrt{11}}}$, N_{3k} equals α or β for $k=4,5,6,7$ (the actual values of α and β are not important to us here), and N_{3k} equals $\gamma=\sqrt{\frac{44-\sqrt{11}}{770}}$ or $\delta=-\gamma$ for $k=8,9,\ldots,12$. Let n_1, n_2, n_3, and n_4 now denote the number of columns whose third element has the value α, β, γ, and δ, respectively. Taking the dot product of the third row with the first and second rows, respectively, we find that

$$(N_{33}+n_1\alpha+n_2\beta)+(n_3\gamma+n_4\delta)=0$$
$$a(N_{33}+n_1\alpha+n_2\beta)+b(n_3\gamma+n_4\delta)=0 \qquad \text{(A10)}$$

from which we obtain $(b-a)(n_3\gamma+n_4\delta)=0$. Since $b\neq a$ and $\delta=-\gamma\neq 0$, we conclude that $n_3-n_4=0$, which, along with the fact that $n_3+n_4=5$, implies that $n_3=n_4=5/2$, contradicting the fact that n_3 and n_4 are by definition integers.

The proof for $r=10$, which is not included here due to space limitations, is very similar with a contradiction appearing in the fourth row. □

Theorem 5: Let N be the $n\times 2$ matrix whose ith row is given by (24), repeated here

$$N_i=\left[\sqrt{\frac{2}{n}}\cos\left(\frac{\pi(i-1)}{n}\right)\quad\sqrt{\frac{2}{n}}\sin\left(\frac{\pi(i-1)}{n}\right)\right]$$
$$i=1,2,\ldots,n. \qquad \text{(A11)}$$

Then, N is the null space matrix for an $(n-2)\times n$ matrix J that is optimally fault tolerant for up to two failures.

Proof: First, we need to show that N is a null space matrix that is at least optimally fault tolerant to a single failure. As before, let $\phi_k=\pi k/n$ for $k=0,1,\ldots,n-1$. The fact that $N^TN=I$ follows from

$$\begin{aligned}N^TN&=\frac{2}{n}\sum_{k=0}^{n-1}\begin{bmatrix}\cos\phi_k\\\sin\phi_k\end{bmatrix}\left[\cos\phi_k\quad\sin\phi_k\right]\\&=\frac{1}{n}\sum_{k=0}^{n-1}\begin{bmatrix}1+\cos 2\phi_k & \sin 2\phi_k\\ \sin 2\phi_k & 1-\cos 2\phi_k\end{bmatrix}\\&=I\end{aligned} \qquad \text{(A12)}$$

where we have used some standard trigonometric identities including the well-known identity $\sum_{k=0}^{n-1}\cos\frac{2\pi k}{n}=\sum_{k=0}^{n-1}\sin\frac{2\pi k}{n}=0$ for $n>1$. This proves that N is a null space matrix. Clearly, the rows of N each have norm $\sqrt{2/n}$, which proves that N is optimally fault tolerant to a single failure.

Now the adjacent rows of N are at an angle of π/n radians apart while nonadjacent rows would be $l\pi/n$ radians apart for some $l=2,\ldots,n-1$. Consequently, the worst-case relative manipulability index corresponding to N is $r^*=\frac{2}{n}\sin(\pi/n)$.

To see that this is the optimal solution to the problem, suppose that N' is any other $n\times 2$ null space matrix with equal length rows. Recall the earlier observation that postmultiplication of N' by an orthogonal matrix has no effect on its fault-tolerance

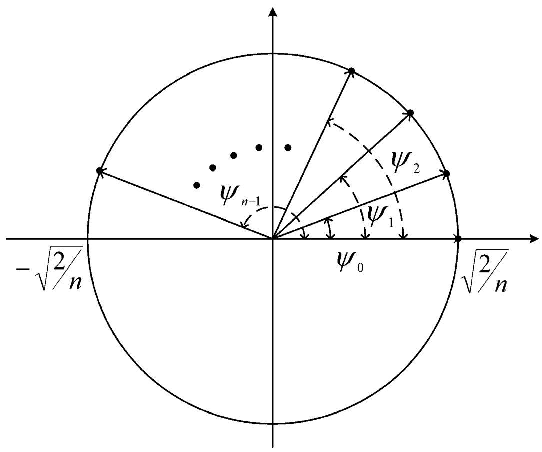

Fig. 8. Illustration showing how the rows of the null space matrix can be spread out to obtain an optimally fault-tolerant manipulator Jacobian.

properties. Consequently, we can take a given null space matrix N', postmultiply it by a suitable rotation matrix R^T to move one of the rows to $(\sqrt{2/n}, 0)$, multiply those rows of $N'R^T$ lying in the lower half plane by -1 and rearrange the rows so that they appear in increasing order of the angle that they make with the x axis when viewed as points on the circle of radius $\sqrt{2/n}$. This would not affect the fault-tolerance properties of the given null space matrix, so we can assume that N' has the desired form. The ordered angles of the columns are then $0 = \psi_0 \leq \psi_1 \leq \cdots \leq \psi_{n-1} \leq \pi$, as depicted in Fig. 8. Thus, the jth row of N' would be $N'_j = \left[\sqrt{\frac{2}{n}} \cos \psi_{j-1} \quad \sqrt{\frac{2}{n}} \sin \psi_{j-1} \right]$. Suppose that the worst case relative manipulability index for N' is better than our proposed solution, i.e., suppose that

$$\min_{1 \leq i < j < n} \frac{2}{n} \sin(\psi_j - \psi_i) > \frac{2}{n} \sin\left(\frac{\pi}{n}\right) = r^*. \tag{A13}$$

Now $\psi_1 > \pi/n$, otherwise

$$\frac{2}{n} \sin(\psi_1 - \psi_0) = \frac{2}{n} \sin \psi_1 \leq \frac{2}{n} \sin\left(\frac{\pi}{n}\right) = r^* \tag{A14}$$

contradicting the assumption given in (A13). Similarly,

$$\psi_j - \psi_{j-1} > \frac{\pi}{n}, \qquad j = 1, 2, \ldots, n-1. \tag{A15}$$

Hence,

$$\begin{aligned}\psi_{n-1} &= \psi_{n-1} - \psi_{n-2} + \psi_{n-2} - \psi_{n-3} \\ &\quad + \cdots + \psi_1 - \psi_0 + \psi_0 \\ &> \frac{\pi}{n} + \cdots + \frac{\pi}{n} + 0 = \frac{(n-1)\pi}{n}. \end{aligned} \tag{A16}$$

Thus, $(n-1)\pi/n < \psi_{n-1} \leq \pi$ so that $(2/n)\sin(\psi_{n-1} - \psi_0) = (2/n)\sin(\psi_{n-1}) < r^*$. In other words, locking the two joints corresponding to angles ψ_0 and ψ_{n-1} results in a relative manipulability index that is less than r^*, which contradicts (A13). This proves that (A11) corresponds to an optimal solution. □

REFERENCES

[1] Y. Chen, J. E. McInroy, and Y. Yi, "Optimal, fault-tolerant mappings to achieve secondary goals without compromising primary performance" *IEEE Trans. Robot.*, vol. 19, no. 4, pp. 680–691, Aug. 2003.

[2] R. Colbaugh and M. Jamshidi, "Robot manipulator control for hazardous waste-handling applications," *J. Robot. Syst.*, vol. 9, no. 2, pp. 215–250, 1992.

[3] J. D. English and A. A. Maciejewski, "Fault tolerance for kinematically redundant manipulators: Anticipating free-swinging joint failures," *IEEE Trans. Robot. Autom.*, vol. 14, no. 4, pp. 566–575, Aug. 1998.

[4] K. N. Groom, A. A. Maciejewski, and V. Balakrishnan, "Real-time failure-tolerant control of kinematically redundant manipulators," *IEEE Trans. Robot. Autom.*, vol. 15, no. 6, pp. 1109–1116, Dec. 1999.

[5] M. L. Leuschen, I. D. Walker, and J. R. Cavallaro, "Investigation of reliability of hydraulic robots for hazardous environment using analytic redundancy," in *Proc. Annu. Rel. Maintain. Symp.*, Washington, DC, Jan. 18–21, 1999, pp. 122–128.

[6] J. E. McInroy, J. F. O'Brien, and G. W. Neat, "Precise, fault-tolerant pointing using a Stewart platform," *IEEE/ASME Trans. Mechatronics*, vol. 4, no. 1, pp. 91–95, Mar. 1999.

[7] L. Notash and L. Huang, "On the design of fault tolerant parallel manipulators," *Mech. Mach. Theory*, vol. 38, pp. 85–101, 2003.

[8] E. C. Wu, J. C. Hwang, and J. T. Chladek, "Fault-tolerant joint development for the space shuttle remote manipulator system: Analysis and experiment," *IEEE Trans. Robot. Autom.*, vol. 9, no. 5, pp. 675–684, Oct. 1993.

[9] Y. Yi, J. E. McInroy, and Y. Chen, "Fault tolerance of parallel manipulators using task space and kinematic redundancy," *IEEE Trans. Robot.*, vol. 22, no. 5, pp. 1017–1021, Oct. 2006.

[10] Y. Yi, J. E. McInroy, and F. Jafari, "Optimum design of a class of fault tolerant isotropic Gough-Stewart platforms," in *Proc. IEEE Int. Conf. Robot. Autom.*, New Orleans, LA, Apr. 26–May 1, 2004, pp. 4963–4968.

[11] A. A. Maciejewski, "Fault tolerant properties of kinematically redundant manipulators," in *Proc. IEEE Int. Conf. Robot. Autom.*, Cincinnati, OH, May 13–18, 1990, pp. 638–642.

[12] R. G. Roberts and A. A. Maciejewski, "A local measure of fault tolerance for kinematically redundant manipulators," *IEEE Trans. Robot. Autom.*, vol. 12, no. 4, pp. 543–552, Aug. 1996.

[13] C. J. J. Paredis and P. K. Khosla, "Designing fault-tolerant manipulators: How many degrees of freedom?," *Int. J. Robot. Res.*, vol. 15, no. 6, pp. 611–628, Dec. 1996.

[14] S. Tosunoglu and V. Monteverde, "Kinematic and structural design assessment of fault-tolerant manipulators," *Intell. Autom. Soft Comput.*, vol. 4, no. 3, pp. 261–268, 1998.

[15] C. Carreras and I. D. Walker, "Interval methods for fault-tree analysis in robotics," *IEEE Trans. Robot. Autom.*, vol. 50, no. 1, pp. 3–11, Mar. 2001.

[16] B. S. Dhillon, A. R. M. Fashandi, and K. L. Liu, "Robot systems reliability and safety: A review," *J. Quality Maintenance Eng.*, vol. 8, no. 3, pp. 170–212, 2002.

[17] B. S. Dhillon and A. R. M. Fashandi, "Safety and reliability assessment techniques in robotics," *Robotica*, vol. 15, no. 6, pp. 701–708, Nov./Dec. 1997.

[18] I. D. Walker and J. R. Cavallaro, "The use of fault trees for the design of robots for hazardous environments," in *Proc. Annu. Rel. Maintain. Symp.*, Las Vegas, NV, Jan. 22–25, 1996, pp. 229–235.

[19] L. Notash, "Joint sensor fault detection for fault tolerant parallel manipulators," *J. Robot. Syst.*, vol. 17, no. 3, pp. 149–157, 2000.

[20] Y. Ting, S. Tosunoglu, and R. Freeman, "Actuator saturation avoidance for fault-tolerant robots," in *Proc. 32nd Conf. Decis. Control.*, San Antonio, TX, Dec.1993, pp. 2125–2130.

[21] Y. Ting, S. Tosunoglu, and D. Tesar, "A control structure for fault-tolerant operation of robotic manipulators," in *Proc. IEEE Int. Conf. Robot. Autom.*, Atlanta, GA, May 2–6, 1993, pp. 684–690.

[22] M. L. Visinsky, J. R. Cavallaro, and I. D. Walker, "A dynamic fault tolerance framework for remote robots," *IEEE Trans. Robot. Autom.*, vol. 11, no. 4, pp. 477–490, Aug. 1995.

[23] S. K. Ralph and D. K. Pai, "Computing fault tolerant motions for a robot manipulator," in *Proc. IEEE Int. Conf. Robot. Autom.*, Detroit, MI, May 10–15, 1999, pp. 486–493.

[24] J. Park, W.-K. Chung, and Y. Youm, "Failure recovery by exploiting kinematic redundancy," in *5th Int. Workshop Robot Human Commun.*, Tsukuba, Japan, Nov. 11–14, 1996, pp. 298–305.

[25] R. S. Jamisola, Jr., A. A. Maciejewski, and R. G. Roberts, "Failure-tolerant path planning for kinematically redundant manipulators

anticipating locked-joint failures," *IEEE Trans. Robot.*, vol. 22, no. 4, pp. 603–612, Aug. 2006.
[26] C. J. J. Paredis and P. K. Khosla, "Fault tolerant task execution through global trajectory planning," *Rel. Eng. Syst. Safety*, vol. 53, pp. 225–235, 1996.
[27] C. L. Lewis and A. A. Maciejewski, "Fault tolerant operation of kinematically redundant manipulators for locked joint failures," *IEEE Trans. Robot. Autom.*, vol. 13, no. 4, pp. 622–629, Aug. 1997.
[28] R. G. Roberts, "The dexterity and singularities of an underactuated robot," *J. Robot. Syst.*, vol. 18, no. 4, pp. 159–169, 2001.
[29] T. Yoshikawa, "Manipulability of robotic mechanisms," *Int. J. Robot. Res.*, vol. 4, no. 2, pp. 3–9, 1985.
[30] J. W. Burdick, "On the inverse kinematics of redundant manipulators: Characterization of the self-motion manifolds," in *Proc. IEEE Int. Conf. Robot. Autom.*, Scottsdale, AZ, May 14–19, 1989, pp. 264–270.
[31] C. S. Ukidve, J. E. McInroy, and F. Jafari, "Orthogonal Gough–Stewart platforms with optimal fault tolerant manipulability," in *Proc. IEEE Int. Conf. Robot. Autom.*, Orlando, FL, May 15–19, 2006, pp. 3801–3806.
[32] J. E. McInroy and F. Jafari, "Finding symmetric orthogonal Gough–Stewart platforms," *IEEE Trans. Robot.*, vol. 22, no. 5, pp. 880–889, Oct. 2006.
[33] Y. Yi, J. E. McInroy, and F. Jafari, "Generating classes of orthogonal Gough–Stewart platforms," in *Proc. IEEE Int. Conf. Robot. Autom.*, Apr. 26–May 1, 2004, pp. 4969–4974.
[34] Y. Yi, J. E. McInroy, and F. Jafari, "Generating classes of locally orthogonal Gough–Stewart platforms," *IEEE Trans. Robot.*, vol. 21, no. 5, pp. 812–820, Oct. 2005.
[35] Z. Guo, J. E. McInroy, and F. Jafari, "Realization of micromanipulating Gough–Stewart platforms with desired dynamics," in *Proc. IEEE Int. Conf. Robot. Autom.*, Orlando, FL, May 15–19, 2006, pp. 655–660.
[36] F. Jafari and J. E. McInroy, "Orthogonal Gough-Stewart platforms for micromanipulation," *IEEE Trans. Robot. Autom.*, vol. 19, no. 4, pp. 595–603, Aug. 2003.
[37] C. Klein and T. A. Miklos, "Spatial robotic isotropy," *Int. J. Robot. Res.*, vol. 6, no. 2, pp. 72–83, 1987.
[38] K. E. Zanganeh and J. Angeles, "Kinematic isotropy and the optimum design of parallel manipulators," *Int. J. Robot. Res.*, vol. 16, no. 2, pp. 185–197, Apr. 1997.
[39] P. I. Corke, "A robotics toolbox for MATLAB," *IEEE Robot. Autom. Mag.*, vol. 3, no. 1, pp. 2–32, Mar. 1996.
[40] K. L. Doty, C. Melchiorri, E. M. Schwartz, and C. Bonivento, "Robot manipulability," *IEEE Trans. Robot. Autom.*, vol. 11, no. 3, pp. 462–468, Jun. 1995.
[41] R. G. Roberts, "On the local fault tolerance of a kinematically redundant manipulator," *J. Robot. Syst.*, vol. 13, no. 10, pp. 649–661, Oct. 1996.

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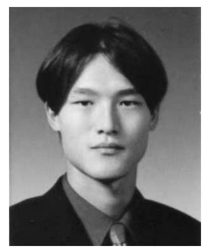

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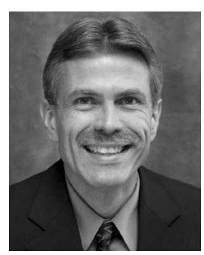

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