Kinetic Limitations on the Use of Redundancy in Robotic Manipultors

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Abstract

The kinematic specification of motion for redundant manipulators has relied primarily on a formulation which decomposes joint velocity solutions into a pseudoinverse component and a homogeneous solution component. While such a formulation is conceptually appealing since it treats the redundant degrees of freedom as independent from those required to maintain a desired end effector trajectory, it has been shown to be physically inaccurate when applied to the kinetic behavior of redundant manipulators. In this work, the kinetic behavior of the homogeneous solution component is analyzed in order to specify realistic limitations on the use of redundancy. It is shown that the equations which govern these limitations are related to the conditions for guaranteeing stability of the local torque minimization formulation.

I. Introduction

The vast majority of efforts to utilize redundancy in robotic manipulators have been focused on the resolution of redundancy at the kinematic level. The kinematics of manipulators is frequently represented by

$$\dot{x} = J\dot{\theta} \tag{1}$$

where $\dot{x}$ is an m dimensional vector specifying the end effector velocity, $\dot{\theta}$ is an n dimensional vector denoting the joint velocities, and J is the m by n Jacobian matrix. For redundant manipulators $n > m$ so that the general solution to (1) is typically presented in the form

$$\dot{\theta} = J^+\dot{x} + (I - J^+J)\dot{\phi} \tag{2}$$

where $^+$ denotes the pseudoinverse, $(I - J^+J)$ is a projection operator onto the null space of J, and $\dot{\phi}$ is an arbitrary vector in $\dot{\theta}$ space. The second term in (2) is the homogeneous solution to (1) since it results in no end effector velocity and will be denoted here by $\dot{\theta}_H$. This homogeneous solution is frequently used to optimize some secondary criterion under the constraint of the specified end effector velocity by choosing $\dot{\phi}$ to be the gradient of some function of θ [10]. Alternative formulations for instantaneously optimizing a secondary criterion by augmenting the Jacobian matrix have also been presented [1,2]. Some of the secondary criteria that have been applied include joint range availability [9], singularity avoidance [12,14], various measures of dexterity [3,4,7,15,16], and obstacle avoidance[13,14]. The homogeneous solution can also be used to optimize secondary criteria defined in Cartesian space, either to impose a priority to the manipulation variables [13] or to avoid obstacles [11], by using

$$\dot{\phi} = [J_2(I - J^+J)]^+(\dot{x}_2 - J_2J^+\dot{x}) \tag{3}$$

where the subscript 2 refers to the secondary criterion. The overall solution is then given by substituting (3) into (2) to obtain

$$\dot{\theta} = J^+\dot{x} + [J_2(I - J^+J)]^+(\dot{x}_2 - J_2J^+\dot{x}) \tag{4}$$

which has been simplified by taking advantage of the fact that the projection operator is Hermetian and idempotent [11].

In all of the above techniques, the specified end effector trajectory is the implicit primary criterion. Unfortunately, the specification of an arbitrary homogeneous joint velocity may result in unrealistic demands on manipulator performance. These difficulties were first illustrated in [8] where the dynamic performance of a redundant manipulator showed significant end effector tracking errors when a secondary criterion was imposed. A more dramatic difficulty with using homogeneous solutions is the instability illustrated in [5] when redundancy is resolved at the acceleration level to instantaneously minimize joint torque. In this case, the joint acceleration is related to the end effector acceleration by differentiating (1) to obtain

$$\ddot{x} = J\ddot{\theta} + \dot{J}\dot{\theta} \tag{5}$$

where once again the general solution is expressed in the form

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\dot{\theta}) + (I - J^+J)\ddot{\phi}. \tag{6}$$

The dynamic equations of a manipulator can be written in closed form as

$$\tau = H\ddot{\theta} + \dot{\theta} \cdot C \cdot \dot{\theta} + g \tag{7}$$

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where τ is the vector of joint torques, H is the inertia matrix, C is a matrix of Coriolis and centrifugal coefficients, and g is the gravity vector. If $\tilde{\tau}$ is denoted as the torque due to the pseudoinverse solution

$$\tilde{\tau} = HJ^{+}(\ddot{x} - \dot{J}\dot{\theta}) + \dot{\theta} \cdot C \cdot \dot{\theta} + g \tag{8}$$

then it is easy to show that the minimum joint torque in a least squares sense is given by

$$\tau = \tilde{\tau} - H[H(I - J^{+}J)]^{+}\tilde{\tau} \tag{9}$$

which results in a joint acceleration of

$$\ddot{\theta} = J^{+}(\ddot{x} - \dot{J}\dot{\theta}) - [H(I - J^{+}J)]^{+}\tilde{\tau}. \tag{10}$$

It has been shown that this joint acceleration can induce large joint velocities which may require physically unrealizable joint torques in order to maintain the desired end effector trajectory.

In this work, in order to place realistic limitations on the use of redundancy, the kinetic effects of a homogeneous solution will be analyzed. It will be shown that an arbitrary homogeneous solution cannot be used without potentially affecting the primary constraint of a desired end effector trajectory. In addition, conditions for identifying the instability of the torque minimization technique will be presented. It will be shown that these conditions are only a function of a manipulator's configuration and thus can be used to determine desirable regions of operation.

II. Kinetic Effects of a Homogeneous Solution

In order to explicitly consider the kinetic effects of a homogeneous velocity, only the case where $\dot{x} = 0$, that is, the desired end effector trajectory requires the hand to remain stationary at a given position and orientation, will be considered. This case occurs in practice whenever a reconfiguration of the manipulator is required to avoid a moving obstacle or as the result of a residual homogeneous velocity after the desired end effector trajectory is completed. Under these conditions, there will in general be an acceleration at the end effector due to the rate of change of J as given by (5). The resulting joint angle acceleration required to maintain the desired configuration of the end effector is given by (6), which if one assumes that there is to be no acceleration along the homogeneous solution results in

$$\ddot{\theta} = -J^{+}\dot{J}\dot{\theta}_H. \tag{11}$$

If the discussion is restricted to manipulators with a single degree of redundancy, then the acceleration given by (11) will result in a constant magnitude of homogeneous velocity which will trace a curve in joint space that corresponds to all the possible manipulator configurations that can be reached without moving the end effector.

The joint acceleration given by (11) has a simple physical interpretation in that it is inversely related to the radius of curvature of this homogeneous solution space curve. In particular, if the radius of curvature is denoted by ρ then (11) can be written as

$$\ddot{\theta} = \frac{\|\dot{\theta}_H\|^2}{\rho}\hat{r} \tag{12}$$

where $\hat{r}$ is a unit vector directed from the homogeneous solution curve toward the center of curvature. The two important points to note about this acceleration are that, first, it's magnitude is proportional to the square of the magnitude of the homogeneous joint velocity, and second, that it's direction is independent of not only the magnitude of $\dot{\theta}_H$ but also of the direction of $\dot{\theta}_H$ around the homogeneous solution curve. Therefore, the direction of the joint acceleration required to maintain the desired end effector trajectory is a function of only the manipulator configuration.

The torque required to maintain a homogeneous joint velocity is given by (8), which, if one neglects the velocity independent gravity term, can be written as

$$\tilde{\tau} = \|\dot{\theta}_H\|^2[\frac{-H\hat{r}}{\rho} + v_n \cdot C \cdot v_n] \tag{13}$$

where v_n is a unit vector along $\dot{\theta}_H$. In the case of a single degree of redundancy v_n corresponds to the nth output singular vector specifying the null space of J. Clearly, the torque given by (13) must be physically achievable in order to maintain the desired end effector trajectory. Now consider the case where an acceleration along the homogeneous solution is allowed. Such an acceleration, denoted by $\ddot{\theta}_H$ and given by the second term in (6), will affect the torque requirements in two ways: directly, through the inertial torque required to achieve the acceleration, and indirectly, through the increase or decrease of the homogeneous velocity. When using the instantaneous torque minimization formulation, the direct effect of $\ddot{\theta}_H$ is used to decrease the torque requirements by applying the acceleration given by (10). In this case the homogeneous acceleration term would be given by

$$\ddot{\theta}_H = \|\dot{\theta}_H\|^2[H(I - J^{+}J)]^{+}t \tag{14}$$

where

$$t = [\frac{H\hat{r}}{\rho} - v_n \cdot C \cdot v_n]. \tag{15}$$

While this homogeneous acceleration term will minimize the instantaneous torque requirement, it tells us nothing about the indirect effect on future torque requirements. In order to obtain this information one must look at the direction of the homogeneous acceleration relative to the homogeneous velocity. Mathematically, if

$$v_n^T\ddot{\theta}_H > 0 \tag{16}$$

then the homogeneous acceleration term will increase the magnitude of the homogeneous velocity and subsequently increase the torque requirements. This, in effect, amounts to a positive feedback system and results in the instability of local torque minimization noted in [5].

In order to guarantee global stability when using the local torque minimization formulation the homogeneous acceleration must not be applied when (16) is true. It is possible to identify regions of stability and instability for this formulation by evaluating the conditions for which (16) holds. Substituting (14) into the left hand side of (16) results in

$$v_n^T \ddot{\theta}_H = v_n^T \|\dot{\theta}_H\|^2 [H(I - J^+J)]^+ t. \tag{17}$$

It is easy to show that

$$[H(I - J^+J)]^+ = \frac{v_n v_n^T H}{v_n^T H^2 v_n} \tag{18}$$

so that (17) becomes

$$v_n^T \ddot{\theta}_H = \|\dot{\theta}_H\|^2 \frac{v_n^T H}{v_n^T H^2 v_n} t. \tag{19}$$

Since only the sign of $v_n^T \ddot{\theta}_H$ is of concern, (19) can be simplified to

$$\text{sign}(v_n^T \ddot{\theta}_H) = \text{sign}(v_n^T H t) \tag{20}$$

since H is positive definite. There are two important observations concerning (20) which should be pointed out. First, the magnitude of the homogeneous joint velocity does not in any way affect the sign of $v_n^T \ddot{\theta}_H$. Second, the vector Ht is independent of the direction of the homogeneous velocity, being solely a function of the manipulator configuration. As a result of the second point, it is possible to determine regions of operation for which the local torque minimization method is inherently stable or unstable. The following section will present a specific example.

In addition to providing conditions for the stability of the instantaneous torque minimization formulation, the above discussion also pertains to the use of homogeneous solutions for realizing other secondary criteria. The torque requirements of a homogeneous velocity given by (13) along with the condition expressed by (16) identify when and to what degree a desired homogeneous joint velocity solution can be induced without resulting in unrealistic torque requirements. In particular, if the torque required to maintain a desired homogeneous velocity is approaching its physical limitation, (16) can be used to determine whether the magnitude of the homogeneous velocity should be reduced immediately or whether the manipulator should wait for a configuration where both the torque and velocity can be reduced simultaneously.

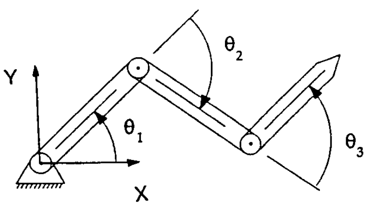

Fig. 1 Geometry of the planar three-link manipulator used in the examples.

III. A Two-Dimensional Example

The issues presented in the previous section will be illustrated for the planar three degree-of-freedom manipulator depicted in Fig. 1. The links are all identical and are modeled as thin uniform rods with lengths of 1 m and masses of 10 kg. The homogeneous solution curves for this manipulator, which have been previously presented [6], are shown in Fig. 2. These curves are plotted for end effector positions ranging from $x = 1.00$ m to $x = 3.00$ m. The curves are generated by selecting the desired end effector position, specifying a homogeneous joint velocity of 1 rad/sec, and then applying the acceleration given by (11). An alternative technique for generating

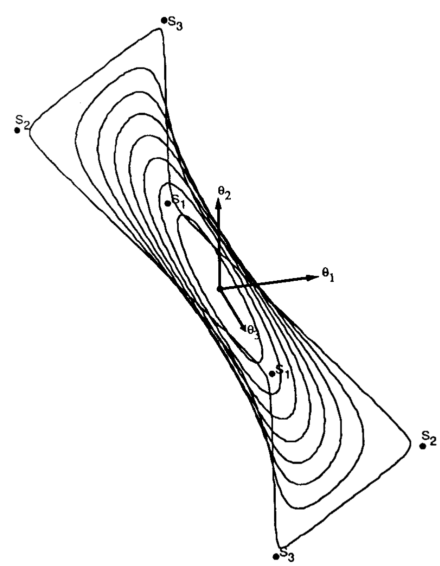

Fig. 2 A parallel projection of the homogeneous solution curves for the manipulator in Fig. 1 plotted in 3D θ space.

these curves is presented in [7]. An important observation concerning these curves is the wide variation in the radius of curvature. This is particularly noticeable for the homogeneous solution curve which goes through the three internal singular configurations labeled S_1 to S_3. Near these singular configurations the radius of curvature approaches zero, resulting in accelerations (and torques) which approach infinity in order to maintain a constant end effector position. The other sections of this curve, however, are nearly linear and thus require virtually no acceleration in order to maintain the desired end effector position. Another view of these homogeneous solution curves is presented in Fig. 3 where the viewing direction is along the vector $\theta^T = [3\ 2\ 1]$. This view is chosen because it tends to more accurately reflect the shape of these curves, particularly at reaches farther from the base, since these curves tend to lie in a plane with a surface normal of [3 2 1]. This orientation is due to using equal link lengths which results in a 3 : 2 : 1 ratio in the columns of the J when the manipulator is fully extended.

In Fig. 4 are shown plots of the acceleration, inertial torque, and total torque required to maintain a homogeneous velocity of 1 rad/s when the end effector is commanded to be at a position 2.00 m from the base. Since the norm of the joint velocity is unity, the norm of the joint acceleration is also equal to the inverse of the radius of curvature of the homogeneous solution curve. The maximum and minimum accelerations denoted A to D can be shown to correspond with the maximum and minimum radiuses of curvature for the curve in Fig. 3. From these plots it is clear that while the inertial torques due to the radius of curvature are the dominant characteristic in determining the overall torque requirements, the Coriolis and centripetal torques do play a significant

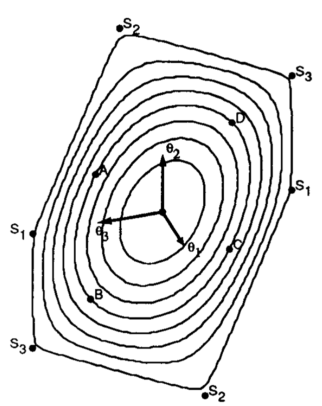

Fig. 3 Another view of the homogeneous solution curves from Fig. 2.

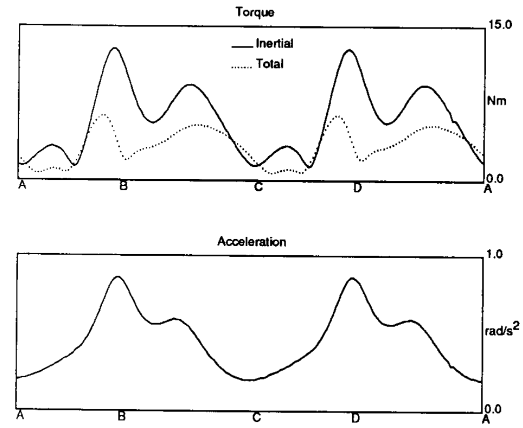

Fig. 4 Graphs of the acceleration, inertial torque, and total torque required to maintain a homogeneous velocity of 1 rad/sec around the homogeneous solution curve for $x = 2.00$ m.

role, typically mediating the effect of the inertial torques, and therefore cannot be ignored. It is important to note once again that the total torque required to maintain the homogeneous velocity, $\bar{\tau}$ given by (13), is independent of the direction of that velocity. In other words, in terms of Fig. 3, $\bar{\tau}$ is independent of whether the velocity is clockwise or counterclockwise.

The direction of the homogeneous acceleration, $\ddot{\theta}_H$, required to reduce the magnitude of the joint torques can be obtained by evaluating (14). The plot in Fig. 5 graphically depicts the sign of $\ddot{\theta}_H$, by illustrating whether it will require a clockwise or counterclockwise acceleration around the homogeneous solution curve. This plot determines whether $\ddot{\theta}_H$ can be used to reduce the magnitude of the homogeneous velocity while simultaneously reducing the joint torque. Note that this is identical to the conditions guaranteeing stability for the local torque minimization scheme. If the manipulator is in a configuration where the homogeneous velocity is in the same direction as the acceleration given by Fig. 5, then the local torque minimization scheme should not be applied.

The data plotted in Fig. 5 can also be used to identify possibly desirable regions of operation. In particular, those configurations which are the boundary between clockwise and counterclockwise $\ddot{\theta}_H$ can be classified as inherently stable or unstable depending on the direction of the transition. As an example, consider the manipulator configuration labeled E in Fig. 5. Regardless of the direction of the homogeneous joint velocity, the homogeneous acceleration can always reduce this velocity while simultaneously minimizing the joint torque thus resulting in an inherently stable configuration. The manipulator

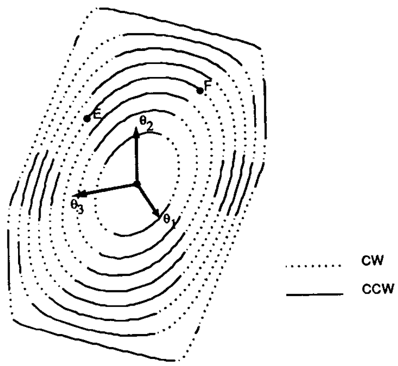

Fig. 5 A plot of the homogeneous solution curves illustrating the direction of the homogeneous acceleration required to reduce the joint torque due to a homogeneous velocity.

configuration labeled F, however, is inherently unstable because regardless of the direction of the homogeneous velocity, reducing the joint torque will always result in a homogeneous acceleration that increases the homogeneous velocity. In some respects it is useful to think of point E being in a valley and point F being on the top of a ridge although the analogy is not perfect.

IV. Simulation Results

To illustrate the characteristics of stable and unstable configurations, simulations were performed in which the manipulator was put into the desired configuration, given an initial homogeneous joint velocity of 1 rad/s, and then commanded to maintain a stationary end effector position. Each simulation was performed twice, once for a homogeneous velocity in the clockwise direction and then again in the counterclockwise direction. In both cases results are shown for the pseudoinverse formulation, given by (11), as a basis of comparison to the torque minimization formulation, which includes (14).

The results for the stable configuration, denoted E, are presented in Fig. 6. As anticipated for this configuration, both the joint velocity norm and joint torque norm can be reduced simultaneously, regardless of the direction of the homogeneous velocity. This characteristic of stable configurations makes them ideal for decelerating a homogeneous velocity when approaching the desired homogeneous space solution for a specific secondary criterion. While the local minimum torque solution cannot in general be used to bring the homogeneous solution to zero velocity, it can be reduced to a point where the non-minimum torque solution is still well within physically achievable limits. For the homogeneous velocity in the counterclockwise direction there is a small hump in the velocity norm which denotes a change in sign of (16). Clearly this hump could be removed by not applying the homogeneous acceleration under these conditions as discussed above.

The results for the unstable configuration, denoted F, are presented in Fig. 7. From these results one can see that the local torque minimization scheme does initially result in a decrease of the torque required to maintain the desired end effector position. However, this decrease is short-lived due to the buildup of the homogeneous velocity which eventually results in physically unachievable torque requirements. It is important to note that these characteristics are not dependent on the end effector trajectory which has brought the manipulator into the unstable configuration. It should also be pointed out that these unstable configurations are not inherently undesirable. In particular, if the torque minimization formulation is only applied for limited periods of time one can "shave" the peaks from the torque curve while waiting for a stable configuration in which to remove the induced homogeneous velocity.

There are two final points which should be addressed concerning the characteristics of homogeneous space solutions. The first is that stable configurations are in no way correlated with globally optimal minimum torque solutions. The second point relates to the fact that unless the secondary criterion induces a large homogeneous velocity, the kinetic requirements of the primary constraint of a specified end effector trajectory will tend to dominate the dynamic behavior of the manipulator. This will usually be true until the manipulator starts to approach the

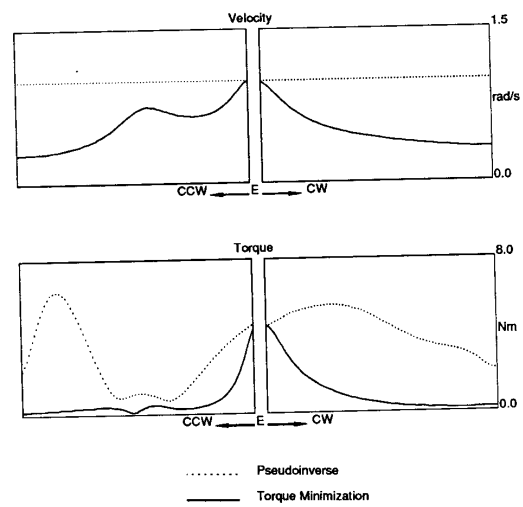

Fig. 6 Simulation results showing the joint velocity norm and torque norm for the manipulator starting in an inherently stable configuration.

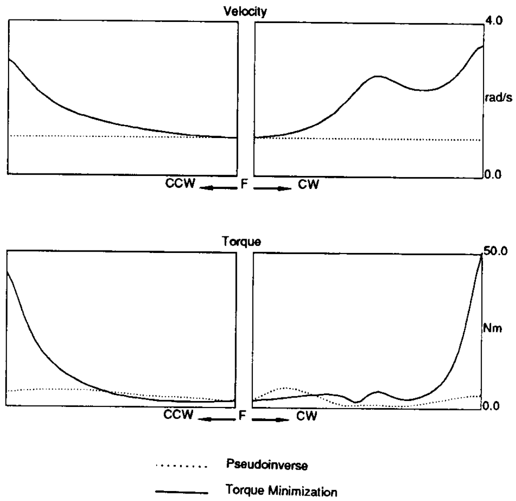

Fig. 7 Simulation results showing the joint velocity norm and torque norm for the manipulator starting in an inherently unstable configuration.

end of its gross motion trajectory where the desired end effector velocity is reduced. It is at this point that consideration of the kinetic effects of any residual homogeneous velocity is critical since they will tend to dominate the behavior of the manipulator. This is particularly true of those tasks which require fine manipulation after gross motion as is typical of most assembly operations.

V. Conclusions

The kinematic specification of motion for redundant manipulators has relied primarily on a formulation which treats the redundant degrees of freedom as independent from those required to maintain a desired end effector trajectory. While such a formulation is conceptually appealing, it has been shown to be physically inaccurate when applied to the kinetic behavior of redundant manipulators. In this work, the kinetic effects of homogeneous solutions have been analyzed with emphasis on placing realistic limitations on how redundancy can be utilized without adversely affecting the primary goal of a desired end effector trajectory. It has been shown that it is possible to identify manipulator configurations which possess the desirable characteristic of being able to either remove or impart a homogeneous velocity while simultaneously reducing the torque requirements on the manipulator. The conditions which govern these configurations have also been shown to be directly related to the conditions for guaranteeing global stability for the local torque minimization formulation.

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