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Examples of planar robot kinematic designs from optimally fault-tolerant Jacobians

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***Abstract*— It is common practice to design a robot's kinematics from the desired properties that are locally specified by a manipulator Jacobian. It has been recently shown that multiple different physical robot kinematic designs can be obtained from (essentially) a single Jacobian that has desirable fault tolerant properties [1]. Fault tolerance in this case is defined as the post-failure Jacobian possessing the largest possible minimum singular value over all possible locked-joint failures. In this work, a mathematical analysis that describes the number of possible planar robot designs for optimally fault-tolerant Jacobians is presented. Two examples, one that is optimal to a single joint failure and the second that is optimal to two joint failures, are discussed. The paper concludes by illustrating some of the large variability in the global kinematic properties of these designs, despite being generated from the same Jacobian.**

***Index Terms*— redundant robots, robot kinematics, fault-tolerant robots.**

I. INTRODUCTION[1]

The design and operation of fault-tolerant manipulators is critical for applications in remote and/or hazardous environments where routine maintenance and repair are not possible. Example applications include space exploration [2], [3], underwater exploration [4], and nuclear waste remediation [5], [6] where there has been a great deal of research to improve manipulator reliability [7], [8], design fault-tolerant robots [9], [10], and determine mechanisms for analyzing [11], detecting [12], [13], identifying [14]–[16], and recovering [17]–[20] from failures. Typical failure modes that have been considered include locked joint failures [21], where a joint is immobilized either due to the failure itself or due to the application of fail-safe brakes, and free-swinging joint failures [22] where the joint's associated actuator is no longer able to generate a force or torque.

A large body of work on fault-tolerant manipulators has focused on the properties of kinematically redundant robots, both in serial or parallel form [23]–[27]. These analyses have been performed both on the local properties associated with the manipulator Jacobian [28]–[31] as well as the global characteristics such as the resulting workspace following a particular failure [32]–[35]. (Clearly both local and global kinematic properties are related, e.g., workspace boundaries correspond to singularities in the Jacobian.) In this work it is assumed that one is given a set of local performance constraints that require a manipulator to function in a configuration that is optimal under normal operation and after an arbitrary single joint fails and is locked in position. Specifically, the desired Jacobian matrix must be isotropic, i.e., possess all equal singular values prior to a failure, and have equal minimum singular values for every possible single column being removed. However, one can then use global characteristics to distinguish between multiple manipulators that meet the local design constraints.

The remainder of this paper is organized in the following manner. A local definition of failure tolerance centered on desirable properties of the manipulator Jacobian is mathematically defined in the next section. In Section III, the Gram matrix is used to describe all Jacobians with the same optimal fault tolerance properties. These results are used to present illustrative examples of optimally fault-tolerant Jacobian in Section IV. The conclusions of this work are then presented in Section V.

II. BACKGROUND ON OPTIMALLY FAULT-TOLERANT JACOBIANS[1]

The dexterity of manipulators is frequently quantified in terms of the properties of the manipulator Jacobian matrix which relates end-effector velocities to joint angle velocities. The Jacobian will be denoted by the $m \times n$ matrix J where m is the dimension of the task space and n is the number of degrees-of-freedom of the manipulator. For redundant manipulators $n > m$ and the quantity $n - m$ is the degree of redundancy. The manipulator Jacobian can be written as a collection of columns

$$J_{m\times n} = \begin{bmatrix} j_1 & j_2 & \cdots & j_n \end{bmatrix} \tag{1}$$

where j_i represents the end-effector velocity due to the velocity of joint i. For an arbitrary single joint failure at joint f, assuming that the failed joint can be locked, the resulting m by $n-1$ Jacobian will be missing the fth column, where

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[1]Sections I and II are very similar to those in [1], and are included here to provide the background to make this paper self-contained.

f can range from 1 to n. This Jacobian will be denoted by a preceding superscript so that in general

$$^{f}J_{m\times(n-1)} = \begin{bmatrix} j_1 & j_2 & \cdots & j_{f-1} & j_{f+1} & \cdots & j_n \end{bmatrix}. \quad (2)$$

The properties of a manipulator Jacobian are frequently quantified in terms of the singular values, denoted σ_i, which are typically ordered so that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$. Most local dexterity measures can be defined in terms of simple combinations of these singular values such as their product (determinant) [37], sum (trace), or ratio (condition number) [38]–[40]. The most significant of the singular values is σ_m, the minimum singular value, because it is by definition the measure of proximity to a singularity and tends to dominate the behavior of both the manipulability (determinant) and the condition number. The minimum singular value is also a measure of the worst-case dexterity over all possible end-effector motions.

The definition of failure tolerance used in this work is based on the worst-case dexterity following an arbitrary locked joint failure. Because $^{f}\sigma_m$ denotes the minimum singular value of ^{f}J, $^{f}\sigma_m$ is a measure of the worst-case dexterity if joint f fails. If all joints are equally likely to fail, then a measure of the worst-case failure tolerance is given by

$$\mathcal{K} = \min_{f=1}^{n}(^{f}\sigma_m). \quad (3)$$

To insure that manipulator performance is optimal prior to a failure, an optimally failure tolerant Jacobian is further defined as having all equal singular values due to the desirable properties of isotropic manipulator configurations [38]–[40]. Under these conditions, to guarantee that the minimum $^{f}\sigma_m$ is as large as possible they should all be equal. It is easy to show that the worst-case dexterity of an isotropic manipulator that experiences a single joint failure is governed by the inequality

$$\min_{f=1}^{n}(^{f}\sigma_m) \leq \sigma\sqrt{\frac{n-m}{n}} \quad (4)$$

where σ denotes the norm of the original Jacobian. The best case of equality occurs if the manipulator is in an optimally failure tolerant configuration. The above inequality makes sense from a physical point of view because it represents the ratio of the degree of redundancy to the original number of degrees of freedom.

Using the above definition of an optimally failure tolerant configuration one can identify the structure of the Jacobian required to obtain this property [36]. In particular, one can show that the optimally failure tolerant criteria requires that each joint contributes equally to the null space of the Jacobian transformation [30]. Physically, this means that the redundancy of the robot is uniformly distributed among all the joints so that a failure at any joint can be compensated for by the remaining joints. Therefore, in this work an optimally failure tolerant Jacobian is defined as being isotropic, i.e. $\sigma_i = \sigma$ for all i, and having a maximum worst-case dexterity following a failure, i.e. one for which $^{f}\sigma_m = \sigma\sqrt{\frac{n-m}{n}}$ for all f. The second condition is equivalent to having the columns of the Jacobian have equal norms.

The simplest example of an optimally failure tolerant configuration is given by the following Jacobian for a three degree-of-freedom planar manipulator:

$$J = \begin{bmatrix} j_1 & j_2 & j_3 \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}. \quad (5)$$

The null space at this configuration is given by $\begin{bmatrix}\sqrt{1/3} \, \sqrt{1/3} \, \sqrt{1/3}\end{bmatrix}^T$ which illustrates that each joint contributes equally to the null space motion, thus distributing the redundancy proportionally to all degrees of freedom. Geometrically, it is easy to see that the three vectors j_1, j_2, and j_3 are all $120°$ apart, which results in a balanced coverage of the planar workspace. If the three possible joint failures are considered, one can show that

$$^{f}\sigma_2 = \sqrt{\frac{1}{3}} \quad (6)$$

for $f = 1$ to 3, which satisfies the optimally failure tolerant criterion. Given this example of an optimally failure tolerant J, one might be interested in designing the kinematics for a manipulator that would possess these qualities. In the next section, the Gram matrix is used to analyze the different number of manipulator kinematics that can result from a given fault tolerant Jacobian.

III. Fault Tolerance and the Gram Matrix

The Gram matrix,

$$G = J^T J, \quad (7)$$

provides insight into the geometry and fault tolerance of a manipulator design. Here, the Jacobian J can be the positional, orientational, or the manipulator Jacobian. Some care concerning units should be exercised in the case of the manipulator Jacobian or when there is a mixture of revolute and prismatic joints. When a Jacobian is isotropic, the Gram matrix takes on a particularly simple form: if the singular values of J are equal to 1, then $G = J^T J = I - NN^T$ where the $n \times (n-m)$ matrix N consists of $(n-m)$ orthonormal null vectors of J. In the case of a manipulator with a single degree of redundancy, $G = I - \hat{\mathbf{n}}_J\hat{\mathbf{n}}_J^T$, where $\hat{\mathbf{n}}_J$ is the unit length null vector when J is in a non-singular configuration. The requirement for optimal fault tolerance specifies further conditions on the null space matrix N. Specifically, the rows of N must all have the same norm $\sqrt{\frac{n-m}{n}}$ and be spread out in a sense that will be made precise later.

Once an optimal Gram matrix is determined, an obvious and important question is to characterize all the corresponding Jacobians and Denavit and Hartenberg (DH) parameters for the corresponding manipulators. Clearly, a simple change in the base frame orientation through rotation and/or reflection will not change the basic robot structure. The difference in this case is simply a pre-multiplication of the

Jacobian by an orthogonal matrix. For the sake of discussion, we will say that two configurations are *equivalent* if their corresponding Jacobians differ only by a pre-multiplication by an orthogonal matrix Q. It can be shown that two full rank planar n-R Jacobians J and J' are equivalent if and only if $(J')^T J' = J^T J$, i.e., if their Gram matrices are equal.

Two planar n-R manipulators with equivalent Jacobians have essentially the same DH parameters, so the corresponding robot configurations can be considered to be the same in that sense. This is because when there is a change in the orientation of the base frame, either through a rotation or a combination of a rotation and reflection, the new Jacobian merely differs from the original by a multiplication by an orthogonal matrix. This is nicely illustrated for a planar 3R manipulator, which has a Jacobian of the form

$$J(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} - a_3 s_{123} & -a_2 s_{12} - a_3 s_{123} & -a_3 s_{123} \\ a_1 c_1 + a_2 c_{12} + a_3 c_{123} & a_2 c_{12} + a_3 c_{123} & a_3 c_{123} \end{bmatrix}, \tag{8}$$

where the fixed a_i's are the link lengths, the variable θ_i's are the joint angles, and the remaining DH parameters have values equal to zero. (The notation s_{ijk} and c_{ijk} indicates $\sin(\theta_i + \theta_j + \theta_k)$ and $\cos(\theta_i + \theta_j + \theta_k)$, respectively.) The DH parameters are uniquely determined for a given Jacobian (8), for example, by subtracting different columns to isolate specific terms in (8). If the base frame is changed by a rotation, represented here by a 2×2 rotation matrix $R(\phi)$, the manipulator's Jacobian becomes

$$J'(\theta_1, \theta_2, \theta_3) = R(\phi) J(\theta_1, \theta_2, \theta_3) = J(\theta_1 + \phi, \theta_2, \theta_3), \tag{9}$$

where $R(\phi)$ is the standard rotation matrix corresponding to a counter-clockwise rotation of ϕ radians about the z-axis. The DH parameters of the robot corresponding to the new Jacobian $J'(\theta_1, \theta_2, \theta_3)$ are the same as they were for J with the exception that θ_1 is now replaced with $\theta_1 + \phi$. Consider now the reflection matrix $F = \mathrm{diag}(-1, 1)$, which corresponds to a reflection about the y-z plane. Then the modified Jacobian resulting from pre-multiplying by F is

$$J'(\theta_1, \theta_2, \theta_3) = F J(\theta_1, \theta_2, \theta_3) = J(-\theta_1, -\theta_2, -\theta_3). \tag{10}$$

The new DH parameters are the same except that the joint angles are the negatives of the original joint angles, giving a left-handed version of the same robot. More generally, any orthogonal matrix Q can be written in the form $Q = R(\phi)F$ for a suitable angle ϕ so that pre-multiplying (1) by Q results in the Jacobian

$$QJ(\theta_1, \theta_2, \theta_3) = J(-\theta_1 + \phi, -\theta_2, -\theta_3). \tag{11}$$

Because optimal fault tolerance can be formulated in terms of the Gram matrix, it is desirable to identify the family of DH parameter sets that result in optimally fault tolerant configurations. The unique DH parameters for planar 3R robot are easily obtained from (8) by examining the matrix $\begin{bmatrix} j_1 - j_2 & j_2 - j_3 & j_3 \end{bmatrix}$, e.g., the column norms of this new matrix are equal to the corresponding a_i values. This observation generalizes for any planar n-R robot. One could also obtain the values for a_i from the Gram matrix by noting that for $i = 1, 2, \ldots, n-1$,

$$\begin{aligned} a_i^2 &= \|j_i - j_{i+1}\|^2 \\ &= \|j_i\|^2 + \|j_{i+1}\|^2 - 2 j_i \cdot j_{i+1} \\ &= g_{ii} + g_{i+1,i+1} - 2 g_{i,i+1} \end{aligned} \tag{12}$$

and

$$a_n^2 = \|j_n\|^2 = g_{nn} \tag{13}$$

where $g_{i,i+1}$ is the $(i, i{+}1)$ element of G. Thus, for planar n-R manipulators, a given Gram matrix G determines a family of equivalent manipulators each with the same set of a_i parameters determined by the square root of a simple linear combination of elements in G.

Another important question is whether one can identify other optimally fault tolerant designs from a given Jacobian that are not equivalent by pre-multiplication by an orthogonal matrix. It is clear from the definition of optimal fault tolerance that rearranging the columns of J or multiplying one or more of the columns of J by -1 will not affect local fault tolerance; however, this will typically result in a very different manipulator. We will say that J and J' are *similar* if one is obtained from the other by permuting and/or multiplying the columns of a Jacobian by -1. In other words, J and J' are similar if $J' = JS$ where S is an $n \times n$ matrix corresponding to the desired signed permutation of the columns of J. For convenience, we will say that J and J' are *nontrivially similar* if $S \neq \pm I$. We are interested in similar Jacobians because they share the same fault tolerance properties but generally correspond to fundamentally different manipulators. The Gram matrix G' corresponding to J' is obtained from the original Gram matrix G simply by applying the same row and column operations that were used to obtain J' from J. Consequently, one can easily obtain the a_i parameters for any similar Jacobian directly from the original G for the case of planar revolute manipulators. This will be illustrated in the next section.

IV. Examples of Manipulators with Optimal Fault-Tolerant Jacobians

As mentioned earlier, the restrictions imposed by this definition of fault tolerance limits the number of possible robot geometries. To see this, consider the problem of identifying all planar 3R manipulators with an optimally fault tolerant Jacobian J. When the 2×3 Jacobian J is isotropic with unit singular values, we have

$$G = J^T J = I - \hat{\mathbf{n}}_J \hat{\mathbf{n}}_J^T. \tag{14}$$

Fault tolerance requires that the components of $\hat{\mathbf{n}}_J$ have the same magnitude. However, replacing $\hat{\mathbf{n}}_J$ with $-\hat{\mathbf{n}}_J$ does not affect (2) so we only need to check the four cases $\hat{\mathbf{n}}_J = 1/\sqrt{3} \begin{bmatrix} 1 & \pm 1 & \pm 1 \end{bmatrix}^T$. These four unit null vectors determine four families of non-equivalent Jacobians, each corresponding to one of the four possibilities for $I - \hat{\mathbf{n}}_J \hat{\mathbf{n}}_J^T$, which together identify all Jacobians that are optimally fault tolerant.

The optimally fault tolerant Jacobian given in (5) corresponds to the case when the elements of $\hat{\mathbf{n}}_J$ are all positive and equal. In this case the Gram matrix corresponding to the positional Jacobian is

$$G = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}. \quad (15)$$

The link length parameters for this particular G are then $a_1 = a_2 = \sqrt{\frac{2}{3} + \frac{2}{3} - 2(\frac{-1}{3})} = \sqrt{2}$ and $a_3 = \sqrt{2/3}$. From the family of similar Gram matrices obtained through permutations and multiplications by -1 as described earlier, one can easily deduce that the only possible link length values for an optimally fault tolerant planar 3R manipulator are $L_l = \sqrt{2}$ and $L_s = \sqrt{2/3}$, which are obtained by using off-diagonal elements that equal $\pm\frac{1}{3}$ and diagonal elements equal to $\frac{2}{3}$. Furthermore, the square root of a diagonal value of G is equal to the distance of the end effector from the corresponding joint. In this case, each joint lies on a circle of radius $\sqrt{2/3}$ centered at the end effector with the two possible link lengths $\sqrt{2}$ and $\sqrt{2/3}$, which necessarily place the joints on the vertices of an inscribed hexagon. The four optimally fault tolerant manipulators are described by the link lengths in Table I and illustrated in Fig. 1.

As a further example of an optimally fault tolerant manipulator, consider a planar 4R manipulator. The requirements for optimal fault tolerance are that the Jacobian is isotropic and that the null space matrix N, which consists of two orthonormal null vectors of J, has the properties that its rows each have a norm of $1/\sqrt{2}$ and that the angles between successive rows are $45°$. Any other null space matrix related to N by a row permutation and/or the multiplication of one or more rows by -1 will also result in an optimally fault tolerant Jacobian. The corresponding Jacobian would be given by applying the same operations to the columns of the original Jacobian. An example of a suitable Jacobian is

$$J = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \quad (16)$$

and its corresponding Gram matrix is

$$G = \begin{bmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & \frac{-1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2\sqrt{2}} \\ \frac{-1}{2\sqrt{2}} & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2} \end{bmatrix}. \quad (17)$$

From the diagonal elements of (17) it follows that the joints of the manipulator are located on a circle of radius $1/\sqrt{2}$ centered at the end effector. The link lengths for this particular G are $a_i = \sqrt{1 - \frac{1}{\sqrt{2}}}$ for $i = 1, 2, 3$ and $a_4 = \frac{1}{\sqrt{2}}$. The four potential link lengths for similar Gram matrices are $L_a = \sqrt{1 - \frac{1}{\sqrt{2}}}$, $L_b = \frac{1}{\sqrt{2}}$, $L_c = 1$, and $L_d = \sqrt{1 + \frac{1}{\sqrt{2}}}$. Consequently, it follows that the joints of an optimally fault tolerant planar 4R manipulator appear on the vertices of an octagon inscribed on a circle of radius $\frac{1}{\sqrt{2}}$ centered at the end effector. The list of all possible manipulators is presented in Table II and depicted in Fig. 2. Not every manipulator with the property that its joints are located in the vertices of this octagon are optimally fault tolerant, but the Gram matrix clearly identifies this requirement for the family of optimally fault tolerant manipulators.

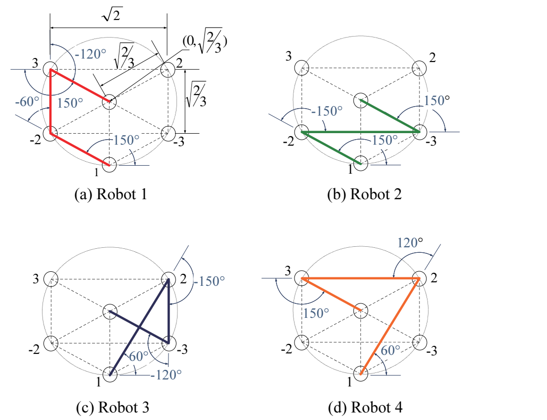

Fig. 1. A simple three degree-of-freedom planar robot that corresponds to the optimal fault-tolerant Jacobian given by (5) is shown in (d). The three other manipulators that have the same properties of the Jacobian in (5) are shown in (a), (b), and (c).

TABLE I
LINK LENGTHS FOR ROBOTS SHOWN IN FIG. 1.

Robot	a_1	a_2	a_3
1	L_s	L_s	L_s
2	L_s	L_l	L_s
3	L_l	L_s	L_s
4	L_l	L_l	L_s

The fact that there are multiple manipulator designs with the same desired local fault tolerance properties, allows one to use other criteria for selecting a preferred design. In particular, while the robots all share the same local properties at the given configuration, they are quite different in terms of their global properties. For example, first consider the 3R robots defined in Table I and Fig. 1. Even when joint limits are not considered, their workspaces are quite different, e.g., the maximum reach will be either $3L_s$, $2L_s + L_l$, or $L_s + 2L_l$. More importantly, if one is concerned with fault-tolerance, the values of the proposed fault-tolerance measure vary significantly for these four robot designs.

To determine how the fault tolerance measure $\mathcal{K}$ varies as a robot moves away from the configuration that has the optimal Jacobian, the optimal value of $\mathcal{K}$ were computed for every location within each of the four robot's workspaces. Because $\mathcal{K}$ is not a function of θ_1, it is sufficient to compute its maximum value as a function of distance from the base of the manipulator. The maximum value of $\mathcal{K}$ is determined by computing all possible robot configurations for each distance, and calculating $\mathcal{K}$ for the Jacobian at that configuration. The results for two of the four robots are shown in Fig. 3.

The first interesting point to note is that Robot 4in Fig. 3, which is generated form the original jacobian in (5), actually has a configuration with a larger value of $\mathcal{K}$ at the design

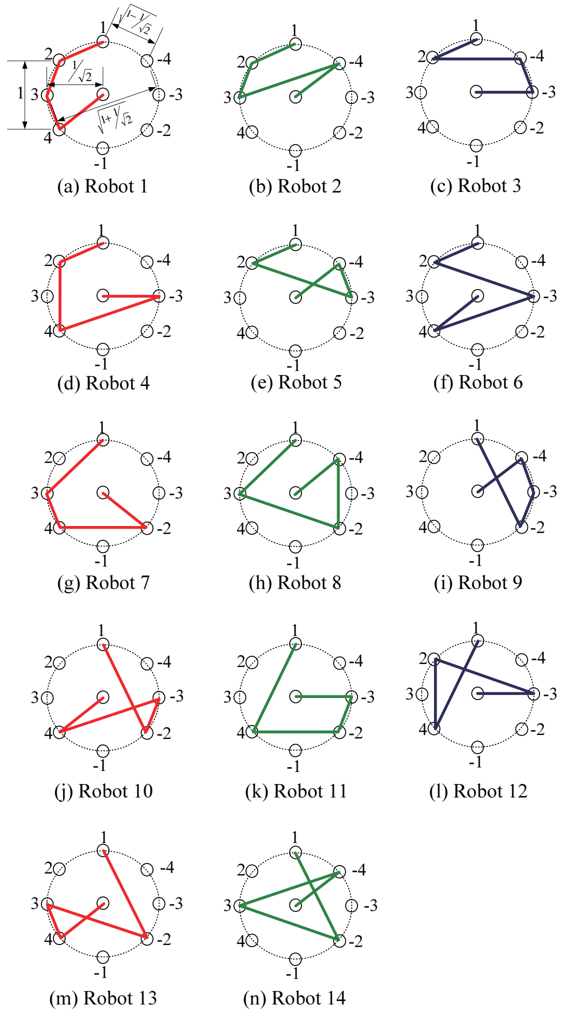

Fig. 2. A simple four degree-of-freedom planar robot that corresponds to the optimal fault-tolerant Jacobian given by (16) is shown in (a). The thirteen other manipulators that have the same properties of the Jacobian in (16) are shown in (b) to (n).

TABLE II

LINK LENGTHS FOR ROBOTS SHOWN IN FIG. 2.

Robot	a_1	a_2	a_3	a_4
1	L_a	L_a	L_a	L_b
2	L_a	L_a	L_d	L_b
3	L_a	L_c	L_a	L_b
4	L_a	L_c	L_d	L_b
5	L_a	L_d	L_a	L_b
6	L_a	L_d	L_d	L_b
7	L_c	L_a	L_c	L_b
8	L_c	L_d	L_c	L_b
9	L_d	L_a	L_a	L_b
10	L_d	L_a	L_d	L_b
11	L_d	L_c	L_a	L_b
12	L_d	L_c	L_d	L_b
13	L_d	L_d	L_a	L_b
14	L_d	L_d	L_d	L_b

point that is a distance of $\sqrt{2/3}$ from the base than that of the optimal value of $\mathcal{K} = \sqrt{1/3}$. This is possible because at this configuration the Jacobian is no longer isotropic, however, its non-isotropy is due to a larger maximum singular value, and so may not be considered undesirable. In addition, the

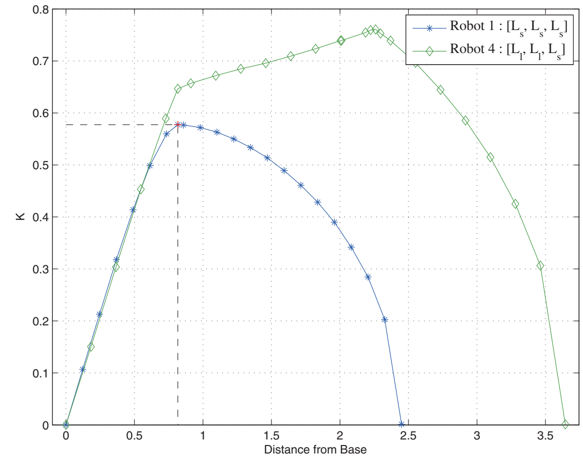

Fig. 3. The relationship between $\mathcal{K}$ and the distance from the base for robots 1 and 4 in Table I.

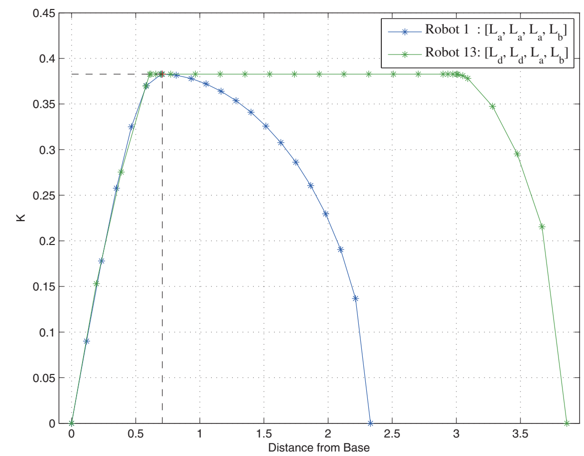

Fig. 4. The relationship between $\mathcal{K}$ and the distance from the base for robots 1 and 13 in Table II.

value of $\mathcal{K}$ is significantly higher than the optimal value for a significant portion of this manipulator's workspace, making it particularly well suited for applications that require failure tolerance.

In contrast, consider Robot 1 in Fig. 3. It has a value of $\mathcal{K} = \sqrt{1/3}$ at the the optimal distance as designed, however, this is its peak value of $\mathcal{K}$, and $\mathcal{K}$ is monotonically decreasing away from this point. Thus, in addition to having the smallest workspace, this manipulator has a significantly smaller tolerance to joint failures throughout its workspace.

Similarly, the fourteen robots in Table II have different workspace properties, e.g. robot 1 has the smallest maximum reach of $3L_a + L_b$ and robot 14 has the largest at $3L_d + L_b$. Fig. 4 illustrates how robots 1 and 13 are also different in terms of the fault tolerance measure with respect to the distance from the base with the case of two joint failures. Robot 1 has a peak in $\mathcal{K}$ at its optimal value of $\frac{1}{2}\sqrt{1 - \frac{1}{\sqrt{2}}}$ at the design point, with $\mathcal{K}$ decreasing relatively rapidly away from this point. In contrast, robots 13 maintains the same optimal value of $\mathcal{K}$ over a large portion of the workspace, i.e., 62% of its total reach.

V. Conclusions

It has been previously shown that there are multiple different robot designs that possess the same desired Jacobian at a specific operating point. This work has presented a mathematical analysis, based on the Gram matrix, that allows one to enumerate all of the possible planar manipulators that possess certain desired fault tolerance properties based on the form of a desired Jacobian. This analysis was illustrated on both a 3R manipulator experiencing a single locked joint failure and a 4R manipulator experiencing two joint failures. It was further shown that there are significant differences in the capabilities of the resulting manipulators, both in terms of pre- and post-failure performance.

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