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Optimal Fault-tolerant Jacobian Matrix Generators for Redundant Manipulators

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***Abstract*— The design of locally optimal fault-tolerant manipulators has been previously addressed via adding constraints on the bases of a desired null space to the design constraints of the manipulators. Then by algebraic or numeric solution of the design equations, the optimal Jacobian matrix is obtained. In this study, an optimal fault-tolerant Jacobian matrix generator is introduced from geometric properties instead of the null space properties. The proposed generator provides equally fault-tolerant Jacobian matrices in R^3 that are optimally fault-tolerant for one or two locked joint failures. It is shown that the proposed optimal Jacobian matrices are directly obtained via regular pyramids. The geometric approach and zonotopes are used as a novel tool for determining relative manipulability in the context of fault-tolerant robotics and for bringing geometric insight into the design of optimal fault-tolerant manipulators.**

I. INTRODUCTION

Fault-tolerant manipulators are essential where dependable robots are required such as robotic manipulators in medical applications, nuclear waste disposal, and in exploration of remote environments like deep sea and space [1]. Fault-tolerant manipulators are capable of accomplishing their tasks despite the presence of partial failures. To accomplish this, there is a great deal of literature addressing the fault-tolerant design, motion planning, and control of manipulators. The present paper focuses on the design of manipulators.

In terms of design, both serial and parallel manipulator structures have been studied [2] with respect to kinematic parameters or optimal configurations for tolerating faults [3]. This paper considers the locally optimal kinematic design of fault-tolerant manipulators. Our aim is to determine a general form for the Jacobian matrix of optimal fault-tolerant positional manipulators. A similar problem was addressed in [4] using an algebraic formulation, whereas here we focus on a geometric approach. There has been extensive study on the design of these manipulators in [5, 6] where the main objective is the determination of optimal fault-tolerant Jacobian matrices. For instance, the optimal Jacobian matrices proposed in [4-7] were obtained from: (a) algebraic or numeric solution of an optimisation problem [7, 8] or (b) algebraic or numeric solution of the constraints associated with properties of the desired null space of the Jacobian matrix [4-7]. It has been shown that there are fundamental limitations to the cases where such properties exist [5].

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The main contributions of this paper are (a) introducing the concepts of zonotopes in the context of fault-tolerant manipulators and (b) giving a geometric understanding to the structure of optimality for fault tolerance in robotic manipulators. This paper builds on the previous work of the authors in [9]. In the present paper, we use the concept of zonotope and we identify pyramids that are more specific than the symmetric geometries of [9]. The novelty of this work is the use of zonotope as a geometric tool for characterizing optimal fault-tolerant Jacobian matrices of manipulators. This tool has leveraged our previous result from fully symmetric geometries [9] to pyramids.

This article is organized as follows: At first in section II, the basic definitions and the measure of the fault tolerance are presented. Then in section III, the properties of pyramids as applied to fault-tolerant geometries are addressed. Next in section IV, regular pyramids are introduced as a generator of optimal fault-tolerant Jacobian matrices that are optimal fault-tolerant for one or two faults, and a general form of the optimal fault-tolerant Jacobian is presented. The consistency of the results is validated by comparison to the literature. Finally the concluding remarks are presented in section V.

II. DEFINITIONS AND BACKGROUND

The forward kinematics of a manipulator is given by

$$\mathbf{x} = \mathbf{f}(\mathbf{q}) \qquad (1)$$

where $\mathbf{x}$ is an m-dimensional vector in the task space and $\mathbf{q}$ is an n-dimensional vector of the joint variables. For kinematically redundant manipulators $n > m$.

Globally optimal fault tolerance requires that one guarantee a solution of the inverse kinematic problem over the entire desired workspace [10]. However, there are a number of applications where a manipulator moves in a relatively small region of the workspace, e.g., a surgical robot. In such cases, instead of global optimality, it may be more appropriate to consider local optimality. A measure of locally optimal fault tolerance for manipulators has been addressed or used in [5, 6, 11].

The local model of forward kinematics is expressed at the velocity level by using

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}} \tag{2}$$

where $\mathbf{J} \in R^{m \times n}$ is the $m \times n$ Jacobian matrix. For a locked joint failure in the k-th joint, the resulting reduced Jacobian is given by

$$^{k}\mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \cdots & \mathbf{j}_{k-1} & \mathbf{j}_{k+1} & \cdots & \mathbf{j}_n \end{bmatrix} \tag{3}$$

$^{k}\mathbf{J} \in R^{m \times (n-1)}$ is called k-th reduced Jacobian matrix where $\mathbf{j}_i \in R^{n}$ is the partial velocity due to the i-th joint variable.

A list of common measures of fault tolerance was presented in [9] that included relative manipulability. The measure was first introduced in [11] and has been used in [5, 6]. The relative manipulability associated with a fault in the k-th joint is

$$^{k}\rho = \frac{\sqrt{\det(^{k}\mathbf{J}\,^{k}\mathbf{J}^{T})}}{\sqrt{\det(\mathbf{J}\mathbf{J}^{T})}} \tag{4}$$

where clearly the value of $^{k}\rho$ is between zero and one.

Physically, this represents how much of the manipulability is maintained after a failure in joint k. One would like to maintain as large a value as possible for any joint failure. In this work, we will assume that all joint failures are equally likely and therefore will refer to a manipulator that has equal values of $^{k}\rho$ for all $1 \le k \le n$ as optimally fault-tolerant to single joint failures.

Some of the properties of the relative manipulability [5, 6] are

1- $$\sum_{k=1}^{n} {^{k}\rho^{2}} = n - m \tag{5}$$

2- $$\min(^{k}\rho) \le \sqrt{\frac{n-m}{n}} \tag{6}$$

The worst case relative manipulability is defined by $\min(^{k}\rho)$. For optimality in fault tolerance, maximizing the worst case relative manipulability was investigated in pervious works [4-6]. Another approach for determining optimal fault-tolerant Jacobian matrices was proposed in [9] based on both optimal worst case dexterity and optimal worse case relative manipulability. The two optimality conditions were achieved from fully symmetric geometries.

The design of an optimal Jacobian matrix can be performed by construction of the Jacobian matrix based on the null space properties. By knowing an orthogonal basis of the null space, one can apply a theorem that relates the relative manipulability to the structure of the null space orthogonal bases [4-6]. The design procedure requires adding the constraints on the null space to any other design constraints on the manipulator. Different form of these null space bases have been proposed in [4-6] that can be used for construction of optimal Jacobian matrices.

III. Optimal Fault Tolerance Jacobian

A. *Examples of optimal fault tolerance Jacobian*

An optimal fault-tolerant Jacobian in R^2 with minimal redundancy can be determined for a *3*DOF planar manipulator. This Jacobian matrix consists of three column vectors given by

$$\mathbf{J} = \delta \begin{bmatrix} c\alpha & -s\alpha \\ s\alpha & c\alpha \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{7}$$

where δ is an arbitrary constant and the first 2×2 matrix is a standard rotation matrix in R^2, furthermore $c\alpha = \cos(\alpha)$ and $s\alpha = \sin(\alpha)$.

This is true because the right hand 2×3 matrix has a unit null vector whose elements are of equal magnitude, and an arbitrary scaling or rotation of the task space will not change this fact. This Jacobian has been discussed in [7, 9, 12] , and is associated with a regular triangle in [9]. The relative manipulabilities for this Jacobian matrix is $\sqrt{\frac{1}{3}}$ for all the possible single joint failures.

A similar study for a *4*DOF manipulator in R^3 is presented in [9] where it is shown that the vectors are the vertices of a regular tetrahedron, i.e., these vectors are the column vectors of

$$\mathbf{J} = \delta\mathbf{T} \begin{bmatrix} 0.8385 & -0.4554 & -0.2813 & -0.1018 \\ 0.2011 & 0.7192 & -0.3631 & -0.5572 \\ 0.0803 & -0.1593 & 0.7342 & -0.6551 \end{bmatrix} \tag{8}$$

where $\mathbf{T}$ is a general 3×3 rotation matrix and once again δ is an arbitrary scaling factor. All the relative manipulabilities for the above matrix are $\sqrt{\frac{1}{4}} = 0.5$ [9].

The Jacobian matrices in (7) and (8) are optimally fault-tolerant for any single joint failure. The problem of interest in this paper is to investigate geometries that represent optimal fault tolerance. The optimal fault tolerance is for up to two locked joint failures. The problem of optimal fault-tolerant Jacobian matrices for two joint failures has been addressed in [4] and an optimal fault-tolerant Jacobian matrix was obtained by incorporating the null space properties. The result of our work here will be compared to that in [4] later of this paper.

B. *Problem statement and propositions*

Problem statement: which geometry represents optimal fault tolerance (for one or two failures) in positioning redundant manipulators? How can one design an optimal fault-tolerant Jacobian based on these geometries?

Our previous study for optimal fault tolerance for a single joint failure in [9] resulted in the following propositions.

Proposition 1- A regular triangle is a generator for an optimal fault-tolerant Jacobian matrix for single joint failure of a *3*DOF planar manipulator in R^2 [9].

Proposition 2- A regular tetrahedron is a generator for an optimal fault-tolerant Jacobian matrix for single joint failures of the *4*DOF manipulator in R^3 [9].

Proposition 3- The number of vertices of the generator geometry is the same as the DOF of the manipulator and the geometry is a polyhedron. The number of faces of the polyhedron is the same as those of vertices [9].

Proposition 4- Fully symmetric hyper polyhedrons are the general form of optimal fault-tolerant Jacobian matrices for single joint failures [9].

Our investigation of different geometries resulted in a pantahedron (square pyramid) and a hexahedron (pentagonal pyramid) as generators for optimal fault-tolerant Jacobian matrices. Based on the properties of the pyramids one can show that there is a *5*DOF manipulator that can be optimal fault-tolerant for any single joint failure. However, there is no square pyramid that can be optimality fault-tolerant for two locked joint failures.

For example, based on the concept of the manipulability of the reduced Jacobian matrices, an optimal fault-tolerant Jacobian was obtained as

$$\mathbf{J}=\begin{bmatrix}0 & 0 & 0 & -0.9129 & 0.9129\\ 0 & 0.9129 & -0.9129 & 0 & 0\\ 1 & -0.4082 & -0.4082 & -0.4082 & -0.4082\end{bmatrix} \quad (9)$$

The plot of the column vectors of the proposed Jacobian in (9) is shown in figure 1. This Jacobian is optimally fault-tolerant for single failures with a worse case manipulabilities of $\sqrt{\frac{2}{5}}$.

The geometry is a square pyramid. The centre of the pyramid is the origin of the vectors pointing to the vertices of the pyramids (notice the lines inside the pyramid).

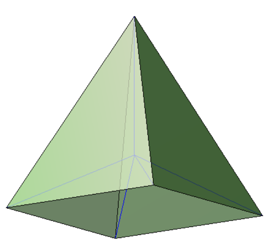

Figure 1 – A square prism obtained from (9), the vectors inside the geometry are the columns of (9).

It has to be noted that, simply assuming any square pyramid will not give the optimal fault-tolerant Jacobian, because the Jacobian matrix must satisfy the conditions of optimality. On the other hand, in the family of pantahedrons one can find other types that are not equally fault-tolerant but instead they exhibits better fault tolerance for some joints rather than others. Two of these polyhedrons based on square pyramid are indicated in figure 2. The geometry on the left side pantahedron of figure 2 is for the Jacobian matrix shown in (10) and on the one in the right is for the Jacobian matrix shown in (11):

$$\mathbf{J}=\begin{bmatrix}0 & 0 & 0 & -1 & 1\\ 0 & 0.8550 & -0.8550 & 0 & 0\\ 1 & -0.5000 & -0.5000 & 0 & 0\end{bmatrix} \quad (10)$$

$$\mathbf{J}=\begin{bmatrix}0 & 0 & 0 & -1 & 1\\ 0 & 0.7070 & -0.7070 & 0 & 0\\ 1 & -0.7070 & -0.7070 & 0 & 0\end{bmatrix} \quad (11)$$

The square of the relative manipulability parameters of the Jacobian matrices shown by (9), (10) and (11) are indicated in Table. 1, to show that (5) holds.

TABLE 1- square of the relative manipulability

Faulty Joint	Square of Relative manipulability for (9)	Square of Relative manipulability for (10)	Square of Relative manipulability for (11)
1	0.4000	0.3333	0.5000
2	0.4000	0.3333	0.2500
3	0.4000	0.3333	0.2500
4	0.4000	0.5000	0.5000
5	0.4000	0.5000	0.5000

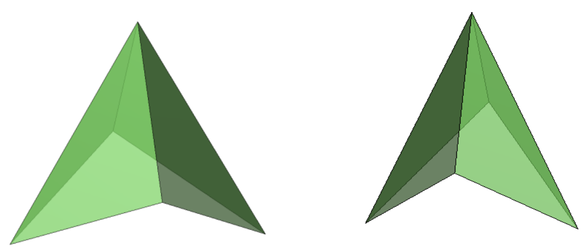

Figure 2 – The geometry on the left is associated with (10) and the geometry on the right is associated with (11)

These geometries can be useful when the possibility of joint failure is not equal for different joints. For example, in the case of (11) joints 2 and 3 of the manipulator, which have less manipulability, could be those joints with lower failure probabilities. None of the geometries in (9)-(11) can be used for optimality of fault tolerance for two failures. It has been previously shown in [4] that there is no Jacobian matrix for a five or fewer degree of freedom positional manipulator that is optimally fault-tolerant for two locked joints failures.

For *6*DOF positional manipulator in R^3 a generator is designed as

$$\mathbf{J}=\begin{bmatrix}0 & 0.8946 & 0.2764 & -0.7236 & -0.7236 & 0.2764\\ 0 & 0 & 0.8507 & 0.5257 & -0.5257 & -0.8507\\ 1 & -0.4472 & -0.4472 & -0.4472 & -0.4472 & -0.4472\end{bmatrix} \quad (12)$$

By calculating the relative manipulabilities for one joint failures, it is easy to show that (12) is an equally fault-tolerant

Jacobian matrix because the worst case relative manipulability is $\sqrt{\frac{1}{2}}$. Other observations associated with this generator:

(a) The Jacobian matrix in (12) is an optimal fault-tolerant Jacobian matrix for two joint failures. The relative manipulability for two joints failures is calculated based on

$$^{ki}\rho = \frac{\sqrt{\det(^{ki}\mathbf{J}\,^{ki}\mathbf{J}^T)}}{\sqrt{\det(\mathbf{J}\mathbf{J}^T)}} \qquad 0 \le k < i < n \qquad (13)$$

where $^{ki}\mathbf{J}$ is obtained by eliminating the k-th and i-th columns of $\mathbf{J}$. The worst case relative manipulability for two joint failures of (12) is $\sqrt{\frac{1}{5}}$.

(b) The Jacobian matrix in (12) results in a regular pentagonal pyramid. This suggest the regular pentagonal pyramid as a generator of the optimal fault-tolerant Jacobian matrix for up to two locked joint failures.

(c) Rotation and scaling of this geometry would not change the geometric properties. Therefore a class of optimally fault-tolerant Jacobian for a *6*DOF positional manipulators is given by:

$$\delta\mathbf{T}\mathbf{J} \qquad (14)$$

where $\mathbf{T}$ is a general *3*D rotation matrix, δ is an arbitrary scaling factor and $\mathbf{J}$ is the Jacobin matrix in (12). The geometric shape resulting from the column vectors is shown in figure 3.

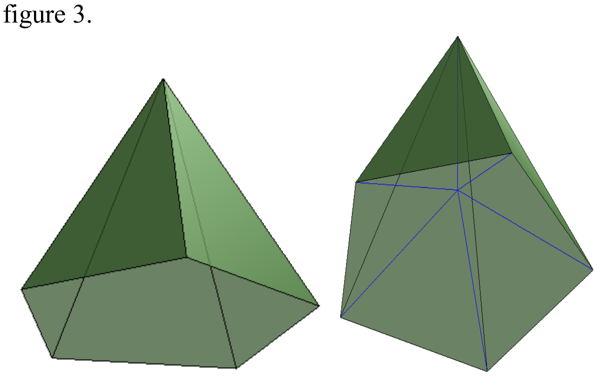

Figure 3 – A pentagonal pyramid as an optimally fault-tolerant Jacobian matrix generator (in left). The vectors pointing to the vertices (in right)are the columns of (12).

C. Discussion

The above results are consistent with the approach that uses constraints on the null space to determine an optimally fault-tolerant Jacobian matrix [4-7]. The columns from the optimal fault-tolerant Jacobian for two joints failure from [4] is shown in figure 4.

The geometries in figures 3 and 4 are identical, i.e., they are hexahedrons with a regular pentagonal base (pentagonal pyramid). This illustrates the strong connection between symmetric geometries and the fault tolerance of the associated manipulators. The geometric approach may provide a more intuitive method for the design of locally optimal fault-tolerant manipulators because of its direct physical interpretation.

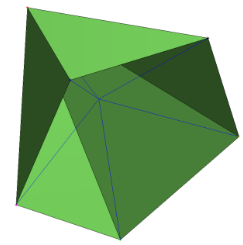

Figure 4 – Vectors of a Jacobian for a *6*DOF manipulator that is optimally fault-tolerant for two faults. The Jacobian matrix of this geometry was shown in [4].

IV. METHOD OF FINDING THE PROPER GEOMETRIES FOR OPTIMAL FAULT TOLERANCE

We noticed the connection between optimal fault tolerance and symmetric geometries in [9]. But the results were limited to single joint failures. In [9] we used fully symmetric geometries (e.g. regular tetrahedron). However in the present paper the geometries in figure 3 are not fully symmetric but they still maintain the optimality condition.

To design the geometries in (9) and (12) the concept of the manipulability of the reduced Jacobian matrices was used. We satisfied the condition when the manipulability of the joint failure associated with the main vertices in figure 1 and figure 3 are equal to those associated with any vertex of the base. In figure 3, by removing any vertex of the pentagonal base the remaining geometry is the same as the others. Therefore the condition of optimality is preserved for the faults associated with the columns of 2-6 of (12). The offset of the base of the pyramid and the position of the main vertex were calculated according to the equality of the manipulability of the reduced matrices. The analysis of these geometries in terms of reduced manipulability is achieved by the concept of zonotopes that is introduced in the following.

A. Zonotopes and optimal relative manipulability

The geometric interpretation of the relative manipulability is obtained from the concept of Zonotopes [13]. First of all, a zonotope associated with the matrix is defined as a Minkovski sum of the column vectors of the matrix [13]. The manipulability of the manipulators is the volume of the zonotope associated to the Jacobian matrix of the manipulator. As the fault in a joint of the manipulator removes the corresponding column of the Jacobian matrix, geometrically, it results in a reduced zonotope obtained from the remaining column vectors (or by eliminating the vertex associated to the removed column). Therefore, proposing the geometry that results in equal-volume reduced zonotopes is equivalent to optimality for any single joint failure. At this stage, this approach is valid for positional manipulators (2D or 3D). This is due to the fact that for spatial manipulators there is an additional constraint associated with the orthogonality of the linear and rotational velocity. In the following, the volume of the zonotopes is used to obtain a general form for optimal fault-tolerant Jacobian matrices. It is shown that this general form is a pyramid with a regular base.

B. Pyramids and optimal fault tolerance

The aim of this section is to summarize the properties of pyramids. Assume a pyramid with a regular base has n vertices. The vertices are referred to as $\mathbf{p}_k$ where $1 \le k \le n$ and the main vertex is called $\mathbf{p}_1$.

For an nDOF serial manipulator, the coordinates of the main vertex of the pyramid is

$$\mathbf{p}_1 = [0 \quad 0 \quad a]^T \tag{15}$$

The positions of vertices of the base of the pyramids are

$$\mathbf{p}_{k+1} = [\cos((k-1)360/(n-1)) \quad \sin((k-1)360/(n-1)) \quad -b]^T \tag{16}$$

where a and b are unknown parameters and $1 \le k \le n-1$.

A normalized Jacobian matrix generator is given by

$$\mathbf{J} = \frac{1}{a}[\mathbf{p}_1 \quad \dots \quad \mathbf{p}_{k-1} \quad \mathbf{p}_k \quad \mathbf{p}_{k+1} \quad \dots \quad \mathbf{p}_n] \in R^{mn} \tag{17}$$

In order to satisfy the condition of equal relative manipulability, the geometric properties of the Jacobian matrix of (17) are used. The zonotope of the Jacobian matrix is indicated by $Z(\mathbf{J})$ [13] and the volume of this zonotope is indicated by $vol(Z(\mathbf{J}))$. The volume is calculated using

$$vol(Z(\mathbf{J})) = \det(\mathbf{J}\mathbf{J}^T) \tag{18}$$

The reduced zonotopes that are associated with the reduced Jacobian are indicated by $Z({}^k\mathbf{J})$, where $1 \le k \le n$. Some properties of the Jacobian matrix in (17) are:

Property 1- The geometric representation of (17) is a pyramid with a regular polygonal base.

Property 2- The regular shape for the zonotope requires all the columns of the Jacobian matrix to be of equal norm, therefore $a^2 = 1 + b^2$.

Property 3- The volume of the zonotope of the Jacobian matrix (17) is $vol(Z(\mathbf{J}))$ and it is obtained using

$$vol(Z(\mathbf{J})) = \det(\mathbf{J}\mathbf{J}^T) = \frac{(n-1)^2}{4}\left(a^2 + (n-1)b^2\right) \tag{19}$$

Property 4- Failure of any joint is modelled by removing the column associated with that joint. For the reduced Jacobian matrices, if the faulty joint is associated with vertices of the base of the pyramid then the volume of the reduced zonotopes will be the same and

$$vol(Z({}^k\mathbf{J})) = vol(Z({}^i\mathbf{J})) \qquad i \ne k \qquad 1 < i,k \le n \tag{20}$$

Property 5- The condition of the optimality of fault tolerance for a single joint failure is

$$vol(Z({}^1\mathbf{J})) = vol(Z({}^k\mathbf{J})) = vol(Z({}^i\mathbf{J})) \qquad i \ne k \qquad 1 < i,k \le n \tag{21}$$

In the fifth property, the equality of $vol(Z({}^k\mathbf{J})) = vol(Z({}^i\mathbf{J}))$ was previously indicated in property 4. The first part of (21) is used to obtain the unknown variables of a and b.

Property 6- The volume of the reduced zonotopes is calculated by direct calculation of the determinant of 3×3 matrices as

$$vol(Z({}^1\mathbf{J})) = \det({}^1\mathbf{J}\,{}^1\mathbf{J}^T) = \frac{(n-1)^3}{4}b^2 \tag{22}$$

$$vol(Z({}^2\mathbf{J})) = \det({}^2\mathbf{J}\,{}^2\mathbf{J}^T) = \frac{(n-3)}{2}\frac{(n-1)}{2}\left(a^2 + (n-2)b^2\right) - \frac{(n-1)}{2}b^2 \tag{23}$$

$$vol(Z({}^k\mathbf{J})) = vol(Z({}^2\mathbf{J})) \qquad 2 \le k \le n \tag{24}$$

Property 7- Using the equality of these volumes on (21) and the constraint on the norm of the columns $a^2 = 1 + b^2$ in property 2, the unknown parameters are calculated as $b = \sqrt{\frac{n-3}{2n}}$ and $a = \sqrt{\frac{3(n-1)}{2n}}$.

By these parameters, the Jacobian matrix (17) is fully identified and the condition of the fault tolerance is maintained.

Property 8- Finally, the optimal fault-tolerant Jacobian matrix is obtained as

$$\mathbf{J} = \sqrt{\frac{2n}{3(n-1)}}\begin{bmatrix} 0 & c(0) & c\left(\frac{2\pi}{n-1}\right) & \dots & c\left(\frac{2(n-2)\pi}{n-1}\right) \\ 0 & s(0) & s\left(\frac{2\pi}{n-1}\right) & \dots & s\left(\frac{2(n-2)\pi}{n-1}\right) \\ \sqrt{\frac{3(n-1)}{2n}} & -\sqrt{\frac{n-3}{2n}} & -\sqrt{\frac{n-3}{2n}} & \dots & -\sqrt{\frac{n-3}{2n}} \end{bmatrix} \tag{25}$$

This is the general form of an optimal fault-tolerant Jacobian matrix for any number of DOF.

Property-9 The relative manipulability of (25) is obtained by using (19)-(24) and results in:

$${}^k\rho = \frac{\sqrt{\det({}^k\mathbf{J}\,{}^k\mathbf{J}^T)}}{\sqrt{\det(\mathbf{J}\mathbf{J}^T)}} = \sqrt{\frac{n-3}{n}} \qquad 1 \le k \le n \tag{26}$$

Corollary: Every pyramid described by (25) is the generator of optimal fault-tolerant Jacobian for any single failure associated to an nDOF redundant manipulator.

Note that (25) is consistent with the Jacobian matrices in (9) for *3*DOF planar manipulator, (10) for *4*DOF, and (11) for *5*DOF manipulators.

C. Investigation of optimal fault tolerance for two faults

The extension for two faults requires the following equality

$$vol(Z({}^{ki}\mathbf{J})) = vol(Z({}^{rs}\mathbf{J})) \qquad i \ne k, r \ne s \quad 1 < i,k,r,s \le n \tag{27}$$

The symmetric form of the prism reduces the number of equalities that need to be checked to the following cases

1- $vol\left(Z\left({}^{12}\mathbf{J}\right)\right) = vol\left(Z\left({}^{23}\mathbf{J}\right)\right)$ (28)

2- $vol\left(Z\left({}^{23}\mathbf{J}\right)\right) = vol\left(Z\left({}^{2i}\mathbf{J}\right)\right) \quad 4 \le i \le n$ (29)

This requires checking of $n-2$ equality conditions. The other equalities in (27) will be valid because of the symmetric form of the pyramid.

The other method to investigate the optimality condition for two joint failures is using of definition of relative manipulability instead of volume of the zonotopes, because the Jacobian matrix is now fully known. The relative manipulability for two joints failure is shown in Table 2. From this table, it is easy to see that the optimality condition for two failures only exists for a *6*DOF positional manipulator. This fact has been observed in [4].

TABLE 2- Min and max of relative manipulability for two joint failures for positional manipulators up to 50DOF

DOF	Min relative manipulability	Max relative manipulability
4	0.0000	0.000
5	0.0000	0.3873
6	0.4472	0.4472
7	0.5079	0.5669
8	0.5649	0.6245
9	0.6138	0.6614
10	0.6546	0.6992
12	0.7167	0.7488
16	0.7932	0.8124
20	0.8377	0.8500
25	0.8722	0.8800
40	0.9220	0.9250
50	0.9381	0.9400

Note that if the number of DOFs is increased then the minimum and maximum of the relative manipulability will come closer to each other. For example, for more than *25*DOF the difference between the minimum and maximum of the relative manipulability is less than 1%. Therefore manipulators with many DOFs will be close to optimal fault tolerance to both one and two failures.

V. CONCLUSION

This paper addressed the design of optimally fault-tolerant Jacobian matrices for redundant manipulators. The proposed method was based on the geometric properties of the Jacobian matrix instead of using the desired null space constraints. Through this approach a class of optimal fault-tolerant manipulator Jacobian was introduced for an *n*DOF positional manipulator. The advantage of the proposed approach is that it directly generated the optimal Jacobian matrix using the geometrical properties of zonotopes. The geometric shape of the column vectors of the generator matrices was a pyramid.

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