Chapter 19

Electrical Properties of Materials

Dr. Zhe Cheng

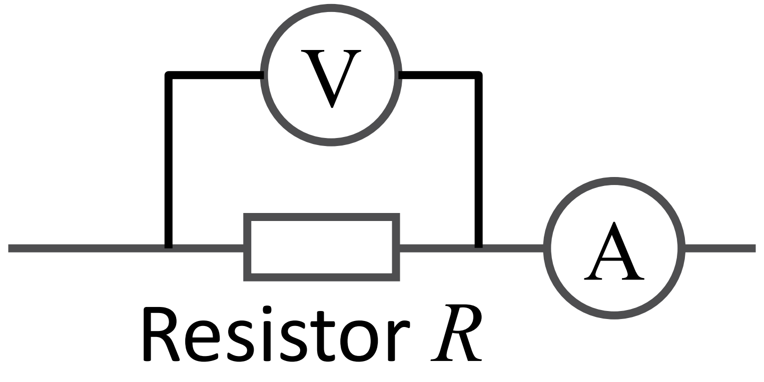
Ohm's Law

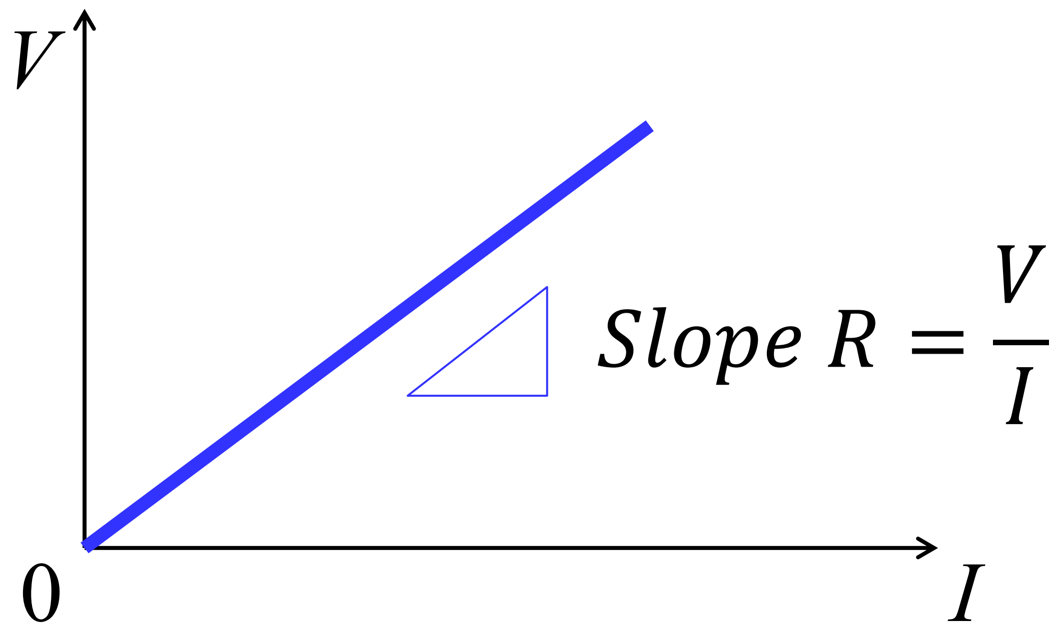
Current – Voltage ($I - V$) Relationship for a <u>resistor</u>

$$V = IR$$

$$I = \frac{V}{R}$$

For a pure resistor, its resistance is voltage across it divided by current through it, and it is a constant





Example

2 V DC voltage is applied across a 10 Ω resistor. What is the current through it?

$$I = \frac{V}{R} = \frac{2\ V}{10\ \Omega} = 0.2\ A$$

Resistance, Resistivity, & Conductivity

- **Relationship** between **resistance** R and the material's property - **resistivity** ρ

$$R = \rho \frac{L}{A}$$

L Length of resistor

A Cross-section area of the resistor

For a given material at a given condition, longer l and smaller A → larger R

- **Resistivity** $\rho = R\frac{A}{L}$ in unit of Ω • m

- **Conductivity** $\sigma = \frac{1}{\rho} = \frac{1}{R} \cdot \frac{L}{A}$ in unit of $\Omega^{-1} \bullet m^{-1}$ or S/m

- Exercise

  If Cu resistivity is 1.72×10^{-8} Ω•m at 20°C, calculate the resistance for a 50 cm long, 1 mm diameter Cu wire

$$R = \rho \frac{L}{A} = 1.72 \times 10^{-8}\, \Omega \cdot m \times \frac{0.5\, m}{3.14 \times (0.5 \times 10^{-3}\, m)^2} = 0.011\, \Omega$$

Another Form of Ohm's Law

From $\sigma = \frac{1}{\rho} = \frac{1}{R} \cdot \frac{L}{A}$ $\qquad$ $R = \frac{1}{\sigma} \cdot \frac{L}{A}$

Therefore $I = \frac{V}{R} = \frac{V}{\frac{1}{\sigma} \cdot \frac{L}{A}} = \sigma \frac{VA}{L}$

$$\frac{I}{A} = \sigma \frac{V}{L}$$

Remember electrical field strength $E = \frac{V}{L}$

Therefore, current density $j = \frac{I}{A} = \sigma E$

Electrical Power & Joule Heating

- Electrical Power $P = VI = RI \cdot I = I^2 R = V\frac{V}{R} = \frac{V^2}{R}$

- Example

For a copper electrical transmission line 2000 m long and carry a current of 100 A, if power loss should not be more than 10^6 W loss in power, what would be the diameter of the copper wire? Knowing copper conductivity is 5.9 x10^7 S/m.

Wire power loss

$$P = I^2 R < 10^6 W \qquad R < \frac{10^6 W}{(100A)^2} = 100\ \Omega$$

On the other hand,

$$R = \frac{1}{\sigma} \cdot \frac{L}{A} \qquad A = \frac{L}{R \cdot \sigma} > \frac{L}{100\ \Omega \cdot \sigma} = \frac{2000\ m}{100\ \Omega \times 5.9 \times 10^7\ \frac{S}{m}} = 3.39 \times 10^{-7}\ m^2$$

Area $A = \frac{\pi d^2}{4}$ Cu wire diameter $d = \sqrt{\frac{4A}{\pi}} > \sqrt{\frac{4 \times 3.39 \times 10^{-7}\ m^2}{3.14}} = 6.6 \times 10^{-4}\ m = \mathbf{0.66\ mm}$

Materials Classification by Conduction Behavior of e^- or $h^\bullet$

- **(Electronic) Conductor**
  - Easily conducts electrons or holes (missing of electrons)
  - Charge carrier mostly electrons, but can be holes
  - Linear behavior in current/voltage relationship
  - Essentially NO band-gap (E_g = 0 eV)

- (Electronic) **Insulator**
  - Does not conduct electrons/holes under normal condition (i.e., not too high T & $E = V / L$)
  - Large band-gap (E_g > ~3 eV)

- **Semiconductor**
  - Between conductors and insulators
  - Conducts electrons/holes at certain condition (e.g., slightly elevated temperature)
  - Intermediate band-gap (E_g ~1-2 eV)

(Electronic) Conductors

- All metals and some ceramics (e.g., graphite, $SrVO_3$, $La_{0.85}Sr_{0.15}MnO_3$, & polymers conduct electrons (or holes) readily
- High total conductivity, often >10^3 S/m or higher at RT
- Conductivity decreases (resistivity increases) with increasing temperature

Resistivity Change w/ Temperature for Electronic Conductors

For electronic conductors, in many cases, resistivity increases linearly with temperature

$$\rho = \rho_{RT}(1 + \alpha_T \cdot \Delta T)$$

Example

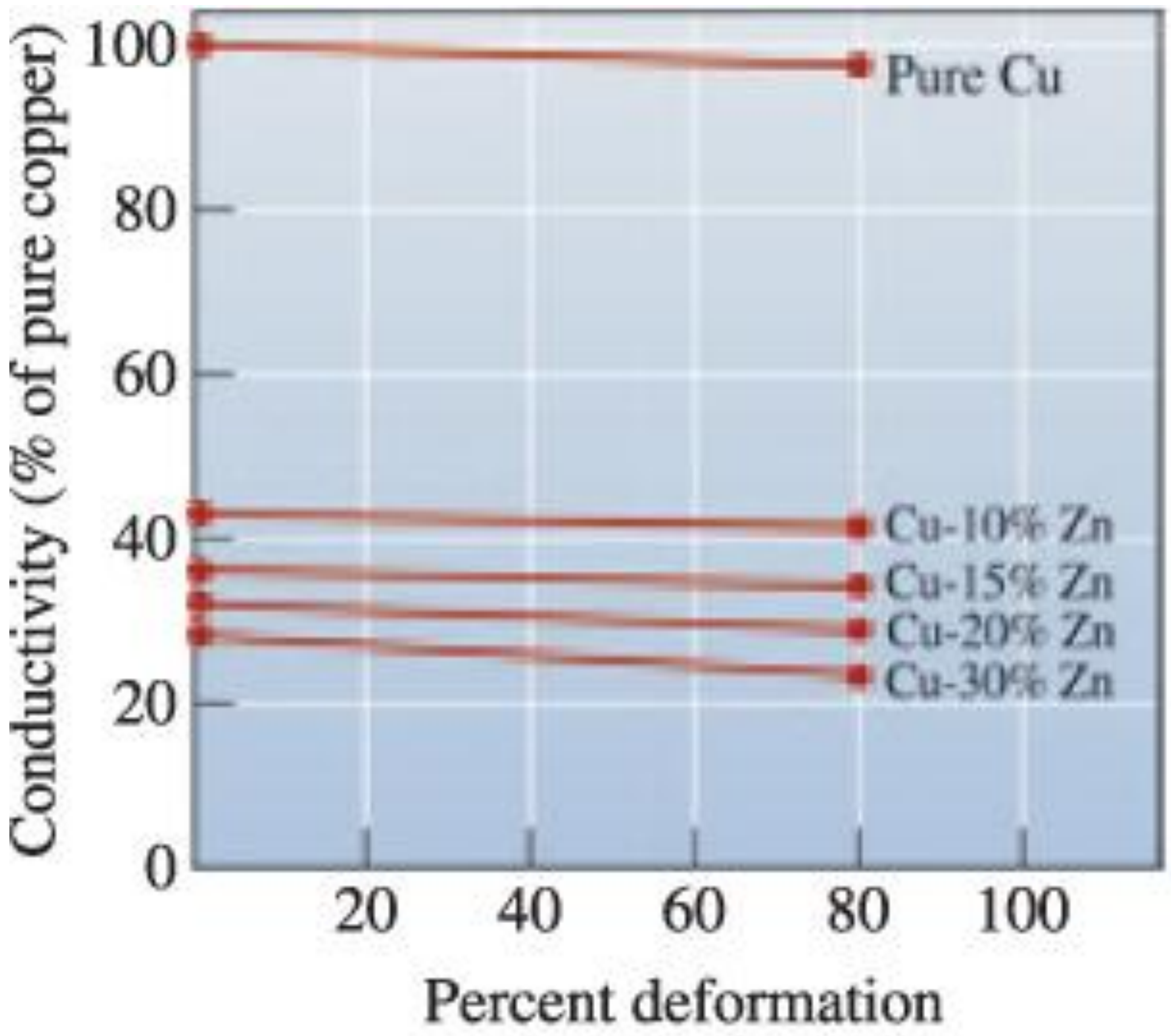
Knowing at room temperature or 25 ^{o}C, copper resistivity is 1.68 x10^{-8} Ω •m, and the temperature resistivity coefficient is 0.0043/^{o}C, please calculate its resistivity for copper at 500^{o}C. What about conductivity?

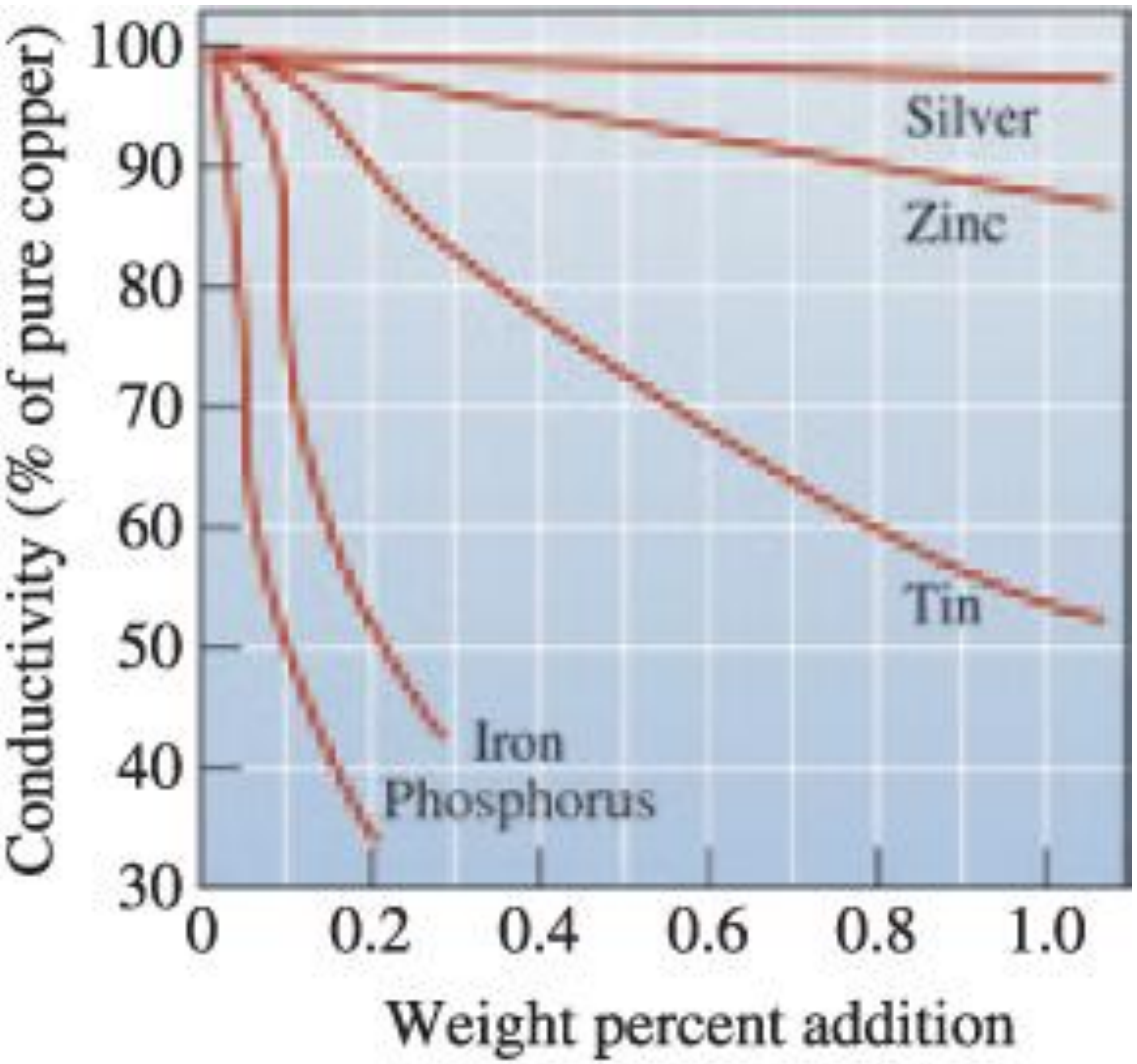
$$\rho(500℃) = 1.68 \times 10^{-8}\Omega \cdot m[1 + 0.0043℃^{-1} \cdot (500℃ - 25℃)] = 5.1 \times 10^{-8}\Omega \cdot m$$

$$\sigma(500℃) = \frac{1}{\rho(500℃)} = \frac{1}{5.1 \times 10^{-8}\Omega \cdot m} = 1.96 \times 10^{7}\frac{S}{m}$$

Conductivity Change w/ Composition (Alloying) & Processing

(Metal electronic) conductivity often changes (mostly decreases) due to doping (adding alloying elements) or cold-working (accumulation of dislocations)





Semiconductor & Conductivity

- Between conductors and insulators, with bandgap of ~1-2 eV
- Generally, total electronic conductivity for a semiconductor given by:

$$\sigma = nq\mu_e + pq\mu_h$$

n Electron concentration, #/m^3
p Hole concentration, #/m^3
q Charge per electron/hole, C
μ_e Mobility for electron, m^2/(V·s)
μ_h Mobility for hole, m^2/(V·s)

Intrinsic & Extrinsic Semiconductor

For intrinsic semi-conductor (e.g., pure Si or Ge without dopant/impurity atom doping):
Electron and hole concentration the same, i.e., $n = p$

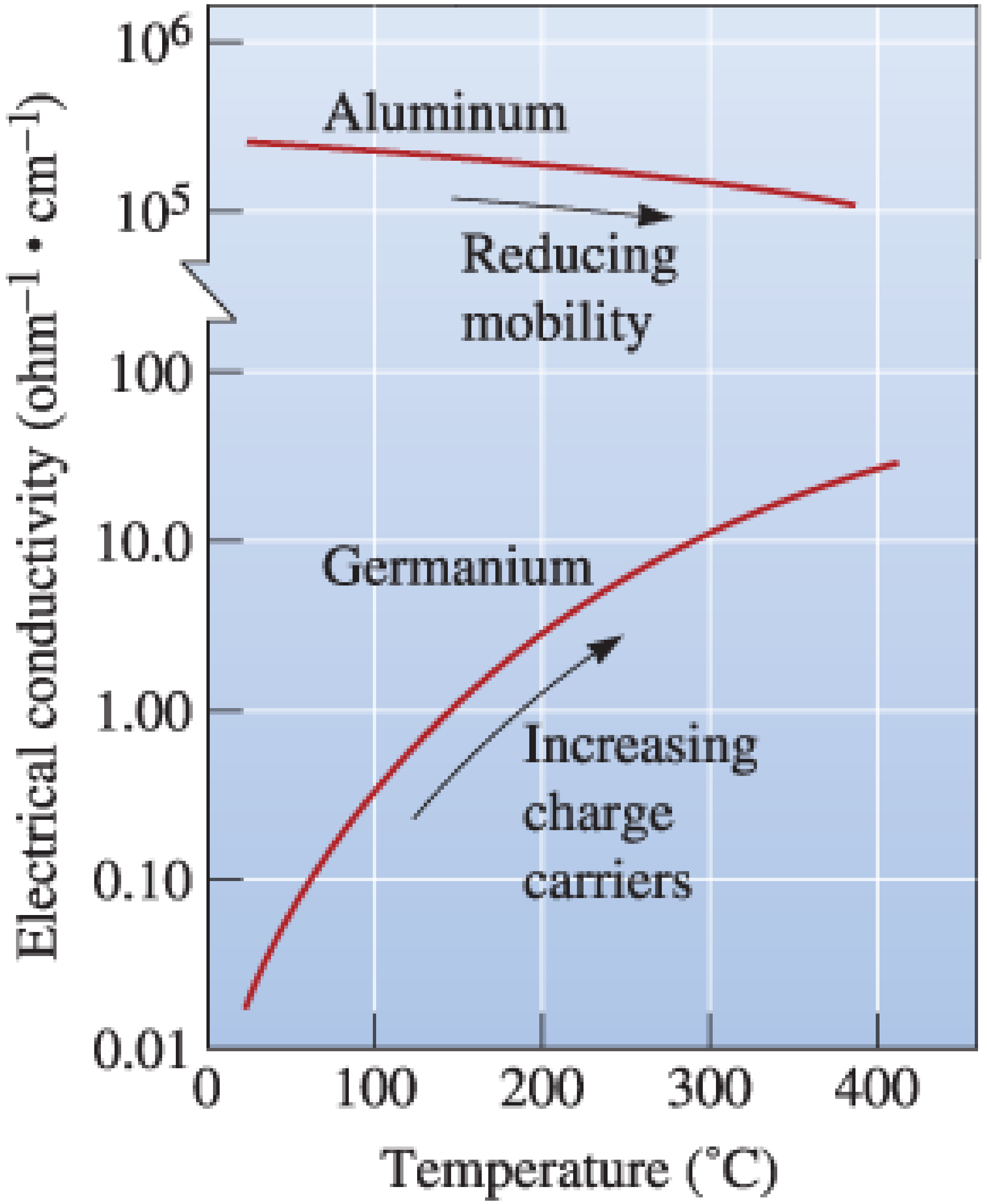
Electron and hole concentration for intrinsic semiconductor:

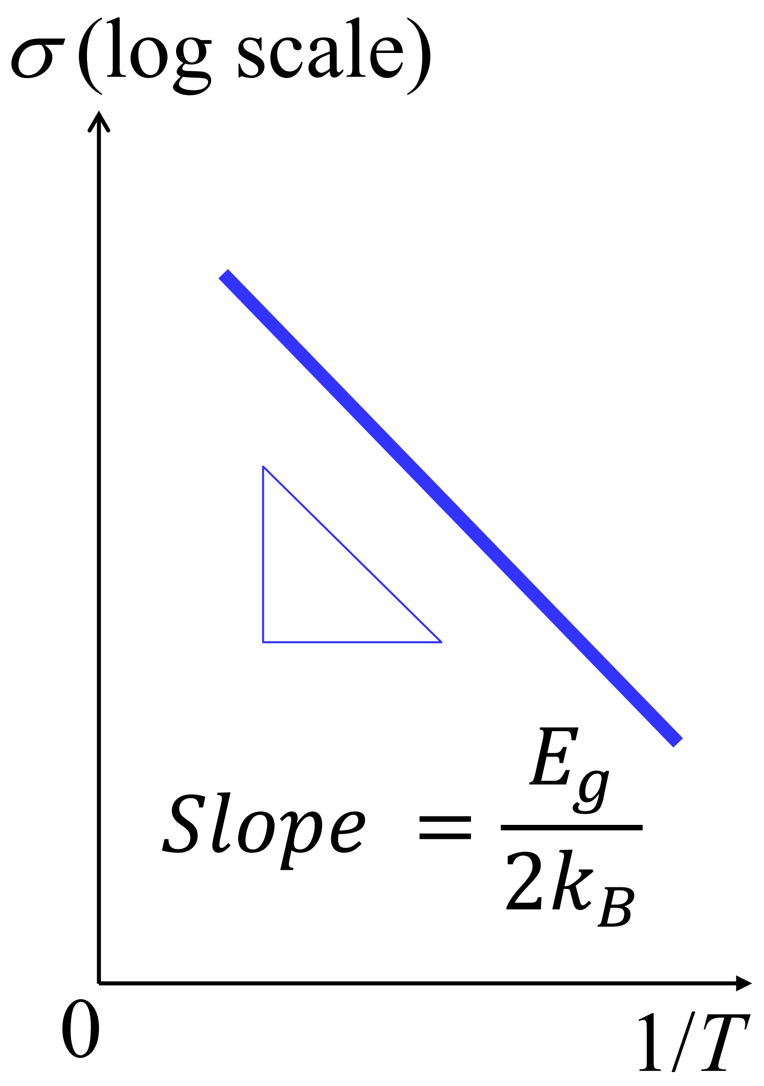
$$n_i = p_i = n_0 \exp\left(-\frac{E_g}{2k_B T}\right)$$

Total electronic conductivity for intrinsic semiconductor:

$$\sigma = n_0\, q\left(\mu_e + \mu_q\right) \exp\left(-\frac{E_g}{2k_B T}\right)$$

- Mobility often treated as constant over small temperature range
- Conductivity increases "exponentially" w/ temperature

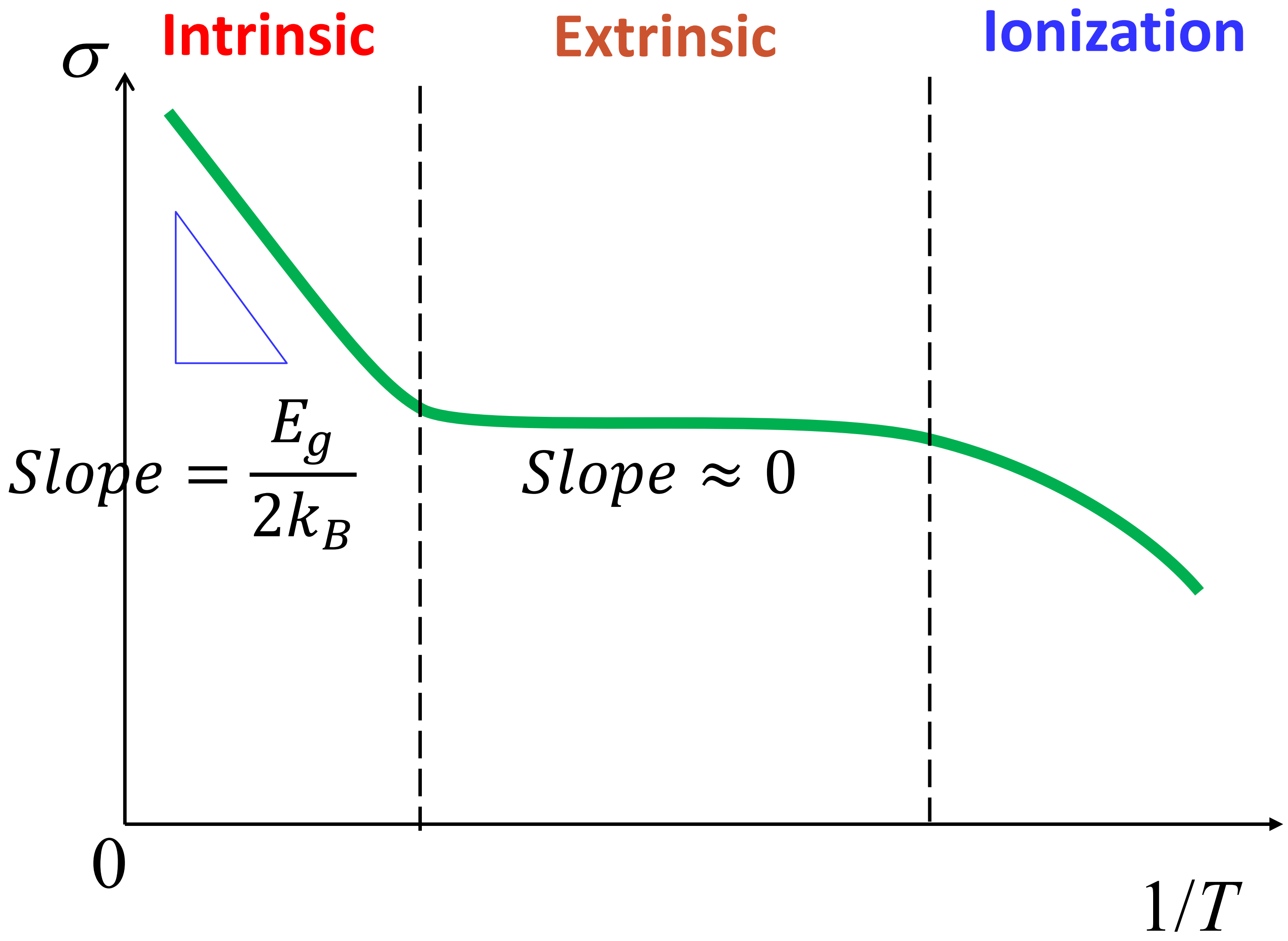




Extrinsic Semiconductor & Conductivity

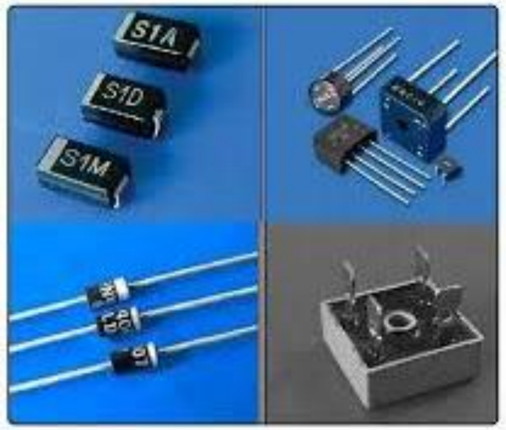
- Material can be semiconducting due to adding/doping of impurity atom, especially at ambient temperature, i.e., without the need to reach elevated temperature
  - Doping phosphorous (P) into Si, generate extra electrons, leading to n-type semiconductor
  - Doping boron (B) into Si, generating extra hole, leading to p-type semiconductor

- Conductivity vs. 1/T shows 3 regions:
  - **Intrinsic region - High T**
    Dominated by e^- and $h^\bullet$ from activation due to temperature
  - **Extrinsic region - Ambient T**
    Dominated by impurity dopants
  - **Ionization region – Low T**
    Dopant atoms (e.g., P in Si or B in Si) start to ionize/dissociate to form relatively free e^- or $h^\bullet$



Application of Semiconductors

- Current regulation (direction or amplitude)
- Logistic/computing
- Memory devices
- Light emitting diodes (LED)
- Solar cells
- Sensors
- …







Insulator & Dielectric Strength

Insulators

Materials with bandgap larger than ~3 eV and high resistivity (often ~10^8 $\Omega \bullet$m or higher) at room temperature

<u>Dielectric strength</u> for insulators

Critical field strength value beyond which electrical breakdown would happen

Material	MV/m (or V/μm)
Air	3
Al_2O_3	6
$BaTiO_3$	10
Diamond	2,000
Polyethylene (PE)	20
Window glass	10

Dielectric Constant & Capacitance

Capacitance C

Charge stored per unit voltage (difference)

$$C = \frac{Q}{V}$$

Higher C means, for the same voltage (difference), more charge (and as a result, electrical energy) can be stored (as separated charge)

Capacitance from dielectric material

$$C = \frac{k\epsilon_0 A}{t}$$

k Relative permittivity/dielectric constant

ε_0 Vacuum permittivity: 8.85 x 10^{-12} F/m

A Area of capacitor

t Thickness of dielectric layer

Material	Relative Permittivity / Dielectric Constant k
Air	1
Al_2O_3	09
$BaTiO_3$	2000
Diamond	5
Polyethylene (PE)	2
Window glass	4

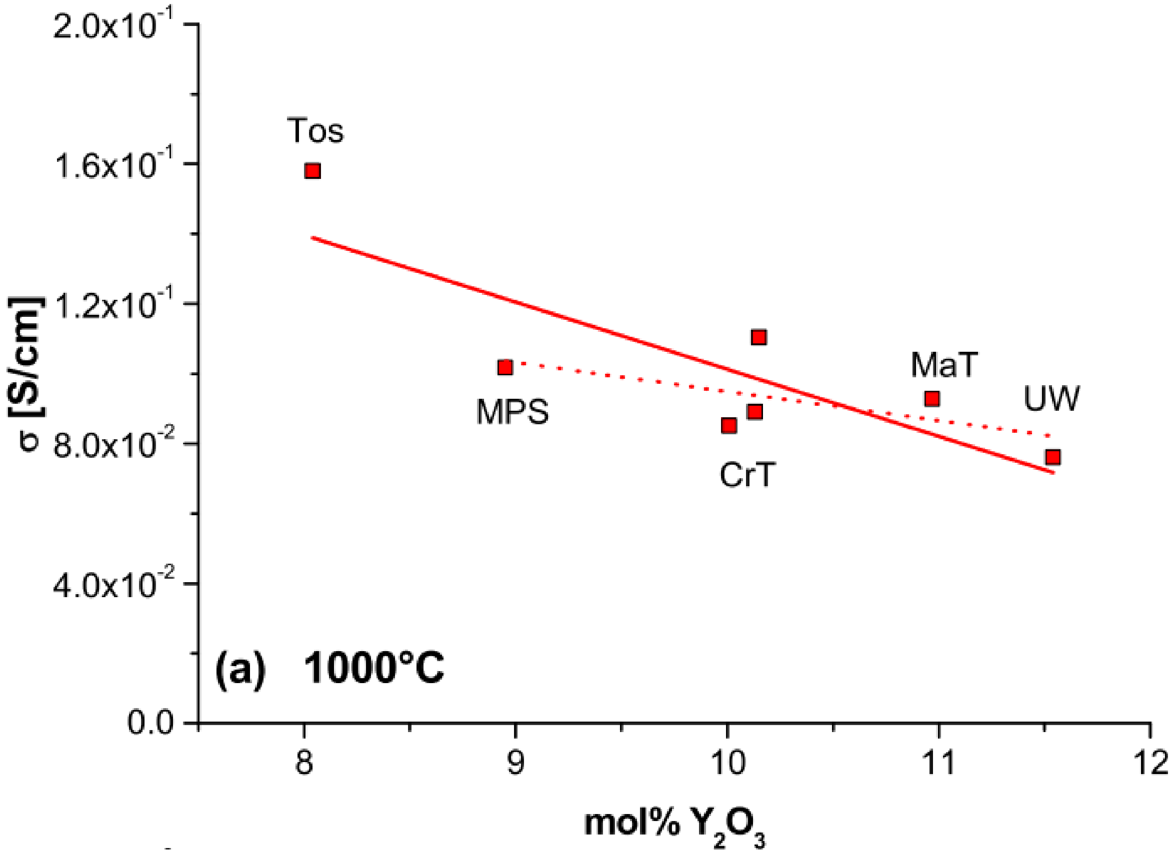
Capacitance increase as dielectric constant or area increases or thickness decreases

Ionic Conducting Materials

- Other than common electronic conducting materials (metals or ceramics like $SrVO_3$), some materials/systems conduct electricity via movement of **ions** (i.e., charged atoms or charged atom groups)
- Example: salt (e.g., NaCl) aqueous solution
- Example: Molten salt (e.g., NaCl) $E_g \approx 8.5$ eV!
- Example: 8 mol.% Y_2O_3 stabilized ZrO_2 (8**YSZ**)
  - Colorless (& transparent) bandgap ~5.8 eV!

    https://doi.org/10.1103/PhysRevMaterials.2.035801
  - Total conductivity ~0.1 S/cm at 1000°C
  - Conductivity due to moving of oxygen vacancy $V_O^{\bullet\bullet}$ at elevated temperature

https://www.msesupplies.com/products/mse-pro-yttria-stabilized-zirconia-ysz-single-crystal-substrates



Aharmer et al. J Electrochem Soc v164, F790 (2017), https://iopscience.iop.org/article/10.1149/2.0641707jes

Conductivity & Mobility for Ionic Conductors

- Total electrical conductivity, assuming multiple types of charge carriers, would be:

$$\sigma = \sum_k c_k q_k \mu_k$$

c_k Concentration for charge carrier k
q_k Net charge for charge carrier k, in C
μ_k Mobility for charge carrier k

- Mobility, in unit of $m^2/(V \cdot s)$, for charge carrier k would be:

$$\mu_k = \frac{Z_k q_e D_k}{k_B T}$$

Z_k Number of electron charge for charge carrier k
q_e Net charge for an electron: 1.6×10^{-19} C
D_k Diffusion coefficient for charge carrier k
k_B Boltzman constant: 1.38×10^{-23} J/K
T Temperature, in K

Conductivity for Ionic Conducting Y_2O_3-doped ZrO_2

If diffusion coefficient for oxygen vacancy at 750^oC is 1.25×10^{-6} cm^2/s (*), and oxygen vacancy concentration is 2.3×10^{21} /cm^3, please estimate the total electrical conductivity for 8YSZ material. Assuming oxygen vacancy is the predominant charge carrier contributing to total electrical conductivity. (* estimated from oxygen diffusion coefficient of 5×10^{-8} cm^2/s & oxygen vacancy fraction X_{OV} = 4% for $Zr_{0.84}Y_{0.16}O_{1.92}$)

Mobility for oxygen vacancy:

$$\mu_{OV} = \frac{Z_{OV} q_e D_{OV}}{k_B T} = \frac{2 \times 1.6 \times 10^{-19} C \times 1.25 \times 10^{-6} \frac{cm^2}{s}}{1.38 \times 10^{-23} \frac{J}{K} \times (750 + 273) K} = 2.83 \times 10^{-5} \frac{cm^2}{V \cdot s}$$

Total electrical conductivity

$$\sigma = \sum_k c_k q_k \mu_k = c_{OV} q_{OV} \mu_{OV} = 2.3 \times 10^{21} \frac{1}{cm^3} \times (2 \times 1.6 \times 10^{-19} C) \times 2.83 \times 10^{-5} \frac{cm^2}{V \cdot s}$$

$$\sigma = 0.021 \frac{C}{V \cdot s \cdot cm} = 0.021 \frac{A \cdot s}{V \cdot s \cdot cm} = 0.021 \frac{1}{\frac{V}{A} \cdot cm} = 0.021 \frac{1}{\Omega \cdot cm} = \mathbf{0.021 \frac{S}{cm}}$$

END

Homework 19.0

Carefully review chapter 19 lecture slides and, if time allows, read textbook sections of Askeland 19.1, 19.3-19.4, 19.9-19.10 (some numerical example problems such as 19-2, 19-4, 19-5, 19-7 could be omitted) and give an honor statement confirming the reading

Homework 19.1

Calculate the resistance and resistivity for a conductor wire with cross-section area of 1.25 mm^2 and length of 20 cm subject to voltage of 220 V, if the current density is 1 A/cm^2. Assume the wire is uniform.

Homework 19.2

A current of 15 A passes through a 2 mm diameter, 500 m long aluminum wire. Please calculate the power loss. (Aluminum electrical conductivity is 3.77×10^5 S/cm)

Homework 19.3

A 10 A current flows through a 2.54 mm diameter copper wire. If the power lost should be no more than 200 W, what is the max length of the wire? (Assume copper conductivity of 5.98×10^5 S/cm.)

Homework 19.4

Calculate the electrical conductivity for Ag and Pt at 600^oC. Knowing room temperature resistivity for Ag and Pt is $1.59 \times 10^{-6}\ \Omega \bullet cm$ and $9.85 \times 10^{-6}\ \Omega \bullet cm$, respectively, while the temperature resistivity coefficient for Ag and Pt is 0.0041/^oC and 0.0039/^oC, respectively.

Homework 19.5

If diffusion coefficient for oxygen vacancy at 1000°C is $7.5x10^{-6}$ cm^2/s (*), and oxygen vacancy concentration is $2.3x10^{21}$ $/cm^3$, please estimate the total electrical conductivity for 8YSZ material. Assuming oxygen vacancy is the predominant charge carrier contributing to total electrical conductivity. (* estimated from oxygen diffusion coefficient of $3x10^{-7}$ cm^2/s & oxygen vacancy fraction X_{OV} = 4% for $Zr_{0.84}Y_{0.16}O_{1.92}$)

Homework 19.6

Knowing a $BaTiO_3$ capacitor has active electrode area of 1 cm^2, the distance between two electrodes (dielectric layer thickness) is 10 μm, and $BaTiO_3$ has a relative permittivity of 2000. Please calculate its capacitance and the amount of charge that can be stored in the $BaTiO_3$ capacitor at 2 V.