

# Chapter 9: Phase Diagrams

## ISSUES TO ADDRESS...

- Concepts of Phase, Component, Equilibrium

- Phase diagram

In particular, if we specify...

--a composition (e.g., wt% Cu - wt% Ni), and

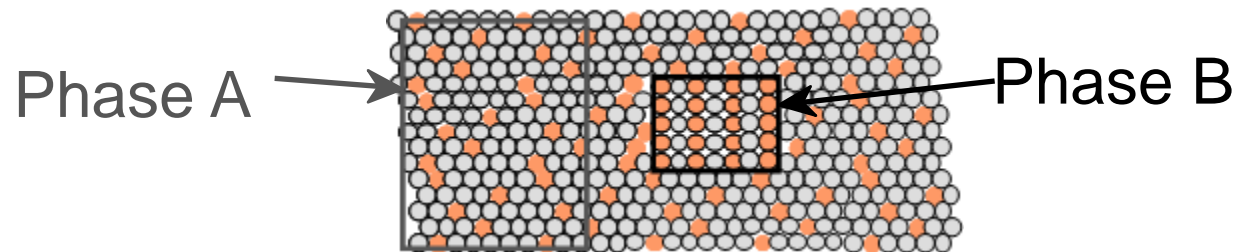
--a temperature ( $T$ )

then...

How many phases do we get?

What is the composition of each phase?

How much of each phase do we get?



○ Nickel atom

● Copper atom



# Phases & Components

- Phase

A homogeneous portion of a materials system that has uniform physical and chemical characteristics

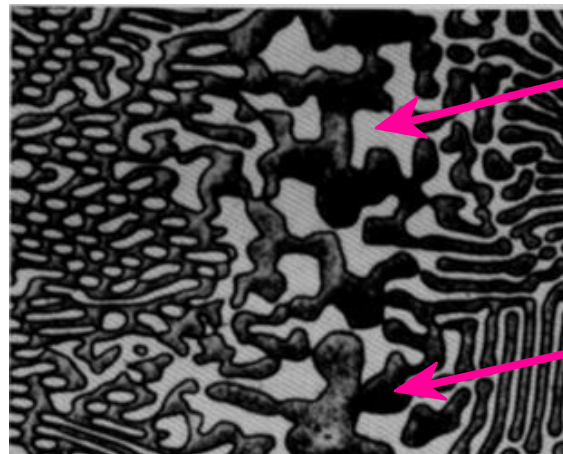
Pure liquid water: a single phase system

Pure iced water: two-phase system - ice and water

Sugary water: single phase system - water with dissolved sugar

- Component

Pure elements or compounds (e.g., Al and Cu)



$\beta$  phase  
(lighter)

$\alpha$  phase  
(darker)

Adapted from  
chapter-opening  
photograph,  
Chapter 9,  
*Callister 3e.*

Aluminum-Copper Alloy



# Images Illustrating Concepts of Phases & Components

## HCl solution

1 phase: liquid

2 components: water & HCl



<http://www.carolina.com/specialty-chemicals-d-l/hydrochloric-acid-in-plastic-coated-safety-bottle-121-m-reagent-ac-grade-25-l/867793.pr>

## Iced Water

2 phases: liquid and solid

1 component: water



<https://www.pinterest.com/pin/265290234275314171/>

## Water-Oil mixture

2 phases: liquid 1(watery) & liquid 2 (oily)

2 components: water & oil



<http://carpinteriavalleyassociation.org/2013/07/oil-and-water-do-not-mix/>



# Class Exercise

- **For the following system, how many phases and how many components are present? What are they**
  - **Water with ice cubes in it?**  
Two phases (water and ice) and one component (water or H<sub>2</sub>O)
  - **Hydrochloric acid solution?**  
One phase (that uniform solution) and two components (H<sub>2</sub>O, HCl)
  - **Sugary water with pure ice cube in it?**  
Two phases (sugary water and ice) and two components (H<sub>2</sub>O, sugar)
  - **Cooking oil dropped into a cup of water and shaken up**  
Two phases (oily and watery) and two components (oil and water)



# Phase Equilibrium

**Equilibrium:** minimum energy state for system at a given T, P, and composition (i.e. equilibrium state will persist indefinitely for a fixed T, P and composition).

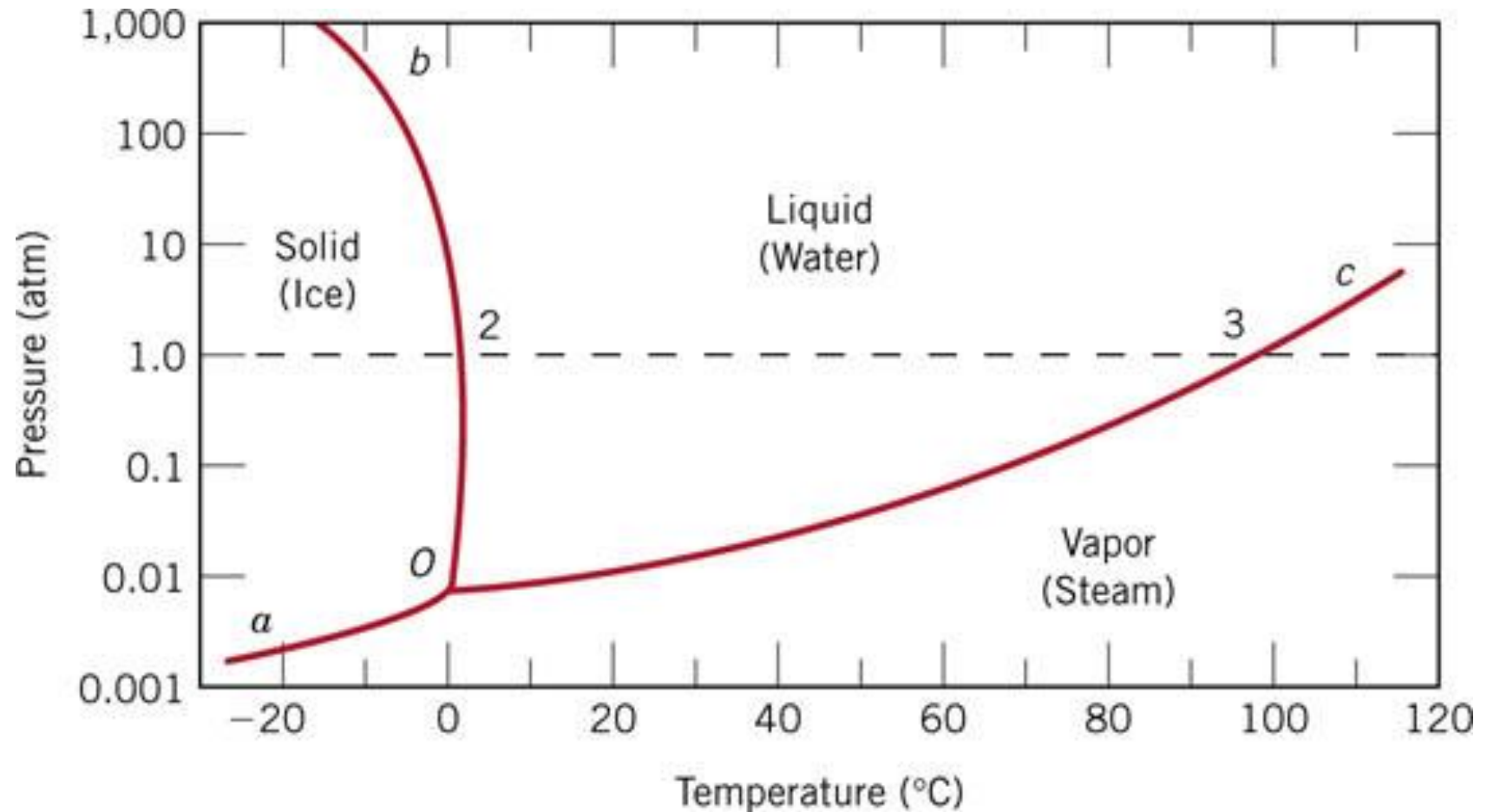
**Phase Equilibrium:** phase characteristics (composition, structure, and relative amount) will stay unchanged over time as long as condition (T, P, composition, etc.) do not change).

**Phase diagrams – diagrams** about the existence of equilibrium phases as a function of T, P and composition (Here, often P is kept constant for simplicity).



# Phase Diagram – Example of a Unary System

*Single component system*



# Binary (Two Components) Phase Diagram

## Temperature – Concentration – Phase(s)

Regions: Solution region – single phase

Mixture region – more than one phase

Adapted from Fig. 9.1,  
Callister 7e.

Boundary line – represents

Solubility Limit, i.e., max concentration below which only a single phase solution exists

Questions: for water-sugar system what is the solubility limit at 20°C?

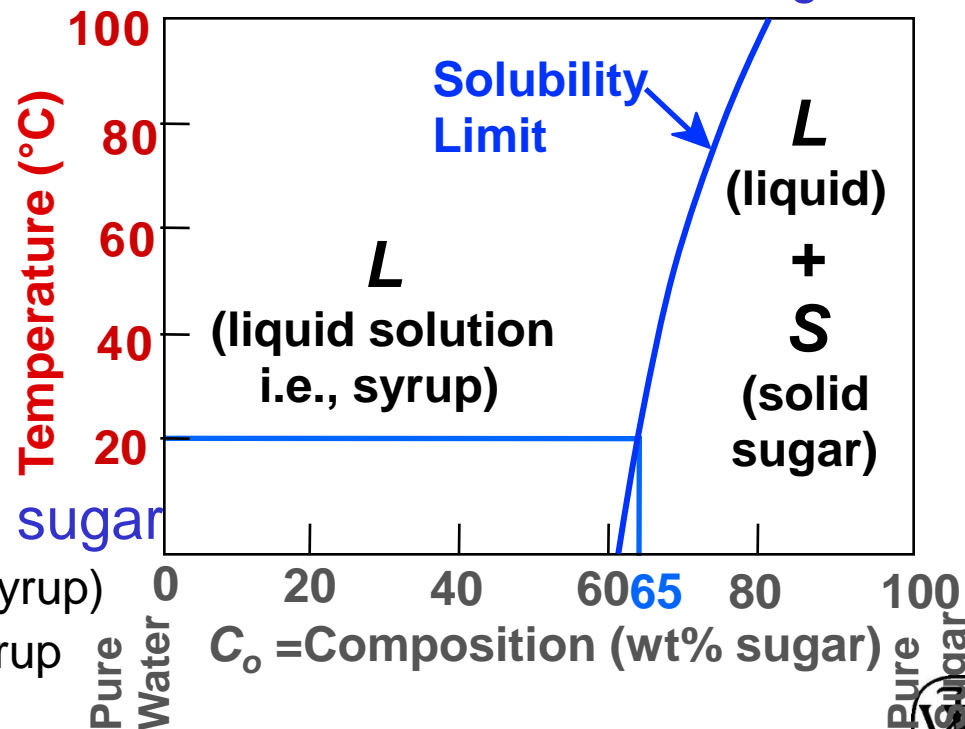
How many phases when sugar concentration below solubility limit?

Answer: solubility (limit) ~ 65 wt% sugar

If  $C_o < 65$  wt% sugar: single phase (syrup)

If  $C_o > 65$  wt% sugar: two phases (syrup + solid sugar)

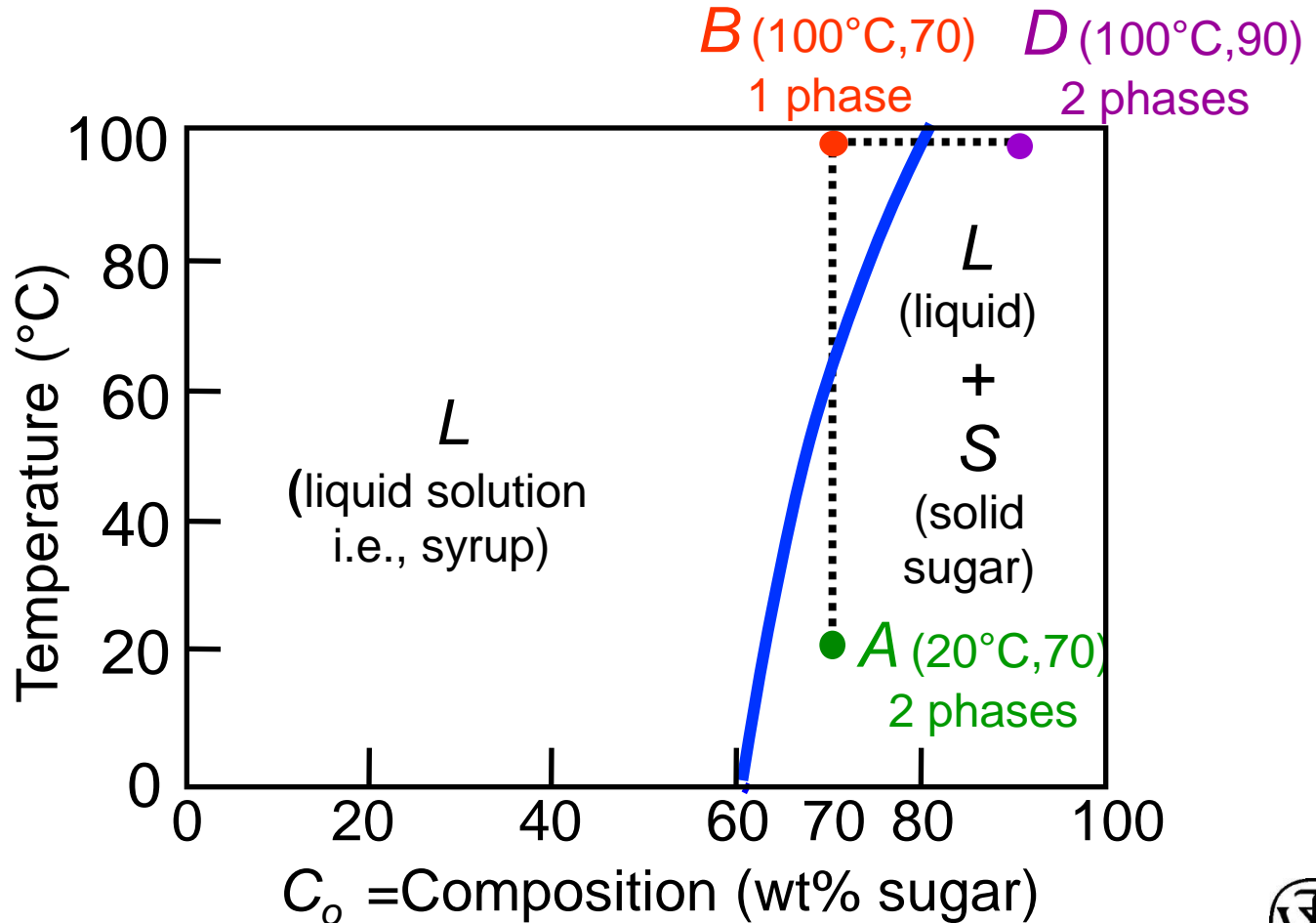
Sucrose/Water Phase Diagram



# Effect of $T$ & Composition ( $C_o$ )

- Changing  $T$  can change # of phases: path  $A$  to  $B$ .
- Changing  $C_o$  (e.g., by adding more sugar) can also change # of phases: path  $B$  to  $D$ .

water-  
sugar  
system



Adapted from  
Fig. 9.1,  
Callister 7e.





# Solid Solution

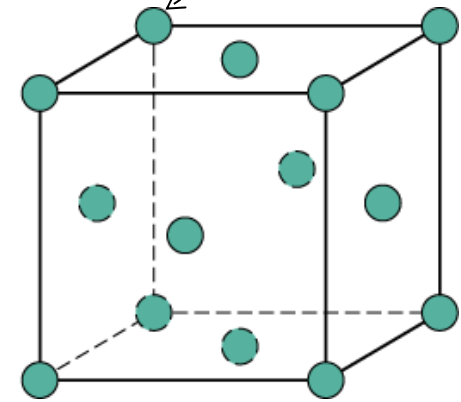
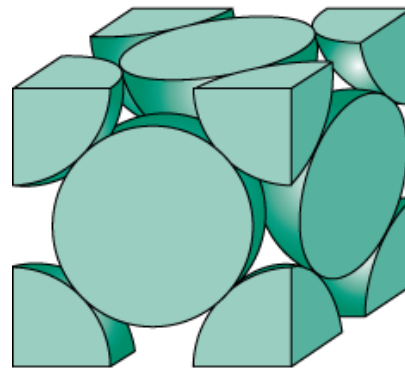
Uniform “solution” for **solid state** material (e.g., Ni-Cu alloy), in which atoms mixed randomly in lattice points, often as a single phase

	Crystal Structure	electroneg	$r$ (nm)	Valence
Ni	FCC	1.9	0.1246	+2
Cu	FCC	1.8	0.1278	+1, +2

*For Ni-Cu solid solution alloy, any atom in a lattice site can be either Ni or Cu*

## W. Hume – Rothery rules

- Same crystal structure (e.g., FCC)
- Similar electronegativities
- Similar atomic radii
- Similar valence state

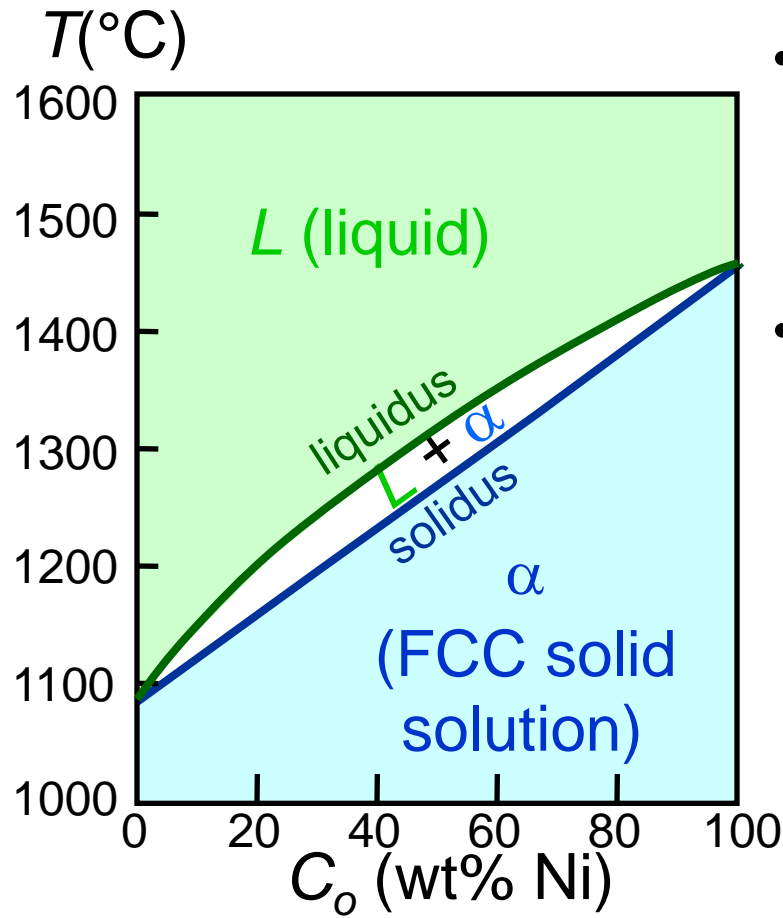


- Ni and Cu are totally miscible in all proportions and form a continuous single phase solid solution

# Binary Phase Diagrams for Solid Binary (Isomorphous) System

- Phase diagram - indicate phases as function of  $T$ ,  $C_o$ , and  $P$ . under **equilibrium** condition
- Binary system: **2 components**: Cu & Ni
  - independent variables:  $T$  and  $C$  ( $P = 1$  atm is almost always used).

• Phase Diagram for Cu-Ni system



- **Phases possible:**

$L$  (liquid phase)

$\alpha$  (FCC solid solution phase)

- **3 regions** (or fields):

$L$  (Single phase region)

$L + \alpha$  (Two phase region)

$\alpha$  (Single phase region)

Adapted from Fig. 9.3(a), Callister 7e.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



# Phase Diagrams: # and types of phases at a given $T$ , $P$ , & sytem composition

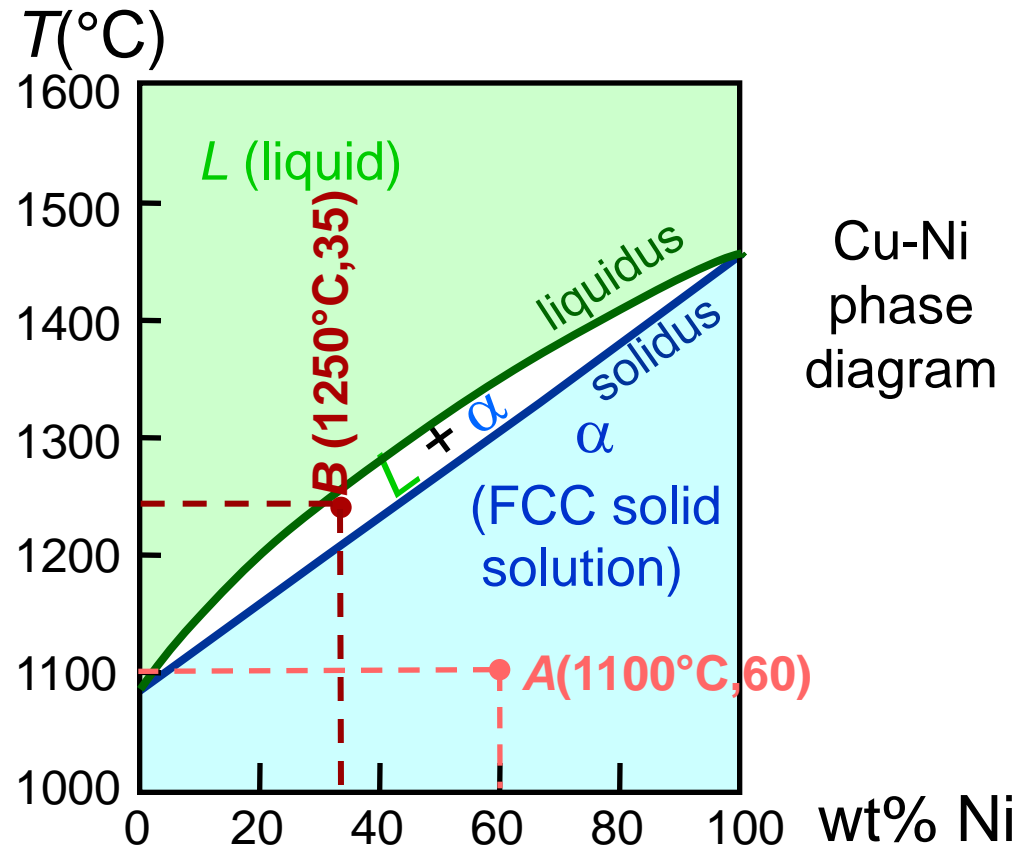
- If we know  $T$  and  $C_o$ , then we know:
  - the # and types of phases present.

- Examples:

$A(1100^{\circ}\text{C}, 60 \text{ wt\% Ni})$ :  
1 phase:  $\alpha$

$B(1250^{\circ}\text{C}, 35 \text{ wt\% Ni})$ :  
2 phases:  $L + \alpha$

Adapted from Fig. 9.3(a), *Callister 7e*.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991).



Cu-Ni  
phase  
diagram



# Phase Diagrams:

## composition of each of the phases

- If we know  $T$  and  $C_o$ , then we know:
  - the composition of each phase present

- Examples:

$C_o = 35 \text{ wt\% Ni}$

At  $T_A = 1320^\circ\text{C}$ :

Only Liquid ( $L$ ) phase

$C_L = C_o (= 35 \text{ wt\% Ni})$

At  $T_D = 1190^\circ\text{C}$ :

Only Solid ( $\alpha$ ) phase

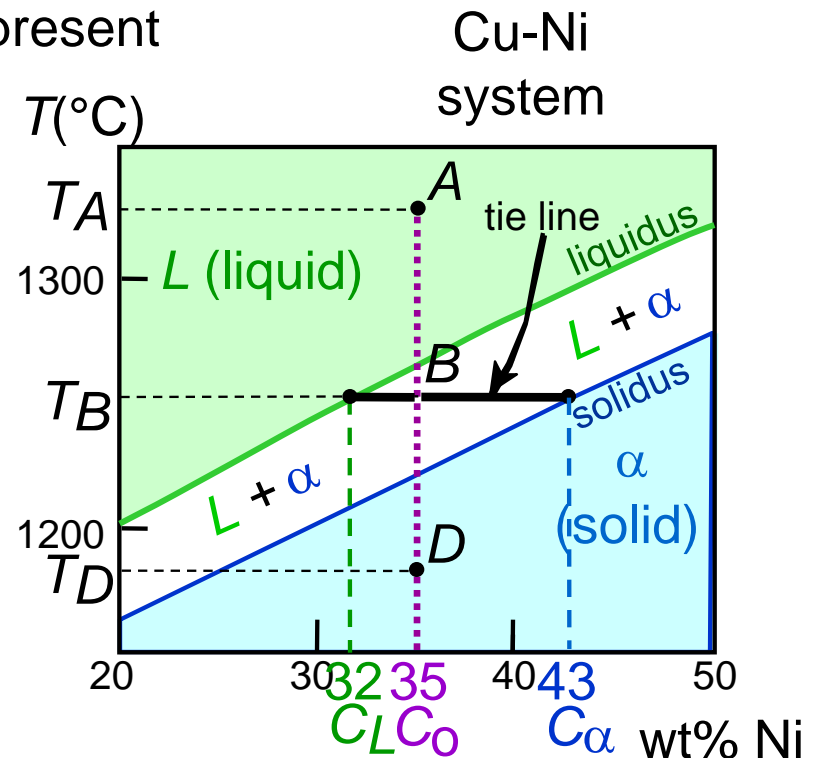
$C_\alpha = C_o (= 35 \text{ wt\% Ni})$

At  $T_B = 1250^\circ\text{C}$ :

Both  $\alpha$  and  $L$  phases

$C_L = C_{\text{liquidus}} (= 32 \text{ wt\% Ni here})$

$C_\alpha = C_{\text{solidus}} (= 43 \text{ wt\% Ni here})$



Adapted from Fig. 9.3(b), *Callister 7e*.  
 (Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)



# Class Exercise:

## Composition of each phase in phase diagram

- For an alloy with 24 wt% Ni, at **G** (1320 °C), **H** (1210 °C), **I** (1160 °C) three points in Cu-Ni phase diagram, give the phase(s) present, and estimate the composition (in terms of wt% Ni) in each of the phase(s)

$$C_0 = 24 \text{ wt\% Ni}$$

At  $T_G = 1320^\circ\text{C}$ :

Only Liquid (L)

$$C_L = C_0 = 24 \text{ wt\% Ni}$$

At  $T_I = 1160^\circ\text{C}$ :

Only Solid ( $\alpha$ )

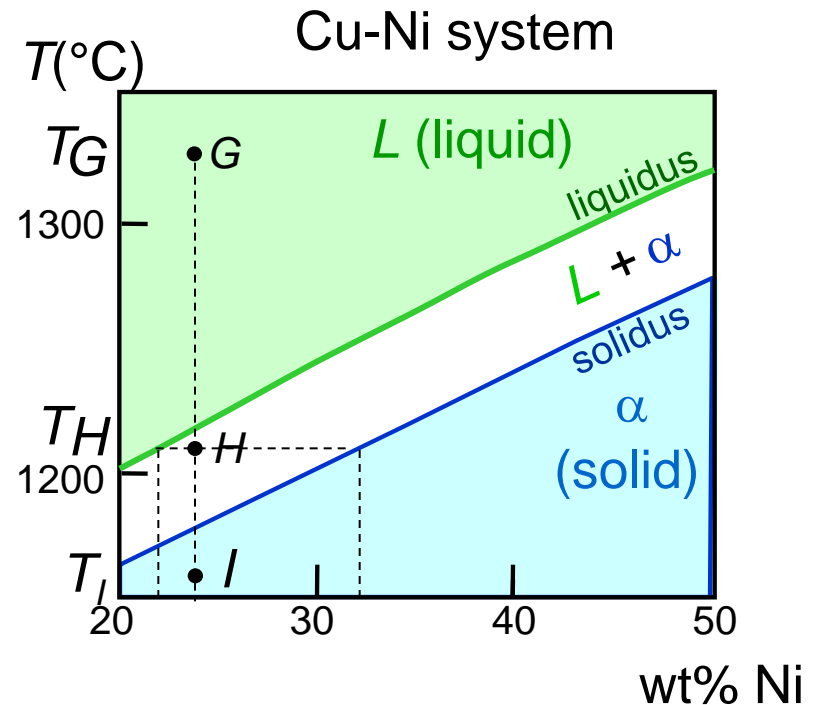
$$C_\alpha = C_0 = 24 \text{ wt\% Ni}$$

At  $T_H = 1210^\circ\text{C}$ :

Both  $\alpha$  and L

$$C_L = C_{\text{liquidus}} \approx 22 \text{ wt\% Ni}$$

$$C_\alpha = C_{\text{solidus}} \approx 32 \text{ wt\% Ni here}$$

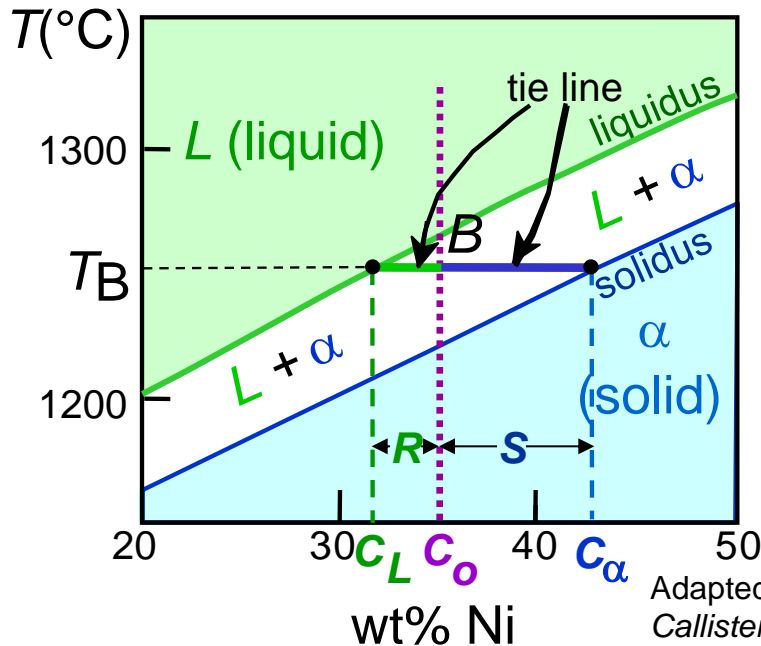


Adapted from Fig. 9.3(b), *Callister 7e*.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)

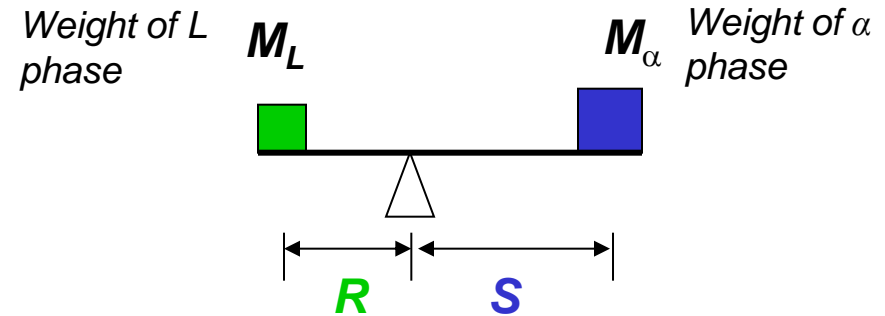


# Relative Amount of Each Phase in Two Phase Region - The Lever Rule

- Tie line – connects the phases in equilibrium with each other – under **isothermal** (same temperature) condition



In two phase region, what is the relative weight fraction ( $W$ , in unit of wt%) of each phase? → “Lever” rule



$$M_{\alpha} \cdot S = M_L \cdot R$$

Weight fraction (in unit of wt%) of the liquid L phase

$$W_L = \frac{M_L}{M_L + M_{\alpha}} = \frac{S}{R + S} = \frac{C_{\alpha} - C_0}{C_{\alpha} - C_L}$$

Weight fraction (in unit of wt%) of  $\alpha$  solid solution phase

$$W_{\alpha} = \frac{R}{R + S} = \frac{C_0 - C_L}{C_{\alpha} - C_L}$$



# Lever Rule: Derivation

Since we have only 2 phases:

$$W_L + W_\alpha = 1 \quad (1)$$

Conservation of mass requires that:

Amount of Ni in  $\alpha$ -phase + amount of Ni in liquid phase = total amount of Ni

or

$$W_\alpha C_\alpha + W_L C_L = C_o \quad (2)$$

From (1), we have:

$$W_\alpha = 1 - W_L$$

Therefore (2) becomes:  $(1 - W_L)C_\alpha + W_L C_L = C_o$

Solving for  $W_L$  and  $W_\alpha$  gives :

$$W_L = \frac{C_\alpha - C_o}{C_\alpha - C_L}$$

$$W_\alpha = \frac{C_o - C_L}{C_\alpha - C_L}$$



# Phase Diagram:

## Class example

- Examples: For points A, B, and D, specify the phase(s) present, the composition (in terms of wt% Ni) of each phase, and the relative weight fraction of each phase

$$C_0 = 35 \text{ wt\% Ni}$$

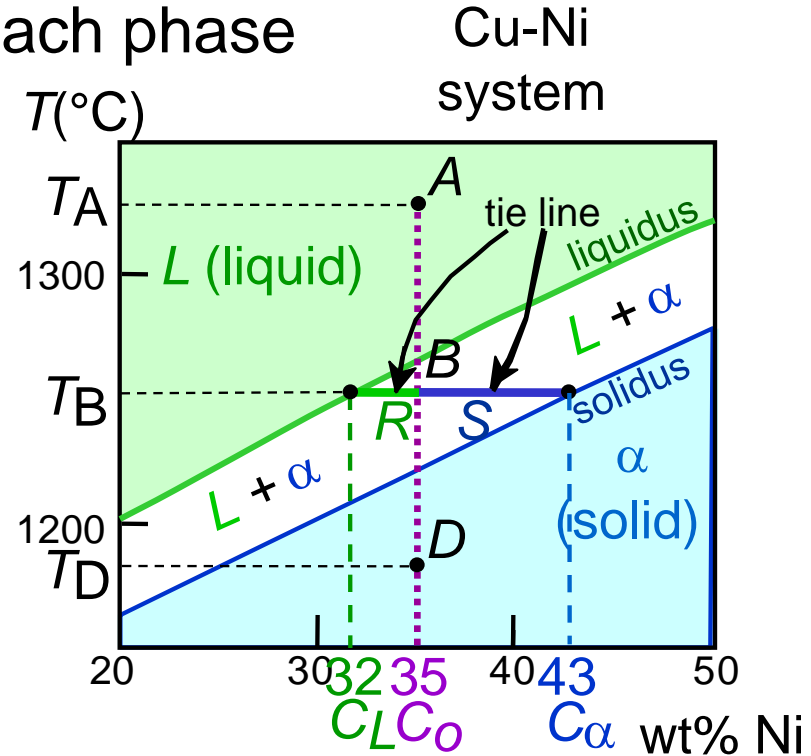
At  $T_A$ : Only Liquid (L)  
 $C_L = 35 \text{ wt\% Ni}$   
 $W_L = 100 \text{ wt\%}, W_\alpha = 0$

At  $T_D$ : Only Solid ( $\alpha$ )  
 $C_\alpha = 35 \text{ wt\% Ni}$   
 $W_L = 0, W_\alpha = 100 \text{ wt\%}$

At  $T_B$ : Both  $\alpha$  and L  
 $C_L = 32 \text{ wt\% Ni}$     $C_\alpha = 43 \text{ wt\% Ni}$

$$W_L = \frac{S}{R+S} = \frac{43 - 35}{43 - 32} = 73 \text{ wt\%}$$

$$W_\alpha = \frac{R}{R+S} = 27 \text{ wt\%}$$



Adapted from Fig. 9.3(b), Callister 7e.  
 (Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)





# Summary

- **Phase diagrams** are useful tools to determine:
  - the number and types of phases,
  - and the **composition** of each phase
  - the weight fraction relative amount of each of the phasesfor a given  $T$ ,  $P$  (*often fixed*) and composition of the system under equilibrium condition

