

# ECE/MATH 522 – Fall 2020

## Random Walks

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**Meeting time:** Tuesdays and Thursdays, 2:00 – 3:15, Stadium 1215

**Course website:** <http://info.canvas.colostate.edu/>

**Office hours:** Fridays 9:30 am or other times by appointment

**Course Objectives:** This course will provide analytical tools to students working on stochastic processes and stochastic models that involve diverse areas such as tracking, relaxation processes, scattering, signal analysis, systems biology, and network traffic.

**Textbook:**

J. Klafter and I. M. Sokolov, First Steps in Random Walks: From Tools to Applications, Oxford University Press, 2011

Students will also receive journal articles and excerpts from the book by W. Feller, An Introduction to Probability Theory and Its Applications (Vols. 1 & 2), 3rd Ed., Wiley 1968

**Pre-requisites:** Probability theory, Fourier and Laplace transforms

**Description:** Mathematical aspects of random walks and diffusion processes. Stochastic modeling of complex systems.

## Topics to be covered:

- Review of characteristic functions and Laplace transforms
- Fluctuations in coin tossing. Reflection principle; returns to the origin
- Brownian motion
- Generalized central limit theorem; Lévy stable distributions
- Generating functions; first passage times (first hitting time, survival models)
- Renewal theory; continuous time random walks (CTRW)
- Lévy walks and Lévy flights
- Master equations
- Long-range dependence (long-memory) processes
- Generalized Fokker –Planck equation
- Fractional calculus (fractional derivatives); fractional diffusion equation
- Graph theory and networks
- Percolation and fractals

## Learning objectives

1. Apply concepts of probability theory to stochastic and fractional processes.
2. Utilize transforms to find statistics of stochastic processes.
3. Understand the relation between random walks and diffusion processes.
4. Employ random walks to model sub- and super-diffusion.
5. Write and solve the Master equation for different Markov chains.
6. Understand the connection between first passage times and occupation statistics.
7. Model complex stochastic processes using different types of random walks including percolation, continuous time random walks, and Lévy flights.
8. Understand basic concepts of percolation theory and its applications.

## Examples of applications:

- Biophysics
- Climate studies
- Dielectric relaxation in complex materials
- Electrical networks, electrical power grids
- Epidemiology, spread of disease in a population
- Finance, insurance, stock prices and gambling
- Genetic drift, biological evolution
- Information networks, social networks, neural networks
- Photon scattering and absorption
- Physical chemistry, chemical kinetics
- Polymer physics
- Time series analysis
- Scale-free networks
- Spatial search and foraging
- Systems biology

**Method of evaluation:** Assignments: 35%, Midterm exam: 25%, Final project: 30%, Readings: 10%

A+ = 98-100

B+ = 87-89

C+ = 76-79

A = 94-97

B = 84-87

C = 65-75

A- = 90-93

B- = 80-83

D = 50-64

F = 0-49

### **Homework assignments:**

Homework assignments will be posted on the course website. Each assignment will be due at the beginning of a specified class meeting. There will be a total of FIVE assignments during the semester.

First and last name, homework number, and course number must be written in the first page. The homework must be stapled, and solutions to the problems must be in the correct order. Solutions must be clear and the derivations to reach results must be included. Writing only the final solution is not acceptable.

### **Take-home midterm exam:**

One midterm take-home exam will be given. Students can consult any material, but they must work on the exam alone. Students are required to reference all consulted materials.

### **Final project**

In the final project the students will demonstrate skills to model complex stochastic processes. The project is expected to include a critical literature survey, numerical simulations, and analytical tools. Project planning will be done together with the class instructor. Students will give a short presentation on their project and submit a three-page report. It is recommended to start planning early, so you are advised to start a discussion with the instructor before the end of September.

### **Readings**

Journal articles will be regularly assigned as Readings. Students need to write one or two paragraphs summarizing the article and providing a critique about what they consider to be strong and weak points of the article. The summary is submitted directly in Canvas and it is strictly limited to 2500 characters (including spaces and punctuation). It is mandatory to include a character count after the summary.

### **Academic integrity**

The course will adhere to the Academic Integrity Policy of the CSU General Catalog (page 7, <http://www.catalog.colostate.edu/FrontPDF/1.6POLICIES1112f.pdf>) and the Student Conduct Code (<http://www.conflictresolution.colostate.edu/conduct-code>)