



New Acoustic Color and Synthetic Aperture Sonar Processing Methods Using Coherence Analysis

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Outline of Presentation

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- Data Preprocessing and Feature Extraction
 - New Coherence-Based Frequency Subband (CFS) Features
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 - Performance Analysis Using a Single-Aspect Classifier
- Multi-Aspect Classification Systems
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 - Classification Results on Individual Objects and Entire Runs
- Synthetic Aperture Sonar (SAS) Processing
 - Conventional Delay-and-Sum SAS processing
 - New Coherence-Based Blind SAS Processing
- Conclusions and Future Work

Problem: Develop methods for detection and classification of underwater mine-like objects using broadband acoustic sonar data

- **Complicated by many factors including:**
 - Man-made and natural clutter
 - Reverberation
 - Changing operating and environmental conditions
 - Lack of *a priori* knowledge about shape and geometry of new non-mine-like objects
- **Approaches may be broken down into the following areas:**
 - **Sensor development and data acquisition:** Use acoustic source to “ping” the seafloor and capture return signals via hydrophone sensor elements
 - ✓ **Feature extraction:** Extract discriminatory mine-like versus non-mine-like properties from sonar returns
 - ✓ **Detection/classification:** Use extracted features to produce decisions regarding the class of represented objects and environment
 - ✓ **SAS Processing:** Generate synthetic image of seafloor by coherently integrating acoustic sonar data from multiple sensors and pings

Research Objectives and Motivations

Research Objectives:

- 1) Development of a feature extraction method that exploits coherence across two sonar pings in specific frequency subbands
 - Offer a theoretically and intuitively meaningful way of performing acoustic color processing
- 2) Development of new multi-aspect classifiers
- 3) Development of a coherence-based blind SAS processing algorithm
 - Does not require use of vehicle motion parameters
- 4) Demonstrate the effectiveness of these tools on two real sonar databases

Motivations:

- Using multiple sonar pings/aspects can improve classification performance since a single ping may not contain enough discriminatory information
- Ping-to-ping coherence patterns in subbands of an object's frequency response are a better indicator of its type than coherence in different blocks of range cells (i.e. sonar time series)
- Conventional SAS processing requires elaborate platform motion estimation and compensation, and produces images that do not convey information useful for object classification

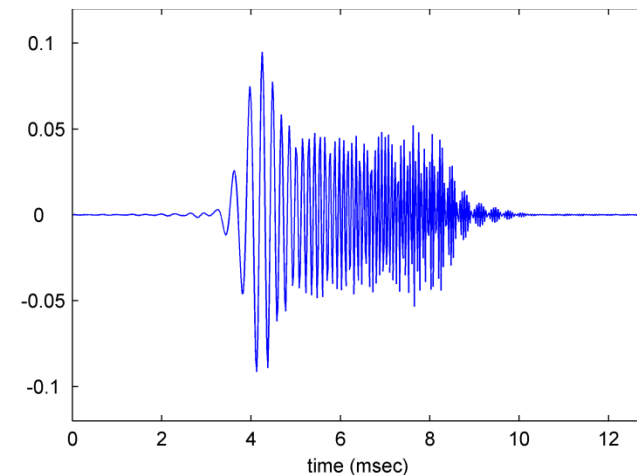
Sonar System

- Data collected by wing Buried Object Scanning Sonar (BOSS)
 - Developed by Florida Atlantic University
- Produces omnidirectional 5 millisecond linear FM transmit signal over 3-19 kHz
- Sonar returns captured by 40 hydrophone elements
 - Uniform linear subarray of 20 hydrophones on each wing of “Bluefin 12” unmanned underwater vehicle (UUV)

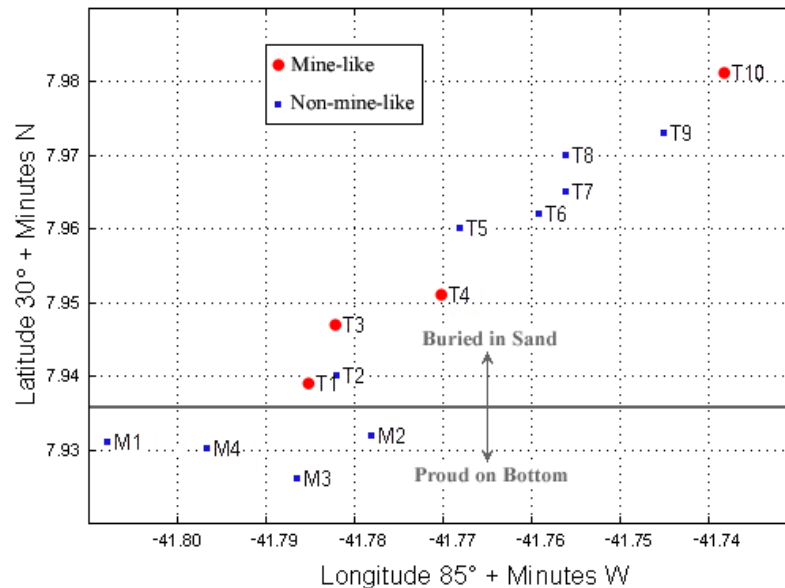
Bluefin 12 UUV with wing BOSS payload



Transmit Signal

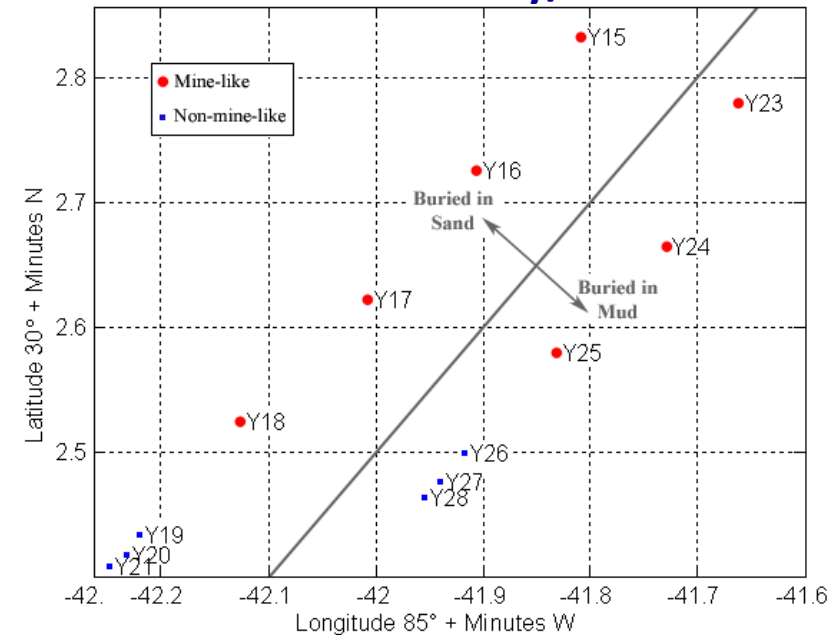


Davis Point Data Set – March 2007



- Ping rate: 25 pings/sec UUV speed: 1.5 m/s UUV altitude: 3 m
- UUV made 'star-shaped' runs centered on objects T1, T3, T4, T5, and T10, and diagonal runs that traversed entire target field
- Each object adequately captured during at least one run
- Objects with properties similar to mines used

Yankee Data Set – May/June 2006



- Ping rate: 20 pings/sec UUV speed: 1.5 m/s UUV altitude: 3 m or 12 m
- UUV made multiple runs over each object
- All objects adequately captured besides Y23 and Y24
- Real mines used, but object characteristics not provided
- Two bottom types: sand and mud

- Sonar return at ping p : $x_p[n] = h_p[n] * s[n] + f_p[n] * s[n] + v_p[n]$

- $s[n]$ = transmit signal, $h_p[n]$ = impulse response of object and bottom

- $f_p[n]$ = impulse response of correlated noise, $v_p[n]$ = uncorrelated noise

- Extract transfer function of object and bottom:

- Apply matched filter to better separate object and bottom return from other returns and correlated noise. In the frequency domain:

$$X_p[k]S^*[k] = H_p[k]|S[k]|^2 + F_p[k]|S[k]|^2 + V_p[k]S^*[k]$$

- Remove effects of transmit signal using inverse filter:

$$\frac{X_p[k]S^*[k]}{|S[k]|^2 + \epsilon} \approx H_p[k] + F_p[k] + \frac{V_p[k]}{S[k]}$$

- Window to remove correlated noise:

$$\hat{H}_p[k] \approx H_p[k] + W[k] * \frac{V_p[k]}{S[k]}$$

- Forms “clean” frequency response of bottom and object within bandwidth of transmit signal (3-19 kHz for BOSS)

Canonical Correlation Analysis (CCA) Review

- Given zero-mean vectors: $\mathbf{x} \in \mathbb{R}^m$ and $\mathbf{y} \in \mathbb{R}^n$ with $m \leq n$

- Composite covariance matrix:
$$E \left[\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \begin{pmatrix} \mathbf{x}^T & \mathbf{y}^T \end{pmatrix} \right] = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix}$$

- Coherence matrix:**
$$C = E[\zeta \boldsymbol{\nu}^T] = E[(R_{xx}^{-1/2} \mathbf{x})(R_{yy}^{-1/2} \mathbf{y})^T] = R_{xx}^{-1/2} R_{xy} R_{yy}^{-T/2}$$

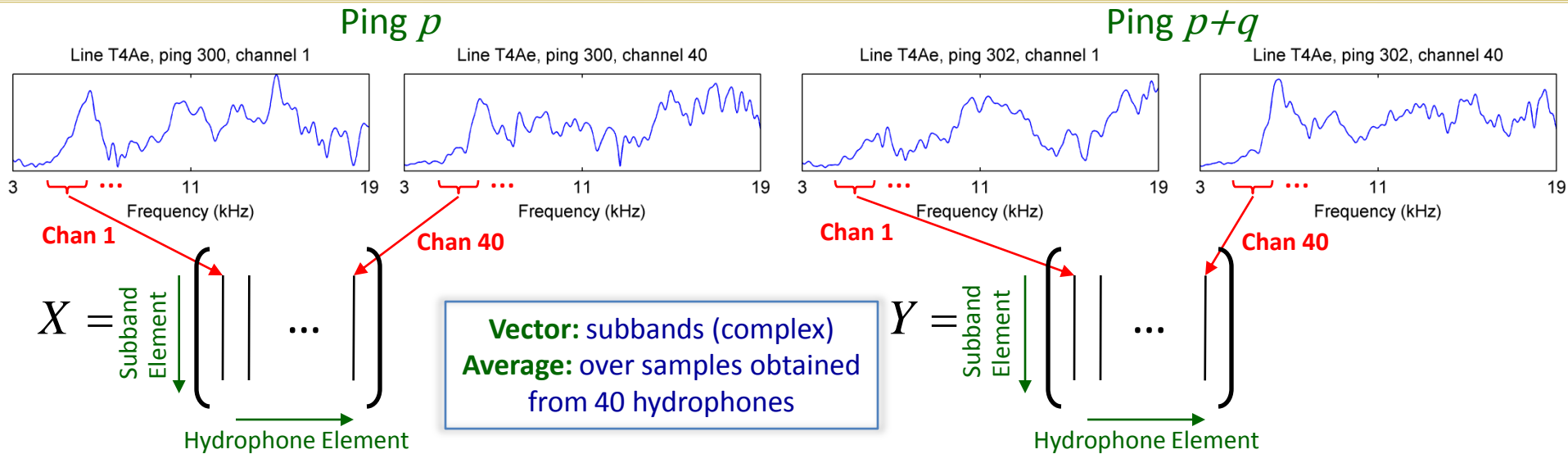
- SVD of Coherence Matrix:
$$C = FKG^T$$
 $F \in \mathbb{R}^{m \times m}$ and $G \in \mathbb{R}^{n \times n}$ orthogonal

- $K = \text{diag}[k_1, k_2, \dots, k_m]$ is the matrix of **canonical correlations** with $1 \geq k_1 \geq k_2 \geq \dots \geq k_m > 0$.

- Canonical coordinates** \mathbf{u} and \mathbf{v} :

$$\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} F^T & \mathbf{0} \\ \mathbf{0} & G^T \end{bmatrix} \begin{bmatrix} \zeta \\ \boldsymbol{\nu} \end{bmatrix} = \begin{bmatrix} F^T & \mathbf{0} \\ \mathbf{0} & G^T \end{bmatrix} \begin{bmatrix} R_{xx}^{-1/2} & \mathbf{0} \\ \mathbf{0} & R_{yy}^{-1/2} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$

- It follows that:
$$K = E[\mathbf{u}\mathbf{v}^T] = F^T C G$$



- CCA used to measure coherence between same subband at pings p and $p+q$ (q is ping separation), thus producing L canonical correlations ($L =$ subband size)
- Dominant canonical correlation ($k_p^{(m)}(1)$) used as feature to represent subband m
- Find features ($k_p^{(m)}(1)$ values) that most effectively discriminate between mine-like and non-mine-like objects using Fisher discrimination measure:

$$J_m = \frac{|\mu_1^{(m)} - \mu_2^{(m)}|}{\sigma_1^{(m)} + \sigma_2^{(m)}}, \quad m \in [0, M - 1]$$

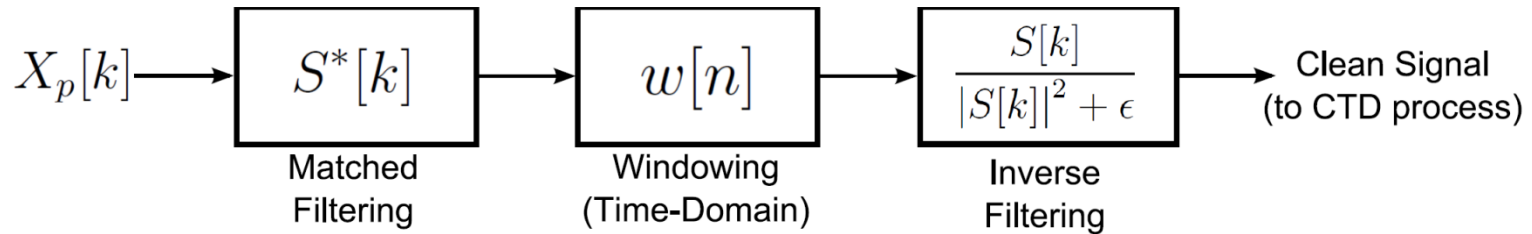
$\mu_i^{(m)}$: mean of k_1 values from m th subband of class i pings

$\sigma_i^{(m)}$: std. deviation of k_1 values from m th subband of class i pings

- 20 features with largest J_m used to form feature vector representing ping p

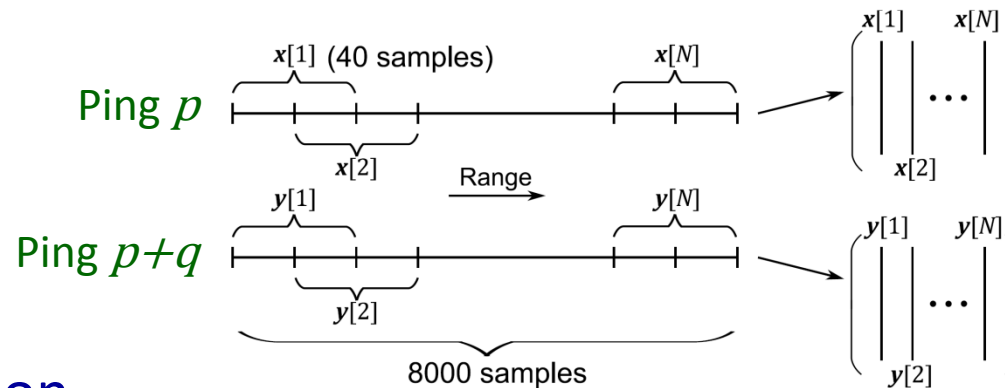
Preprocessing

- Matched filtering, windowing, and inverse filtering still employed:



Data Setup

- Recovered times series (200 samples) from each receiver (40 total) are concatenated
- Concatenated signals partitioned into overlapping blocks of size 40 samples with 50% overlap



Feature Extraction

- CCA performed between the two data channels (sonar returns with some separation)
- Produces 40 canonical correlations with 20 dominant canonical correlations used to form feature vector to represent ping p

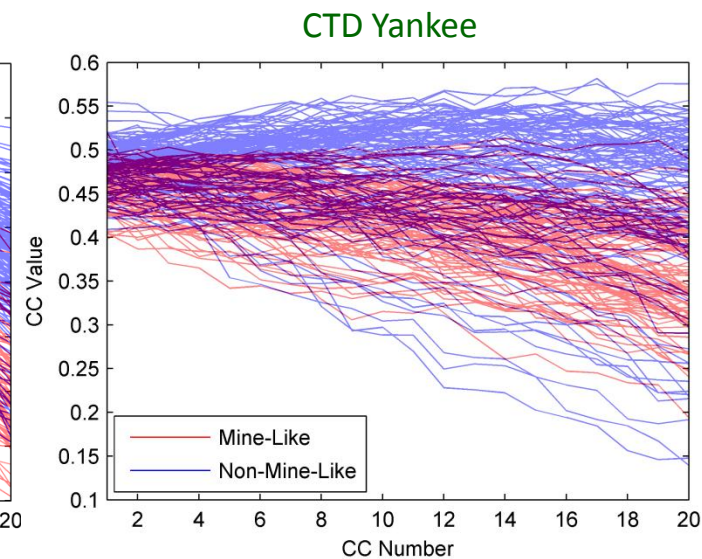
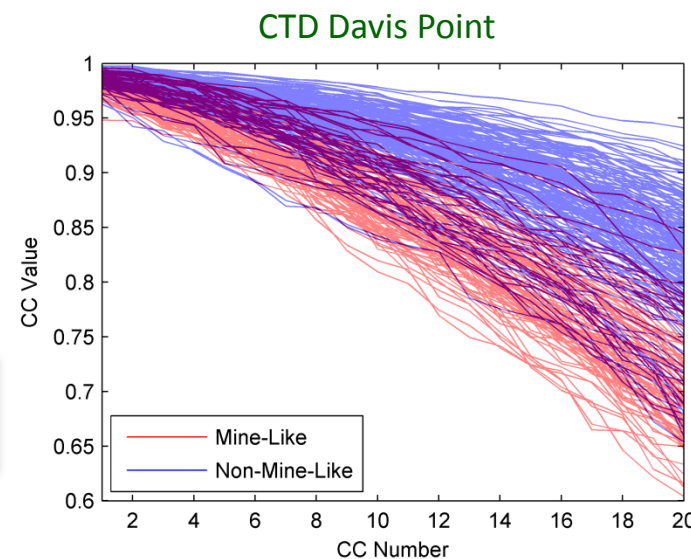
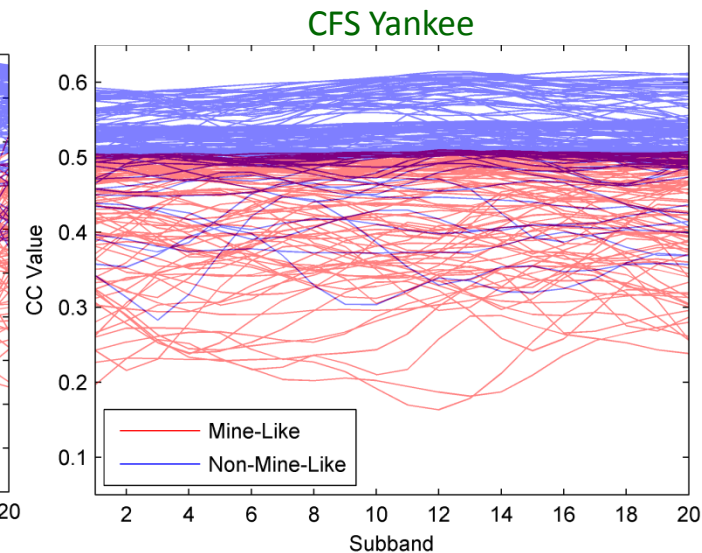
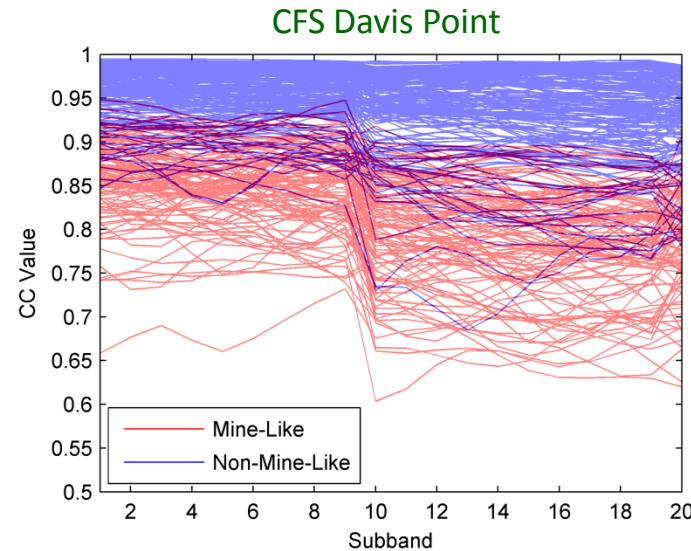
Data Preprocessing and Feature Extraction: Feature Space Properties of Data Sets

Normalization

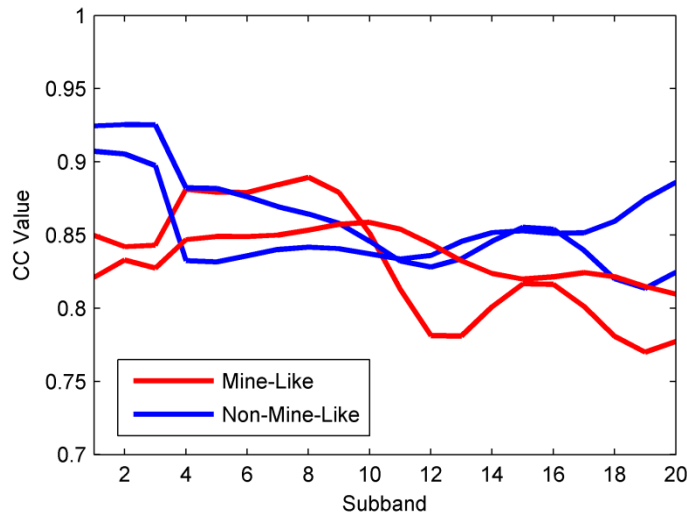
Feature Vector Plots:

- Both CFS and CTD feature spaces show some separation
- Fewer instances where CFS non-mine-like features show similar levels of coherence to CFS mine-like features (compared to CTD)
- To make Yankee feature vectors from different runs compatible, they were normalized

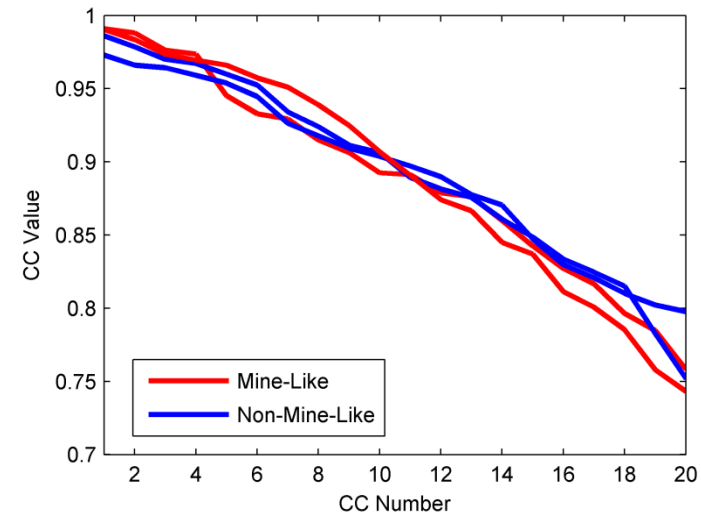
Ping Separation: $q = 2$ for both feature types and data sets



CFS Features

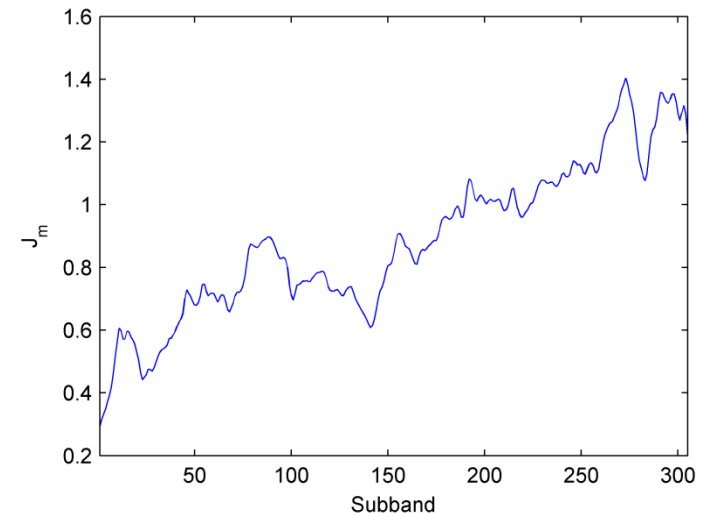


CTD Features



- CFS coherence patterns more useful for classification
 - CFS feature vectors that overlap are more discernable
 - CTD feature vectors are monotonically non-increasing regardless of object type
- High frequency subbands are more useful for classification of objects in Davis Point data set
 - Features extracted using samples in ranges 17.16-17.27 kHz and 18.11-18.95 kHz used to form each feature vector

Fisher Measure for Davis Point



Data Preprocessing and Feature Extraction: Single-Aspect Classification Results

- Back-propagation neural network (BPNN) used to classify individual feature vectors
 - Highlights performance differences between CFS and CTD feature extraction methods

Davis Point Data: Percent Incorrect Classification

	Validation	Testing 1	Testing 2
CFS Features	8.0%	7.9%	10.7%
CTD Features	12.0%	13.9%	21.4%
Reduction	33.3%	43.2%	50.0%

- Pings in training and validation set come from runs in 'star-shaped' groups
 - Contains sonar returns from objects **T1, T3, T4, T5, T6, T9, T10**, and **M2**
- Pings in first testing set come from different runs in 'star-shaped' groups
 - Contains sonar returns from objects **T1, T2, T3, T4, T5, T6, T10**, and **M3**
- Pings in second testing set come from long Southwest-Northeast runs
 - Contains sonar returns from objects **T1, T3, T5, T6, T7, T8, T9, T10, M1**, and **M4**
- Objects captured from different aspects

Yankee Data: Percent Incorrect Classification

	Validation	Testing 1	Testing 2
CFS Features	6.8%	6.8%	12.7%
CTD Features	0%	18.6%	25.5%
Reduction	-	63.4%	50.2%

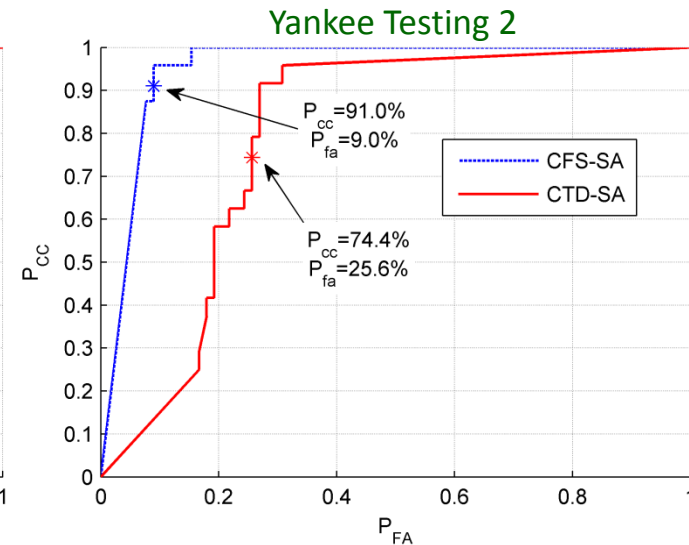
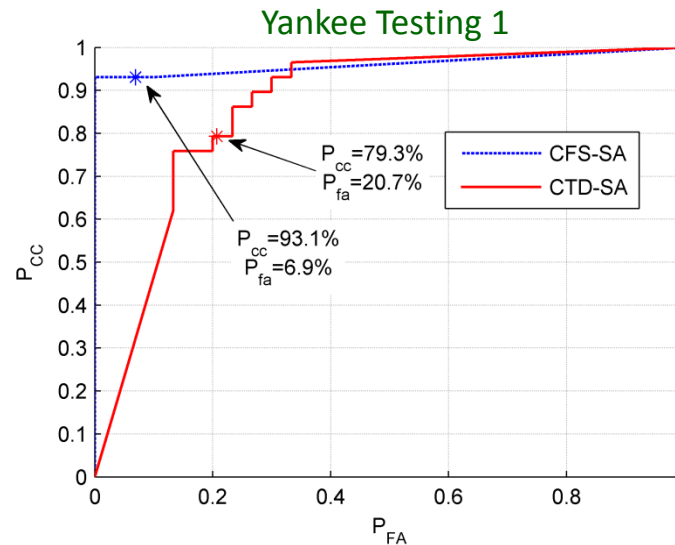
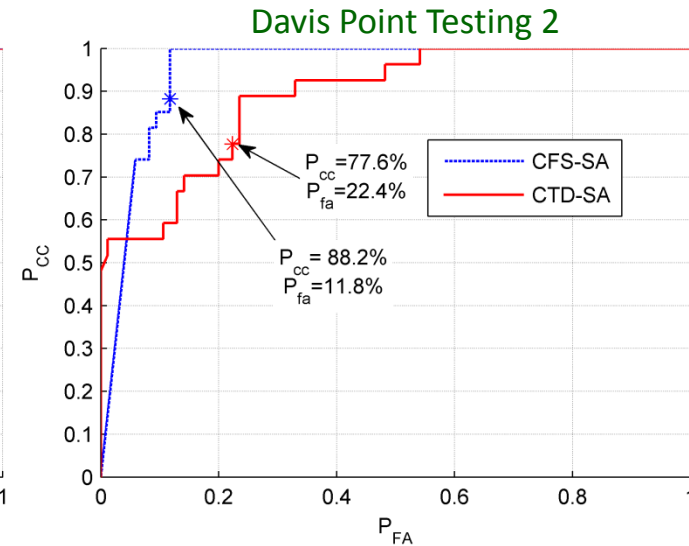
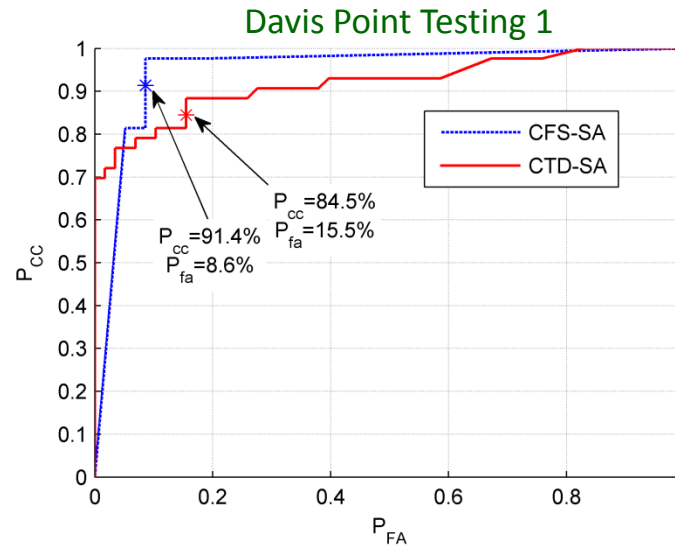
- Pings in training and validation set come from objects buried in sand (**Y15, Y16** (12 m altitude), **Y17, Y19, Y20**, and **Y21**)
- Pings in first testing set come from different runs over objects buried in sand (**Y15, Y17, Y18, Y19, Y20**, and **Y21**)
- Pings in second testing set come from objects buried in mud (**Y25, Y26, Y27**, and **Y28**)
- Pings used in different data sets capture different aspects of each object when possible
- CFS features perform worse on validation set due to misclassification of single ping off center of each mine-like object

Data Preprocessing and Feature Extraction: Single-Aspect Classification Results

Receiver Operating Characteristic (ROC) Curves:

Using CFS features provides:

- Higher correct classification rates in almost every case
- Better performance as measured by ROC curves
- Superior generalization ability

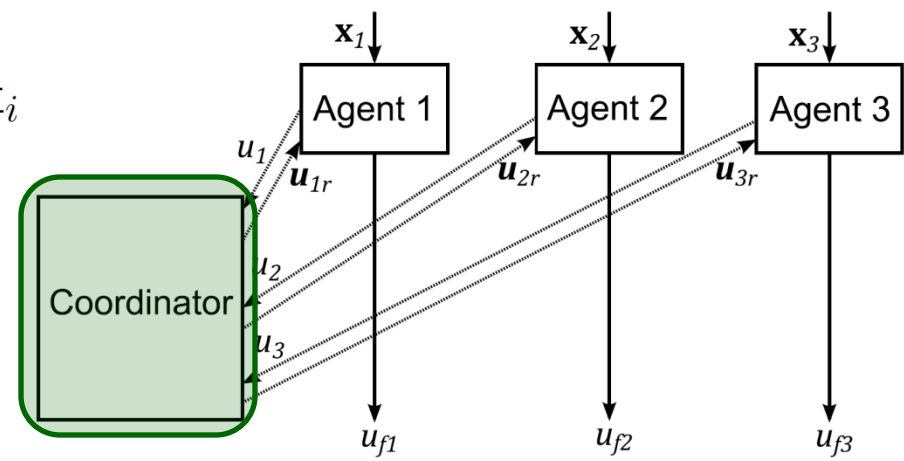


CFS-SA = CFS single-aspect classifier
CTD-SA = CTD single-aspect classifier

Classifier Designs

Collaborative Multi-Aspect Classifier (CMAC)

- N agents, where each makes a preliminary decision, u_i , on a separate feature vector \mathbf{x}_i using a probabilistic neural network (PNN)
- Each agent shares its decision with all other agents via a coordinator.
- BPNN in each agent estimates class conditional probabilities of other agent's decisions (elements of \mathbf{u}_{ir})
- i th agent makes final decision u_{fi} in data fusion center using:

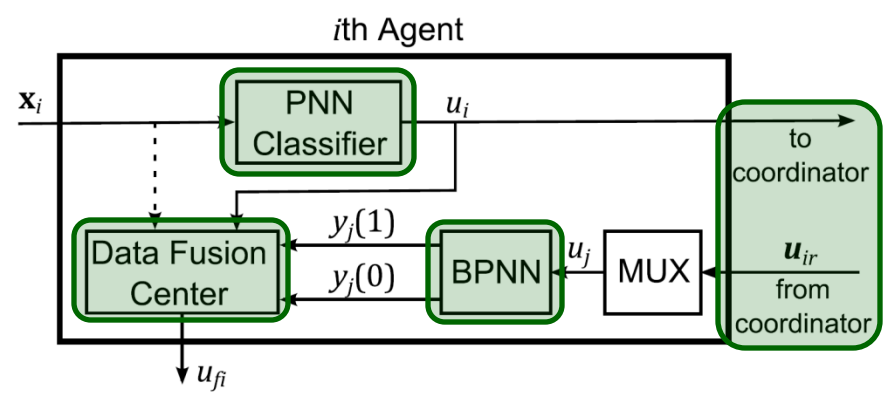


$$u_{fi} = \begin{cases} 1 & \frac{p(C_1|\mathbf{x}_i)}{p(C_0|\mathbf{x}_i)} \geq 1 \\ 0 & \frac{p(C_1|\mathbf{x}_i)}{p(C_0|\mathbf{x}_i)} < 1 \end{cases}$$

$$P(C_1) \prod_{j=1, j \neq i}^{N-1} p(C_0|u_j) [J_{10} - J_{00}]$$

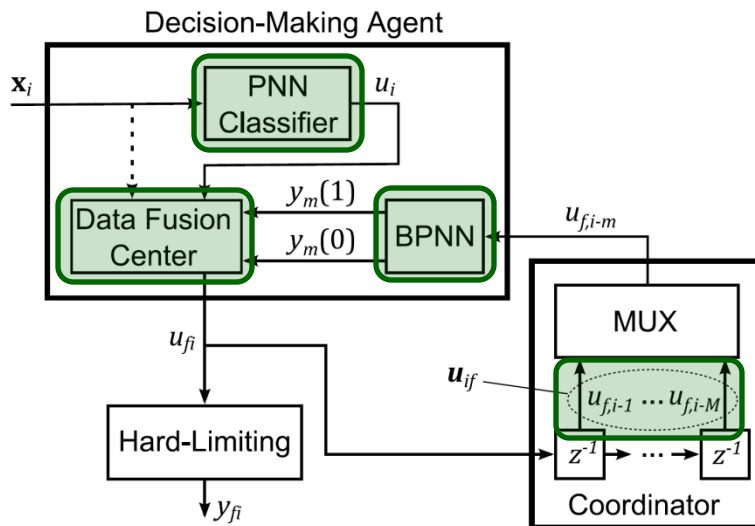
$$P(C_0) \prod_{j=1, j \neq i}^{N-1} p(C_1|u_j) [J_{01} - J_{11}]$$

Labels: PNN Outputs, Prior Probabilities, BPNN Outputs, Decision Costs



- Minimizes the expected cost of making an incorrect decision

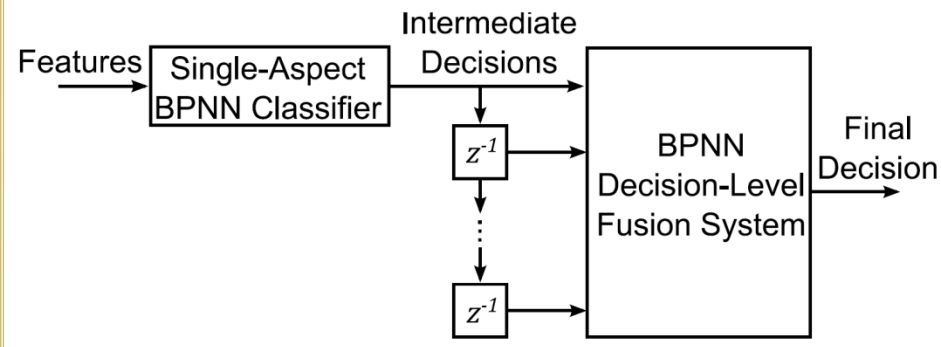
Decision Feedback (DF) Based on One-Agent CMAC



- Generates final decision u_{fi} using:
 - Preliminary decision on current ping
 - Class conditional probabilities of final decisions at M previous pings
 - Likelihood ratio:

$$u_{fi} = \frac{P(C_0)^M p(C_1 | \mathbf{x}_i) \prod_{m=1}^M p(C_1 | u_{f,i-m})}{P(C_1)^M p(C_0 | \mathbf{x}_i) \prod_{m=1}^M p(C_0 | u_{f,i-m})}$$

Nonlinear Decision-Level Fusion (NDLF)



- N intermediate decisions on N separate features produced using single-aspect BPNN
- Final decision formed by fusing intermediate decisions using second BPNN
 - Trained using sets of N single-aspect decisions
 - Incapable of using variable number of pings

Multi-Aspect Classification Results

Davis Point Data: Percent Incorrect Classification

	Validation	Testing 1	Testing 2
CMAC	4.0%	2.0%	6.2%
DF	8.0%	4.0%	8.0%
NDLF	6.0%	8.9%	8.0%
Reduction (CMAC vs. NDLF)	33.3%	77.5%	22.5%

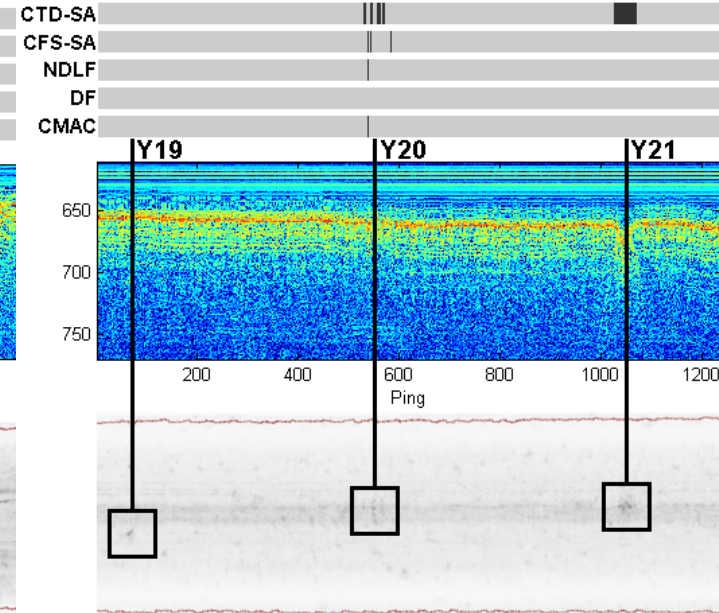
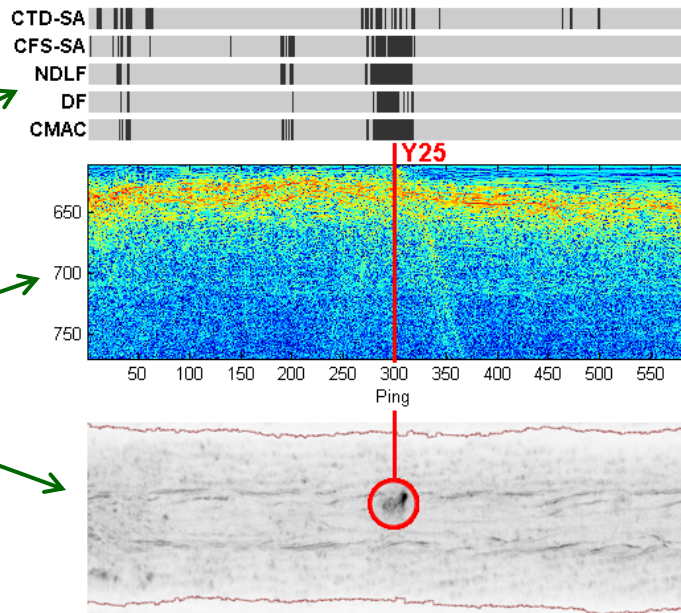
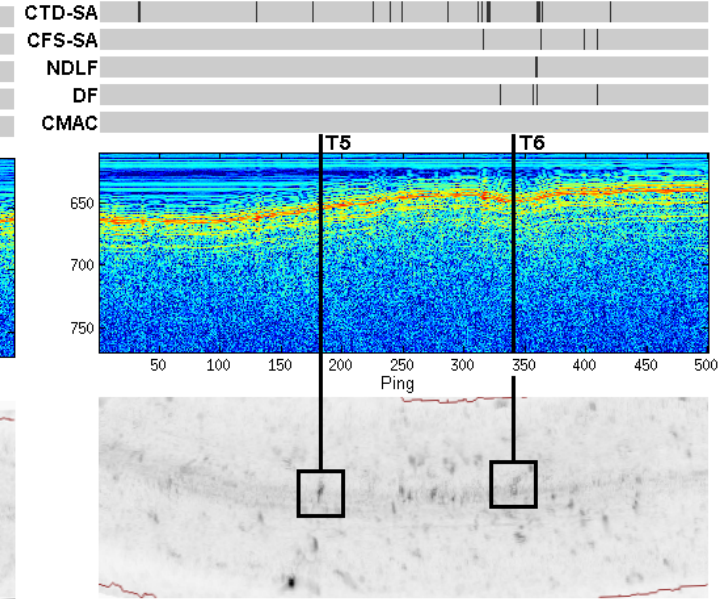
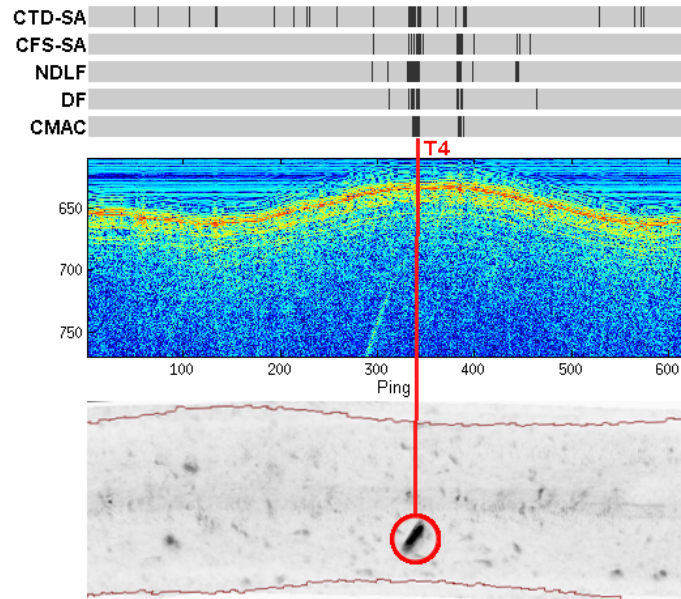
Yankee Data: Percent Incorrect Classification

	Validation	Testing 1	Testing 2
CMAC	0%	3.4%	5.9%
DF	2.3%	5.1%	8.8%
NDLF	0%	5.1%	8.8%
Reduction (CMAC vs. NDLF)	0%	33.3%	33.0%

- CFS feature vectors used
- Same training, validation, and testing sets used for single-aspect classifier results are used for multi-aspect classifier results
- **CMAC**: 3 agents, **DF**: 2 previous decisions, **NDLF**: 3 intermediate decisions
- CMAC provides highest correct classification rate in every case
 - Corrects nearly every instance of misclassification of center pings off mine-like objects produced by single-aspect classifier
 - Only one object (**Y27** in Yankee data set) had majority of pings misclassified

Multi-Aspect Classification Systems: Simultaneous Detection and Classification

- Two Davis Point runs (top) and two Yankee runs (bottom)
- Largest improvement when using CFS over CTD feature vectors
- CMAC provides most accurate classification



[Additional Runs](#)

Detection/classification strips where potential mine-like objects are shown by black vertical bars

Matched filtered image

SAS image showing relative target locations

Targets in red

Non-targets in black

Synthetic Aperture Sonar Processing: Delay-and-Sum SAS Processing

- Several 3-D SAS frames generated by coherently integrating data from M pings and N hydrophones
 - Frame pixels correspond to specific focal points of seafloor
 - Time sample selected from sonar return collected by each hydrophone at each ping that corresponds to focal point
 - SAS frame $A(x, y, z, k)$ generated using:

$$A(x, y, z, k) = \sum_{n=1}^N \sum_{m=k}^{k+M-1} s_{m,n} [t_{m,n}^{(\mathbf{p}_f)}] d_m r_{m,n}$$

$s_{m,n}[t]$ = sonar return for ping m and hydrophone n

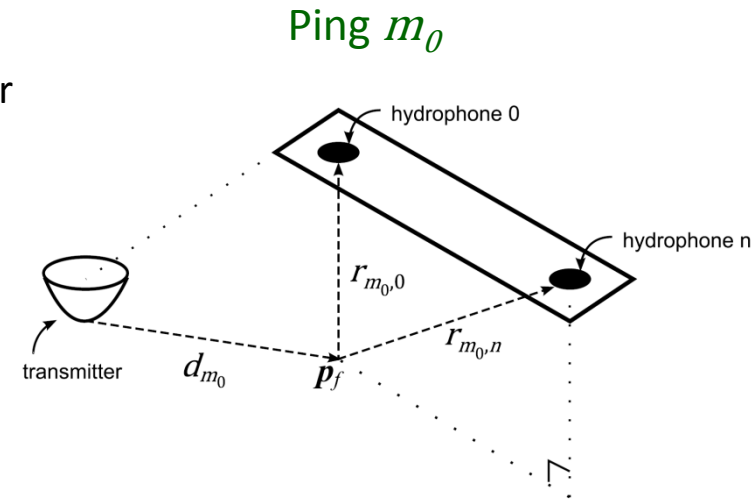
$t_{m,n}^{(\mathbf{p}_f)}$ = time sample for focal point $\mathbf{p}_f = [p_x, p_y, p_z]$ $\Rightarrow x = \frac{p_x}{\Delta p_x}, y = \frac{p_y}{\Delta p_y}, z = \frac{p_z}{\Delta p_z}$

- SAS frames combined by overlapping and using max intensity pixel at each focal point:

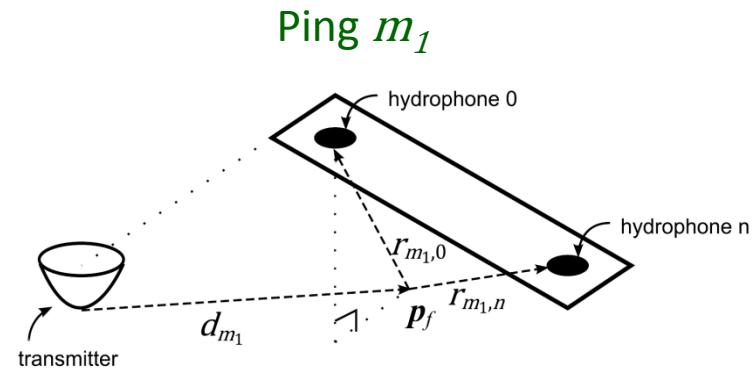
$$B(x, y, z) = \max_k [A(x, y, z, k)]$$

- 2-D SAS image generated using max intensity range value for each point along and across track:

$$P_z(x, y) = \max_z [B(x, y, z)]$$



Focal Point Spacing



Coherence-Based Blind SAS Processing

- Generates image that displays information in ping-frequency (fp) plane
 - Images useful for determining objects' along track location **and class label**

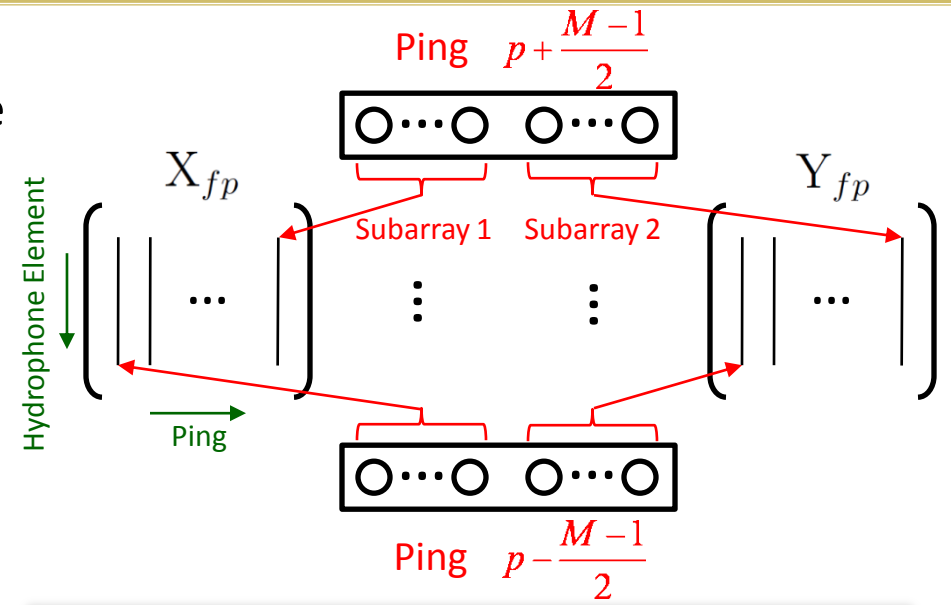
For each frequency sample from 3-19kHz (10.5 Hz per bin):

- Perform CCA between data from two subarrays (one for each wing of BOSS)
 - Vector: hydrophone subarray (N/2 elements)
 - Average: over M pings
- Find canonical coordinate samples:

$$\mathbf{U}_{fp} = \mathbf{F}_{fp}^H (\mathbf{R}_{XX}^{(fp)})^{-1/2} \mathbf{X}_{fp} = \begin{bmatrix} \mathbf{u}_{fp}^{(1)} \\ \vdots \\ \mathbf{u}_{fp}^{(N/2)} \end{bmatrix}$$

- Each pixel formed using average of L $\mathbf{u}_{fp}^{(1)}$ samples in center of ping window:

$$S(f, p) = \left| \frac{1}{L} \sum_j \mathbf{u}_{fp}^{(1)}(j) \right|, j \in \left[\frac{1}{2}(M - L + 2), \frac{1}{2}(M + L) \right]$$

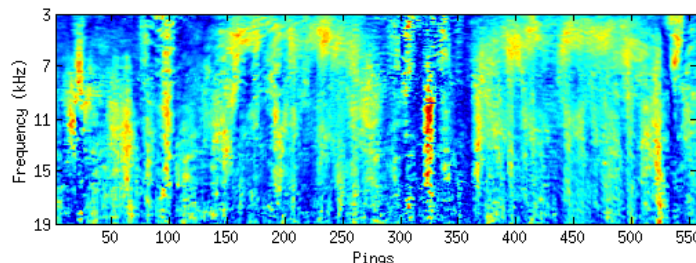


No platform motion estimation required!

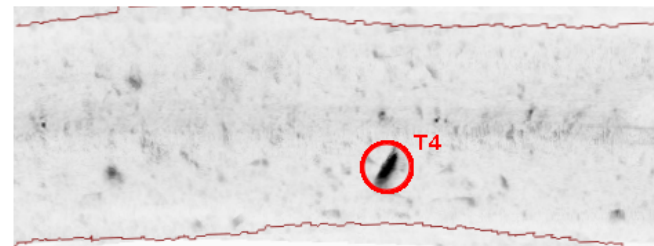
Synthetic Aperture Sonar Processing: SAS Image Comparison

[Additional Runs](#)

Coherence-based SAS-like image

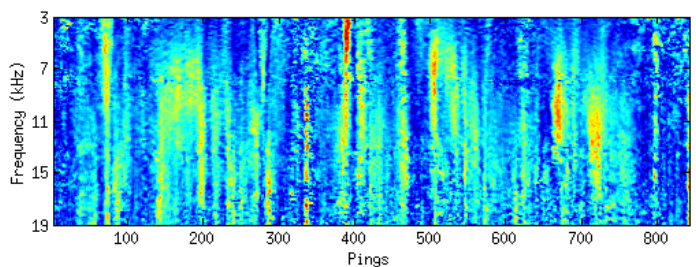


Conventional delay-and-sum SAS image

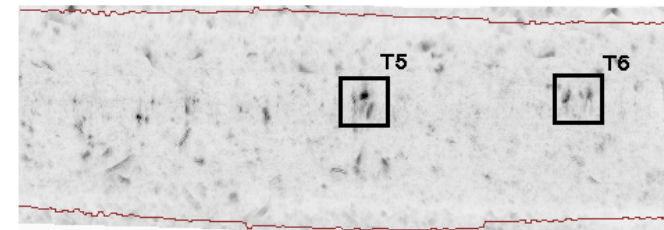
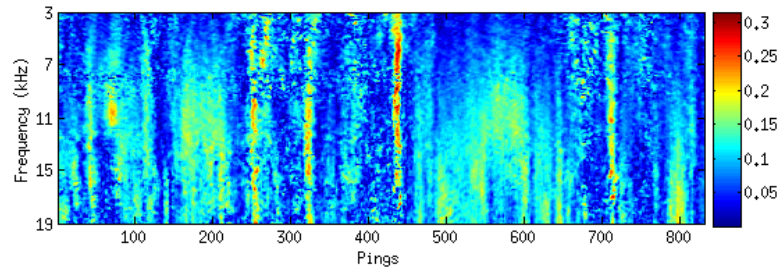
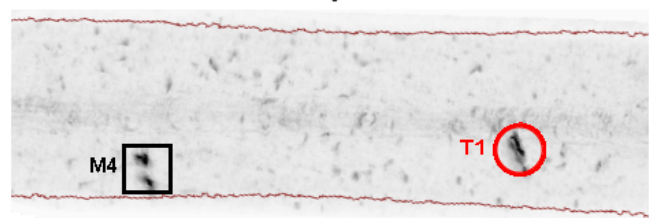


Ping Window: $N = 71$

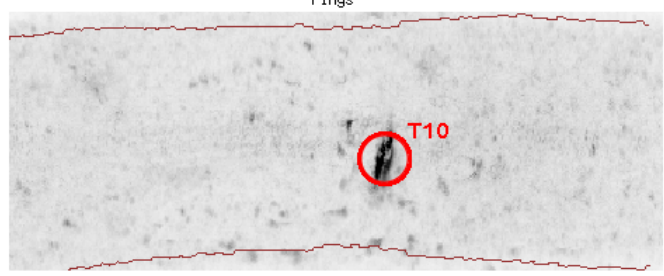
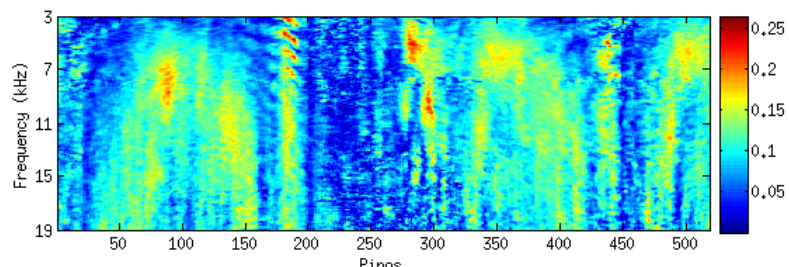
Coordinates samples:
 $L = 5$



Targets in red
Non-targets in black



Magnitude of averaged canonical coordinate samples



Conclusions

- Coherence-based features (CFS and CTD) are robust to changes in vehicle altitude and many types of environmental conditions
- When using the CFS features, higher classification rates and fewer false alarms are generally obtained compared to the CTD method
 - Attributed to the ability of the CFS method to produce theoretically and intuitively more meaningful coherence patterns from the frequency subbands of two sonar pings
- Multi-aspect classification can be used to further improve classification performance
- Conventional SAS images contain information useful for object detection and across track and along track localization
- Coherence-based SAS-like images offer target detection, classification, and along track localization without requiring estimation of platform motion

Future Work

- Conduct an in-depth analysis relating CFS dominant canonical correlation patterns to specific object types and their properties
- Explore different options for normalizing feature vectors in the case where features from different runs are incompatible with each other
- Develop a solid theoretical foundation for the coherence-based blind SAS processing algorithm
- Investigate the applicability of the proposed methods to other detection/classification studies such as problems in signal processing, communications, radar, sensor fusion, etc.

Questions?

Additional Information

- [Literature Review](#)
- [Davis Point Object Descriptions](#)
- [BOSS UUV Paths](#)
- [Subband Decomposition](#)
- [Yankee Feature Normalization](#)
- [CMAC Details](#)
- [DF Details](#)
- [Absolute Classification Rates](#)
- [Multi-Aspect Classifier ROC curves](#)
- [Additional Results on Entire Runs](#)
- [Additional SAS Image Comparison](#)

● Feature extraction:

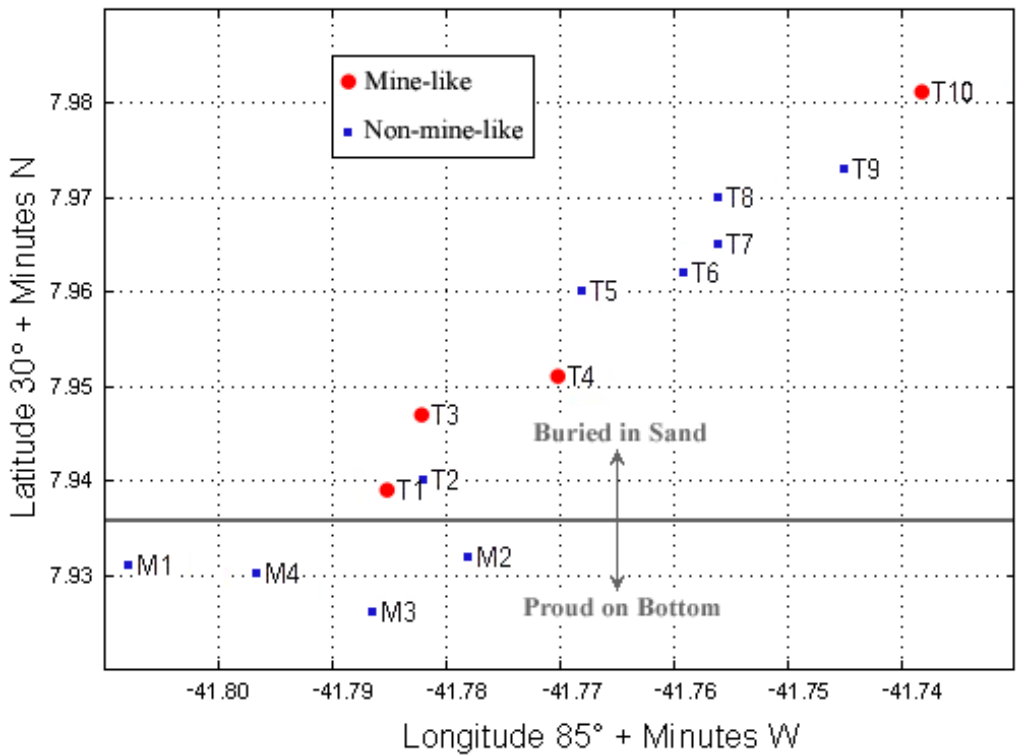
- [Azimi] Two channel canonical correlation analysis and multi-channel coherence analysis applied to sonar returns in time domain
- [Intrator, Miklovic] Acoustic color processing

● Detection/classification:

- [Azimi] Hidden Markov Models (HHMs), decision-level fusion, collaborative multi-aspect classifier (CMAC)
- [Carin] HHMs, wavelet-based feature-level fusion
- [Sternlicht] Fusion of decisions made using image-based and acoustic-based classifiers

● Synthetic Aperture Sonar Processing:

- [Schock] Conventional delay-and-sum SAS processing with BOSS
- [Lo, Solomon] Application of many adaptive beamforming algorithms to sonar data



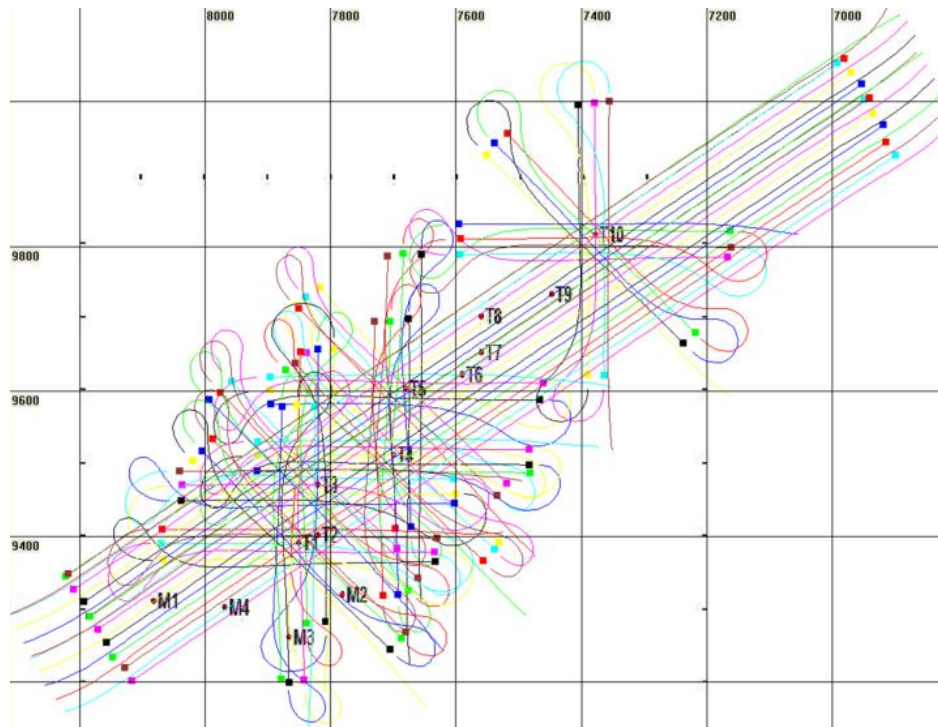
Buried

- T01** SW 6' Bomb-Shaped Marker (18" OD)
- T02** 20" 81mm Artillery Shell
- T03** 5.5' Bomb-Shaped Target (11" OD)
- T04** 6' Iron Cylinder (18" OD)
- T05** 5' Iron Cylinder (7" OD)
- T06** 14" Stainless Steel Sphere (with 80 A-m² Bar Magnet)
- T07** 35" 203mm Artillery Shell
- T08** 14" Stainless Steel Sphere (with 80 A-m² Bar Magnet)
- T09** 2' Iron Cylinder (6" OD)
- T10** NE 6' Bomb-Shaped Marker (18" OD)

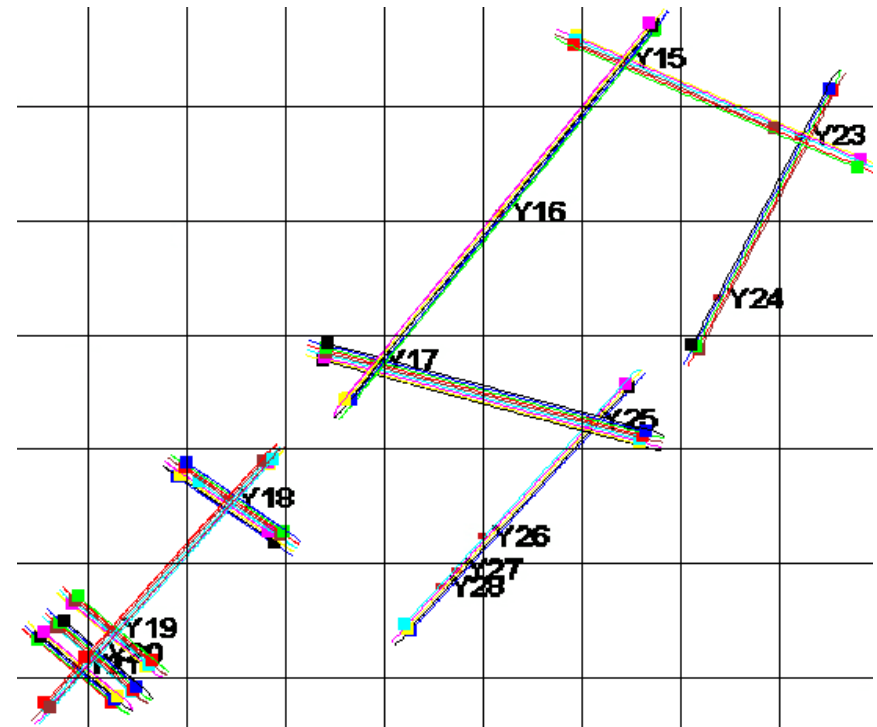
Proud

- M1** 96" Concrete Pipe (18" OD)
- M2** 72" Concrete Pipe (18" OD)
- M3** 72" Concrete Pipe (18" OD)
- M4** 72" Concrete Pipe (18" OD)

Davis Point Data Set – March 2007



Yankee Data Set – May/June 2006



Subband Decomposition

- Extract frequency subbands from target impulse response

$$\hat{\mathbf{h}}_p = \left[\hat{h}_p[0] \dots \hat{h}_p[N-1] \right]^T \text{ for subsequent feature extraction}$$

- Modulation matrix:** $\mathbf{T}_m = \text{diag}(1, W_N^{mL}, W_N^{2mL}, \dots, W_N^{(N-1)mL})$, $m \in [0, M-1]$
where $W_N = e^{-j\frac{2\pi}{N}}$

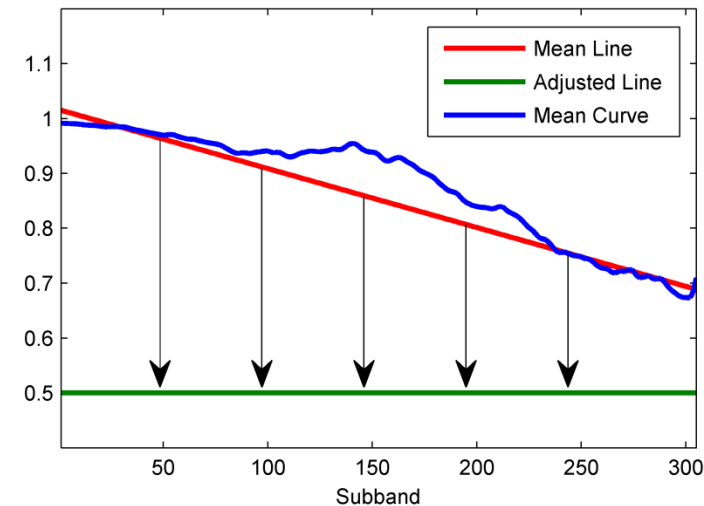
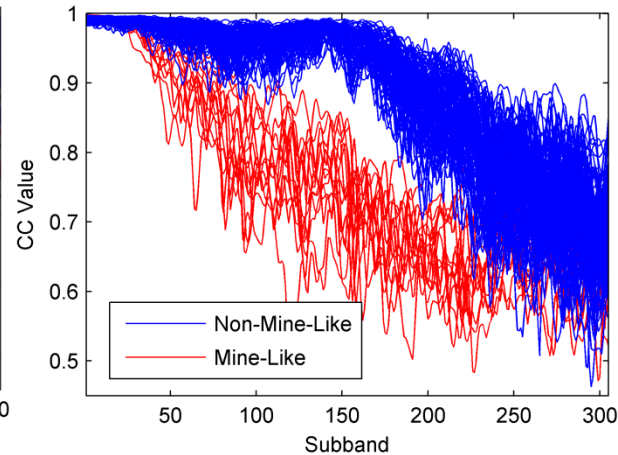
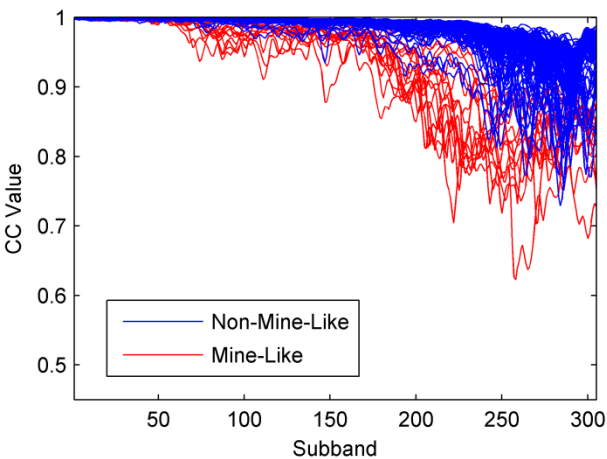
- Circular convolution matrix**
filters out all but L samples:

$$\mathbf{G}_L = \frac{1}{N} \begin{bmatrix} L & \sum_{k=0}^{L-1} W_N^{-(N-1)k} & \dots & \sum_{k=0}^{L-1} W_N^{-k} \\ \sum_{k=0}^{L-1} W_N^{-k} & L & \dots & \sum_{k=0}^{L-1} W_N^{-2k} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=0}^{L-1} W_N^{-(N-1)k} & \sum_{k=0}^{L-1} W_N^{-(N-2)k} & \dots & L \end{bmatrix}$$

- Down-sampling Matrix** (sampling interval M):

$$\mathbf{S}_M = M \begin{bmatrix} 1 & \mathbf{0}_{N-1} \\ \mathbf{0}_M & 1 & \mathbf{0}_{N-M-1} \\ \mathbf{0}_{2M} & 1 & \mathbf{0}_{N-2M-1} \\ \vdots & \vdots & \vdots \end{bmatrix}$$

- m th subband: $\mathbf{y}_p^{(m)} = \mathbf{D}_L \mathbf{S}_M \mathbf{G}_L \mathbf{T}_m \hat{\mathbf{h}}_p$, where \mathbf{D}_L is the $L \times L$ DFT matrix



- Every feature vector from a given run, including empty bottom returns, is modified
- Goal is to modify each feature vector such that the average level of coherence exhibited by features from different runs is similar
- For each of the 305 subbands, a mean value of the dominant canonical correlation samples (each sample is obtained from a different feature vector in the run) is found
- A line is fit to the curve representing the mean of the dominant canonical correlation values using the linear least-squares estimate
- All of the dominant canonical correlation samples in a subband are shifted by the difference between the value of the line fit to the mean curve at that subband, and a horizontal line at a magnitude of 0.5

- Final decision rule minimizes overall expected cost of misclassification for each agent
- Assume:

- Features are conditionally independent given class C_k : $p(\mathbf{x}_1, \dots, \mathbf{x}_N | C_k) = \prod_{i=1}^N p(\mathbf{x}_i | C_k)$

- Local preliminary decision made using: $u_i = \gamma_i(\mathbf{x}_i)$

- Final decision made at each agent using: $u_{fi} = \gamma_{fi}(\mathbf{x}_i, \mathbf{u}_{ir})$

- Cost of making incorrect decision greater than cost of making correct decision:

$$J_{m=n \ k} \geq J_{m=k \ k}, \quad n \in \{0, 1\}$$

- Final decision more likely to agree with preliminary decision than disagree:

$$p(|u_{fi} - u_l| \leq \epsilon | \mathbf{u}_{ir}, \mathbf{x}_i) \geq p(|u_{fi} - u_l| > \epsilon | \mathbf{u}_{ir}, \mathbf{x}_i), \quad i, l \in [1, N]; l \neq i,$$

- Solving $\text{Min}\{E[J\{\gamma_{fi}(\mathbf{x}_i, \mathbf{u}_{ir}), C_k\}] = J_{ik}\}$ subject to the above yields:

$$\begin{array}{l} u_{fi} = 1 \\ \frac{p(\mathbf{x}_i | C_1)}{p(\mathbf{x}_i | C_0)} \geq \frac{P(C_0) \prod_{j=1, j \neq i}^N p(u_j | C_0) [J_{10} - J_{00}]}{P(C_1) \prod_{j=1, j \neq i}^N p(u_j | C_1) [J_{01} - J_{11}]} \\ < \\ u_{fi} = 0 \end{array}$$

- Final decision rule derived from likelihood ratio test given by:

$$u_{fi} = \Lambda(\mathbf{x}_i) = \frac{p(C_1|\mathbf{x}_i, \mathbf{u}_{if})}{p(C_0|\mathbf{x}_i, \mathbf{u}_{if})} = \frac{p(\mathbf{x}_i, \mathbf{u}_{if}|C_1)P(C_1)}{p(\mathbf{x}_i, \mathbf{u}_{if}|C_0)P(C_0)}$$

- Assume previous decisions and \mathbf{x}_i are conditionally independent:

$$p(\mathbf{x}_i, \mathbf{u}_{if}|C_k) = p(\mathbf{x}_i|C_k) \prod_{m=1}^M p(u_{f,i-m}|C_k)$$

- Likelihood ratio can be written as:

$$u_{fi} = \frac{P(C_1)p(\mathbf{x}_i|C_1) \prod_{m=1}^M p(u_{f,i-m}|C_1)}{P(C_0)p(\mathbf{x}_i|C_0) \prod_{m=1}^M p(u_{f,i-m}|C_0)} = \frac{P(C_0)^M p(C_1|\mathbf{x}_i) \prod_{m=1}^M p(C_1|u_{f,i-m})}{P(C_1)^M p(C_0|\mathbf{x}_i) \prod_{m=1}^M p(C_0|u_{f,i-m})}$$

Single-Aspect: Davis Point Data Set

	Validation	Testing 1	Testing 2
CFS Features	92.0%	92.1%	89.3%
CTD Features	88.0%	86.1%	78.6%
Improvement	+4.0%	+6.0%	+10.7%

Single-Aspect: Yankee Data Set

	Validation	Testing 1	Testing 2
CFS Features	93.2%	93.2%	87.3%
CTD Features	100.0%	81.4%	74.5%
Improvement	-6.8%	+11.8%	+12.8%

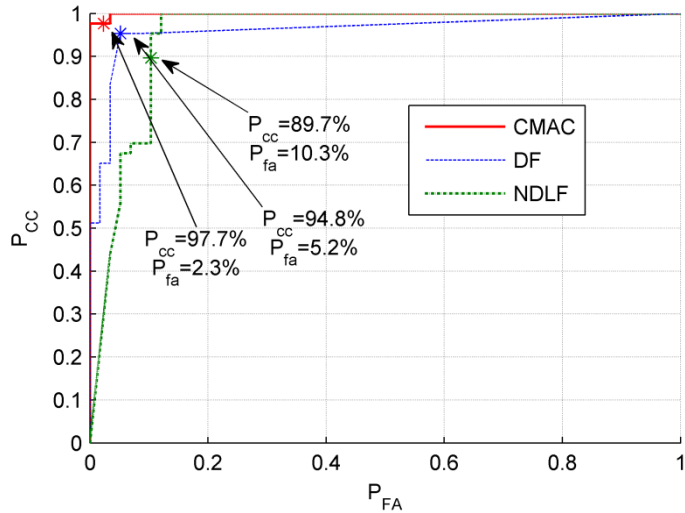
Multi-Aspect: Davis Point Data Set

	Validation	Testing 1	Testing 2
CMAC	96.0%	98.0%	93.8%
DF	92.0%	96.0%	92.0%
NDLF	94.0%	91.1%	92.0%

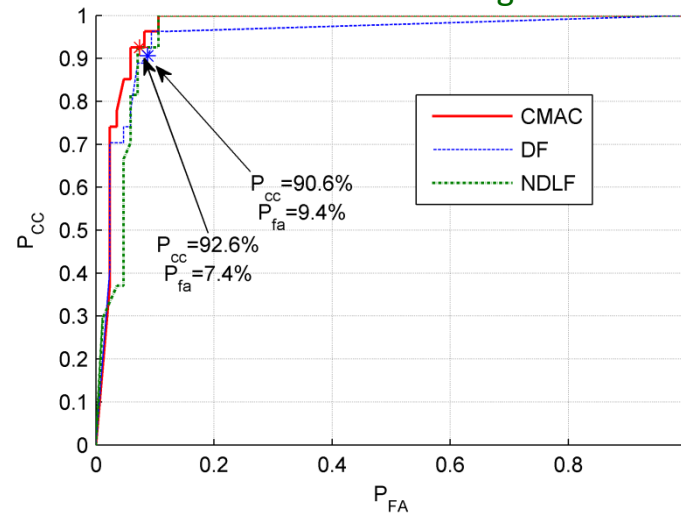
Multi-Aspect: Yankee Data Set

	Validation	Testing 1	Testing 2
CMAC	100.0%	96.6%	94.1%
DF	97.7%	94.9%	91.2%
NDLF	100.0%	94.9%	91.2%

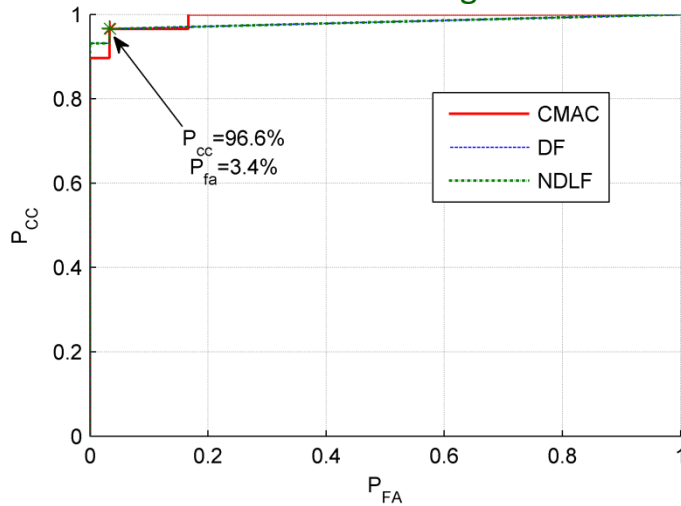
Davis Point Testing 1



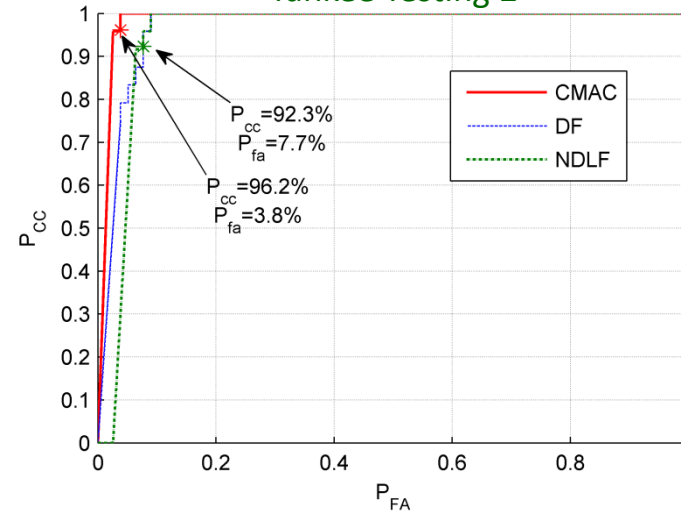
Davis Point Testing 2



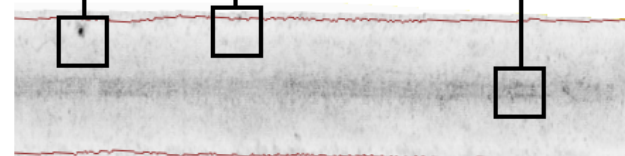
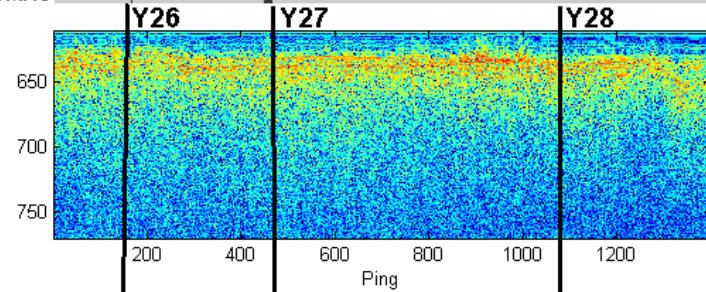
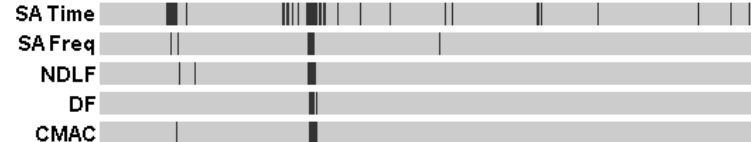
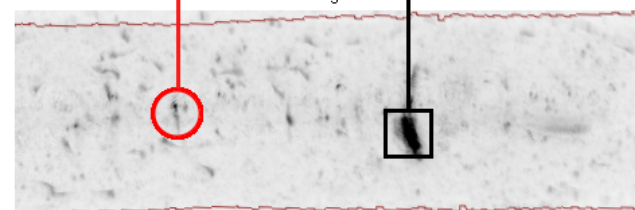
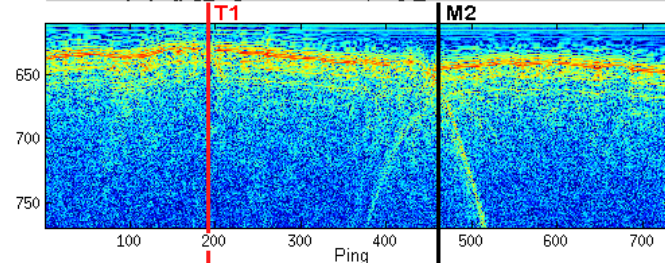
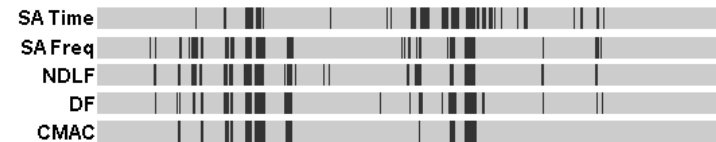
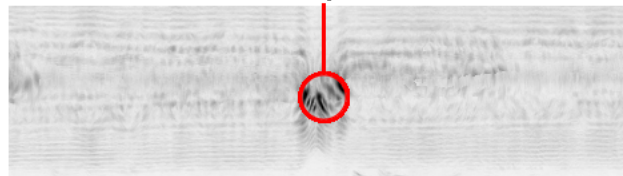
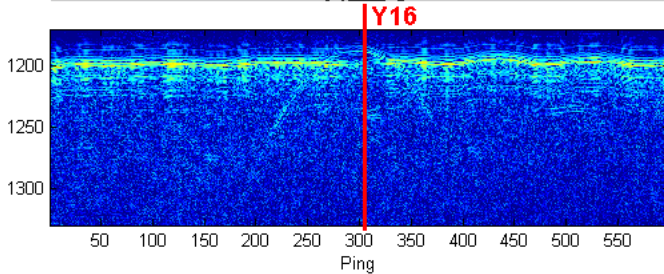
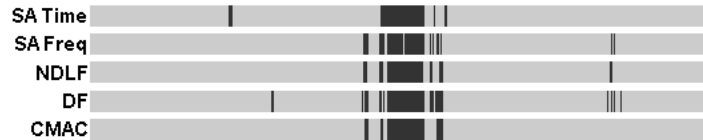
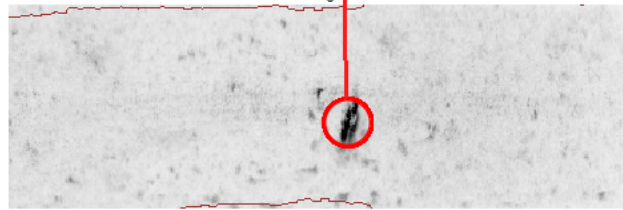
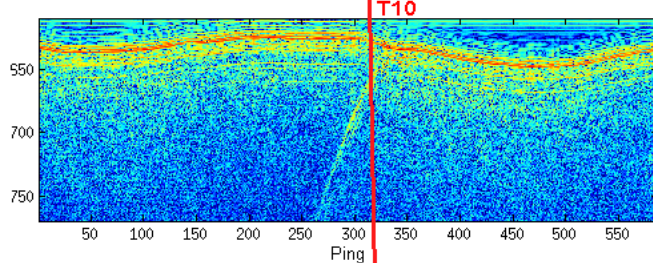
Yankee Testing 1



Yankee Testing 2



Additional Results on Entire Runs



Additional Information: Additional SAS Image Comparison

[Original Runs](#)

[Backup Slides](#)

