

# **Perfect Tracking for Non-minimum Phase Systems: with Applications to Biofuels from Microalgae**

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# Overview of Presentation

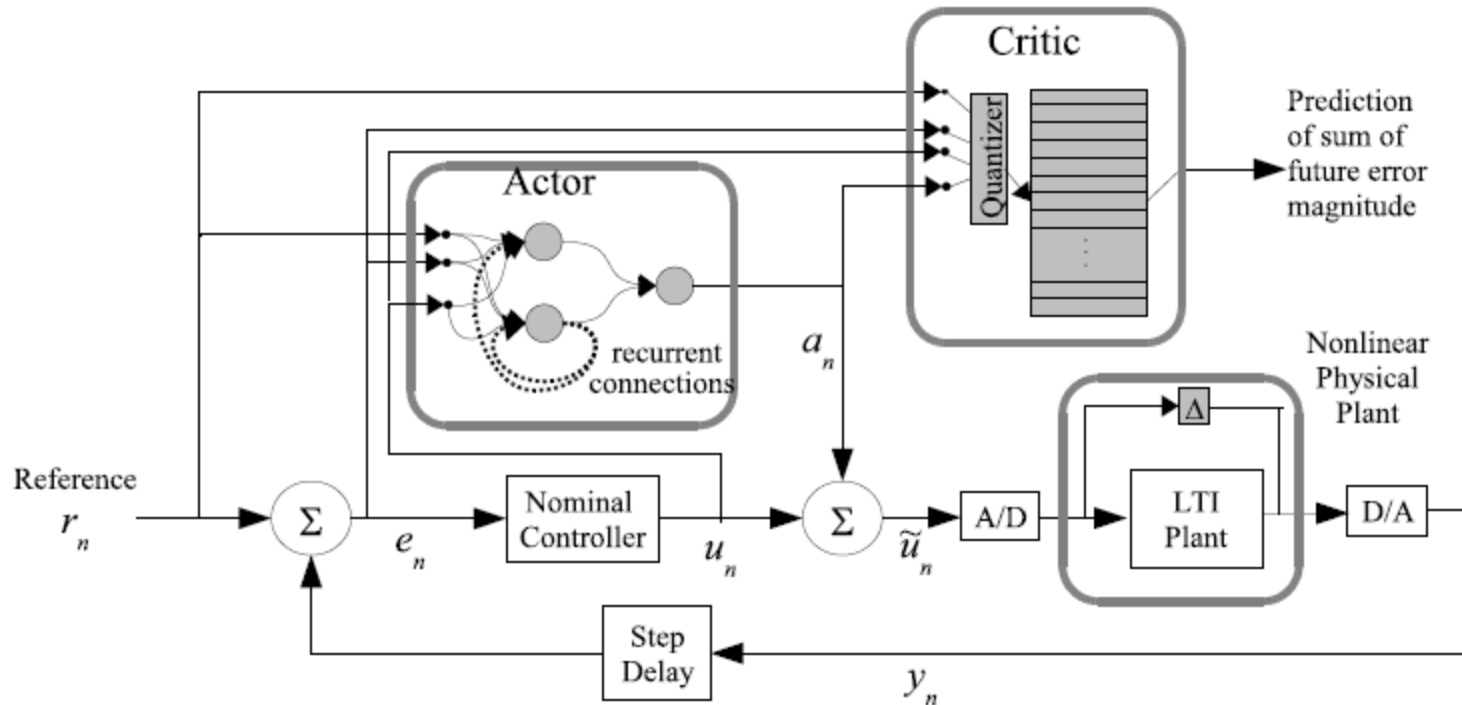
- History of project / Motivation
- Perfect tracking control of non-minimum phase systems
- Biofuels from microalgae
- Conclusions / Future Directions

# Commercial HVAC



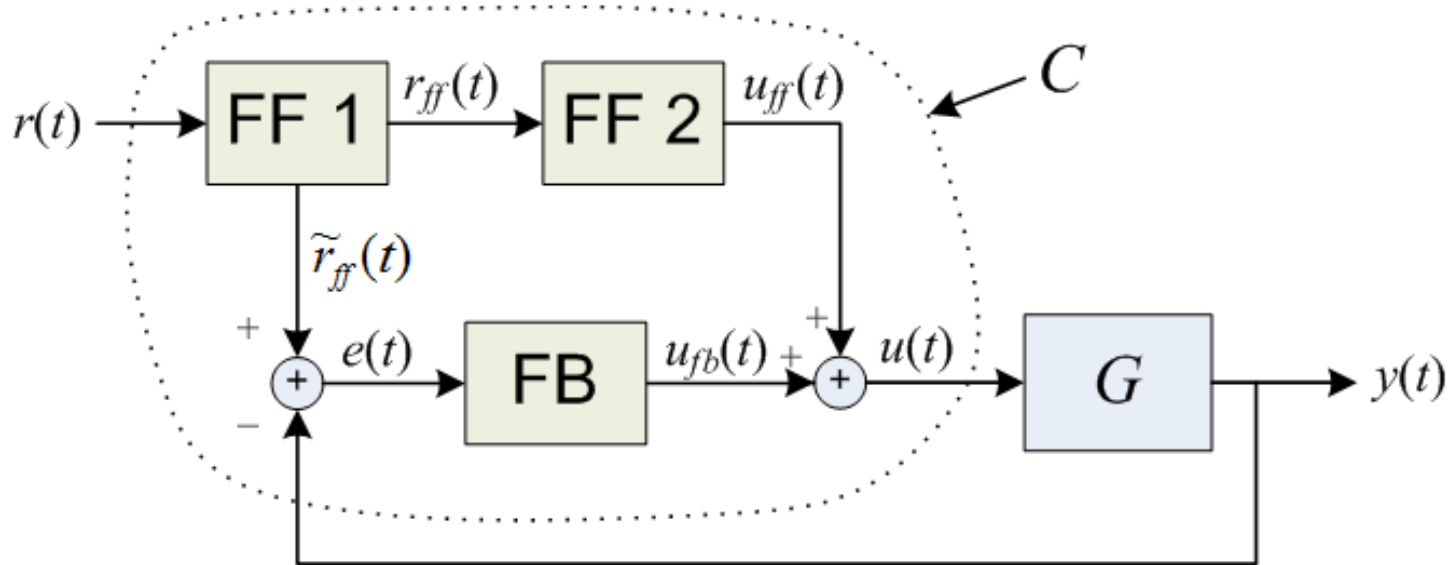
- Difficult to model a whole building
- Knowledge of “disturbances”
- Reinforcement learning control
- Adaption (done inside the feedback loop)

# Robust Reinforcement Learning Control



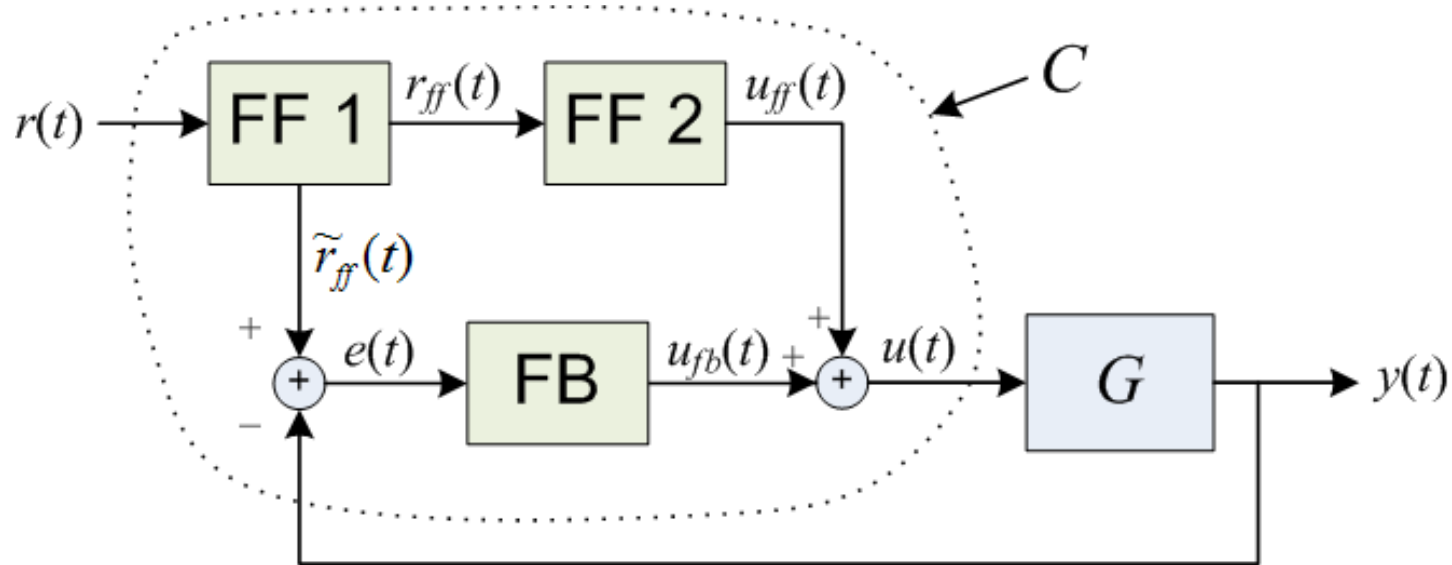
- Guaranteeing stability  $\sim$  Computationally intensive
- Based on IQCs (Integral Quadratic Constraints)
- Can adapt to both reference and disturbance inputs

# Reduce Stability Complexity



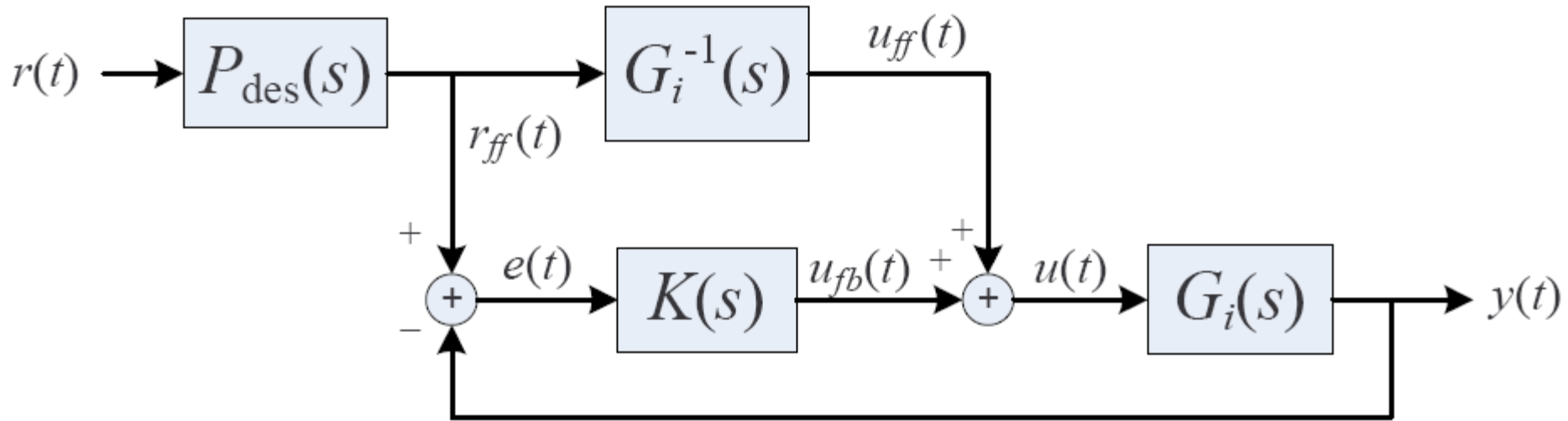
- Fix the feedback (FB) controller
- Adapt the feedforward (FF) controllers
- Closed-loop stability ~ Provide by FB
- FF adaptation ~ Anything (provided bounded)

# Neuromuscular Actuation Systems



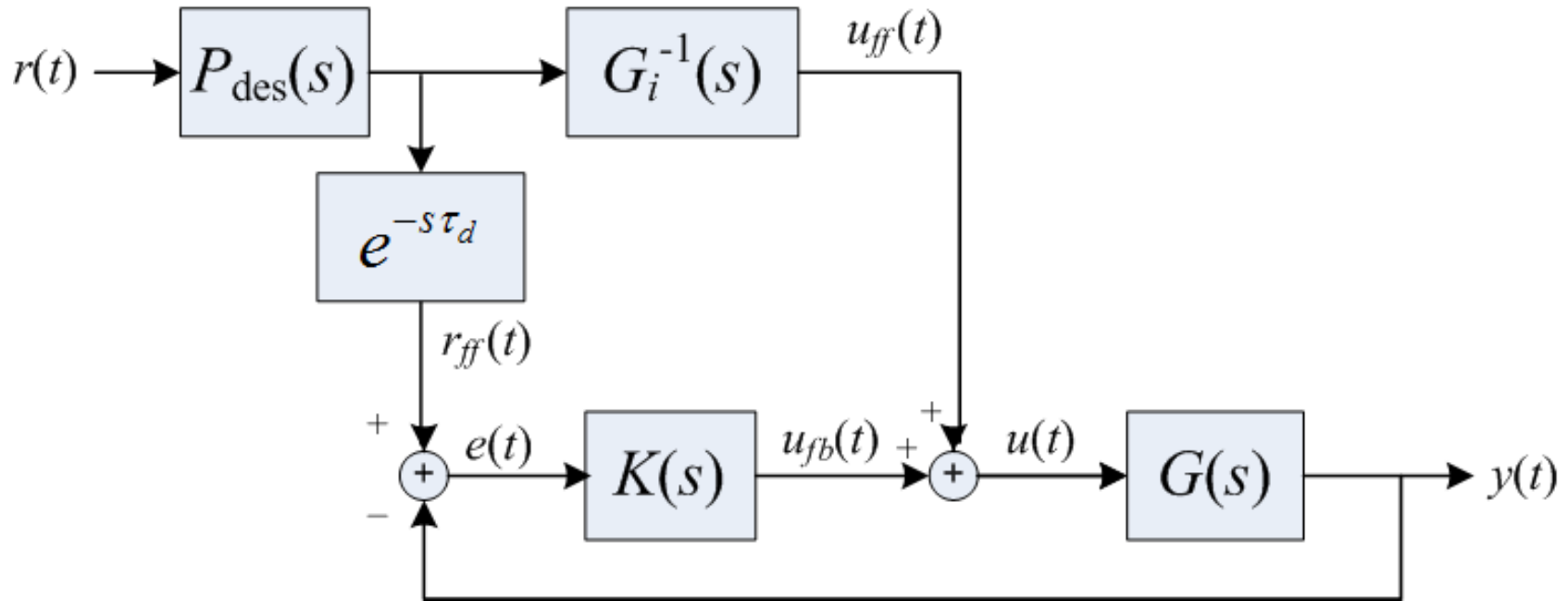
- Calculate desired path (FF calculation)
- Ballistic response (FF Control)
- Dynamic corrections to ballistic response (FB control on small error signals)

# Standard Architecture



- Can provide perfect tracking for minimum-phase systems
- Unstable plants allowed
- Cannot (stably and causally) handle time delays and right-half plane zeros

# Extension to Time Delays



- Delay reference input
- Later – extension to (RHP zeros)
- Previously not presented in literature



# Motivation: Growing Microalgae



- Application: Biofuels from microalgae
- Utilize externally supplied CO<sub>2</sub> to produce more microalgae.
- CO<sub>2</sub> source and microalgae physically separated (transport delays)
- Sensors are expensive and unreliable (FF control desirable)

# Perfect Tracking Control of Non-minimum Phase Systems

# Perfect Tracking

- What trajectory can the plant actually follow?
- What control signal will drive the plant along this trajectory?
- Characterize the class of signals that may be tracked in the nominal case (i.e., when the plant model is perfect) with no external disturbances.
- Extensions to non-minimum phase systems.

# Contributions

- Two controller architectures that provide perfect tracking for a larger class of systems (particularly, systems with time delays)
  - Dual Feedforward Predictive Control (DFFPC)
  - Dual Feedforward Smith Predictor (DFFPC)
    - Clarify limitations of Smith predictor
- Robustness tools
- Feedforward controller design methodologies
- Adaptation techniques

# Plant Factorization

$$G(s) = \frac{K_{\text{DC}} N_{nmp}(s) N_{mp}(s)}{D_s(s) D_u(s)} e^{-s\tau_d}$$

- Minimum-phase and stable:  $\text{Re}(s) < 0$
- Non-minimum phase and unstable  $\text{Re}(s) \geq 0$
- Non-minimum phase includes RHP zeros and time delays
- Normalize via  $K_{\text{DC}}$  such that:
  - $N_{nmp}(0) = N_{mp}(0) = D_s(0) = D_u(0) = 1$
- Could extend to integrators (include in  $D_u(s)$ )

# Plant Decomposition

$$G(s) = G_{noi}(s)G_i(s)$$

Split plant into non-invertible and invertible part

$$G_{noi}(s) = N_{nmp}(s)e^{-s\tau_d}$$

Non-invertible part contains non-minimum phase components

$$G_i(s) = \frac{K_{DC}N_{mp}(s)}{D_s(s)D_u(s)}$$

Invertible part contains minimum-phase components

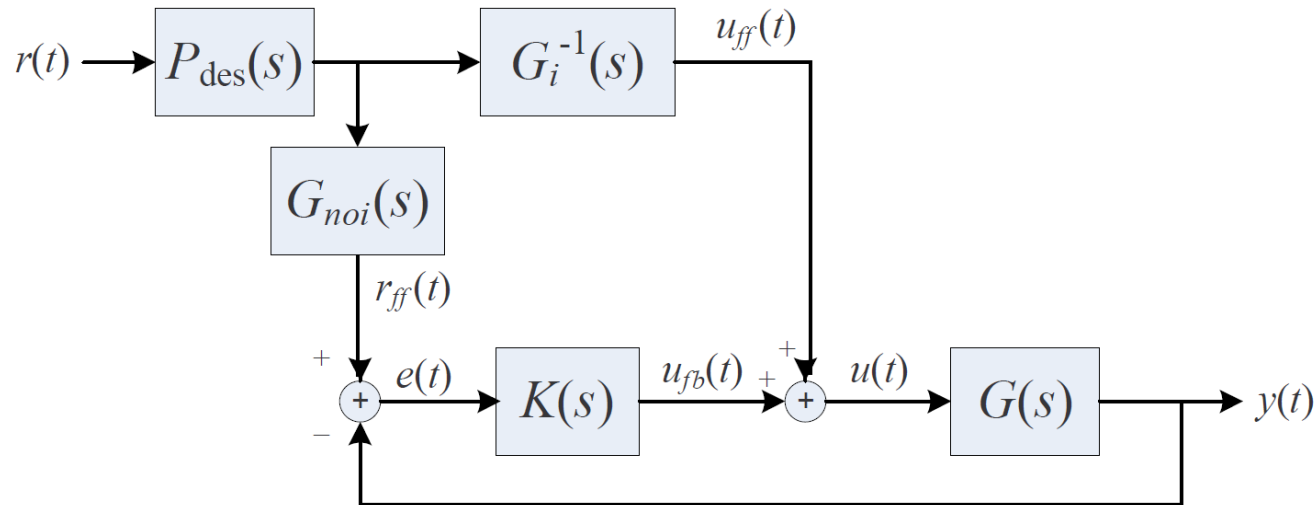
$$G_i^{-1}(s) = \frac{D_s(s)D_u(s)}{K_{DC}N_{mp}(s)}$$

NB: In general,  $G_{noi}(s)$  and  $G_i^{-1}(s)$  are not proper transfer functions

# DFFPC Overview

- Restricted to causal SISO systems (i.e., no prior knowledge of the reference input).
- Define a class of signals that can be perfectly tracked by non-minimum phase LTI systems.
  - Perfect tracking of a filtered reference input in the nominal case with no external disturbances
- Robustness tools for evaluating robust performance on a physical system.
- Stable adaptation techniques to improve performance. (Addressed Later)
- Can handle unstable systems

# Dual Feedforward Predictive Control

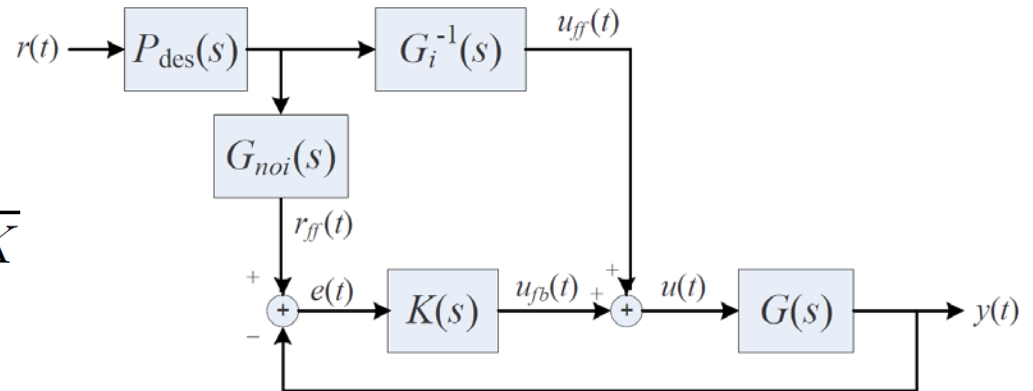


- At steady-state,  $r_{\text{ff}}(t) = r(t)$  or  $P_{\text{des}}(0)G_{\text{noi}}(0) = 1$
- The feedforward transfer functions must be proper:  $P_{\text{des}}(s)G_{\text{noi}}(s)$  and  $P_{\text{des}}(s)G_i^{-1}(s)$



# Nominal Case

$$S = \frac{1}{1 + GK} \quad \text{and} \quad T = \frac{GK}{1 + GK}$$



$$\begin{aligned} S_{\text{DFFPC}} &= P_{\text{des}} G_{\text{noi}} S - P_{\text{des}} G_i^{-1} G S \\ &= P_{\text{des}} G_{\text{noi}} S - P_{\text{des}} G_{\text{noi}} S \\ &= 0. \end{aligned}$$

- Perfect Tracking

$$\begin{aligned} M_{\text{DFFPC}} &= \frac{P_{\text{des}} G_{\text{noi}} GK}{1 + GK} + \frac{P_{\text{des}} G_i^{-1} G}{1 + GK} \\ &= P_{\text{des}} G_{\text{noi}} \frac{1 + GK}{1 + GK} \\ &= P_{\text{des}} G_{\text{noi}} \end{aligned}$$

- Desired closed-loop response contains non-minimum phase dynamics

# Example: Perfect Tracking of a NMP System

$$G(s) = 75 \frac{-s + 2}{(s + 5)(s + 10)} = 3 \frac{\frac{-s}{2} + 1}{\left(\frac{s}{5} + 1\right)\left(\frac{s}{10} + 1\right)}$$

$$G_{noi}(s) = \frac{-s}{2} + 1, \quad G_i^{-1}(s) = \frac{1}{3} \left(\frac{s}{5} + 1\right) \left(\frac{s}{10} + 1\right)$$

$$P_{des}(s) = \frac{1}{\left(\frac{s}{\alpha_{des}} + 1\right)^2}$$

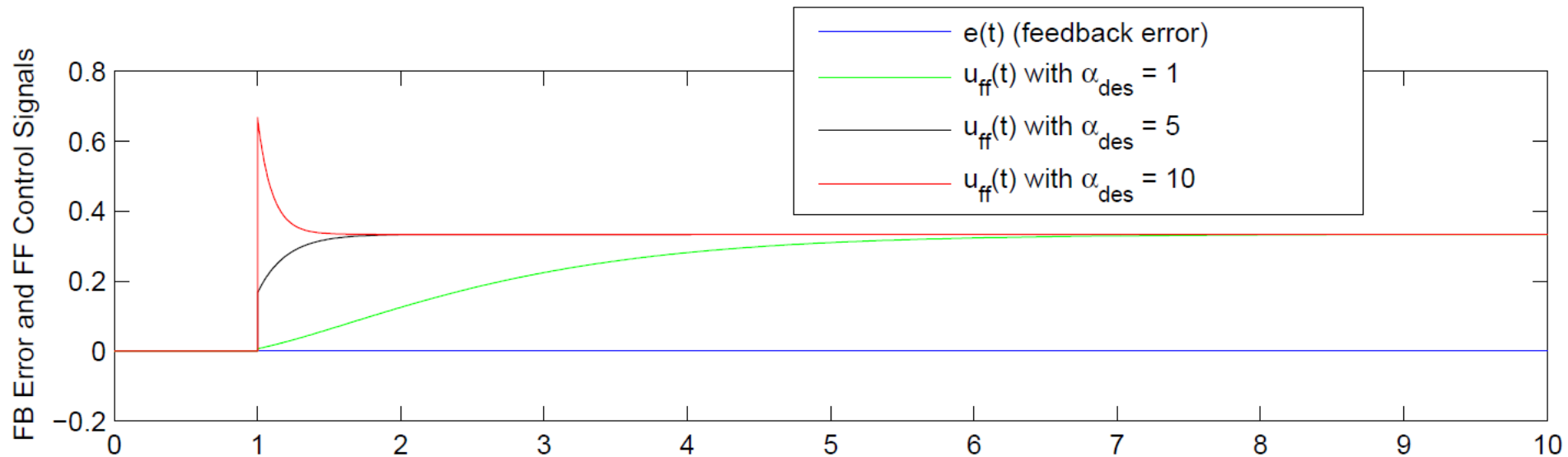
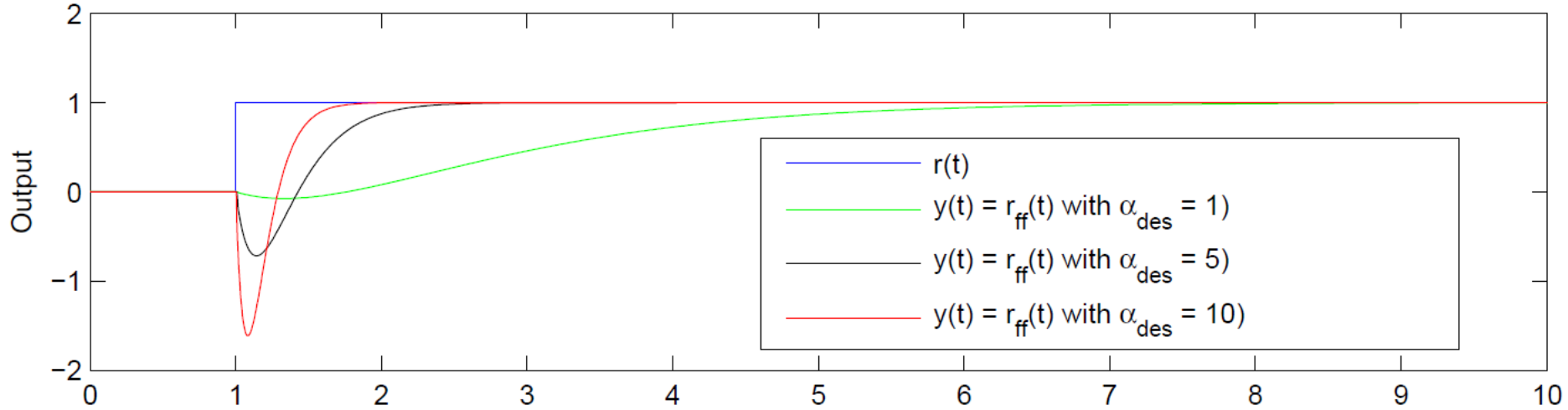
Picked a  $P_{des}(s)$  that will make the two feedforward controllers (below) proper

$$FF1(s) = P_{des}(s)G_{noi}(s) = \frac{\frac{-s}{2} + 1}{\left(\frac{s}{\alpha_{des}} + 1\right)^2}$$

$$FF2(s) = P_{des}(s)G_i^{-1}(s) = \frac{1}{3} \frac{\left(\frac{s}{5} + 1\right)\left(\frac{s}{10} + 1\right)}{\left(\frac{s}{\alpha_{des}} + 1\right)^2}$$

# Simulation Results

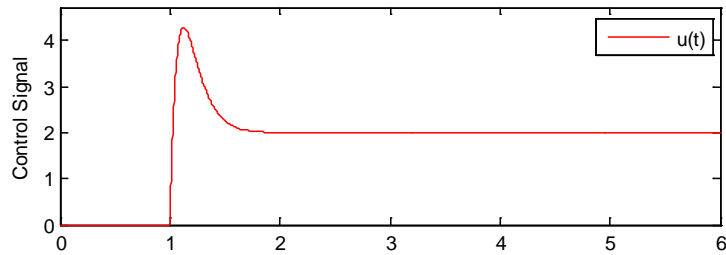
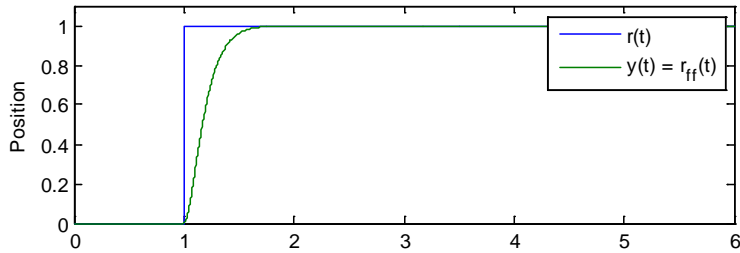
Step Response (Nominal Case)



# Plant Implications

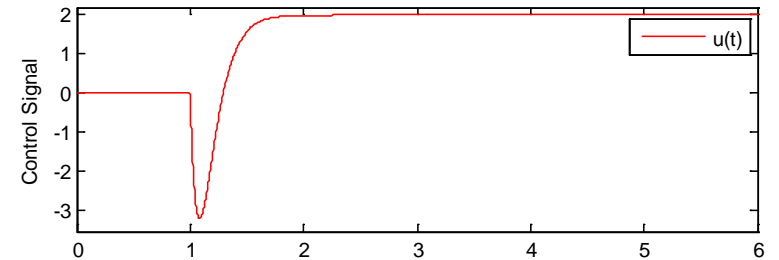
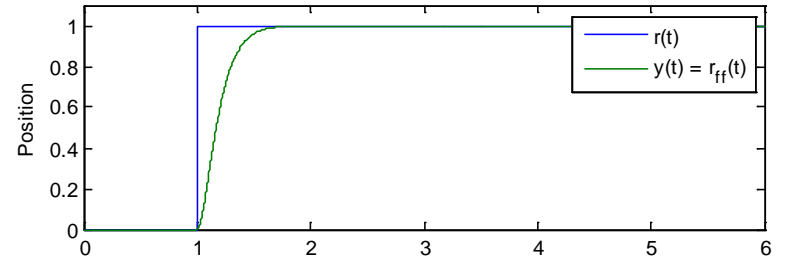
## Stable

Stable Minimum-phase Plant

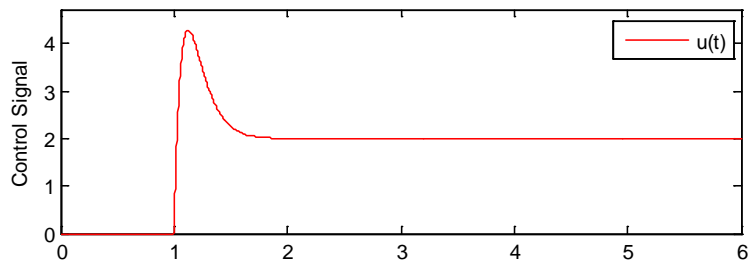
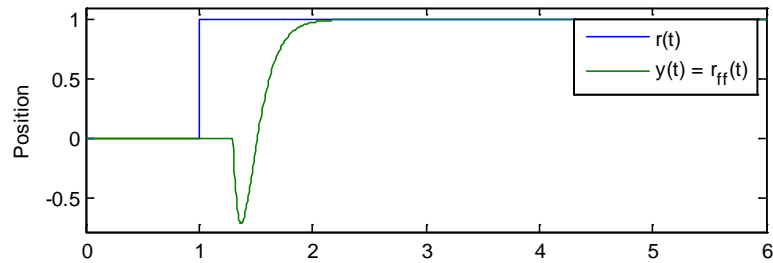


## Unstable

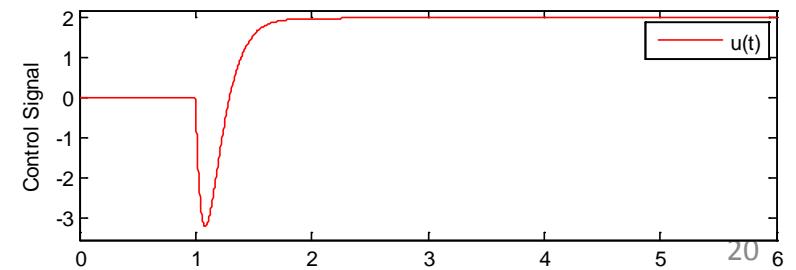
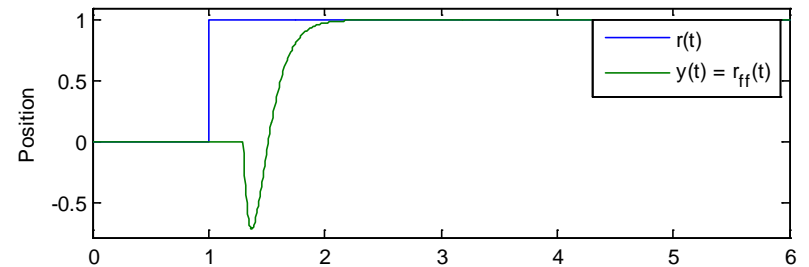
Unstable Minimum-phase Plant



Stable Non-minimum Phase Plant



Unstable Non-minimum Phase Plant



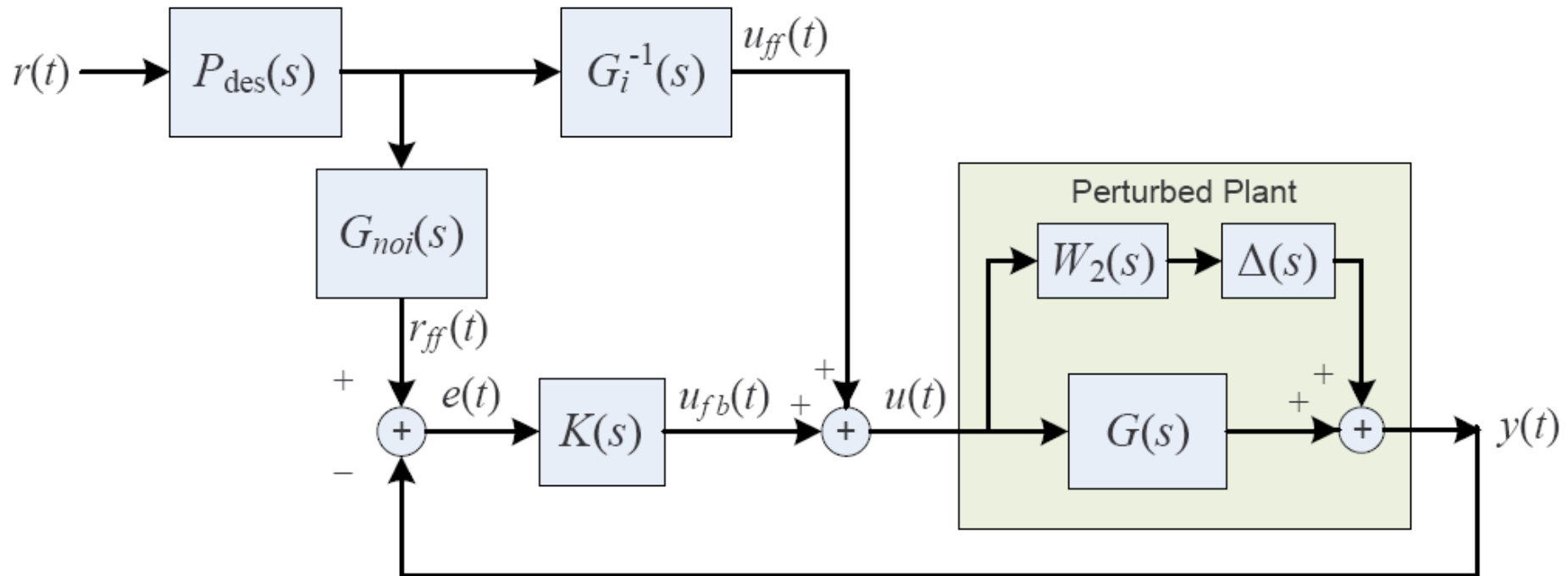
Min Phase

NMP

# Robustness Analysis:

## Additive Uncertainty

$$\tilde{G} = G + W_2\Delta$$



# Robustness Analysis:

## Additive Uncertainty

$$\tilde{G} = G + W_2 \Delta$$

$$\begin{aligned} \tilde{S}_{\text{DFFC}} &= \frac{P_{\text{des}} G_{noi}}{1 + (G + \Delta W_2)K} - \frac{P_{\text{des}} G_i^{-1} (G + \Delta W_2)}{1 + (G + \Delta W_2)K} \\ &= \frac{P_{\text{des}} G_{noi} - P_{\text{des}} G_{noi} - P_{\text{des}} G_i^{-1} \Delta W_2}{1 + (G + \Delta W_2)K} \\ &= - \frac{P_{\text{des}} G_i^{-1} \Delta W_2 / (1 + GK)}{(1 + GK + \Delta W_2 K) / (1 + GK)} \\ &= - \frac{P_{\text{des}} G_i^{-1} \Delta W_2 S}{1 + \Delta W_2 K S} \end{aligned}$$

# Robustness Analysis Cont'

$$\tilde{S}_{\text{DFFC}} = -\frac{P_{\text{des}} G_i^{-1} \Delta W_2 S}{1 + \Delta W_2 K S}$$

Assuming that  $\|\Delta\|_{\infty} \leq 1$ , robust stability requires that

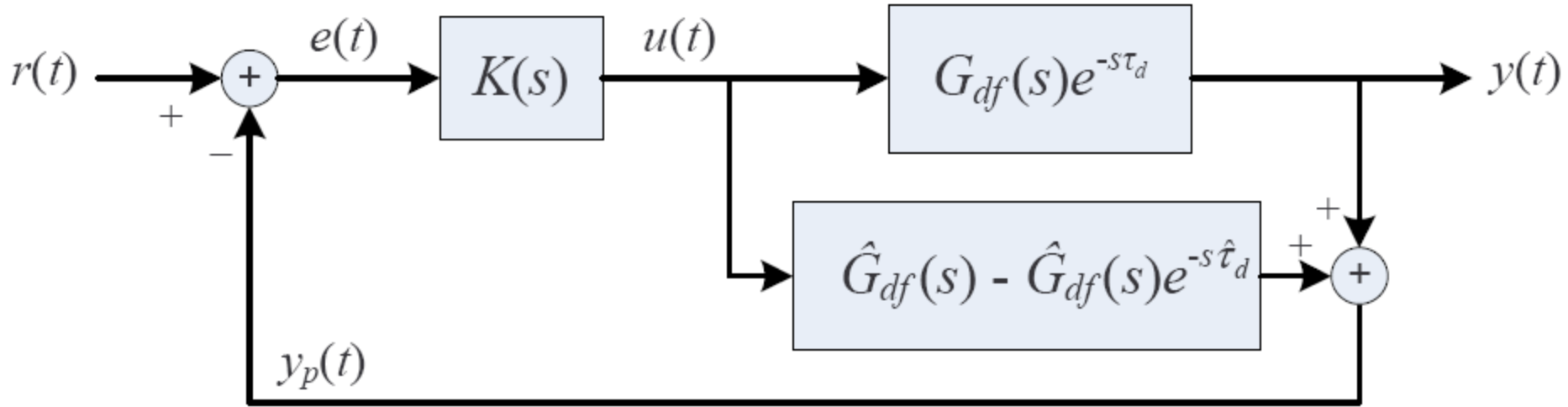
$$\|W_2 K S\|_{\infty} < 1.$$

- Necessary and sufficient condition for robust performance with additive uncertainty is:

$$\| |W_1 P_{\text{des}} G_i^{-1} W_2 S| + |W_2 K S| \|_{\infty} < 1$$

- Similar result for multiplicative uncertainty.

# Original Smith Predictor

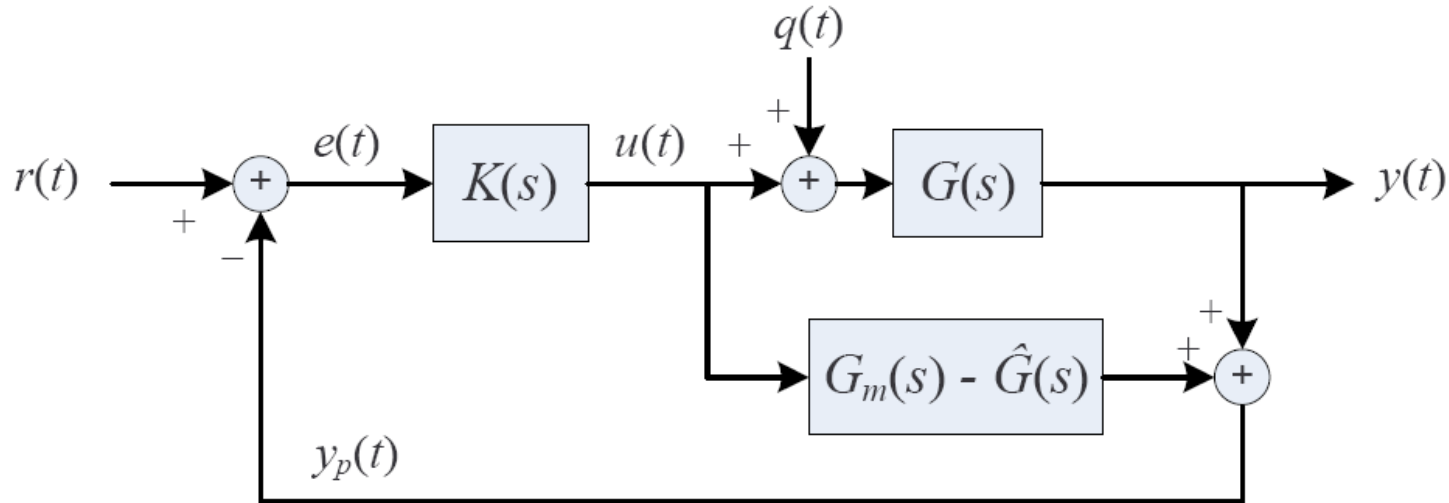


$$M(s) = \frac{G(s)K(s)}{1 + K(s)(G(s) + G_{df}(s) - G(s))} = \frac{G(s)K(s)}{1 + G_{df}(s)K(s)}$$

- Eliminate time delay from feedback loop (better nominal tracking performance)



# Modified Smith Predictor



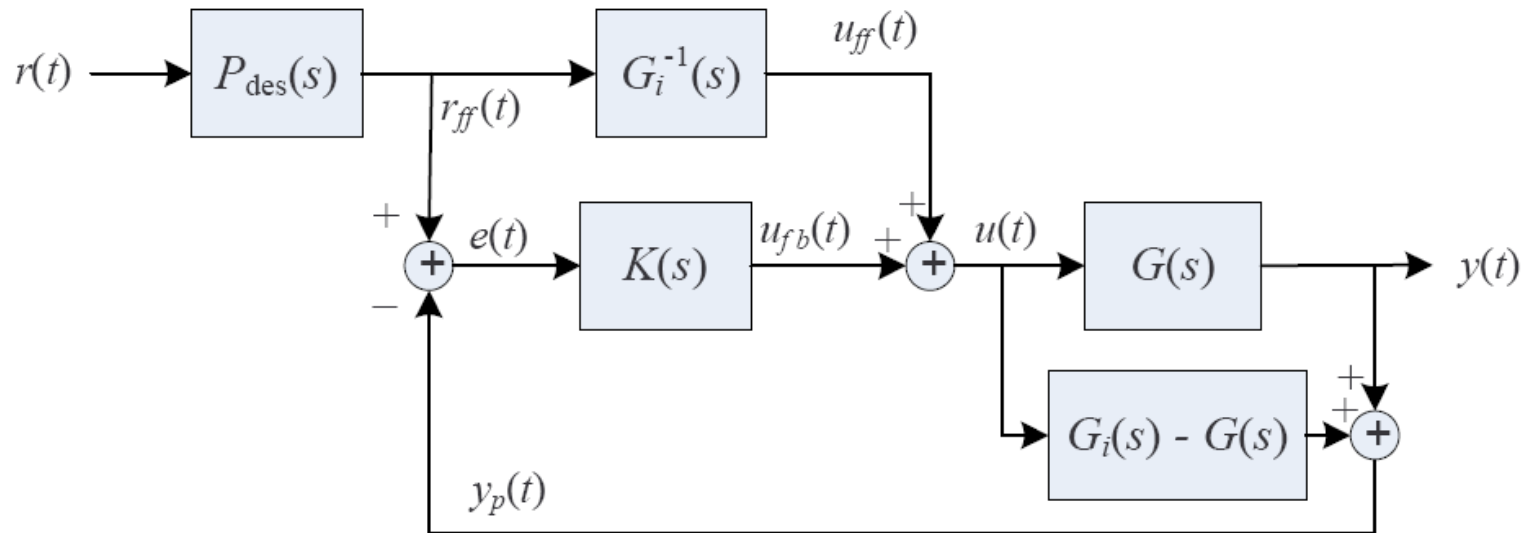
$$\begin{aligned}
 \frac{Y(s)}{Q(s)} &= \frac{G(s)(1 + G_m(s)K(s) - G(s)K(s))}{1 + G_m(s)K(s)} \\
 &= G(s) \left( 1 - \frac{G(s)K(s)}{1 + G_m(s)K(s)} \right) \\
 &= \frac{N_G(s)}{D_G(s)} \left( \frac{D_G(s)D_m(s)D_K(s) + N_m(s)N_K(s)D_G(s) + N_G(s)N_K(s)D_m(s)}{D_G(s)D_m(s)D_K(s) + N_m(s)N_K(s)} \right)
 \end{aligned}$$

- Improved disturbance rejection
- Not suitable for unstable systems

# DFFSP Overview

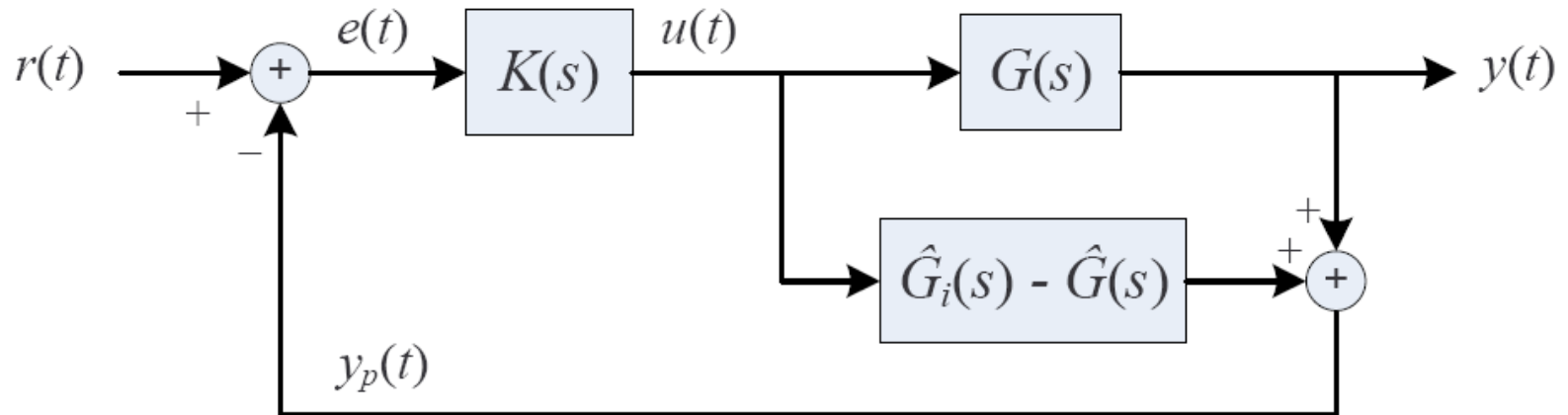
- Restricted to causal SISO systems (i.e., no prior knowledge of the reference input).
- Define a class of signals that can be perfectly tracked by stable non-minimum phase LTI systems.
  - Perfect tracking of a filtered reference input in the nominal case with no external disturbances
- Robustness tools for evaluating robust performance on a physical system.
- Stable adaptation techniques to improve performance. (Addressed Later)
- Cannot handle unstable systems

# Dual Feedforward Smith Predictor



- At steady-state,  $r_{ff}(t) = r(t)$  or  $P_{des}(0) = 1$
- The feedforward transfer functions must be proper:  $P_{des}(s)$  and  $P_{des}(s)G_i^{-1}(s)$
- $G_{noi}(s)$  handled in feedback loop

# (Nominal) Smith Predictor

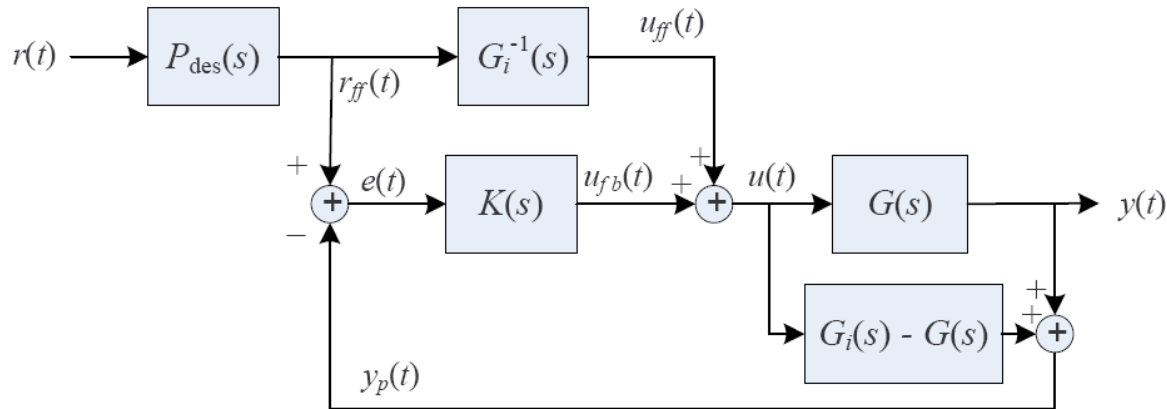


$$S_i(s) = \frac{E(s)}{R(s)} = \frac{1}{1 + G_i(s)K(s)}$$

$$T_i(s) = 1 - S_i(s) = \frac{G_i(s)K(s)}{1 + G_i(s)K(s)} = \frac{Y_p(s)}{R(s)}$$

$$M_i(s) = \frac{Y(s)}{R(s)} = \frac{G(s)K(s)}{1 + G_i(s)K(s)} = T_i(s)G_{noi}(s)$$

# Nominal DFFSP



$$\begin{aligned}
 S_{\text{DFFSP}}(s) &= P_{\text{des}}(s)S_i(s) - P_{\text{des}}(s)G_i^{-1}(s)G_i(s)S_i(s) \\
 &= P_{\text{des}}(s)S_i(s) - P_{\text{des}}(s)S_i(s) \\
 &= 0,
 \end{aligned}$$

- Perfect Tracking

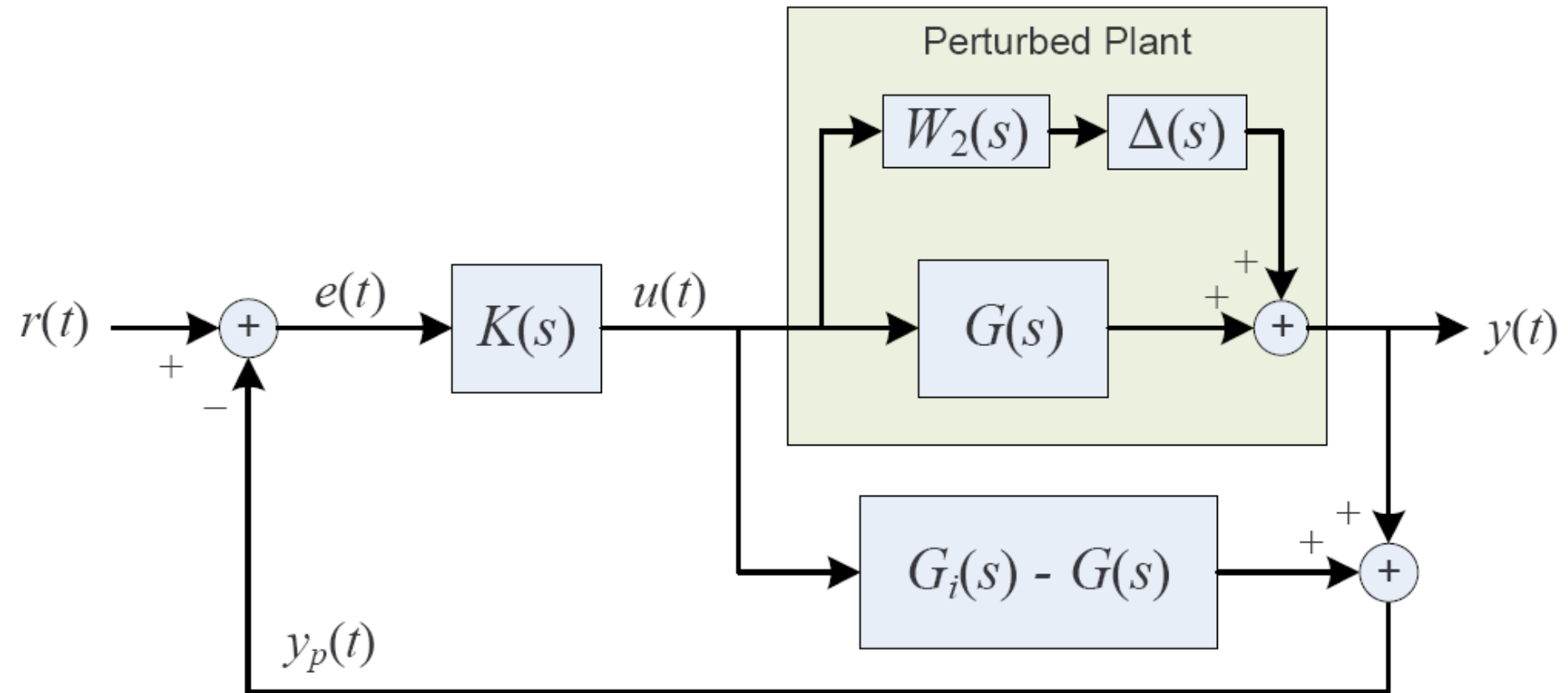
$$\begin{aligned}
 M_{\text{DFFSP}}(s) &= \frac{P_{\text{des}}(s)G(s)K(s)}{1 + G_i(s)K(s)} + \frac{P_{\text{des}}(s)G_i^{-1}(s)G(s)}{1 + G_i(s)K(s)} \\
 &= \frac{P_{\text{des}}(s)G_{\text{noi}}(s)G_i(s)K(s) + P_{\text{des}}(s)G_{\text{noi}}(s)}{1 + G_i(s)K(s)} \\
 &= P_{\text{des}}(s)G_{\text{noi}}(s) \frac{1 + G_i(s)K(s)}{1 + G_i(s)K(s)} \\
 &= P_{\text{des}}(s)G_{\text{noi}}(s),
 \end{aligned}$$

- Desired closed-loop response contains non-minimum phase dynamics

# Robustness Analysis:

## Additive Uncertainty

$$\tilde{G} = G + W_2\Delta$$



# Robustness Analysis:

## Additive Uncertainty

$$\tilde{G} = G + W_2\Delta$$

$$\begin{aligned}\tilde{S}_{\text{DFSP}} &= \frac{P_{\text{des}}}{1 + (G_i + \Delta W_2)K} - \frac{P_{\text{des}}G_i^{-1}(G_i + \Delta W_2)}{1 + (G_i + \Delta W_2)K} \\ &= \frac{P_{\text{des}} - P_{\text{des}} - P_{\text{des}}G_i^{-1}\Delta W_2}{1 + (G_i + \Delta W_2)K} \\ &= -\frac{P_{\text{des}}G_i^{-1}\Delta W_2/(1 + G_iK)}{(1 + G_iK + \Delta W_2K)/(1 + G_iK)} \\ &= -\frac{P_{\text{des}}G_i^{-1}\Delta W_2S_i}{1 + \Delta W_2K S_i}\end{aligned}$$

# Robustness Analysis Cont'

$$\tilde{S}_{\text{DFFS}} = -\frac{P_{\text{des}} G_i^{-1} \Delta W_2 S_i}{1 + \Delta W_2 K S_i}$$

robust stability requires  $\|\Delta W_2 K S_i\|_{\infty} < 1$  for all  $\|\Delta\|_{\infty} \leq 1$

- Necessary and sufficient condition for robust performance additive uncertainty is:

$$\| |W_1 P_{\text{des}} G_i^{-1} W_2 S_i| + |W_2 K S_i| \|_{\infty} < 1$$

- Similar result for multiplicative uncertainty



# DFFPC vs. DFFSP

- DFFPC can handle unstable systems, DFFSP cannot
- DFFSP will result in a higher order controller due to  $(G_i(s)-G(s))$
- Smith predictor improved disturbance rejection properties are not applicable here  $(G_m(s) = G_i(s))$
- Adapting the DFFSP architecture is less straightforward than the DFFPC architecture

# Comments on Methods

- Valid for both continuous-time and discrete-time implementations
- Perfect control for a wide class of systems
  - Minimum-phase and non-minimum phase
  - Stable and Unstable\*
  - Biproper and strictly proper systems
  - Systems with or without time delays
- Robustness tools
- Design Methodologies (Next)
- Adaptation (Later)

\* (for DFFPC only)

# Feedforward 2 ( $FF2(s)$ ) Design

- $P_{\text{des}}(s)$  (and  $FF2(s)$ ) common to feedforward plus feedback architectures presented here
- Direct design
- Robust and optimal design

# Direct Design

- Design for specific closed-loop characteristics
  - e.g., specific rise time with no overshoot

$P_{\text{des}}(s)$  Design Constraints:

- Relative degree of  $P_{\text{des}}(s) \geq$  Relative degree of  $G_i(s)$  (assuming  $G(s)$  is proper)
- $P_{\text{des}}(0) = 1$  (assuming that  $G_{\text{noi}}(0) = 1$  from the problem formulation).

# Direct Design Example

- Minimum-phase Plant
- Closed-loop is  $P_{\text{des}}(s)$

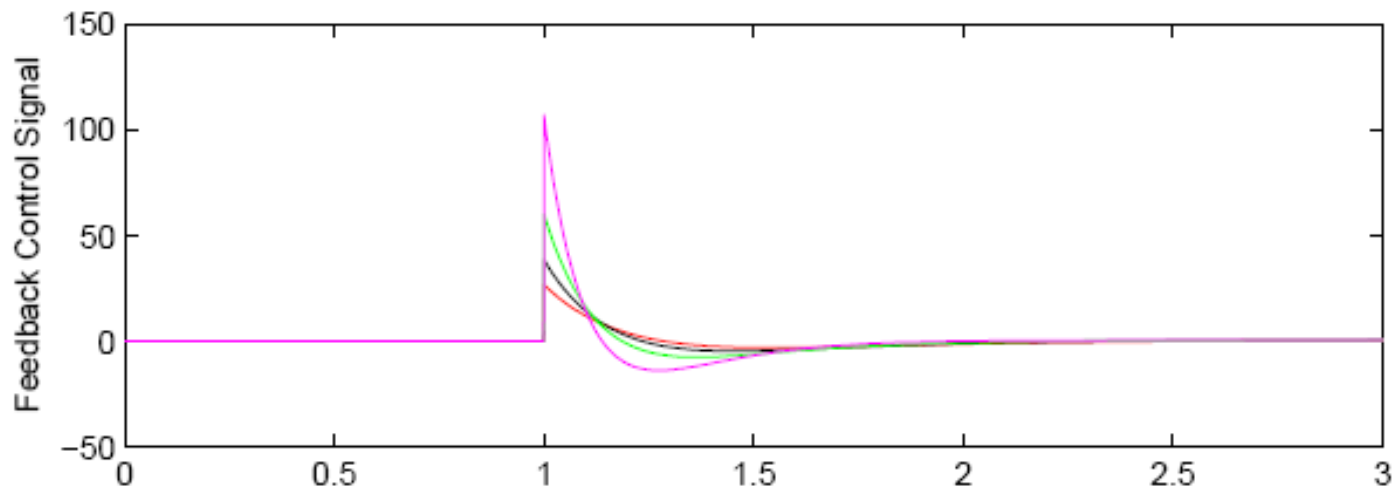
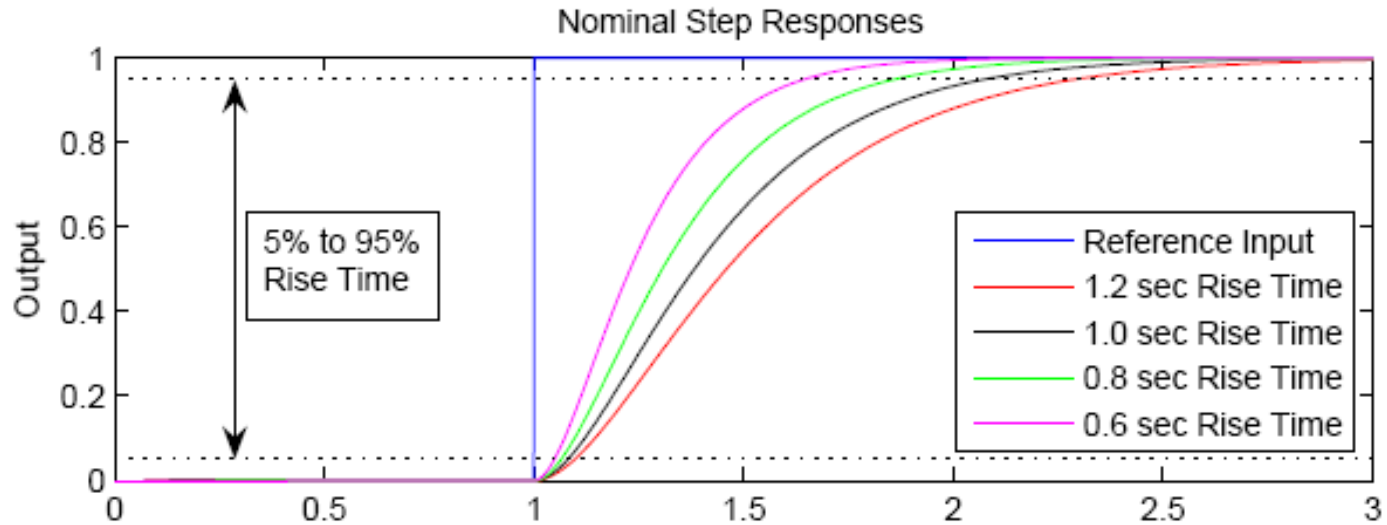
- Example: 
$$P_{\text{des}}(s) = \frac{1}{(\tau s + 1)^2}$$

- Design for rise time (with no overshoot):

$$\tau = \frac{t_{r(\text{des})}}{4.3885}$$

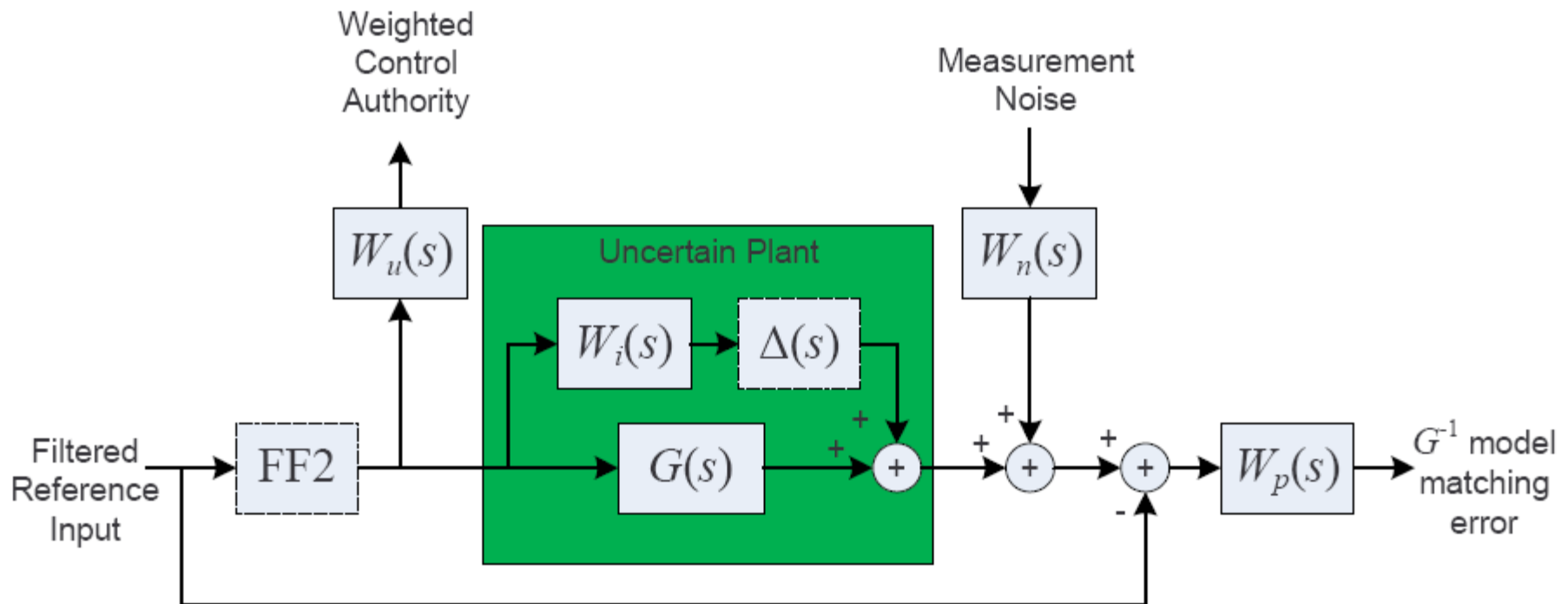
- NB: Numerical solution to above equation.

# Nominal Step Responses



# Robust and Optimal FF Design

- Optimal control – minimize gain from exogenous inputs to exogenous outputs
- Robust control – optimal control in the presence of uncertainty



# Design Method

NB:  $FF2(s) = P_{\text{des}}(s) G_i^{-1}(s)$

1. Design a robust/optimal  $FF2(s)$ .
2. Define  $Z(s) = \frac{1}{K_\mu} FF2(s) G_i(s)$  (where  $K_\mu$  is selected to make  $Z(0) = 1$ ).
3. Use a minimum realization and model reduction techniques to reduce the order of  $Z(s)$ .
4. Define  $P_{\text{des}}(s) = Z(s)F(s)$ , where  $F(s)$  is a stable and proper transfer function with  $F(0) = 1$ . NB:  $F(s) = 1$  is a valid (and common) choice here.



# FF Design Summary

- Based on an existing example (Faanes 2003)
- Provided more investigation into the resulting feedforward controller structure
  - Cancellations and near misses (lower order  $P_{\text{des}}(s)$  designs)
- Extensions may be made to unstable plants
- Option to design on either  $G(s)$  or  $G_i(s)$

# Numerical Example

- Plant definition:

$$\begin{aligned} G(s) &= 25 \frac{-s + 6}{(s + 5)(s + 10)} e^{-0.7s} \\ &= 3 \frac{\frac{-s}{6} + 1}{\left(\frac{s}{5} + 1\right)\left(\frac{s}{10} + 1\right)} e^{-0.7s}. \end{aligned}$$

$$K_{\text{DC}} = 3$$

$$N_{nmp}(s) = \left(\frac{-s}{6} + 1\right), \quad N_{mp}(s) = 1$$

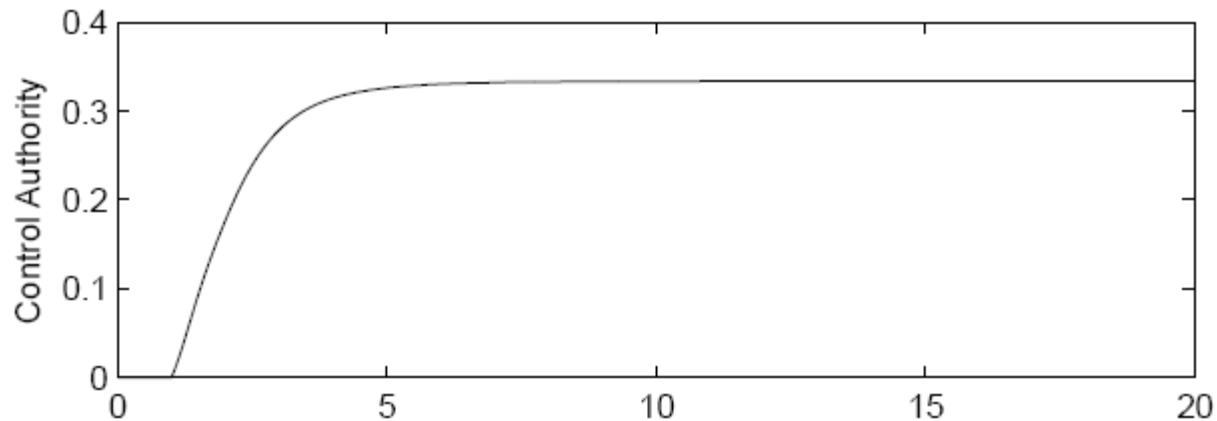
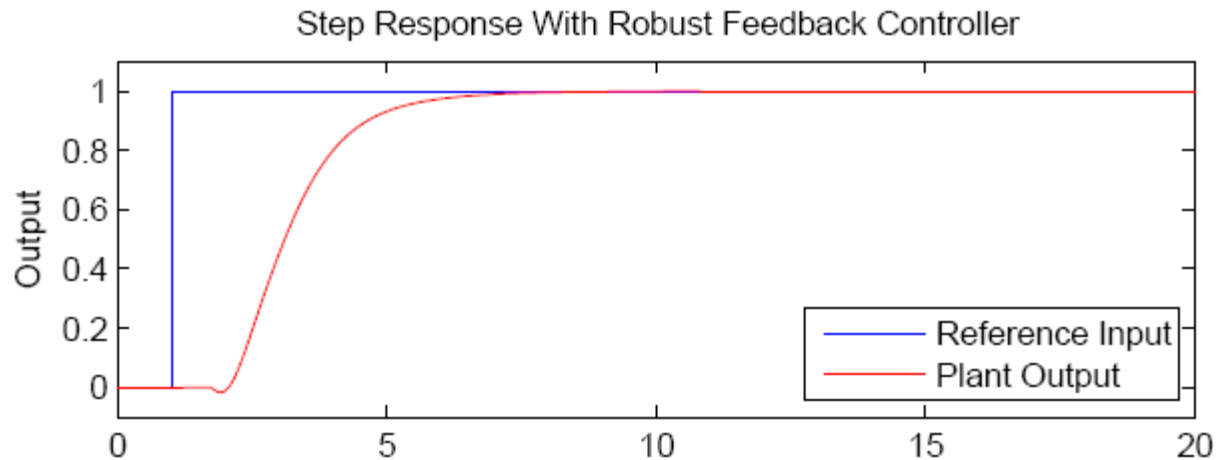
$$D(s) = \left(\frac{s}{5} + 1\right)\left(\frac{s}{10} + 1\right)$$

$$G_{noi}(s) = \left(\frac{-s}{6} + 1\right) e^{-0.7s}$$

$$G_i^{-1}(s) = \frac{1}{3} \left(\frac{s}{5} + 1\right)\left(\frac{s}{10} + 1\right)$$

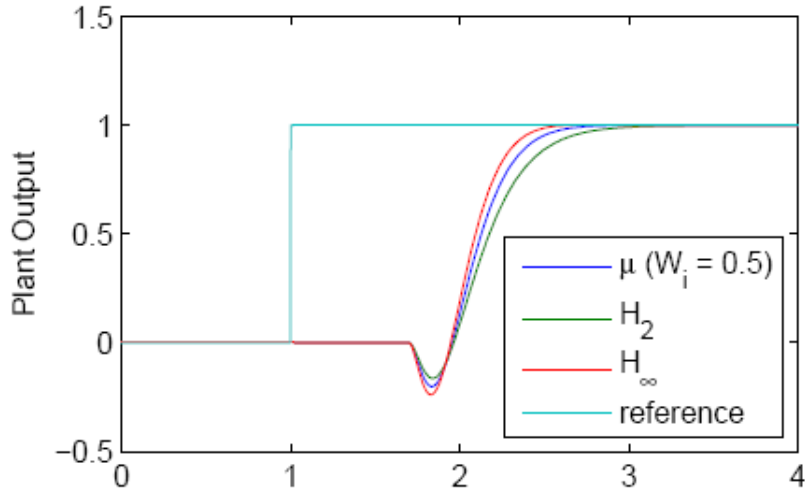
# Robust Feedback Controller Design

$$K(s) = \frac{0.13089(s + 8)(s + 10)(s + 5)(s + 2.857)(s + 1.061)}{s(s + 10.61)(s + 6.804)(s + 1.058)(s^2 + 4.579s + 14.65)}$$

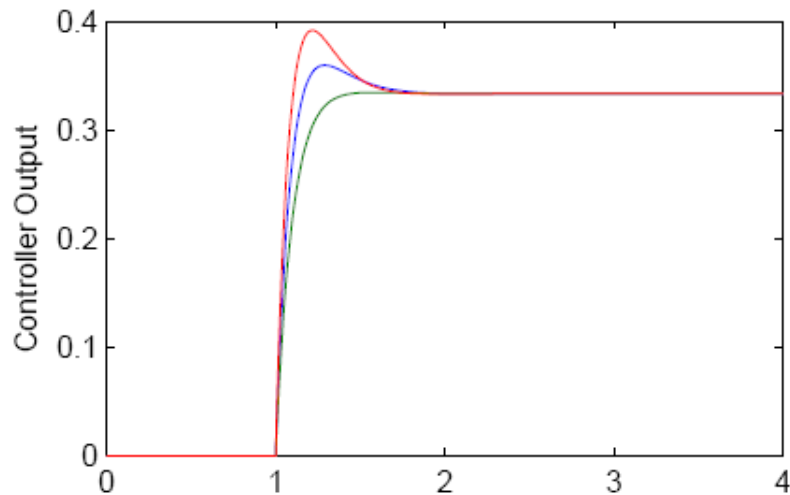
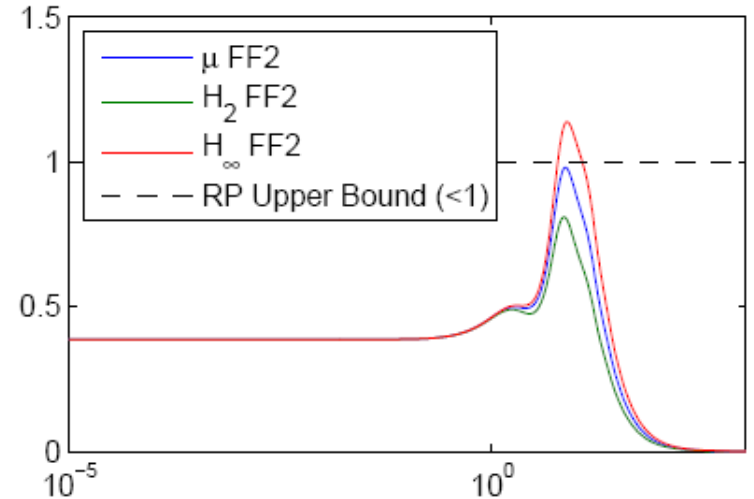


# Feedforward Design

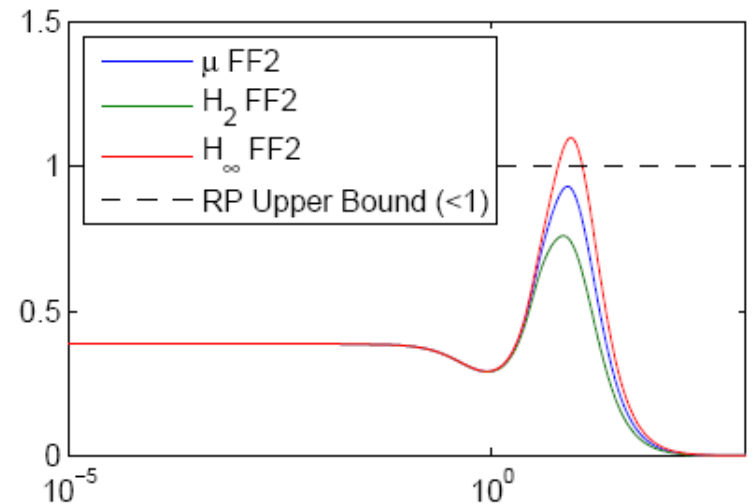
Nominal Closed-Loop (Perfect Tracking) Step Responses  
 $W_r = 1$ ,  $W_u = 10$ , WP DC gain = 1, WP Ginv BW = 10



DFFPC RP  $\Rightarrow || |W_1 * P_{des} * G_{noi} * W_2 * S| + |W_2 * M| ||_\infty <$

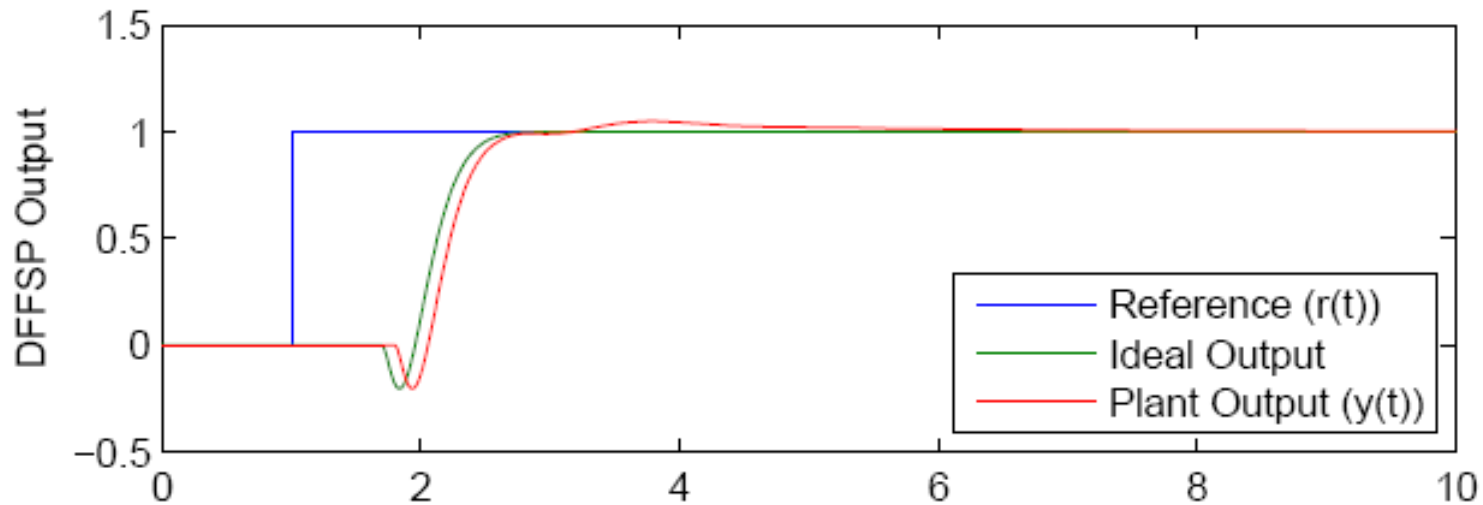
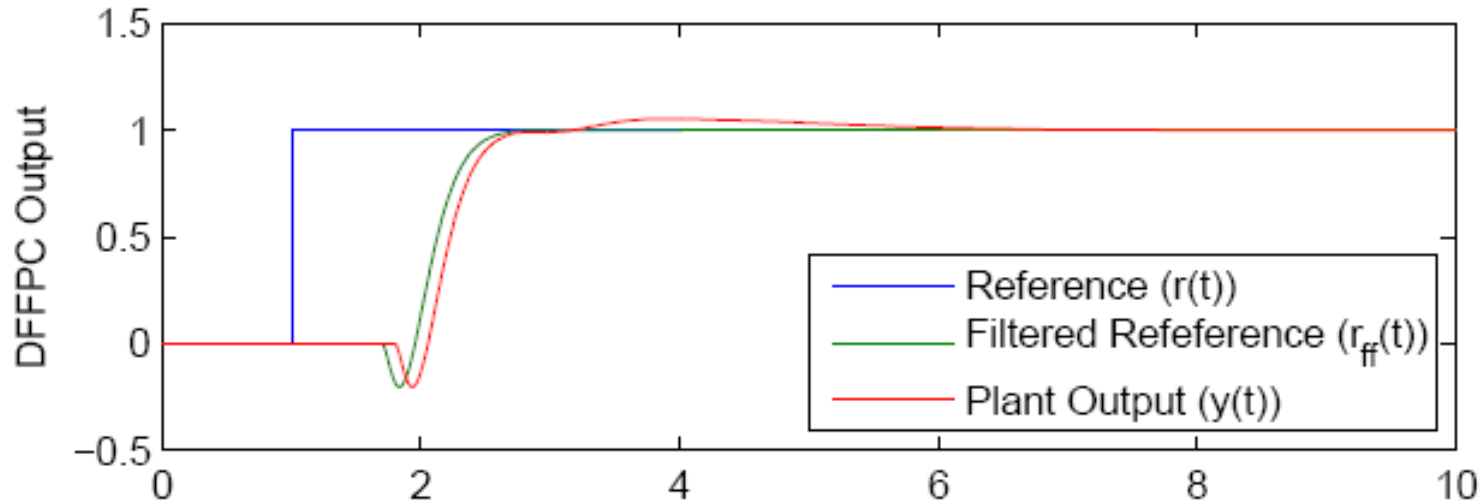


DFFSP RP  $\Rightarrow || |W_1 * P_{des} * G_{noi} * W_2 * S_i| + |W_2 * M_i| ||_\infty <$



# Example Robust Performance

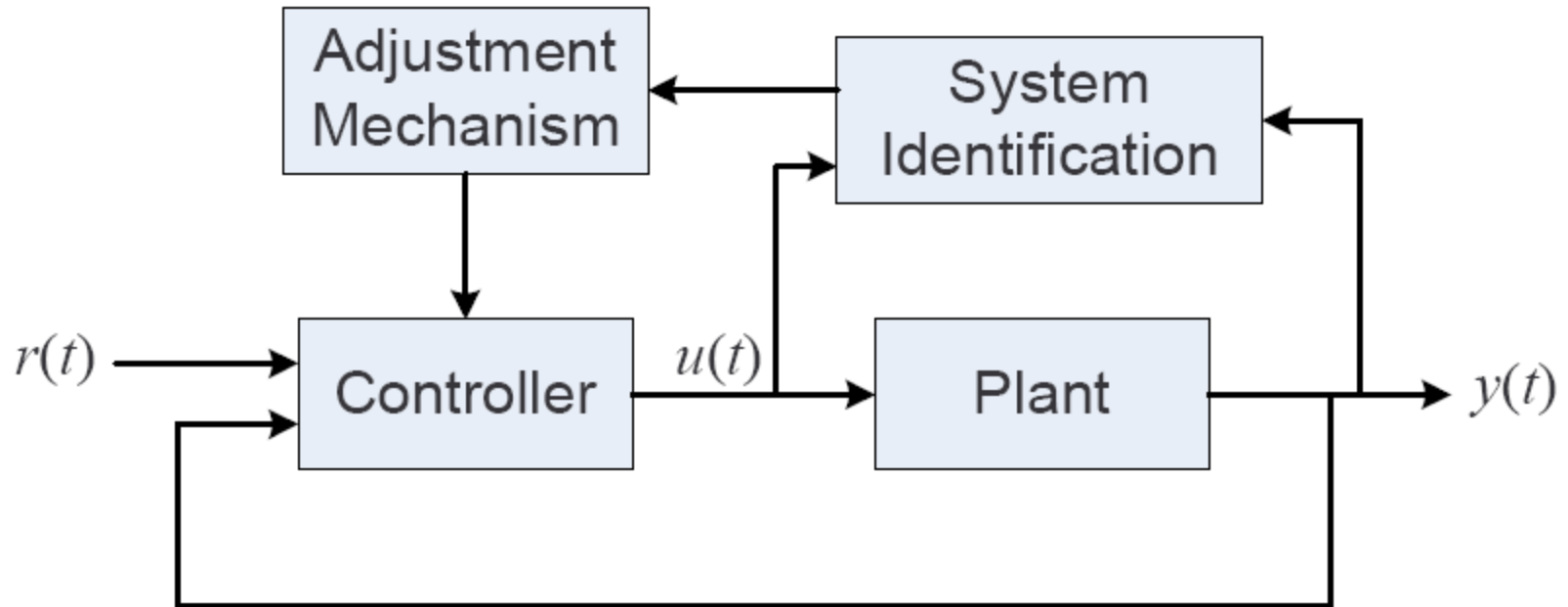
Step Response with Delay Mismatch = 0.1 (sec)



# Adaptation

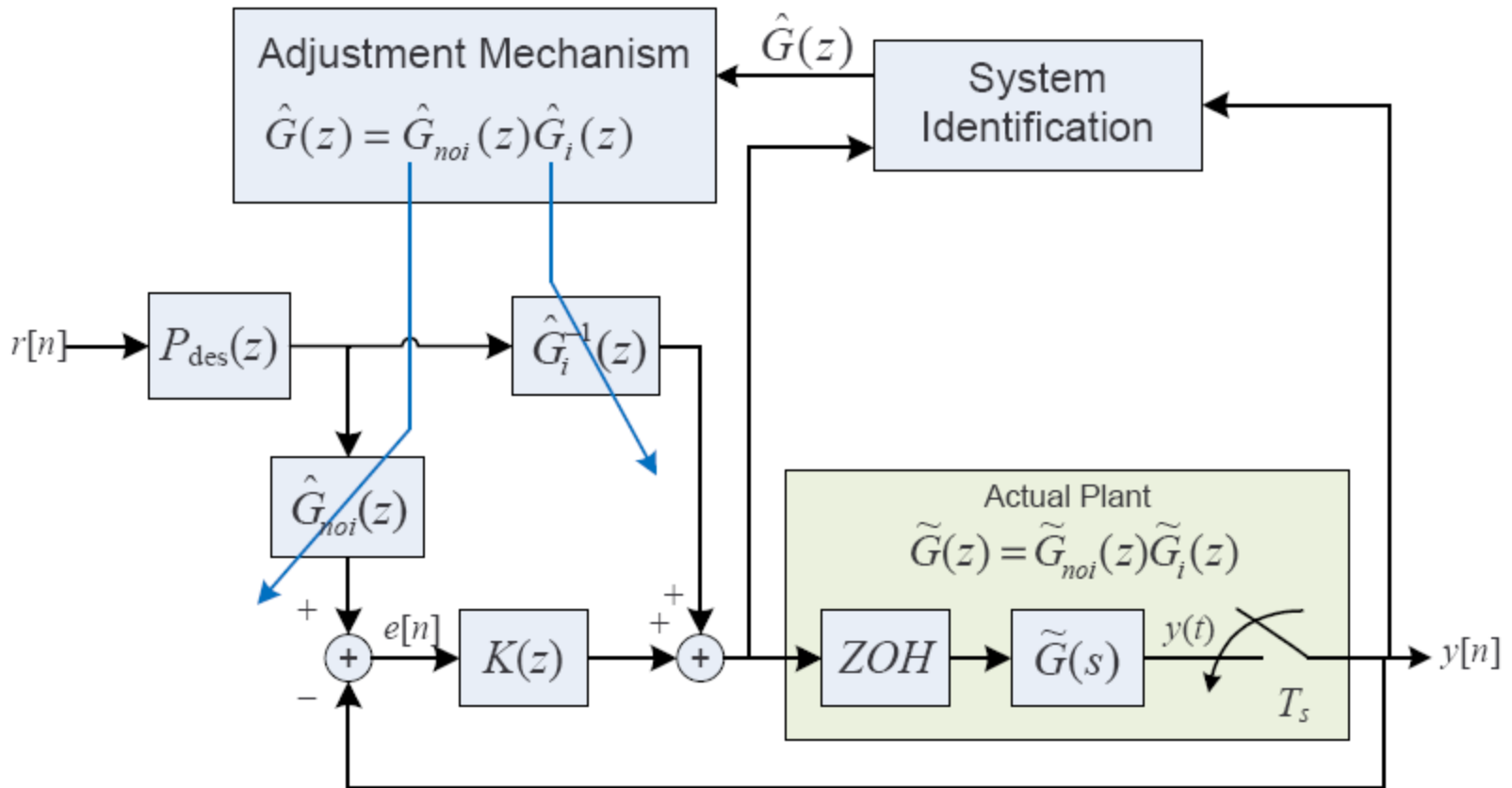
- Stability provided by only adapting FF pieces
- Model identification adaptive control
  - Identify LTI plant models
  - Update feedforward controller blocks directly
- Reinforcement learning control
  - Original motivation
  - Echo state networks

# Model Identification Adaptive Control



- For stability, only adapt feedforward controllers
- Architectures provide a unique and straightforward way to update feedforward controllers

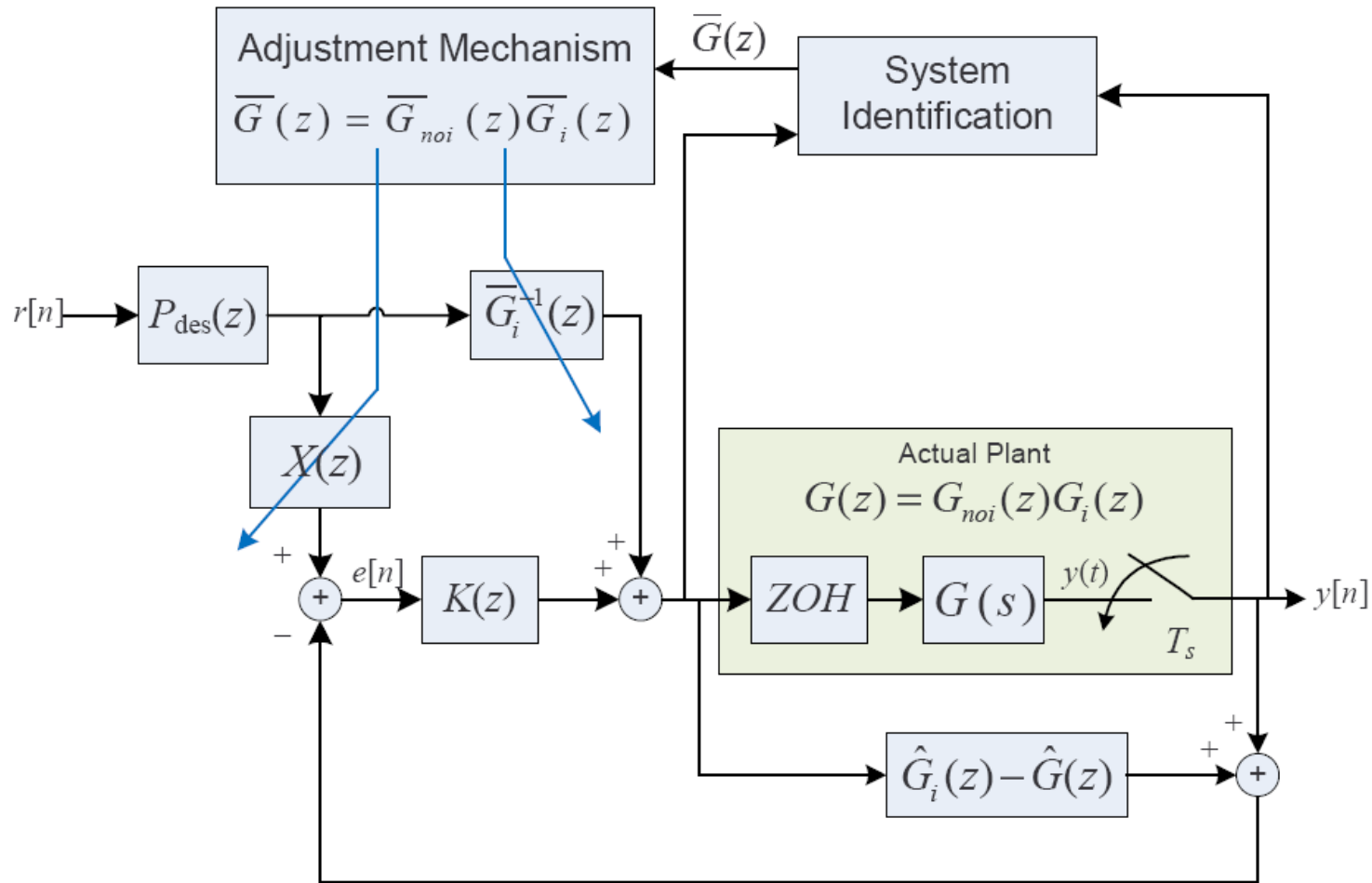
# MIAC DFFPC



- Natural split of identified plant to FF controllers
- Stability guaranteed (even if plant is unstable)



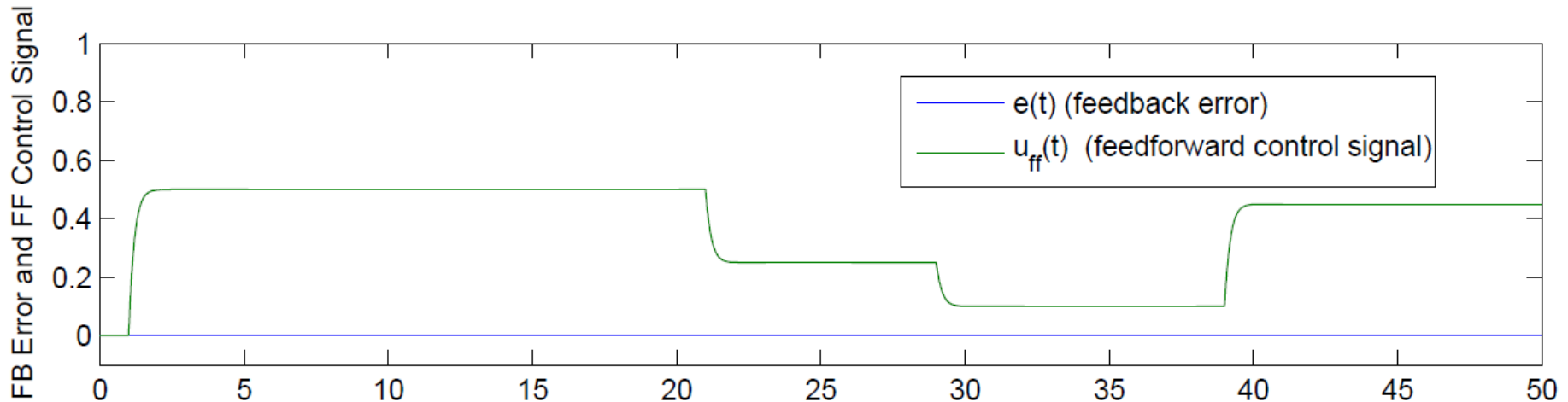
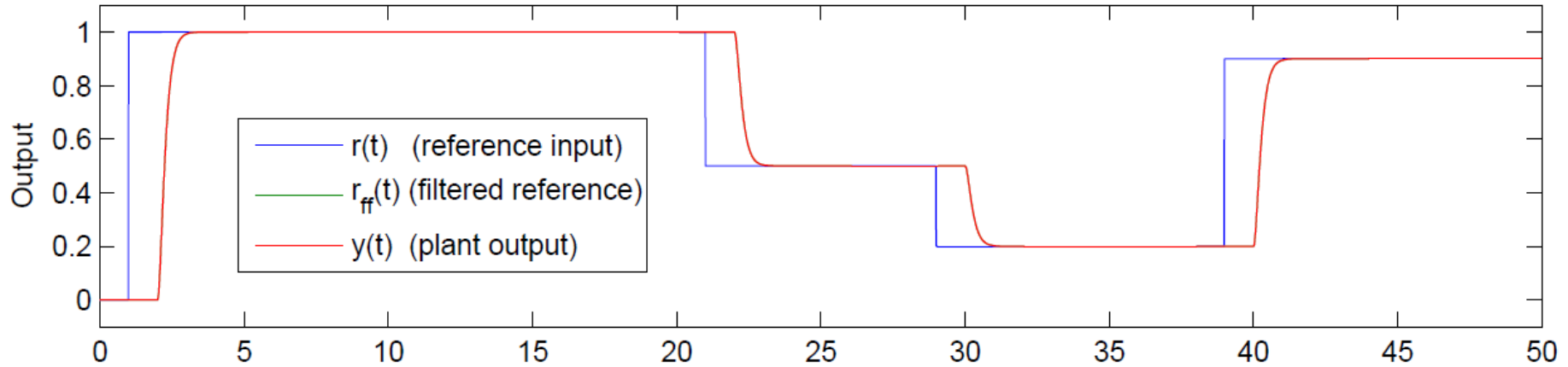
# MIAC DFFSP



- Augmentation required to restore perfect tracking
- Smith predictor still requires a stable plant

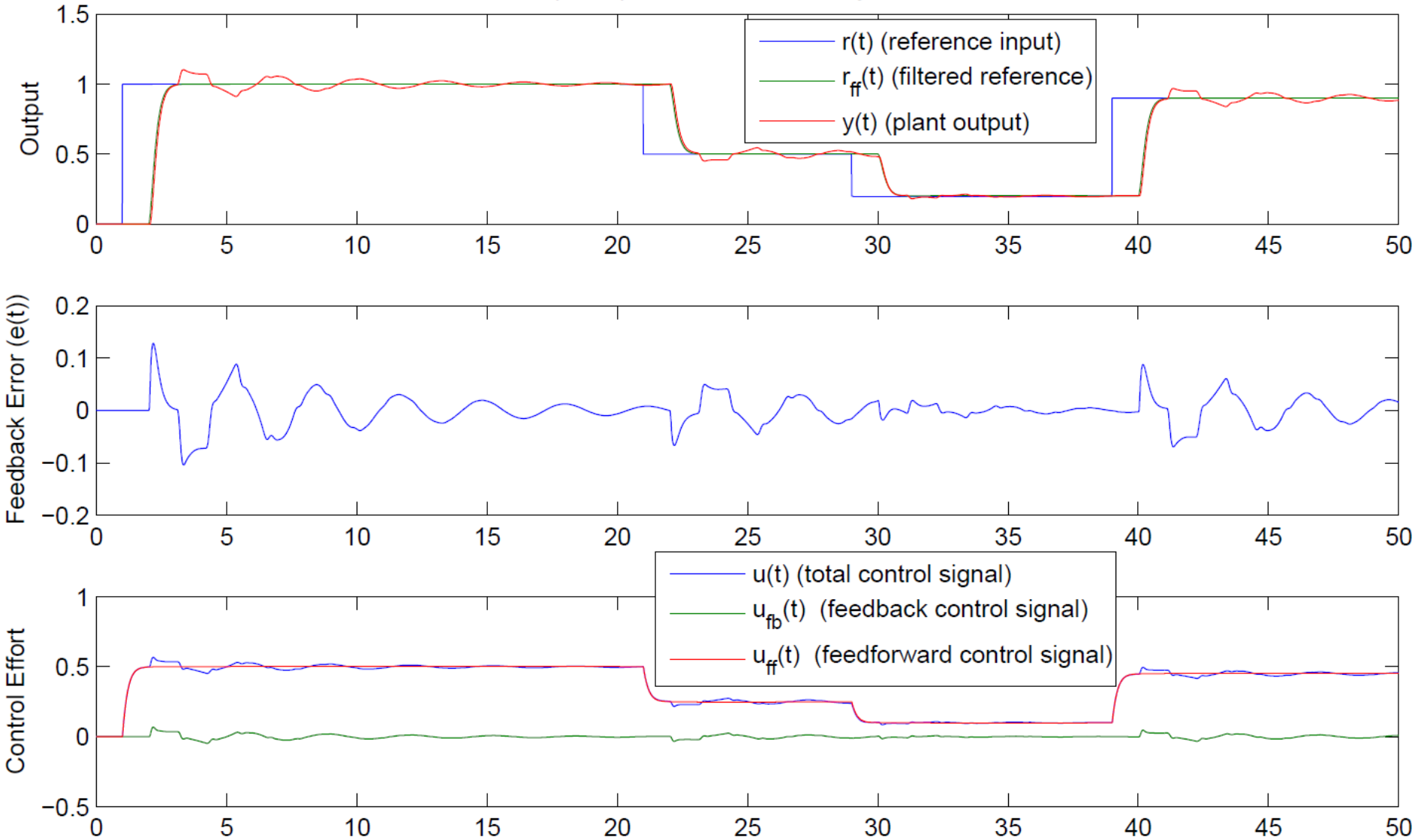
# Adaptation Example: Nominal Simulation

Nominal Step Response with  $\alpha_{\text{des}} = 7$

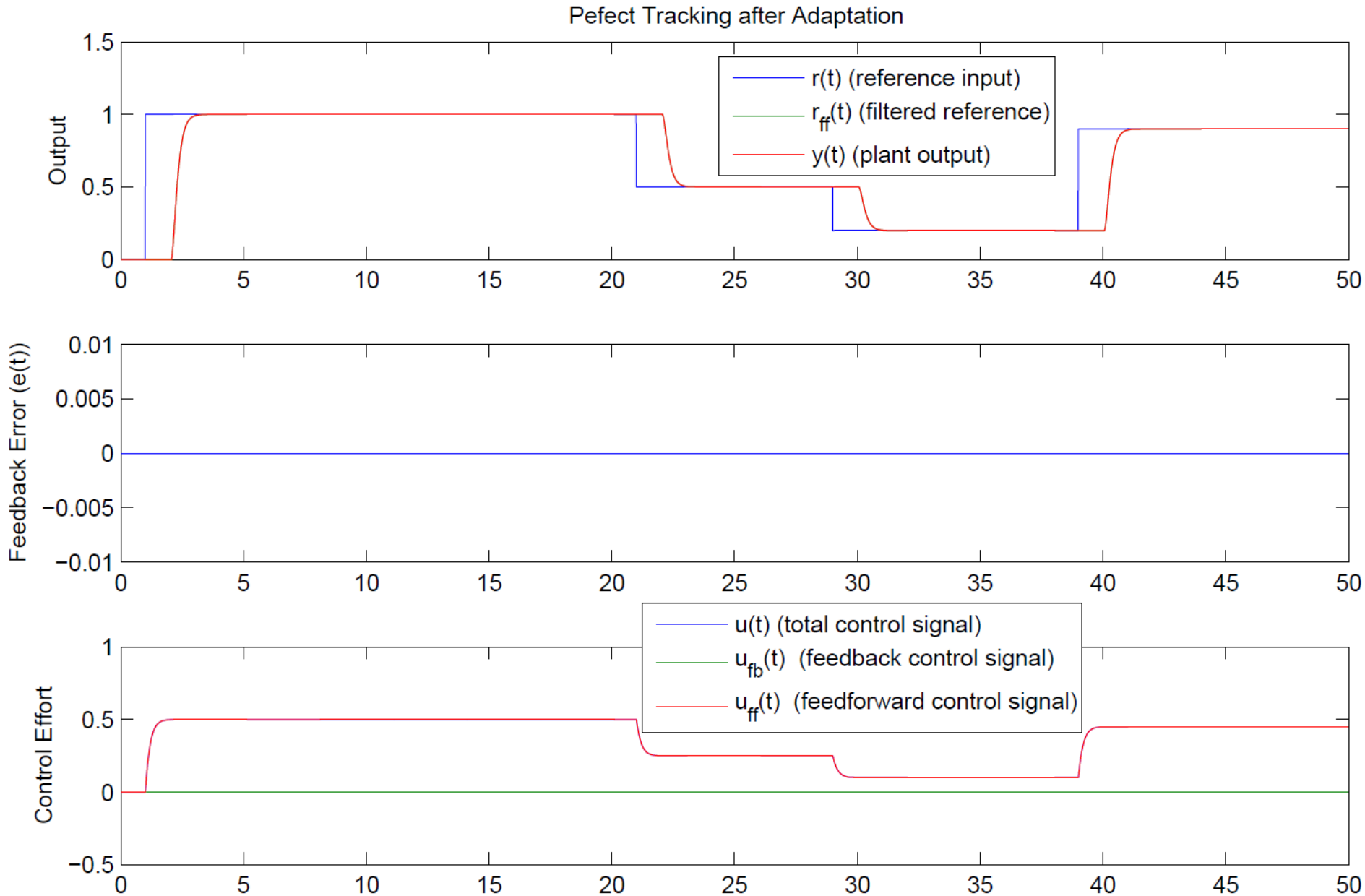


# Perturbed Simulation (Time Delay Mismatch of 0.05 seconds)

Step Response with Time Delay Mismatch



# Perfect Tracking Restored after Adaptation



# Reinforcement Learning Control

- Augment (or replace) existing feedforward structure
- Echo state networks
  - Ability to guaranteed stability of recurrent connections
  - Allows for larger networks (one-time stability analysis)

# Microalgae Modeling and Control

# Microalgae Overview

- Algae can convert excess carbon from human activity into biofuels
- Produce an estimated 7000-15000 gallons per acre per year<sup>1</sup>
- Supply 50% of the US fuel needs while using only 1.1% to 2.5% of the existing cropping land<sup>2</sup>
  - Much of the land around existing power plants is open and would be suitable for algae farms
  - Does not compete with agricultural land

<sup>1</sup> NREL (1998)

<sup>2</sup> Chisti (2007)

# Research Challenges

- Design and operate PBRs at a large scale
  - Problems with dissolved oxygen (DO) removal
  - Problems with efficient CO<sub>2</sub> delivery (i.e., achieving mass transfer)
  - Efficient utilization of all available light (PAR)
  - Requires a mechanical, biological, chemical, electrical, and controls solution
- Developing models and controllers based on the physics of the reactor that scale to commercial size reactors

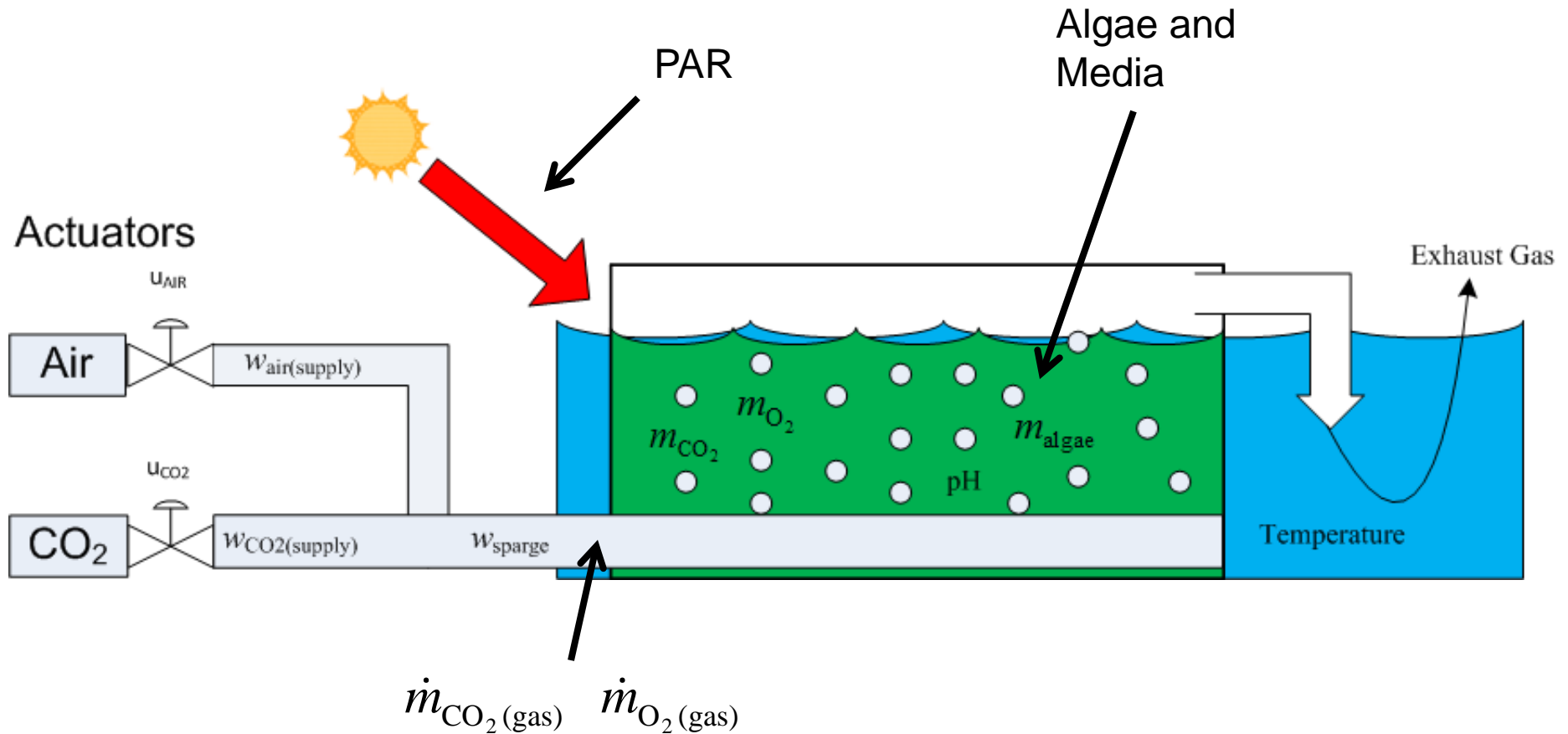


# Reactor Test Bed

- Extended surface flat panel reactor provides efficient sun utilization
- CO<sub>2</sub> rich gas bubbled through the flat panels to deliver CO<sub>2</sub> and remove produced dissolved O<sub>2</sub>



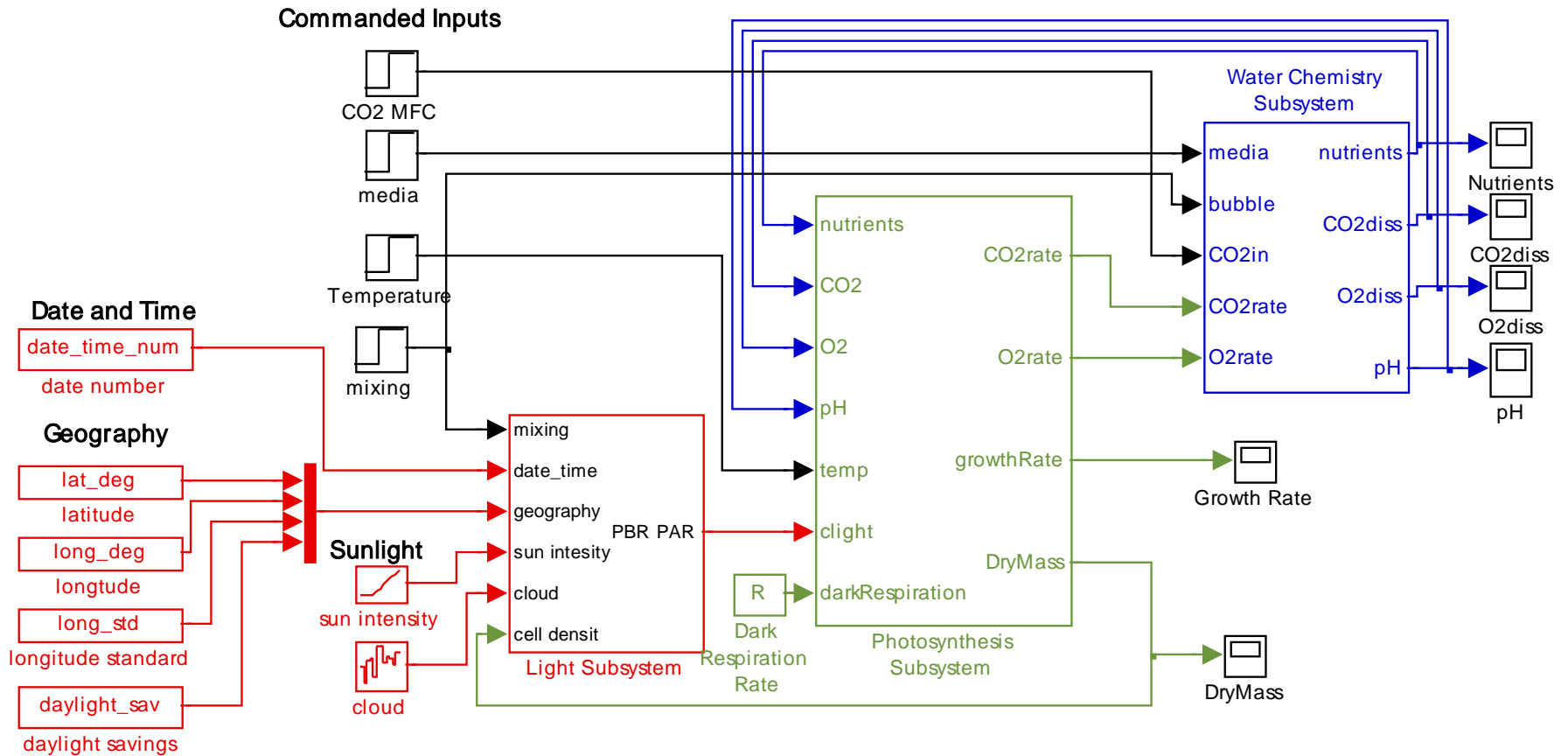
# Reactor Setup



# Modeling and Control Overview

- Develop a component based model that isolates the three components
  - Light subsystem (algebraic model)
  - Photosynthesis subsystem (algae dynamics)
  - Water chemistry subsystem (media dynamics)
- Use model to provide feedforward CO<sub>2</sub> delivery to maintain pH.

# Model Block Diagram



# Light Subsystem

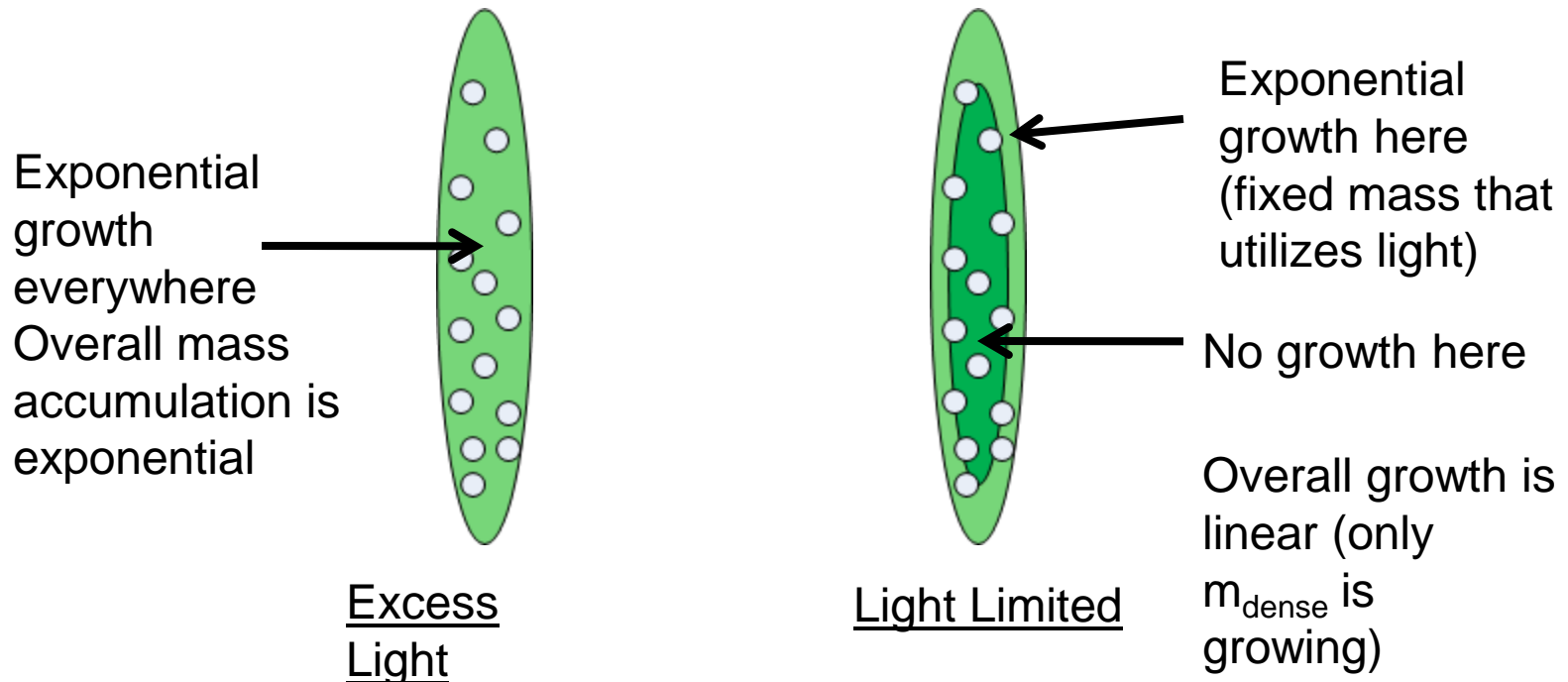
- About 45% of the full spectrum of light is photosynthetically active radiation (PAR)
- A fraction of the direct light will enter the bath and the rest will reflect
- Amount of light that reaches the algae is function of direct light and reactor orientation
- Mixing affects the amount of light the algae can utilize

# Photosynthesis Subsystem

- Models growth as a function of incident PAR
- Growth measures
  - Biomass produced
  - CO<sub>2</sub> consumed
  - O<sub>2</sub> produced
- Based on densities
  - Allows for reactor comparison independent of scale

# Growth Model Description

- Growth – driven by photosynthesis
  - Exponential when excess light
  - Linear above critical density ( $m_{\text{dense}}$ )
  - Exponential Respiration (during both light and dark)



# Growth Model Equations

$$\dot{m}_{\text{algae}} = K_{\text{PAR}} I_{\text{PAR}} \bar{m}_{\text{algae}} - R m_{\text{algae}}$$

$$\bar{m}_{\text{algae}} = \min(m_{\text{algae}}, m_{\text{dense}})$$

$$m_{\text{dense}} = f(m_{\text{algae}}, \text{mixing, geometry})$$

$I_{\text{PAR}}$  : PAR reading from the light subsystem

$K_{\text{PAR}}$  : Sun Utilization Parameter

$R$  : Rate of respiration in the dark

$m_{\text{algae}}$  : The density of algae (g/L or OD units/L)

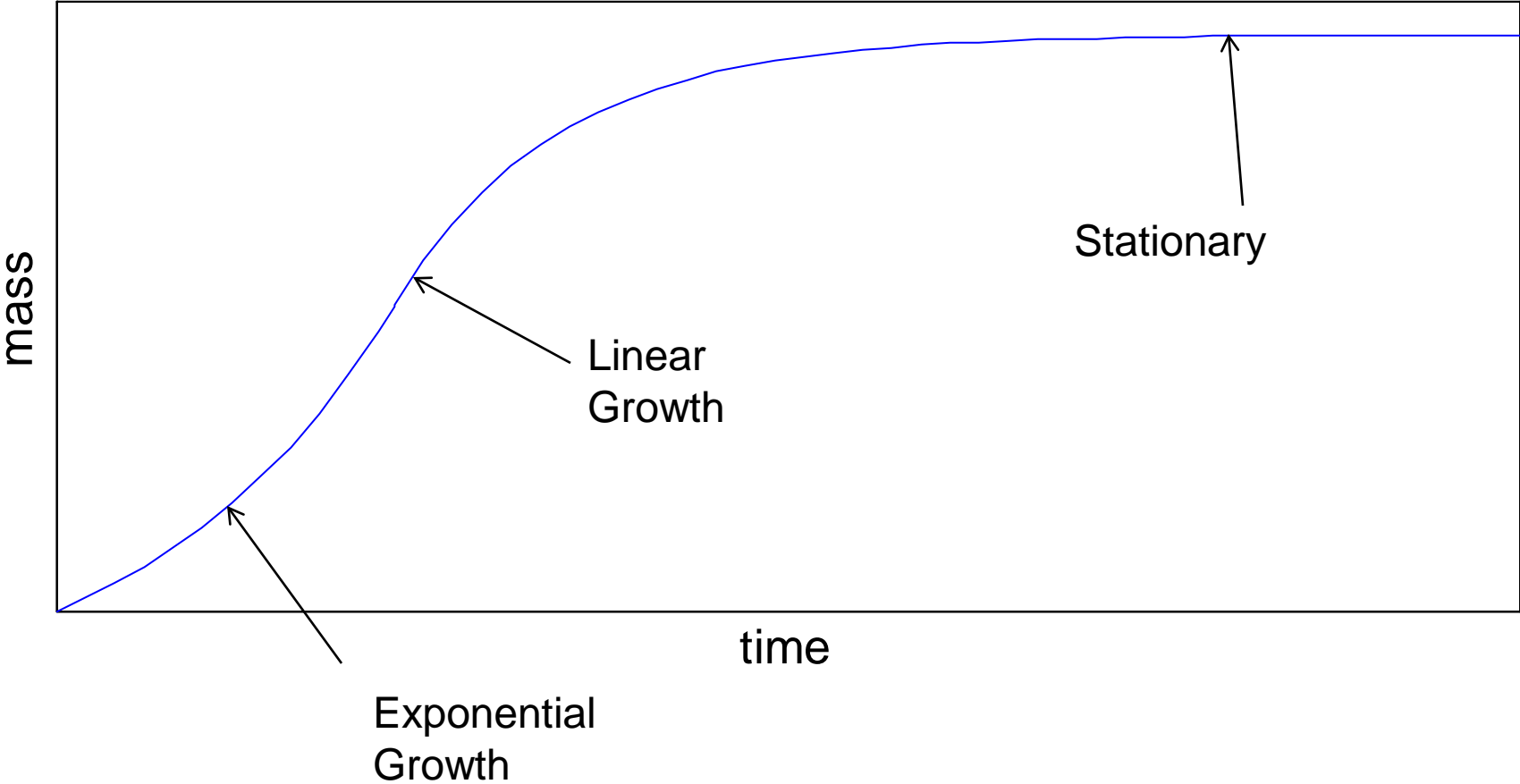
## Gas Consumption/Production Measures

$$\dot{m}_{\text{CO}_2} = K_{\text{CO}_2} \dot{m}_{\text{algae}} \quad \text{CO}_2 \text{ Consumption (used later for pH regulation)}$$

$$\dot{m}_{\text{O}_2} = K_{\text{O}_2} \dot{m}_{\text{algae}} \quad \text{O}_2 \text{ Production}$$



# Growth Phases



# Water Chemistry Subsystem

- Models the dissolved gases ( $\text{CO}_2$  and  $\text{O}_2$ ) in the media
- Interactions with algae
  - $\text{CO}_2$  consumed /  $\text{O}_2$  produced during photosynthesis, vice versa during respiration
- Interactions with sparged gas
  - Dissolved  $\text{O}_2$  and dissolved  $\text{CO}_2$  seek equilibrium between bubbles and media
  - First order plus dead time models
- pH used to infer  $\text{CO}_2$  concentration
  - Takes a few seconds for pH to equilibrate from dissolved  $\text{CO}_2$

# Water Chemistry Models

$$\dot{m}_{\text{O}_2(\text{media})}(t) = \frac{W_{\text{sparge}}}{\tau_{\text{O}_2}} \left( m_{\text{O}_2(\text{gas})}(t - \tau_{\text{d,gas}}) - m_{\text{O}_2(\text{media})}(t) \right) + \dot{m}_{\text{O}_2(\text{algae})}(t)$$

$$\dot{m}_{\text{CO}_2(\text{media})}(t) = \frac{W_{\text{sparge}}}{\tau_{\text{CO}_2}} \left( m_{\text{CO}_2(\text{gas})}(t - \tau_{\text{d,gas}}) - m_{\text{CO}_2(\text{media})}(t) \right) - \dot{m}_{\text{CO}_2(\text{algae})}(t)$$

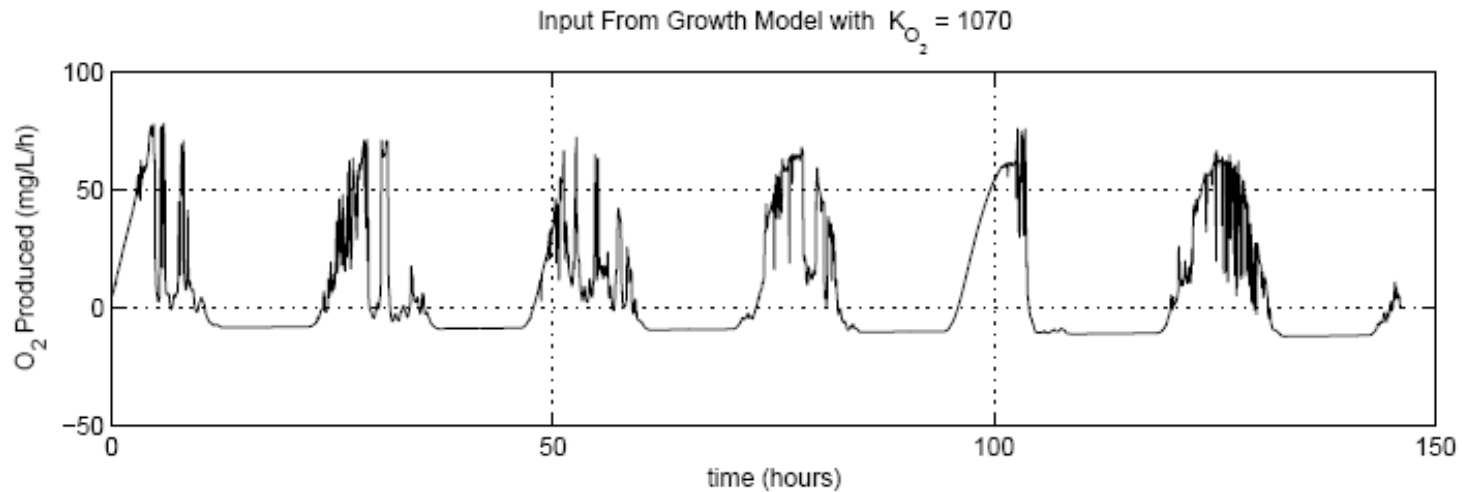
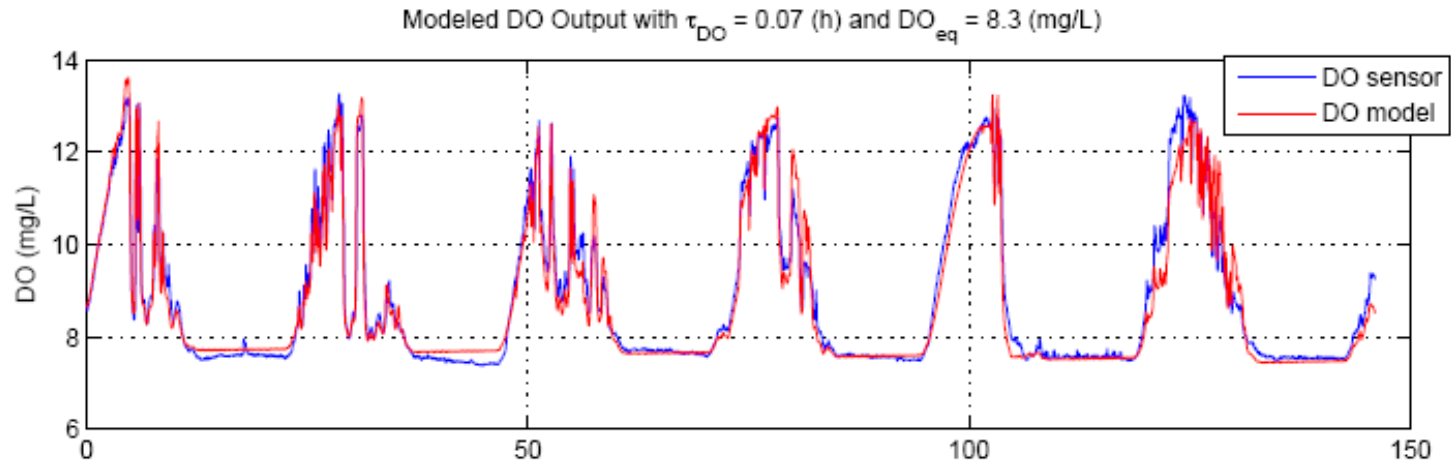
$$\text{pH} \dot{=} = \frac{1}{\tau_{\text{pH}}} \left( K_{\text{pH}} m_{\text{CO}_2(\text{media})}(t) - \text{pH}(t) \right)$$

$W_{\text{sparge}}$  : flow rate of sparge gas (~ number of bubbles )

$\tau_{\text{O}_2}, \tau_{\text{CO}_2}$  : time constant for gas transfer between bubbles and media

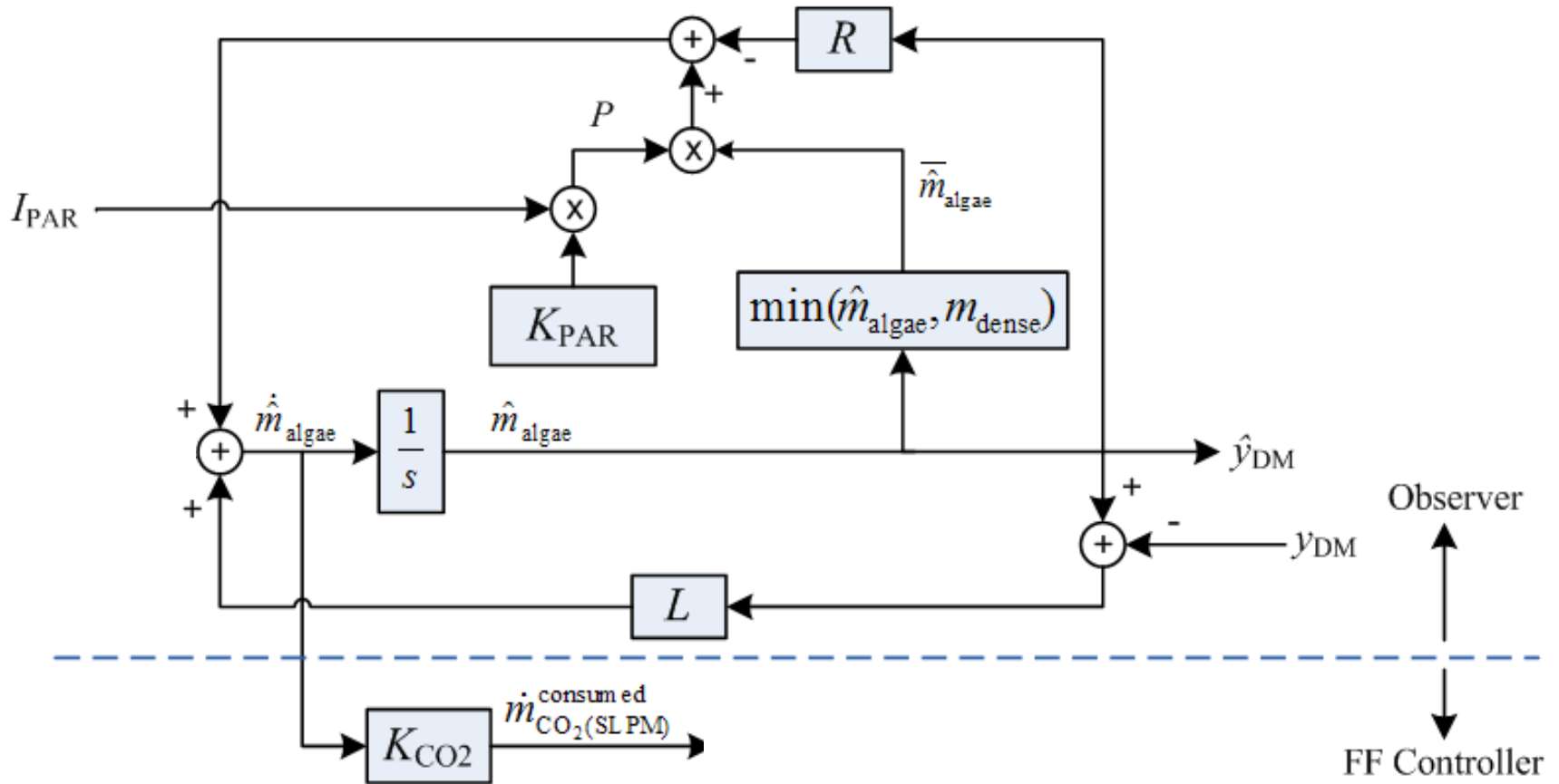
$\tau_{\text{pH}}$  : time constant for pH to settle

# Dissolved Oxygen Model

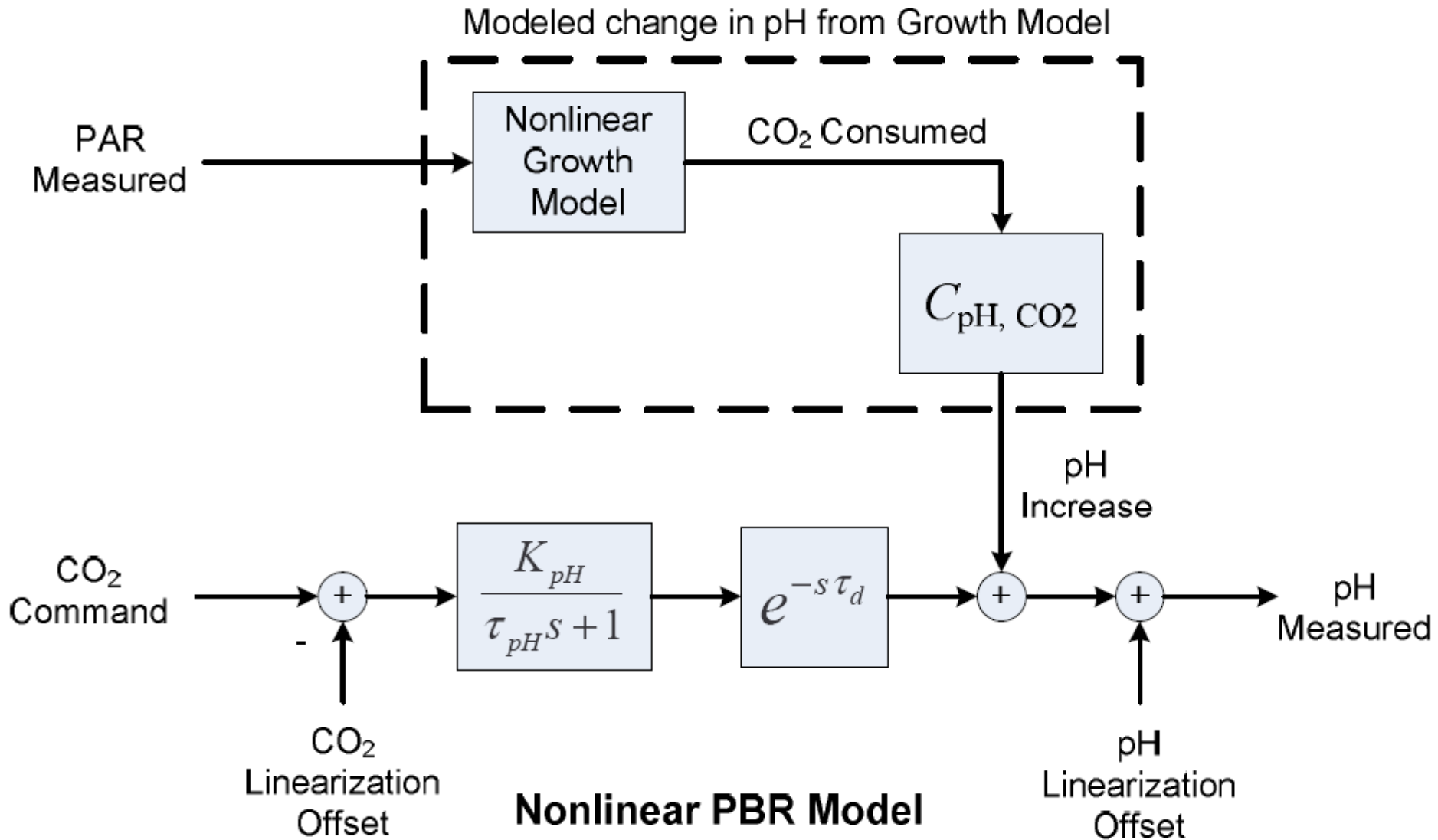


$$K_{O_2}(\text{mgO}_2/\text{g algae}) = 1.07(\text{gO}_2/\text{g algae}) \times 1000 \frac{\text{mg}}{\text{g}} = 1070.$$

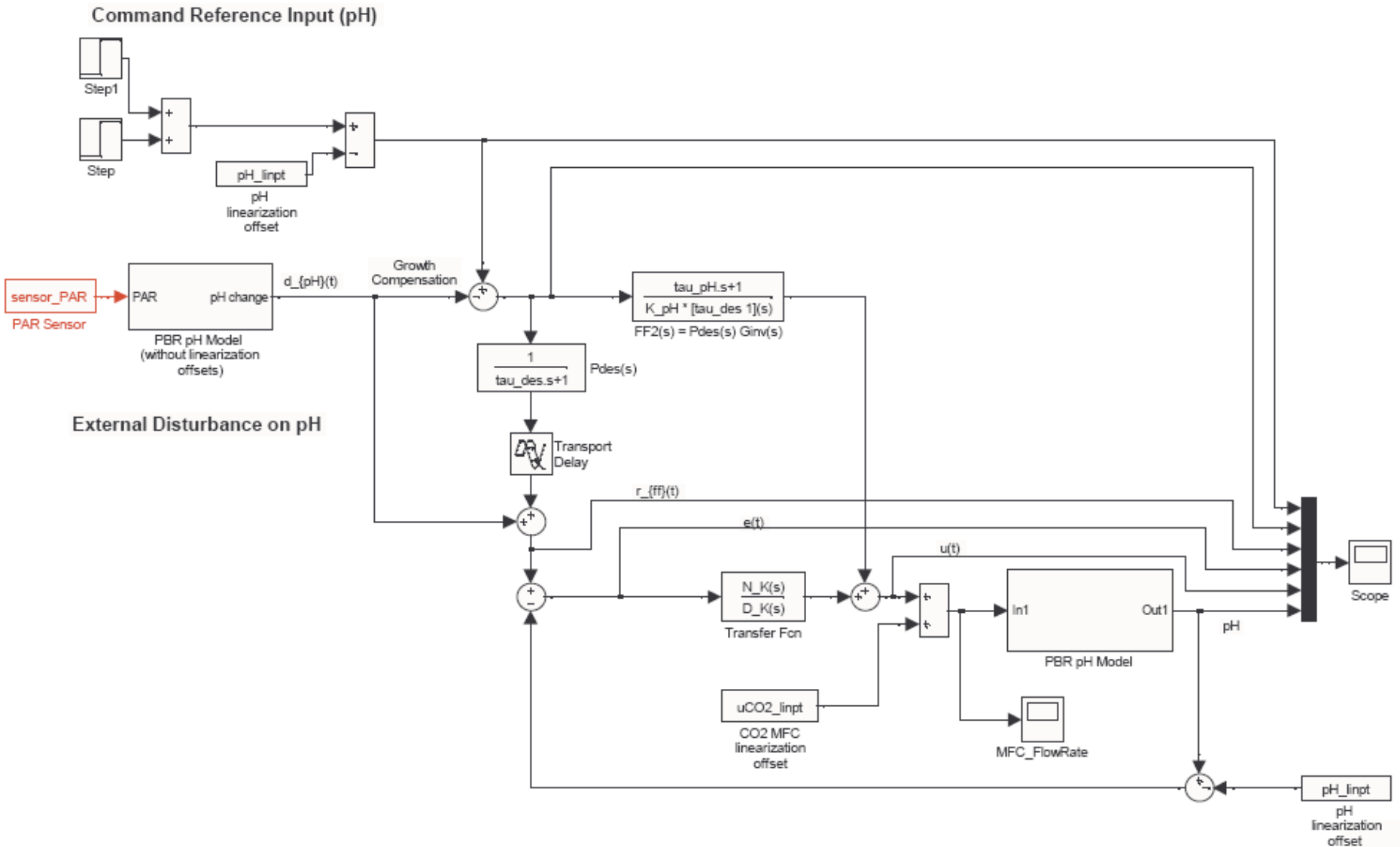
# Observer-Based FF Control



# pH Model (with Growth Dynamics)

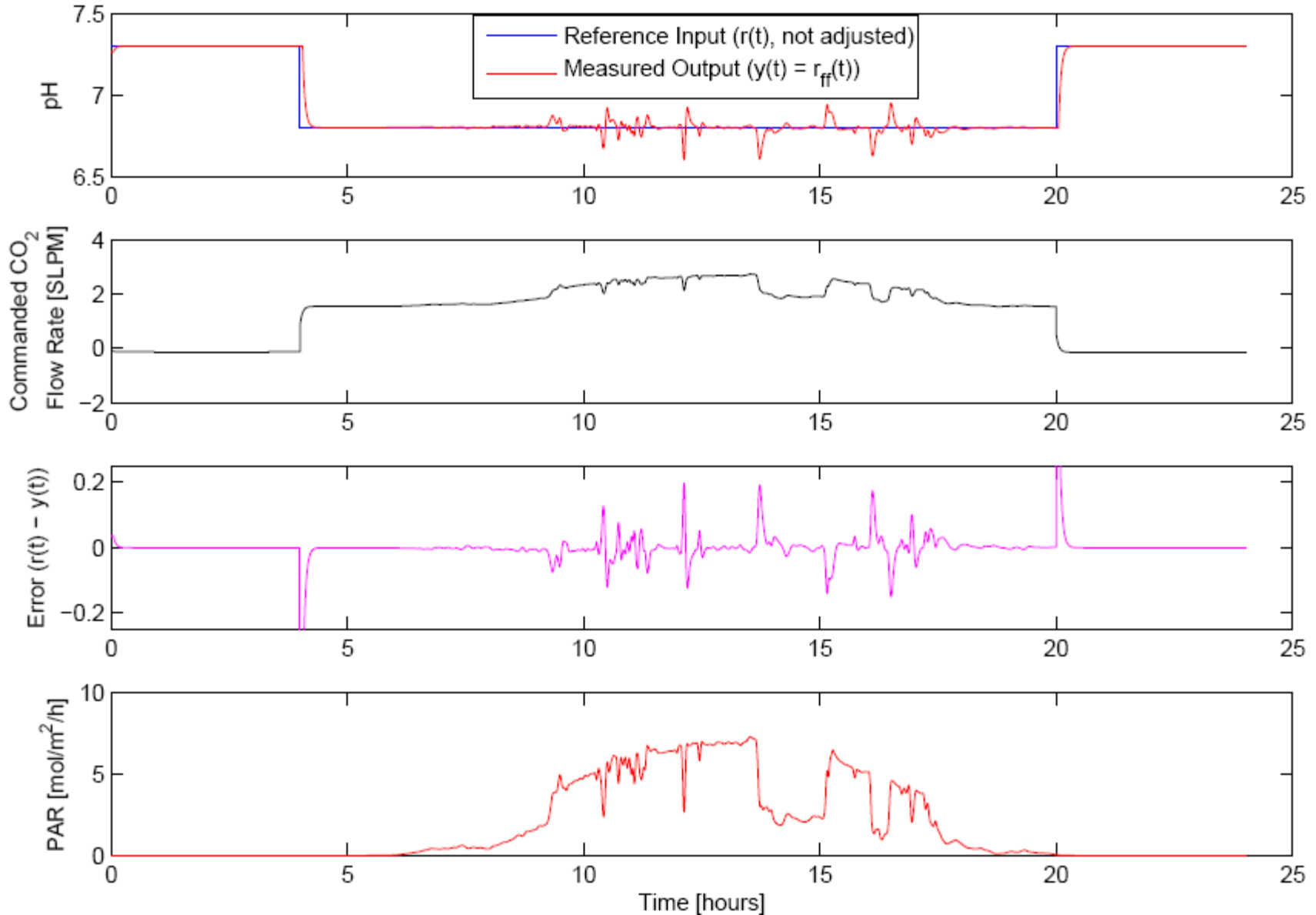


# Modified DFFPC pH Regulation



# Achievable pH Regulation

Perfect pH Tracking of  $r_{ff}(t)$  with  $\alpha_{des} = 20$ ,  $\tau_d = 180$  [sec]





# Conclusions

- Perfect tracking for a larger class of systems
  - Characterize achievable performance
- Robustness tools to analyze performance in the presence of model uncertainty
- Adaption methods for the LTI and NL Feedforward controller(s)
- Physics-based algae model
- Verified algae model on experimental data
- Characterized achievable pH control

# Future Directions

- Feedforward design
  - Design for  $e(t) = y(t) - r(t)$
  - Design perfect tracking of  $r(t)$
- Adaptation
- MIMO systems
- Microalgae modeling and control
  - Lipid model
  - System level optimization
- Control-Structure Interactions

# Conference Publications

- **Buehner, M.R.** and P.M Young. 2010. “Perfect Tracking for Non-minimum Phase Systems”. In: *Proceedings of the 2010 American Control Conference*, Baltimore, Maryland, July, 2010.
- **Buehner, M.R.**, P.M Young, B.D. Willson, D. Rausen, R. Schoonover, G. Babbitt, and S. Bunch. 2009. “Microalgae Growth Modeling and Control for a Vertical Flat Panel Photobioreactor”. In: *Proceedings of the 2009 American Control Conference*, St. Louis, Missouri, pp. 2301-2306, June, 2009.
- **Buehner M.R.**, C.W. Anderson, P.M. Young, K.A. Bush, and D.C. Hittle. 2007. “Improving Performance using Robust Recurrent Reinforcement Learning Control”. In: *Proceedings of the European Control Conference 2007*, Kos, Greece, pp. 1676-1681, July, 2007.

# Journal Publications

- Anderson M.L., **M.R. Buehner**, P.M. Young, D.C. Hittle, C.W. Anderson, J. Tu, and D. Hodgson. 2008. “MIMO Robust Control for Heating, Ventilating, and Air Conditioning (HVAC) Systems”. *IEEE Transactions on Control Systems Technology*, 16(3):475-483.
- Anderson M.L., **M.R. Buehner**, P.M. Young, D.C. Hittle, C.W. Anderson, J. Tu, and D. Hodgson. 2007. “An Experimental System for Advanced Heating, Ventilating, and Air Conditioning (HVAC) Control”. *Energy and Buildings* 39(2):136-147.
- Anderson C.W., P.M. Young, **M.R. Buehner**, J.N. Knight, K.A. Bush, and D.C. Hittle. 2007. “Robust Reinforcement Learning Control using Integral Quadratic Constraints for Recurrent Neural Networks”, *IEEE Transactions on Neural Networks: Special Issue on Neural Networks for Feedback Control Systems* 18(4):993-1002.
- **Buehner M.R.** and P.M. Young. 2006. “A Tighter Bound for the Echo State Property”. *IEEE Transactions on Neural Networks* 17(3):820-824.

# Questions