TRANSIENT ANALYSIS OF CLOSED- AND OPEN-REGION ELECTROMAGNETIC PROBLEMS USING HIGHER ORDER FINITE ELEMENT METHOD AND METHOD OF MOMENTS IN THE TIME DOMAIN

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Research Projects

- Time Domain Computational Electromagnetics (TDCEM)
- **Part 1: Higher Order FEM for Vector Wave Equation Modeling in TD**
- **Part 2: Higher Order MoM for EFIE Modeling in TD**
- **Appendix:**
  A. **Rules for Optimal Simulation Parameters in HO FEM and MoM Analysis**
  B. **MRI Applications**
  C. **SIE Integration: Close Point Projection (CPP)**

Goal: Accurate, stable, and efficient transient solution to real-world applications

PART 1: HIGHER ORDER FEM FOR VECTOR WAVE EQUATION MODELING IN TIME DOMAIN

N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Higher Order Time-Domain Finite Element Method for Microwave Device Modeling with Generalized Hexahedral Elements," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 61, No. 4, pp. 1425-1434, April 2013.

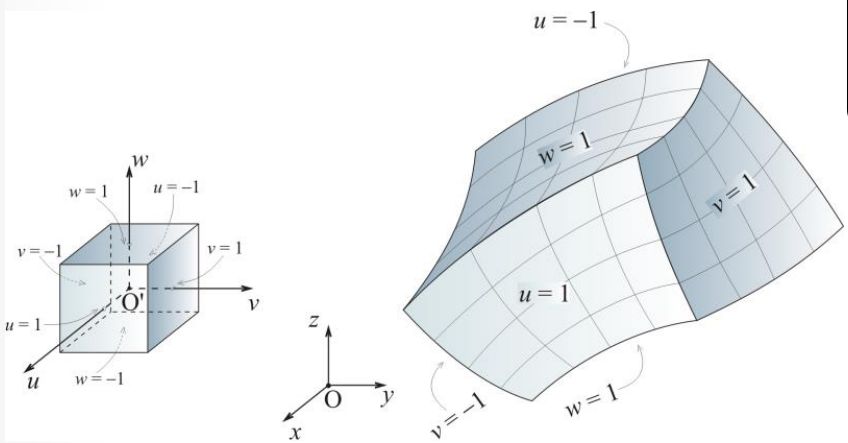
Outline

- HO large-domain Galerkin-type FEM: open- and closed-region problems
  - Discretization of **Vector Wave Equation in TD**
- **TDFEM**, advantages:
  - Analysis of broadband, transient and nonlinear phenomena in a single run
  - Inhomogeneous closed-region problems
  - Focus: microwave waveguide devices
- Theoretical background
- Numerical analysis: waveguide sections with metallic and/or homogeneous and continuously inhomogeneous dielectric discontinuities, single 90º *E*-, *H*-plane curved waveguide bends, cascaded 30º *H*-plane bends (U-, S-type).
- Conclusions

Higher Order TDFEM

- Direct numerical discretization of TD vector wave equation
  - Implicit unconditionally stable time-stepping finite difference scheme known as the Newmark-beta method
- Spatial discretization:
  - Curl-conforming hierarchical polynomial vector basis functions of arbitrary field expansion orders
  - Lagrange-type generalized conformal parametric hexahedral FEs

$$\mathbf{r}(u,v,w)=\sum_{i=1}^{M}\mathbf{r}_i\hat{L}_i^{K_{uvw}}(u,v,w)=\sum_{m=0}^{K_u}\sum_{n=0}^{K_v}\sum_{l=0}^{K_w}\mathbf{r}_{mnl}\,u^m v^n w^l;$$
$$-1\le u,v,w\le 1$$



M. M. Ilić and B. M. Notaroš, "Higher order hierarchical curved hexahedral vector finite elements for electromagnetic modeling," *IEEE Transactions on Microwave Theory and Techniques*, vol. 51, no. 3, pp. 1026–1033, March 2003.

TDFEM Formulation

Curl-curl electric-field vector wave equation

$$\nabla \times \frac{1}{\mu_r} \nabla \times \mathbf{E}(\mathbf{r},t) + \frac{1}{c_0^2} \varepsilon_r \frac{\partial^2 \mathbf{E}(\mathbf{r},t)}{\partial t^2} = 0$$

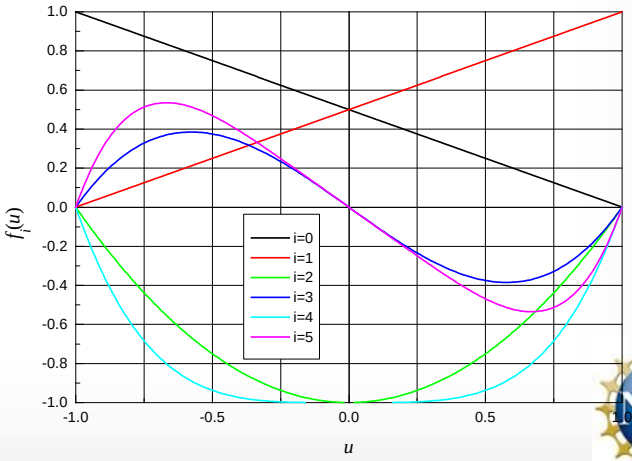
Field expansion

$$\mathbf{E}(\mathbf{r},t) = \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w} \alpha_{uijk}(t)\,\mathbf{f}_{uijk} + \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \sum_{k=0}^{N_w} \alpha_{vijk}(t)\,\mathbf{f}_{vijk} + \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w-1} \alpha_{wijk}(t)\,\mathbf{f}_{wijk}$$

Basis functions - curl-conforming hierarchical polynomials of arbitrary orders

$$\mathbf{f}_{uijk}(u,v,w) = u^i P_j(v) P_k(w)\,\mathbf{a}_u^r$$
$$\mathbf{f}_{vijk}(u,v,w) = P_i(u)\, v^i P_k(w)\,\mathbf{a}_v^r$$
$$\mathbf{f}_{wijk}(u,v,w) = P_i(u) P_j(v)\, w^k \mathbf{a}_w^r$$

$$P_i(u) = \begin{cases} 1-u, & i=0 \\ u+1, & i=1 \\ u^i - 1, & i \ge 2, \text{ even} \\ u^i - u, & i \ge 3, \text{ odd} \end{cases}, \quad -1 \le u,v,w \le 1.$$



Testing - Galerkin method

TDFEM Formulation (cont'd)

Semi-discretized spatial form of vector wave equation

$$[A]\{\alpha(t)\}+\frac{1}{c_0^2}[B]\frac{\mathrm{d}^2\{\alpha(t)\}}{\mathrm{d}t^2}=G$$

$$A_{i,j}=\int_V \frac{1}{\mu_\mathrm{r}}(\nabla\times\mathbf{f}_i)\cdot(\nabla\times\mathbf{f}_j)\mathrm{d}V$$

$$B_{i,j}=\int_V \varepsilon_r\mathbf{f}_i\cdot\mathbf{f}_j\,\mathrm{d}V,\quad i=1,2,\ldots N,\quad j=1,2,\ldots N$$

$$G=-\oint_S \frac{1}{\mu_\mathrm{r}}\mathbf{f}_i\cdot\left(\mathbf{n}\times\left(\nabla\times\mathbf{E}(\mathbf{r},t)\right)\right)\mathrm{d}S$$

Waveguide Port Boundary Condition

Z. Lou and J. M. Jin, "An accurate waveguide port boundary condition for the time-domain finite-element method," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 9, pp. 3014–3023, September 2005.

Waveguide Port Boundary Condition (WPBC)

$$\mathbf{n}\times(\nabla\times\mathbf{E})=\begin{cases}-2\gamma_{10}\mathbf{E}^{\text{inc}}+\gamma_{10}\,\mathbf{E} & \text{(excitation port)}\\ \gamma_{10}\,\mathbf{E} & \text{(receiving ports)}\end{cases}$$

Impulse response of dominant waveguide mode

$$\gamma_{10}=\sqrt{(\pi/a)^2+(s/c_0)^2},\quad s=\mathrm{j}\omega \xrightarrow{\text{ILT}} \Gamma_{10}=\frac{1}{c_0}\frac{\mathrm{d}}{\mathrm{d}t}+h_{10}(t)*$$

Trapezoidal rule approximation

$$G=-\frac{1}{\mu_r}\left(\frac{1}{c_0}\frac{\mathrm{d}}{\mathrm{d}t}+h_{10}(t)*\right)\sum_{j=1}^{N}\alpha_j(t)\left[\oint_{S_1}(\mathbf{n}\times\mathbf{f}_i)\cdot(\mathbf{n}\times\mathbf{f}_j)\mathrm{d}S+\oint_{S_2}(\mathbf{n}\times\mathbf{f}_i)\cdot(\mathbf{n}\times\mathbf{f}_j)\mathrm{d}S\right]$$

$$+\frac{2}{\mu_r}\left(\frac{1}{c_0}\frac{\mathrm{d}}{\mathrm{d}t}+h_{10}(t)*\right)\oint_{S_1}\mathbf{f}_i\cdot\mathbf{E}^{\text{inc}}\mathrm{d}S,\quad i=1,2,\ldots N$$

Unconditionally Stable Two-Step Scheme

Newmark-beta Method

$$[A]\{\alpha(t)\}+\frac{1}{c_0^2}[B]\frac{\mathrm{d}^2\{\alpha(t)\}}{\mathrm{d}t^2}+\frac{1}{c_0\mu_\mathrm{r}}[C]\frac{\mathrm{d}\{\alpha(t)\}}{\mathrm{d}t}+\frac{1}{\mu_\mathrm{r}}[C]\{q_{10}(t)\}=\{f\}$$

$$\{\alpha(t)\}=\frac{1}{4}\left[\{\alpha\}^{n+1}+2\{\alpha\}^{n}+\{\alpha\}^{n-1}\right]$$

$$\frac{\mathrm{d}\{\alpha(t)\}}{\mathrm{d}t}=\frac{1}{2\Delta t}\left[\{\alpha\}^{n+1}-\{\alpha\}^{n-1}\right]$$

$$\frac{\mathrm{d}^2\{\alpha(t)\}}{\mathrm{d}t^2}=\frac{1}{\Delta t^2}\left[\{\alpha\}^{n+1}-2\{\alpha\}^{n}+\{\alpha\}^{n-1}\right]$$

$$C_{p_{i,j}}=\oint_{S_p}(\mathbf{n}\times\mathbf{f}_i)\cdot(\mathbf{n}\times\mathbf{f}_j)\mathrm{d}S,\quad p=1,2;\quad i,j=1,2,\ldots N$$

$$\{f\}=\frac{2E_0}{\mu_\mathrm{r}}\left[\frac{1}{c_0}\frac{\mathrm{d}}{\mathrm{d}t}f^{\mathrm{inc}}(t)+h_{10}(t)*f^{\mathrm{inc}}(t)\right]\oint_{S_1}(\mathbf{n}\times\mathbf{f}_i)\cdot(\mathbf{n}\times\mathbf{e}_{10})\mathrm{d}S$$

$$\{q_{10}\}^n=\frac{\Delta t}{2}\left(h_{10}(0)\{\alpha\}^n+h_{10}(t_n)\{\alpha\}^0\right)+\Delta t\sum_{i=1}^{n-1}h_{10}((n-i)\Delta t)\{\alpha\}^i$$

Unconditionally Stable Two-Step Scheme (cont'd)

$$[D_1]\{\alpha\}^{n+1} = \{f\}^n - [D_2]\{\alpha\}^{n-1} - [D_3]\{\alpha\}^n - \frac{1}{\mu_r}[C]\{q_{10}\}^n$$

$$[D_1] = \frac{1}{4}[A] + \frac{1}{(c_0\Delta t)^2}[B] + \frac{1}{2\Delta t c_0 \mu_r}[C]$$

$$[D_2] = \frac{1}{4}[A] + \frac{1}{(c_0\Delta t)^2}[B] - \frac{1}{2\Delta t c_0 \mu_r}[C]$$

$$[D_3] = \frac{1}{2}[A] - \frac{2}{(c_0\Delta t)^2}[B]$$

N. J. Šekeljić, M. M. Ilić and B. M. Notaroš, "Higher Order Time-Domain Finite-Element Method for Microwave Device Modeling With Generalized Hexahedral Elements," *IEEE Transactions on Microwave Theory and Techniques*, vol. 61, no. 4, pp. 1425-1434, April 2013.

Modal Amplitudes at Waveguide Ports

$$a_{10}(t) = \int_{S_1} \mathbf{e_{10}} \cdot \left[\mathbf{E}(\mathbf{r},t) - \mathbf{E}^{\text{inc}}(\mathbf{r},t)\right]\Big|_{z=z_1} \mathrm{dS} =$$

$$= \int_{S_1} \mathbf{e_{10}} \cdot \mathbf{E}(\mathbf{r},t)\Big|_{z=z_1} \mathrm{dS} - f^{\text{inc}}(t) \int_{S_1} \mathbf{e_{10}} \cdot \mathbf{e_{10}}\Big|_{z=z_1} \mathrm{dS}$$ **(at excitation port)**

$$b_{10}(t) = \int_{S_2} \mathbf{e_{10}} \cdot \mathbf{E}(\mathbf{r},t)\Big|_{z=z_2} \mathrm{dS}$$ **(at receiving port)**

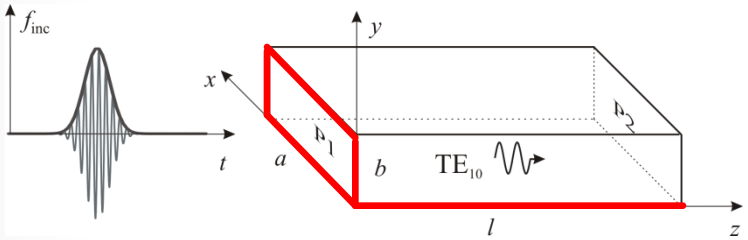
DFT

$$F(f_k) = F(k) = \sum_{n=0}^{N_\mathrm{f}} f(t_n)\, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N_\mathrm{f}} nk}, \quad f_k = k\frac{f_\mathrm{s}}{N_\mathrm{f}}, \quad k = 0, 1, \ldots, N_\mathrm{f} - 1$$

$$S_{11} = \frac{\mathrm{DFT}\{a_{10}(t_n)\}}{\mathrm{DFT}\{f^{\text{inc}}(t_n)\}} = \frac{A_{10}(f_k)}{F^{\text{inc}}(f_k)} \qquad S_{21} = \frac{\mathrm{DFT}\{b_{10}(t_n)\}}{\mathrm{DFT}\{f^{\text{inc}}(t_n)\}} = \frac{B_{10}(f_k)}{F^{\text{inc}}(f_k)}$$

Empty Rectangular Waveguide

$$a = 10\ \text{cm}, \quad b = 5\ \text{cm}, \quad l = 10\ \text{cm}$$

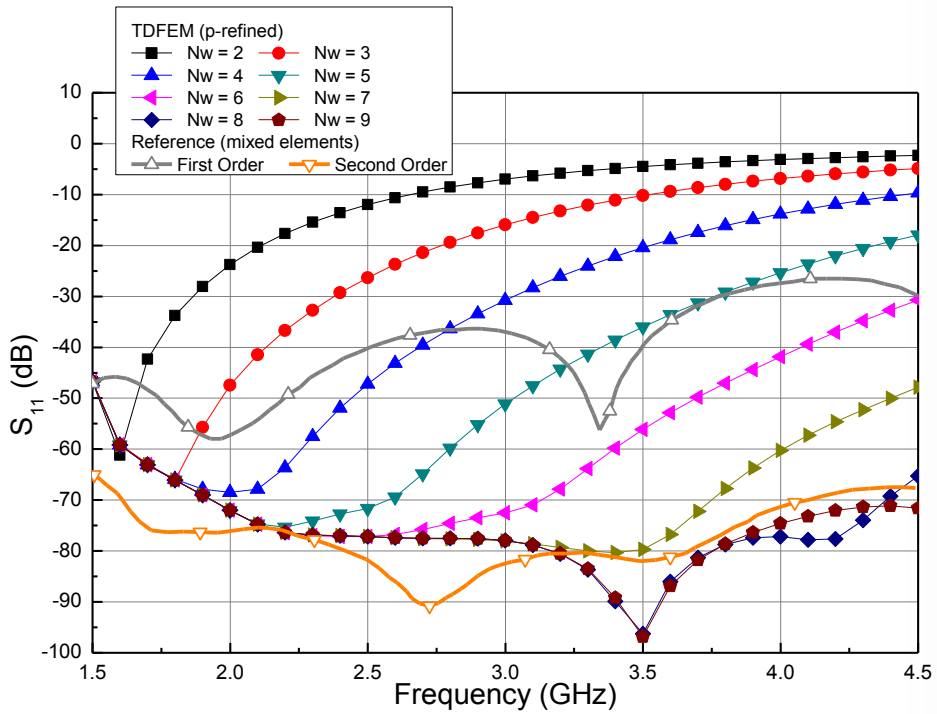


$$N_u = 6, \quad N_v = 4 \qquad N_w = 2 - 9$$

Modulated Gaussian Pulse

$$E_0(t) = e^{-4\left(\frac{t-t_0}{\sigma}\right)^2} \sin[2\pi f_c (t - t_0)] \ \text{V/m}$$

$$f_c = 3\,\text{GHz} \quad \Delta f = 2.5\,\text{GHz} \quad \sigma = 4/(\pi \Delta f) \quad t_0 = 1.4\sigma$$



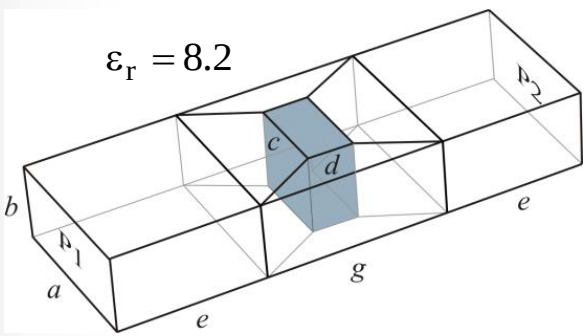
$$\lambda_g = 7.1\,\text{cm}$$

Reference: Z. Lou and J. M. Jin, "An accurate waveguide port boundary condition for the time-domain finite-element method," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 9, pp. 3014–3023, September 2005.

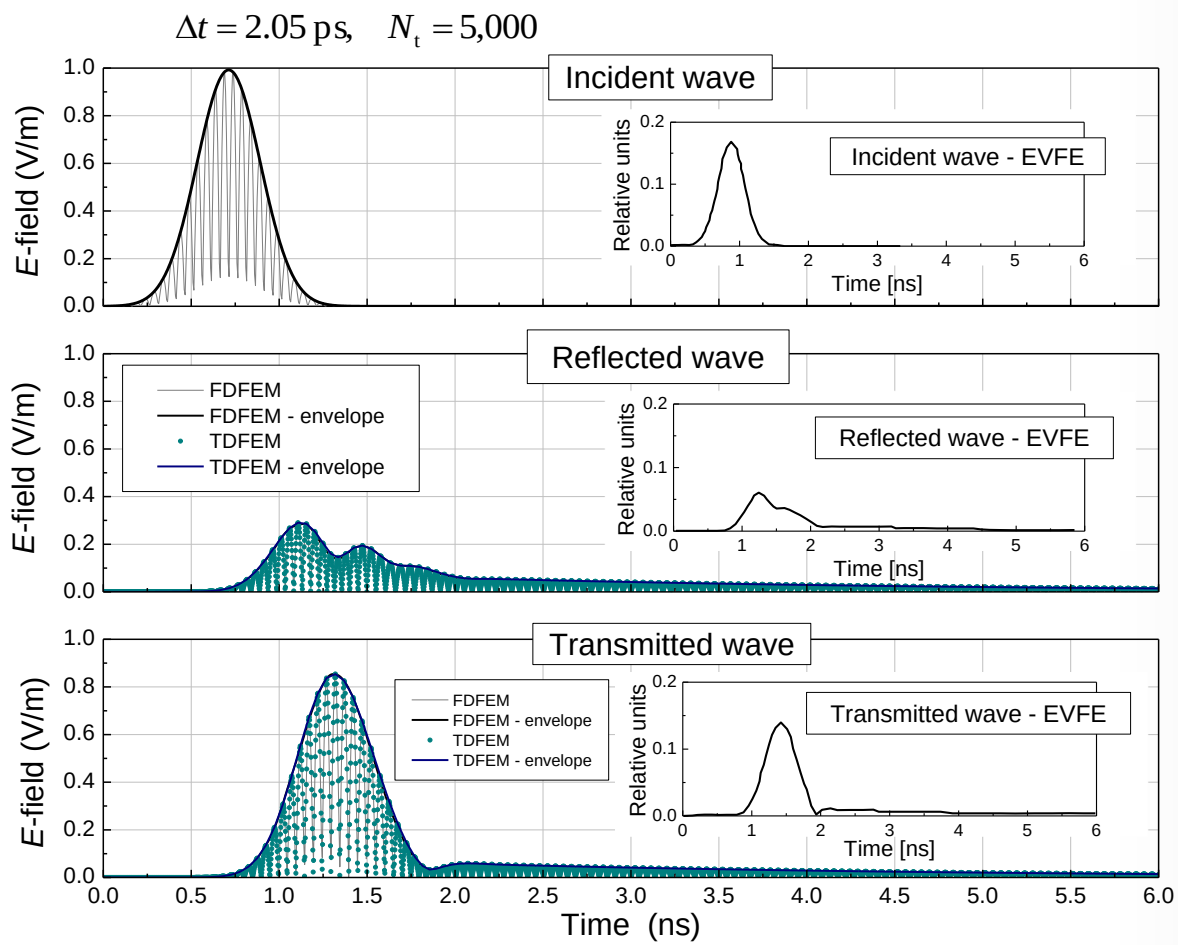
WR-90 Waveguide with Dielectric Post

$a = 22.86\,\text{mm} \quad b = 10.16\,\text{mm} \quad c = 12\,\text{mm}$

$d = 6\,\text{mm} \quad e = 45.72\,\text{mm} \quad g = 24\,\text{mm}$



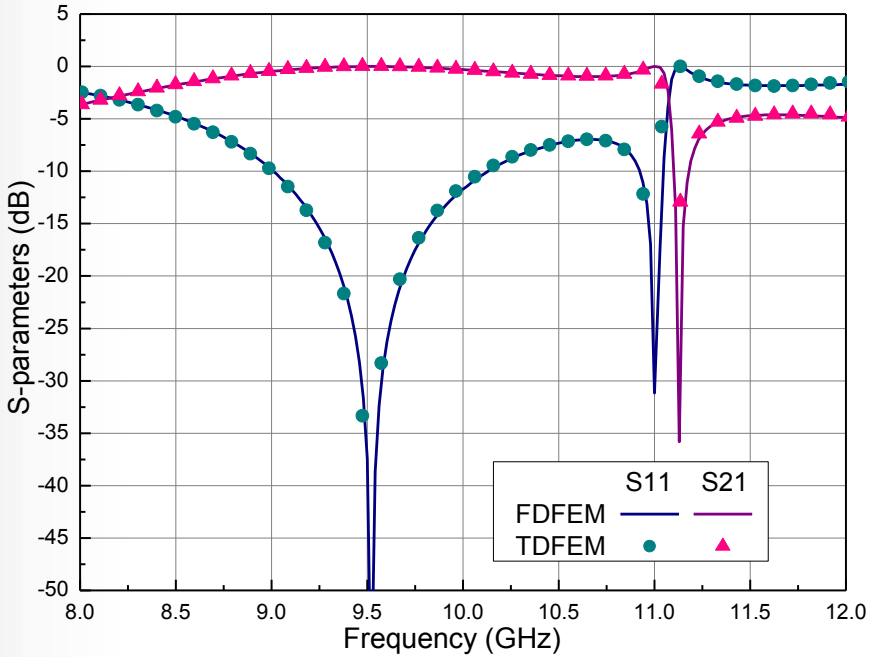
- **Modulated Gaussian pulse**
- **Single mode window**
- **Polynomial orders ranging from 4 to 7**
- **Totally 1,791 FEM unknowns**

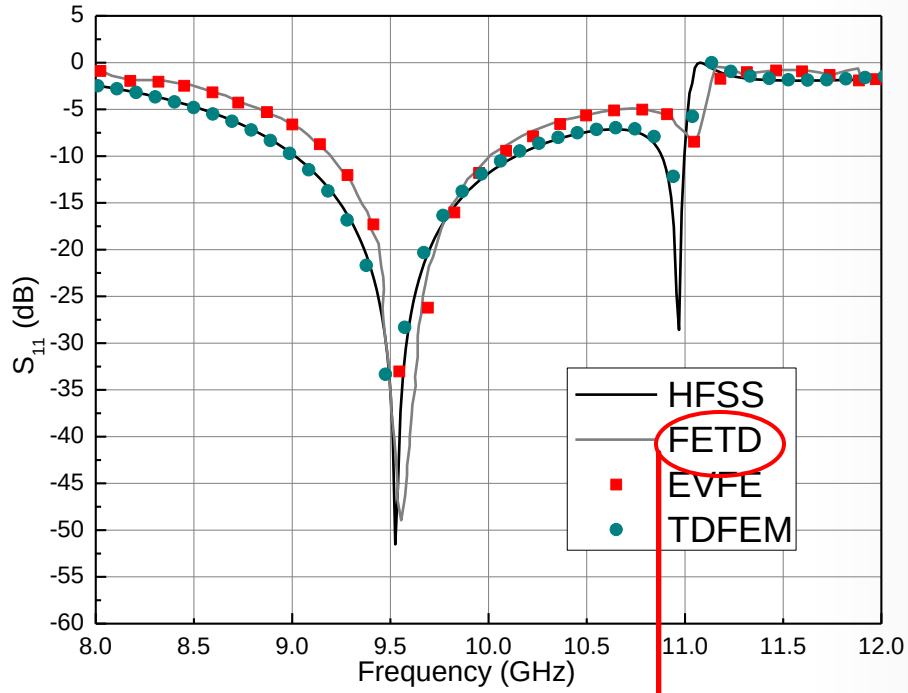


FDFEM: E. M. Klopf, S. B. Manić, M. M. Ilić, and B. M. Notaroš, "Efficient time-domain analysis of waveguide discontinuities using higher order FEM in frequency domain," *Progress In Electromagnetics Research*, vol. 120, pp. 215–234, 2011.

WR-90 Waveguide with Dielectric Post (cont'd)

$$f_s = 1/\Delta t = 488.52\ \text{GHz}, \quad N_f = N_t = 5{,}000$$

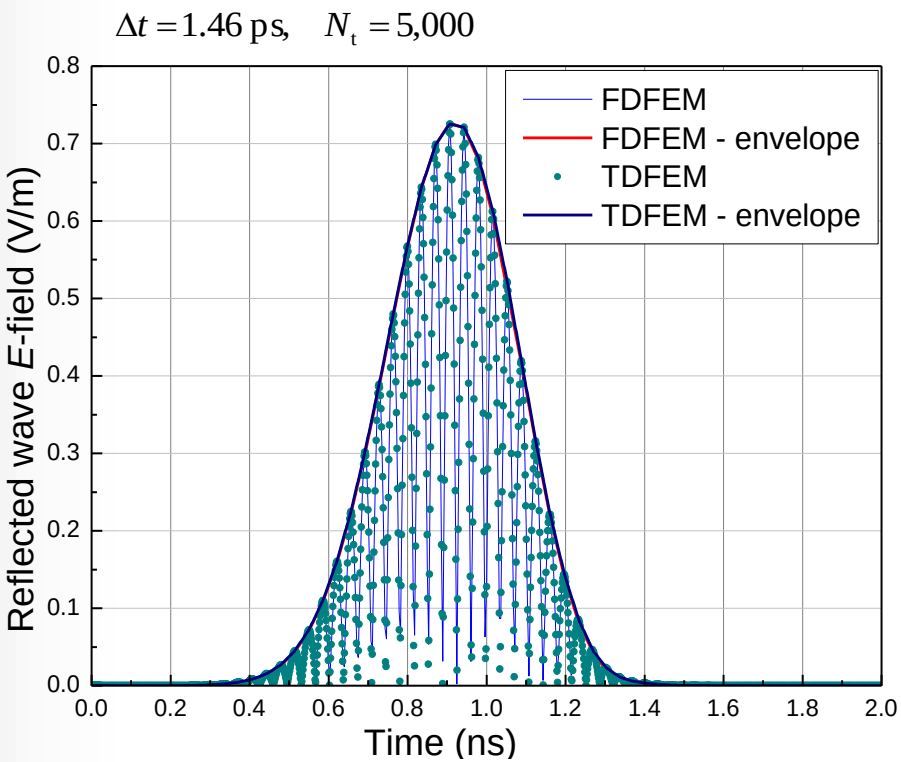


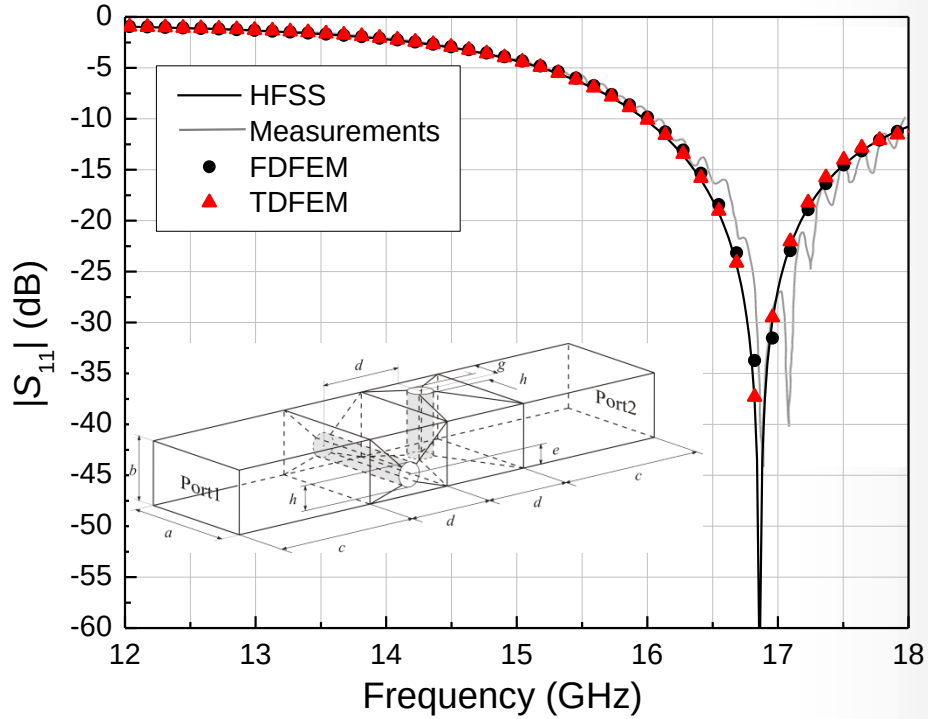


Small domain approximation,
72,373 tetrahedral elements

Reference: H. P. Tsai, Y. Wang, and T. Itoh, "Efficient analysis of microwave passive structures using 3-D envelope-finite element (EVFE)," *IEEE Transaction on Microwave Theory and Techniques*, vol. 50, no. 12, pp. 2721–2727, December 2002.

WR-62 Waveguide with Two Crossed Metallic Posts





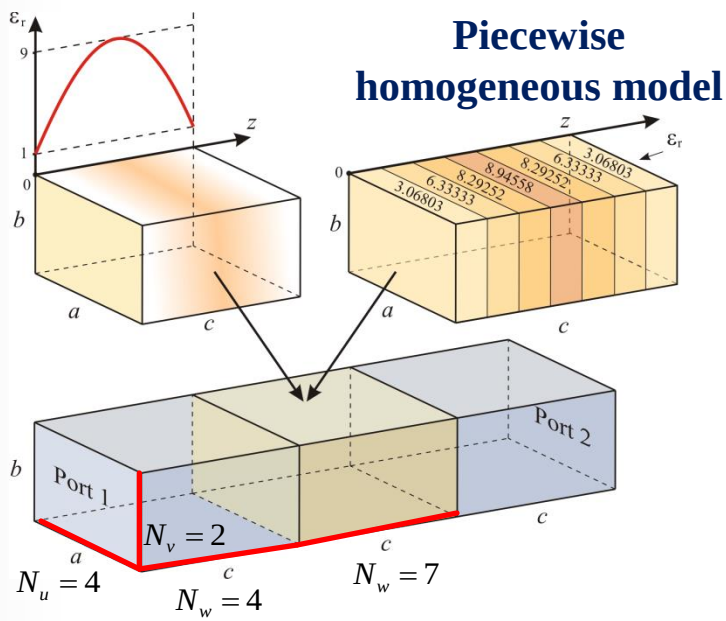
TDFEM: Polynomial orders ranging from 2 to 5, yielding 1,184 unknowns

Measurements: R. Bunger and F. Arndt, "Moment-method analysis of arbitrary 3-D metallic *N*-port waveguide structures," *IEEE Transactions on Microwave Theory and Techniques*, vol. 48, no. 4, pp. 531–537, April 2000.

WR-15 Waveguide with Continuously Inhomogeneous Dielectric Slab

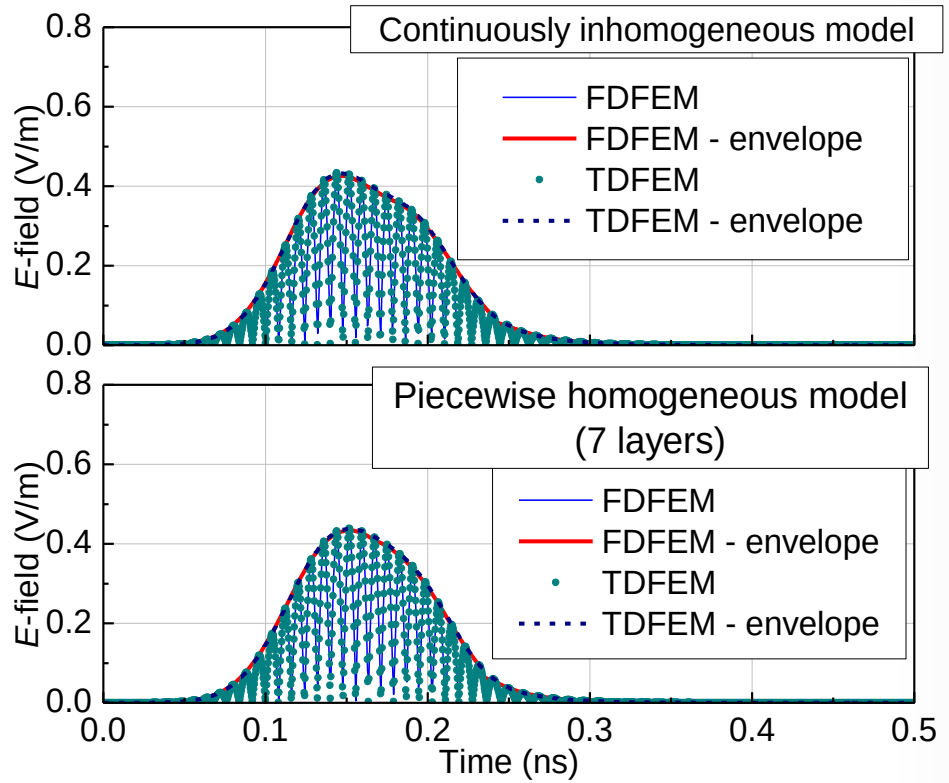
Continuously inhomogeneous model

Piecewise homogeneous model



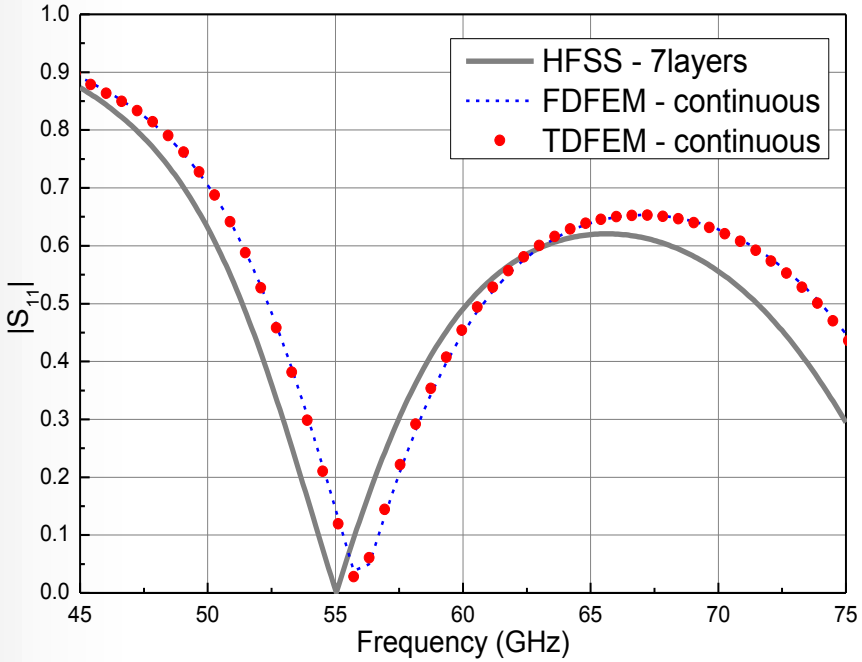
$a = 3.76$ mm, $b = 1.88$ mm, $c = 2.5$ mm

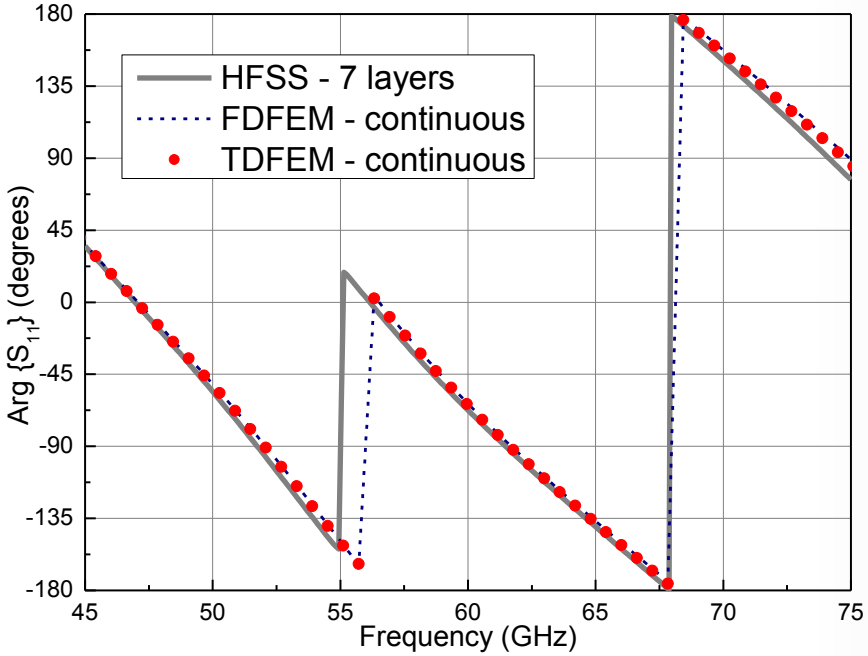
$\Delta t = 0.33$ ps, $N_t = 5{,}000$



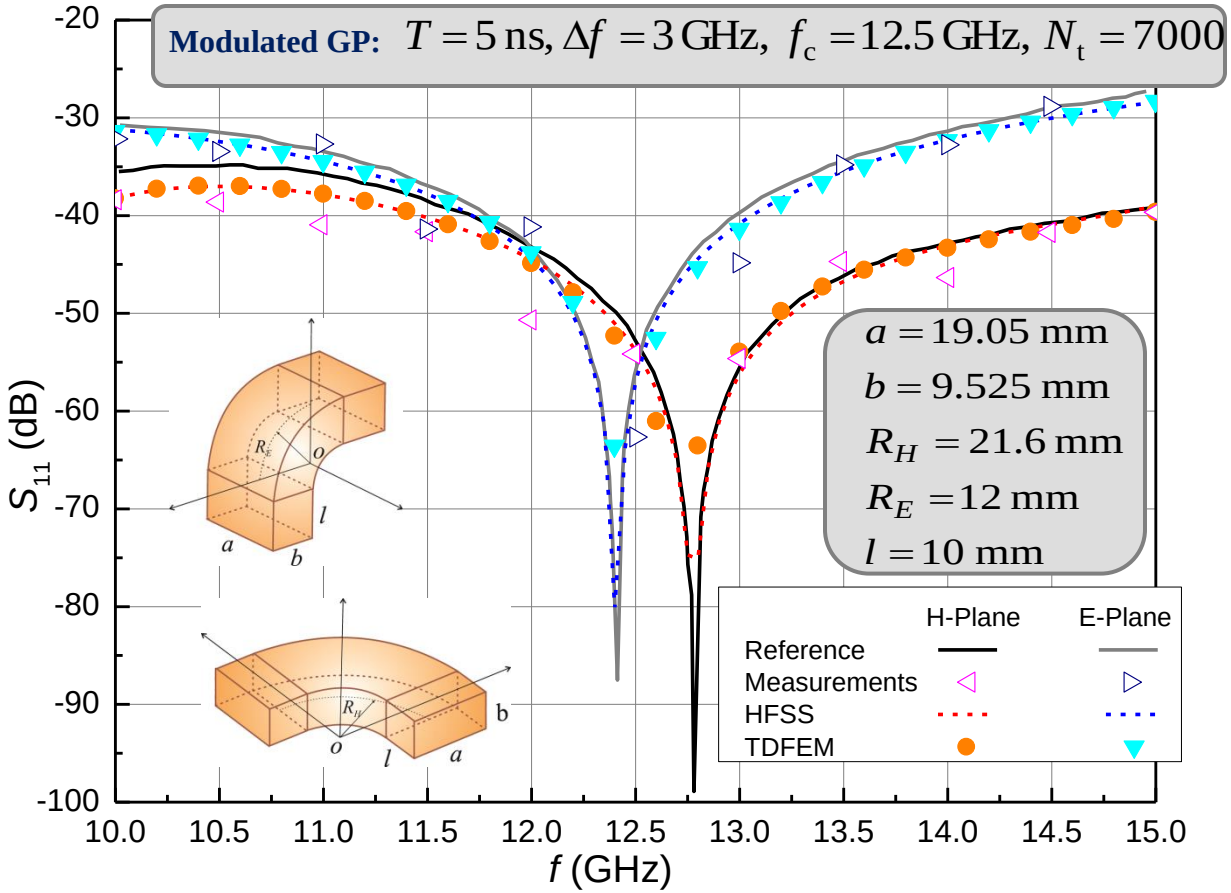
Continuously inhomogeneous model requires 205 unknowns, while piecewise homogeneous model results in 569 unknowns

WR-15 Waveguide with Continuously Inhomogeneous Dielectric Slab (cont'd)





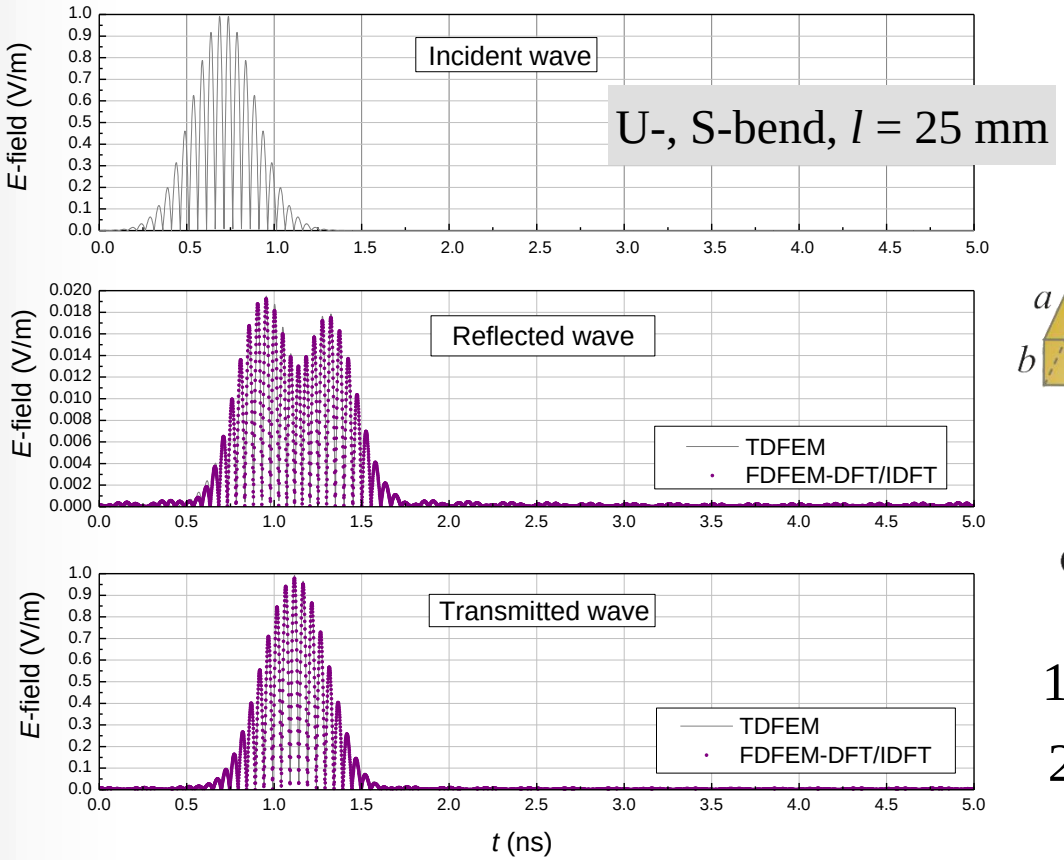
90º *H*-, *E*-Plane WR-75 Bends

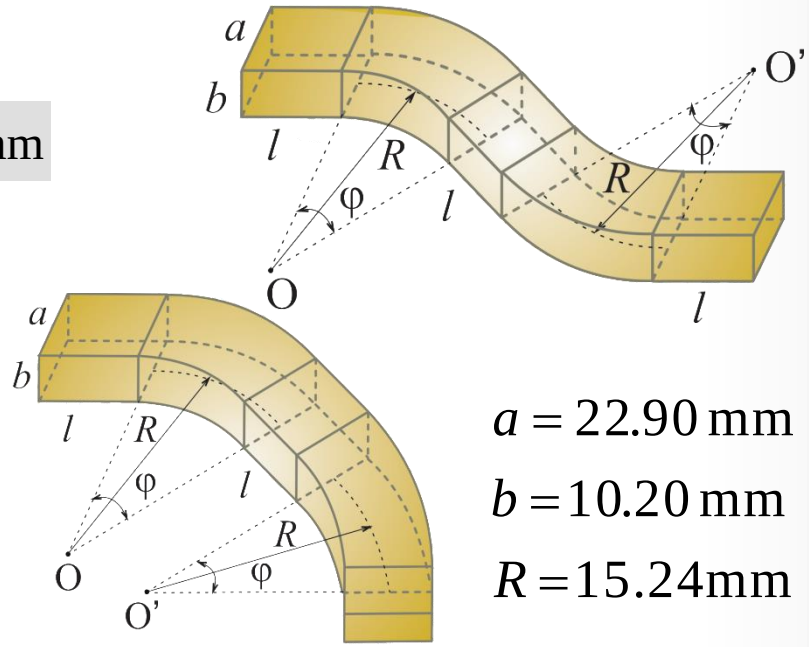


TDFEM: 5 second order hexahedral elements, totally 2,108 unknowns (*N* = 5, 7)
HFSS: 2,293 tetrahedral elements, totally 14,204 unknowns (second order basis functions)

Reference, Measurements: B. Gimeno and M. Guglielmi, "Multimode Equivalent Network Representation for *H*- and *E*-Plane Uniform Bends in Rectangular Waveguide" *IEEE Transactions on Microwave Theory and Techniques*, vol. 44, no. 10, pp. 1679-1687, October 1996.

30º Cascaded *H*-Plane WR-90 Bends





$a = 22.90\,\text{mm}$

$b = 10.20\,\text{mm}$

$R = 15.24\text{mm}$

$\varphi = 30^\circ$

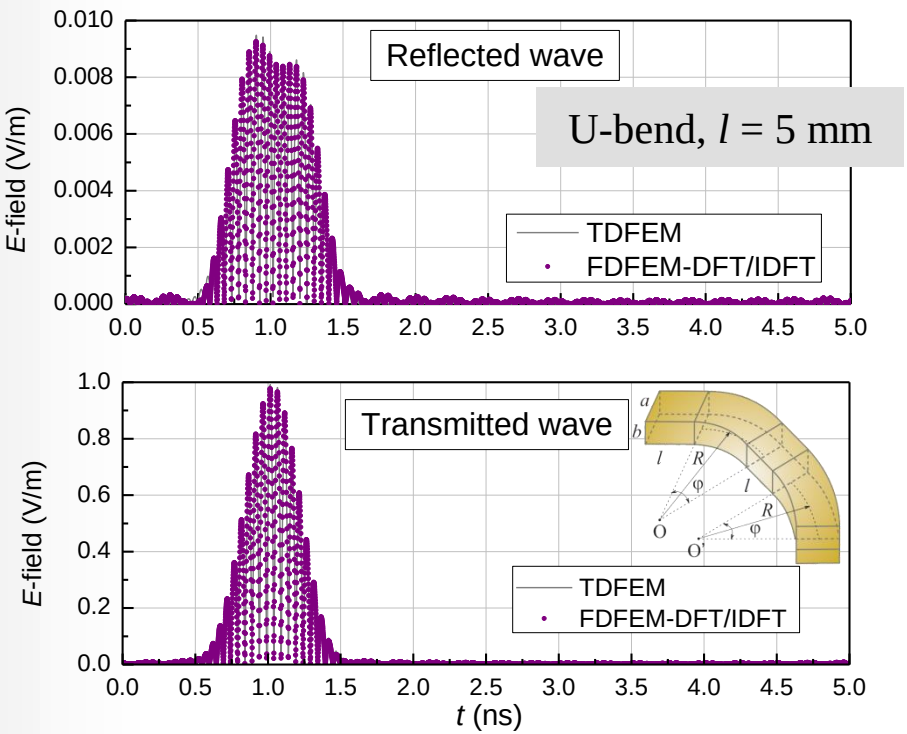
1) $l = 25\,\text{mm}$

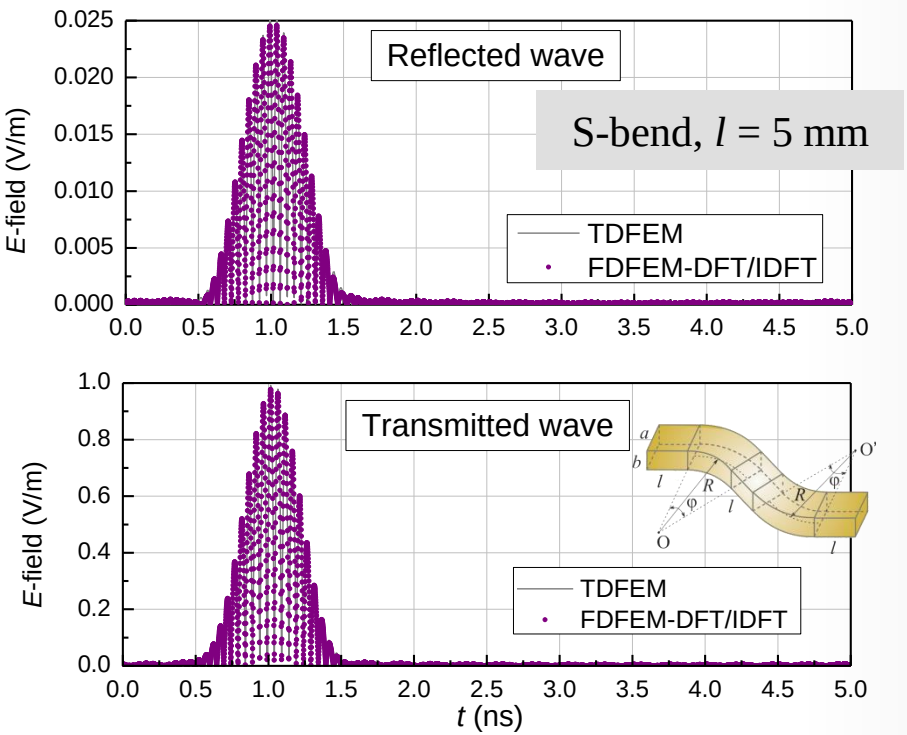
2) $l = 5\,\text{mm}$

Modulated Gaussian Pulse

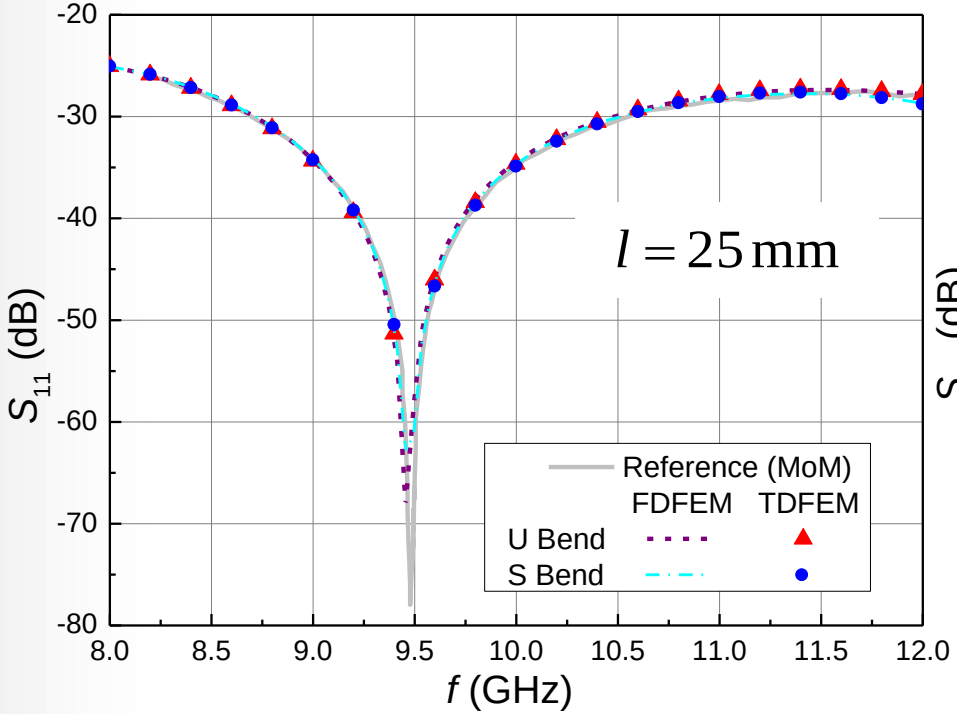
$$T = 5\,\text{ns},\ \Delta f = 2.5\,\text{GHz},\ f_c = 10\,\text{GHz},\ N_t = 5000$$

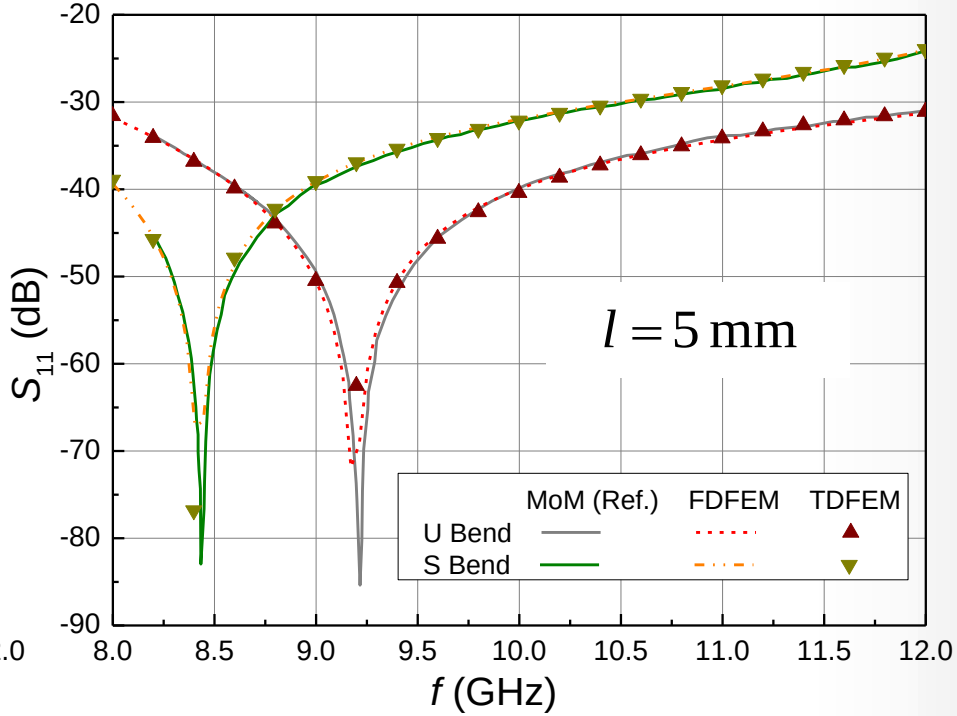
30º Cascaded *H*-Plane WR-90 Bends (cont'd)





30º Cascaded *H*-Plane WR-90 Bends (cont'd)





5 second order hexahedra, totally 2,050 unknowns

Reference: A. Weisshaar, S. M. Goodnick, and V. K. Tripathi, "A Rigorous and Efficient Method of Moments Solution for Curved Waveguide Bends " *IEEE Transactions on Microwave Theory and Techniques*, vol. 40, no. 12, pp. 2200–2206, December 1992.

TD-FEM: Conclusions

- **Novel large-domain *p*-refined Galerkin-type FEM for 3-D modeling in TD**
  - Finite difference time-stepping scheme: Newmark-beta method
  - Mesh truncation: WPBC
  - Generalized curved conformal hexahedral finite elements
- **Arbitrarily loaded and shaped 3-D waveguide structures**
  - Waveguide sections with homogeneous and continuously inhomogeneous dielectric loads
  - Single and cascaded U- and S-type *E*- and *H*-plane waveguide bends
- **Field expansions of orders from 2 to 9, maximum 2,000 unknowns**
- **Number of FEs varies from 1 to 10**
- **Higher order TDFEM solution compared with**:
  - FDFEM-DFT/IDFT
  - Measurements
  - Alternative full-wave numerical solutions in frequency domain
- **Excellent accuracy, efficiency, stability, and convergence**

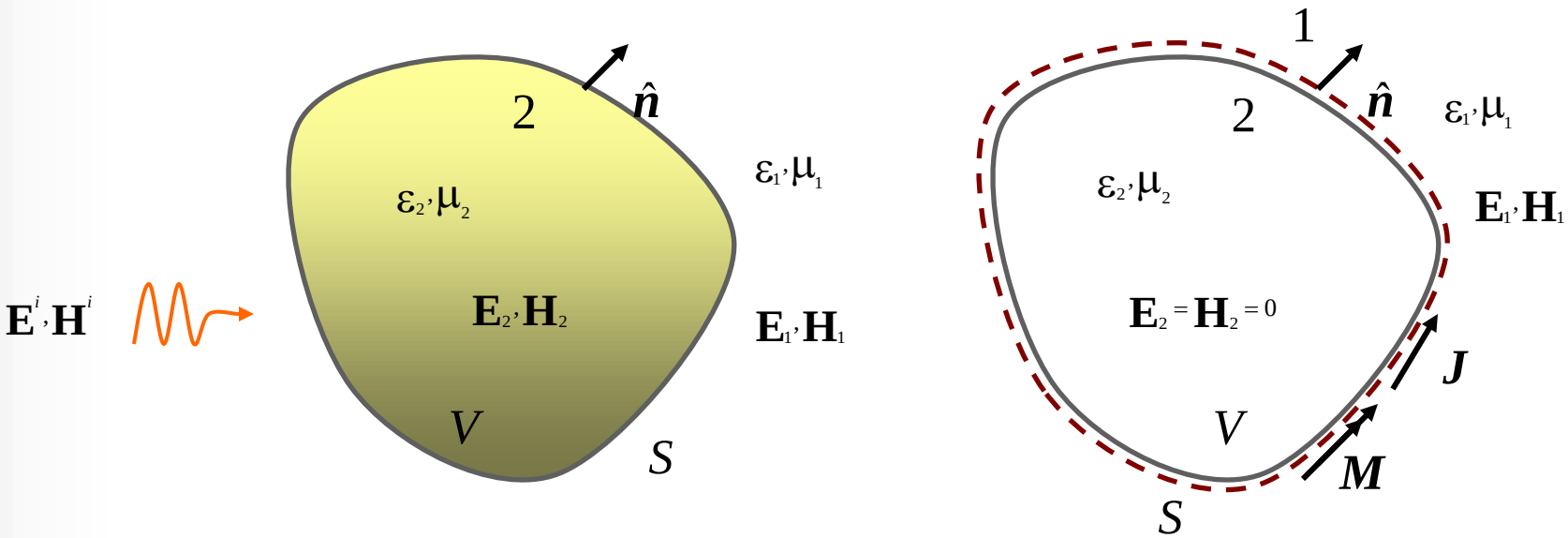
PART 2: HIGHER ORDER MOM FOR EFIE MODELING IN TIME DOMAIN

N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Spatially Large-Domain and Temporally Entire-Domain Electric-Field Integral Equation Method of Moments for 3-D Scattering Analysis in Time Domain," *IEEE Transactions on Antennas and Propagation*, accepted for publication.

Outline

- Higher order large-domain Galerkin-type MoM: open-region problems (scattering and radiation)
  - Discretization of **Surface Integral Equation (SIE) formulation in TD**
- **MoM-TDSIE**, advantages:
  - Analysis of broadband, transient and nonlinear phenomena in a single run (TDFEM)
  - Surface discretization of a structure
  - SIE formulation implicitly satisfies radiation boundary condition
- Theoretical background
- Numerical analysis of PEC scatterers
  - Canonical examples: Cube, Sphere, NASA almond
  - Real-world application: Military tank
- Conclusions

Surface Equivalent Principle



Coupled EFIE/MFIE System of Equations

$$[\mathbf{E}(\mathbf{J}_S(\mathbf{r},t), \mathbf{M}_S(\mathbf{r},t), \varepsilon_1, \mu_1)]_{\text{tang}} + (\mathbf{E}_i(\mathbf{r},t))_{\text{tang}} = [\mathbf{E}(-\mathbf{J}_S(\mathbf{r},t), -\mathbf{M}_S(\mathbf{r},t), \varepsilon_2, \mu_2)]_{\text{tang}},$$

$$[\mathbf{H}(\mathbf{J}_S(\mathbf{r},t), \mathbf{M}_S(\mathbf{r},t), \varepsilon_1, \mu_1)]_{\text{tang}} + (\mathbf{H}_i(\mathbf{r},t))_{\text{tang}} = [\mathbf{H}(-\mathbf{J}_S(\mathbf{r},t), -\mathbf{M}_S(\mathbf{r},t), \varepsilon_2, \mu_2)]_{\text{tang}},$$

$$\mathbf{r} \in S, \quad (\forall)\, t \geq 0$$

MoM-SIE Formulation in TD (MoM-TDSIE)

Coupled EFIE/MFIE System of Equations

$$[\mathbf{E}(\mathbf{J}_S(\mathbf{r},t),\mathbf{M}_S(\mathbf{r},t),\varepsilon_1,\mu_1)]_{\text{tang}}+(\mathbf{E}_i(\mathbf{r},t))_{\text{tang}}=[\mathbf{E}(-\mathbf{J}_S(\mathbf{r},t),-\mathbf{M}_S(\mathbf{r},t),\varepsilon_2,\mu_2)]_{\text{tang}},$$

$$[\mathbf{H}(\mathbf{J}_S(\mathbf{r},t),\mathbf{M}_S(\mathbf{r},t),\varepsilon_1,\mu_1)]_{\text{tang}}+(\mathbf{H}_i(\mathbf{r},t))_{\text{tang}}=[\mathbf{H}(-\mathbf{J}_S(\mathbf{r},t),-\mathbf{M}_S(\mathbf{r},t),\varepsilon_2,\mu_2)]_{\text{tang}},$$

$$\mathbf{r}\in S,\quad (\forall)\, t\geq 0$$

Scattered Electric Field

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}-\nabla\Phi-\frac{1}{\varepsilon}\nabla\times\mathbf{F}$$

Scattered Magnetic Field

$$\mathbf{H}=-\frac{\partial\mathbf{F}}{\partial t}-\nabla U+\frac{1}{\mu}\nabla\times\mathbf{A}$$

Axillary Potentials

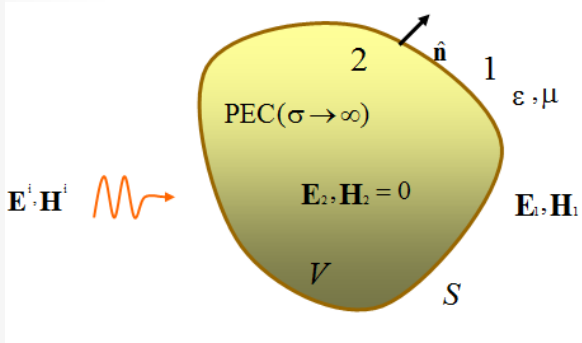
$$\mathbf{A}(\mathbf{r},t)=\frac{\mu}{4\pi}\int_S\frac{\mathbf{J}_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S\qquad \mathbf{F}(\mathbf{r},t)=\frac{\varepsilon}{4\pi}\int_S\frac{\mathbf{M}_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S$$

$$\Phi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_S\frac{\rho_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S=-\frac{1}{4\pi\varepsilon}\int_S\int_0^{t-R/c}\frac{\nabla_s\cdot\mathbf{J}_S(\mathbf{r}',t')}{R}\,\mathrm{d}t'\mathrm{d}S$$

$$U(\mathbf{r},t)=\frac{1}{4\pi\mu}\int_S\frac{\rho_{mS}(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S=-\frac{1}{4\pi\mu}\int_S\int_0^{t-R/c}\frac{\nabla_s\cdot\mathbf{M}_S(\mathbf{r}',t')}{R}\,\mathrm{d}t'\mathrm{d}S$$

MoM-TDEFIE

$$[\mathbf{E}(\mathbf{J}_S(\mathbf{r},t),\varepsilon\ ,\mu\ )]_{\text{tang}} + (\mathbf{E}_i(\mathbf{r},t))_{\text{tang}} = 0, \quad \mathbf{r} \in S, \quad (\forall)\, t \geq 0$$



Scattered Electric Field

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi$$

Magnetic Vector Potential

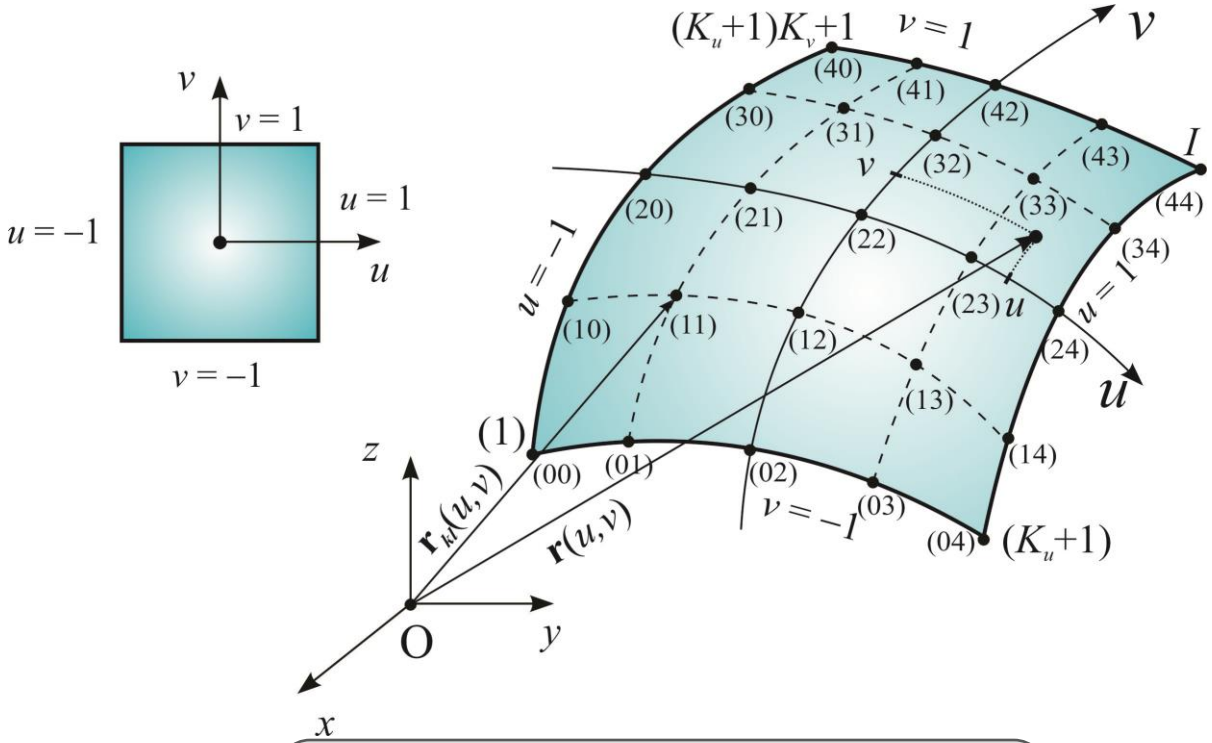
$$\mathbf{A}(\mathbf{r},t) = \frac{\mu}{4\pi} \int_S \frac{\mathbf{J}_S(\mathbf{r}',t-R/c)}{R}\, \mathrm{d}S$$

Electric Scalar Potential

$$\Phi(\mathbf{r},t) = \frac{1}{4\pi\,\varepsilon} \int_S \frac{\rho_S(\mathbf{r}',t-R/c)}{R}\, \mathrm{d}S = -\frac{1}{4\pi\,\varepsilon} \int_S \int_0^{t-R/c} \frac{\nabla_s \cdot \mathbf{J}_S(\mathbf{r}',t')}{R}\, \mathrm{d}t'\mathrm{d}S$$

Higher Order Geometrical Modeling in MoM

Generalized Curvilinear Quadrilateral



$$\boldsymbol{r}(u,v) = \sum_{i=1}^{M} \boldsymbol{r}_i p_i(u,v) = \sum_{k=0}^{K_u} \sum_{l=0}^{K_v} \boldsymbol{r}_{kl} u^k v^l$$

M. Djordjevic and B. M. Notaros, "Double Higher Order Method of Moments for Surface Integral Equation Modeling of Metallic and Dielectric Antennas and Scatterers," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 8, August 2004, pp. 2118-2129.

Higher Order Temporal and Spatial Discretization

Axillary vector function – Hertz Vector

$$\mathbf{J}_S(\mathbf{r},t) = \frac{\partial \mathbf{h}(\mathbf{r},t)}{\partial t}, \qquad \rho_S(\mathbf{r},t) = -\nabla \cdot \mathbf{h}(\mathbf{r},t)$$

Higher Order Spatial Basis Functions

$$\mathbf{h}(u,v,t) = \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} h_{uij}(t)\,\mathbf{f}_{uij}(u,v) + \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} h_{vij}(t)\,\mathbf{f}_{vij}(u,v)$$

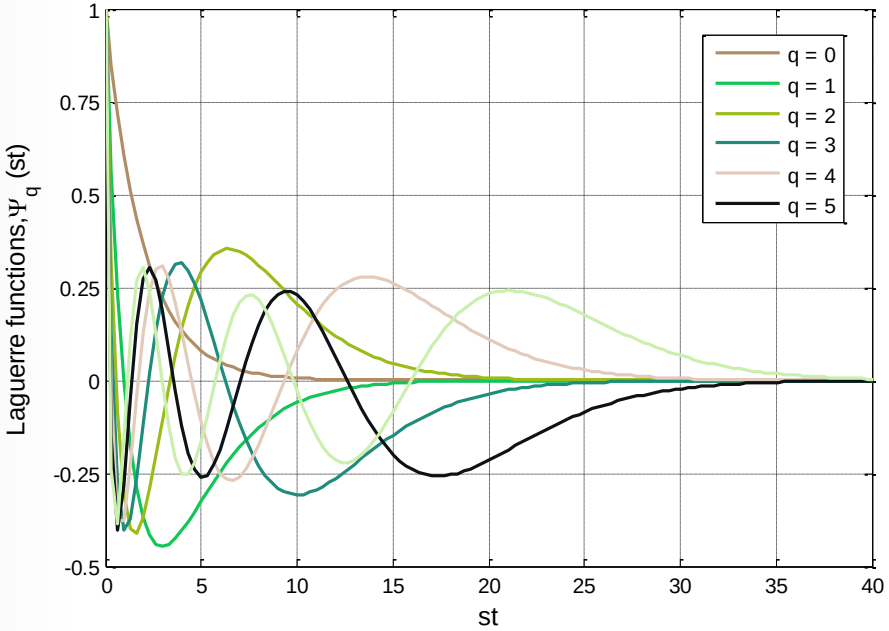
Higher Order Temporal Basis Functions

J. A. Stratton, "Electromagnetic theory", New York: *McGraw-Hill Book*, 1941.

Higher Order Temporal Basis Functions

Associated Laguerre Functions

$$h_{uij}(t) = \sum_{q=0}^{M} h_{uij,q}\,(\Psi_q(st) - 2\Psi_{q+1}(st) + \Psi_{q+2}(st)), \quad \Psi_q(st) = \mathrm{e}^{-st/2} L_q(st)$$



Laguerre Polynomials

$$L_q(x) = \frac{\mathrm{e}^x}{q!}\frac{\mathrm{d}^q}{\mathrm{d}x^q}(x^q \mathrm{e}^{-x}), \quad q \ge 0,\ x \ge 0$$

$$L_0(x) = 1, \quad L_1(x) = 1 - x,$$

$$L_q(x) = \frac{1}{q}((2q-1-x)L_{q-1}(x) - (q-1)L_{q-2}), \quad q \ge 2$$

- ✓ Causality
- ✓ Orthogonality
- ✓ Recursive computation
- ✓ Convergence

T. K. Sarkar and J. Koh, "Generation of a wide-band electromagnetic response through a Laguerre expansion using early-time and low-frequency data", *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 5, pp. 1408-1416, May 2002.

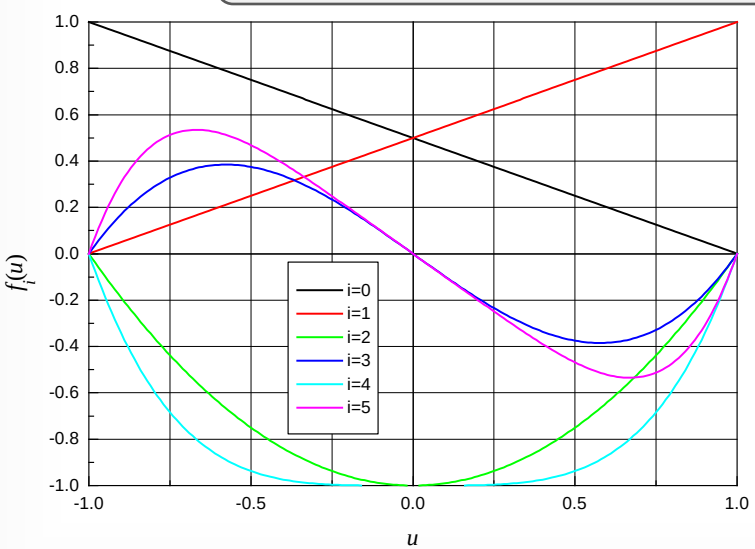
Higher Order Spatial Basis Functions

Polynomial Vector Basis Functions

$$\mathbf{f}_{uij}(u,v) = \frac{P_i(u)v^j}{\Im(u,v)}\mathbf{a}_u(u,v), \qquad P_i(u) = \begin{cases} 1-u, & i=0 \\ u+1, & i=1 \\ u^i-1, & i \geq 2, \text{even} \\ u^i-u, & i \geq 3, \text{odd} \end{cases}, \quad -1 \leq u,v \leq 1$$

$$\mathbf{f}_{vij}(u,v) = \frac{u^i P_j(v)}{\Im(u,v)}\mathbf{a}_v(u,v),$$

$$\mathbf{a}_u(u,v) = \partial\mathbf{r}(u,v)/\partial u, \quad \mathbf{a}_v(u,v) = \partial\mathbf{r}(u,v)/\partial v, \quad \Im(u,v) = |\mathbf{a}_u(u,v)\times\mathbf{a}_v(u,v)|$$



- ✓ **Hierarchical-type**
- ✓ **Divergence-conforming**

M. Djordjevic and B. M. Notaros, "Double Higher Order Method of Moments for Surface Integral Equation Modeling of Metallic and Dielectric Antennas and Scatterers," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 8, August 2004, pp. 2118-2129.

Discretized TDEFIE

$$[\mathbf{E}(\mathbf{J}_S(\mathbf{r},t),\varepsilon\ ,\mu\ )]_{\text{tang}} + (\mathbf{E}_{\text{i}}(\mathbf{r},t))_{\text{tang}} = 0, \quad \mathbf{r} \in S, \quad (\forall)\, t \geq 0$$

Temporal and Spatial Discretization

$$\left[\frac{\mu s^2}{4\pi} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \sum_{q=0}^{M} \left(h_{uij,q} + 2h_{uij,q-1} + h_{uij,q-2} \right) \int_S \frac{1}{4R} \Psi_q(s\tau)\, \mathbf{f}_{uij}(\mathbf{r}')\mathrm{d}S \right.$$
$$\left. - \frac{\nabla}{4\pi\varepsilon} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \sum_{q=0}^{M} \left(h_{uij,q} - 2h_{uij,q-1} + h_{uij,q-2} \right) \int_S \frac{1}{4R} \Psi_q(s\tau) \nabla'_{\text{s}} \cdot \mathbf{f}_{uij}(\mathbf{r}')\mathrm{d}S \right]_{\text{tang}} = (\mathbf{E}_{\text{i}}(\mathbf{r},t))_{\text{tang}}.$$

Full Time-Space MoM Galerkin Testing

Full Time-Space Galerkin Testing

Generalized Galerkin Impedances

$$Z^{A}_{mn,pq} = \frac{\mu}{4\pi}\int_{S_m}\int_{S_n}\int_{sR/c}^{\infty}\frac{1}{R}\mathbf{f}_m(u_m,v_m)\cdot\mathbf{f}_n(u_n,v_n)\Psi_p(st)\Psi_q(s\tau)\,\mathrm{d}(st)\,\mathrm{d}S_n\,\mathrm{d}S_m$$

$$Z^{\Phi}_{mn,pq} = \frac{1}{4\pi\varepsilon}\int_{S_m}\int_{S_n}\int_{sR/c}^{\infty}\frac{1}{R}\left(\nabla\cdot\mathbf{f}_m(u_m,v_m)\right)\left(\nabla\cdot\mathbf{f}_n(u_n,v_n)\right)\Psi_p(st)\Psi_q(s\tau)\,\mathrm{d}(st)\,\mathrm{d}S_n\,\mathrm{d}S_m$$

Temporal Integrals

$$I_{pq}(sR/c) = \int_{sR/c}^{\infty}\Psi_p(st)\Psi_q(st-sR/c)\,\mathrm{d}(st) =$$

$$= \begin{cases} \mathrm{e}^{-sR/(2c)}, & q = p \\ \mathrm{e}^{-sR/(2c)}\left(L_{p-q}(sR/c) - L_{p-q-1}(sR/c)\right), & q < p \\ 0, & q > p \end{cases}$$

Generalized Voltages

$$V^{\mathrm{i}}_{m,p} = \int_{S_m}\mathbf{f}_m\cdot\int_{t=0}^{\infty}\Psi_p(st)\mathbf{E}_{\mathrm{i}}(\mathbf{r},t)\,\mathrm{d}(st)\,\mathrm{d}S_m$$

Higher-order Integrals

$$\xi_i(i_m,j_m,i_n,j_n) = \int_{u_{1m}}^{u_{2m}}\int_{v_{1m}}^{v_{2m}}\int_{u_{1n}}^{u_{2n}}\int_{v_{1n}}^{v_{2n}} u_m^{i_m}v_m^{j_m}u_n^{i_n}v_n^{j_n}g_i(R)\,\mathrm{d}u_n\,\mathrm{d}v_n\,\mathrm{d}u_m\,\mathrm{d}v_m, \quad i = 1,2$$

Unconditionally Stable MOD Solution of MoM-TDEFIE

$$[Z_{mn}]\{h_{n,p}\} = \{V^{\mathrm{i}}_{m,p}\} - [Z^{A,1}_{mn}](2\{h_{n,p-1}\} + \{h_{n,p-2}\}) - [Z^{\Phi,1}_{mn}](\{h_{n,p-2}\} - 2\{h_{n,p-1}\})$$
$$- \sum_{q=0}^{p-1} [Z^{A,2}_{mn,M-p+q}](\{h_{n,q}\} + 2\{h_{n,q-1}\} + \{h_{n,q-2}\})$$
$$- \sum_{q=0}^{p-1} [Z^{\Phi,2}_{mn,M-p+q}](\{h_{n,q}\} - 2\{h_{n,q-1}\} + \{h_{n,q-2}\}),$$

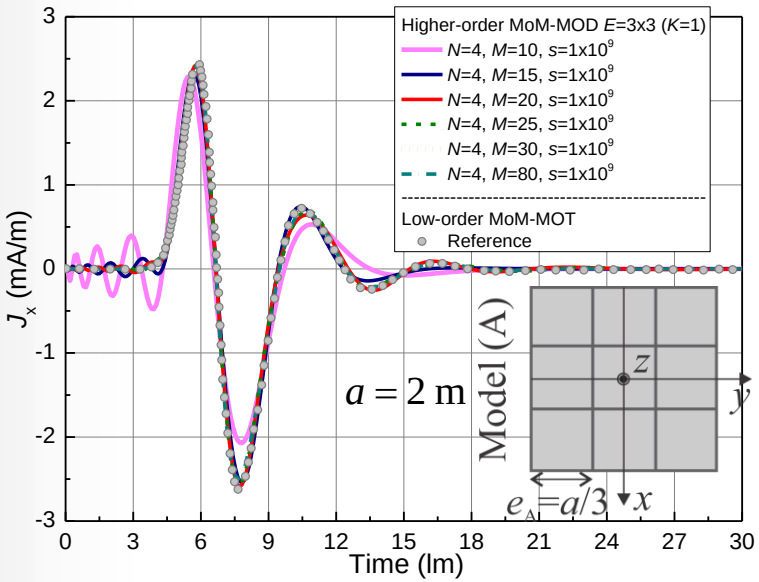
$$p = 0,1,\ldots,M, \quad q = 0,1,\ldots,p-1, \quad m = 1,2,\ldots,N_{\mathrm{MoM}}, \quad n = 1,2,\ldots,N_{\mathrm{MoM}}$$

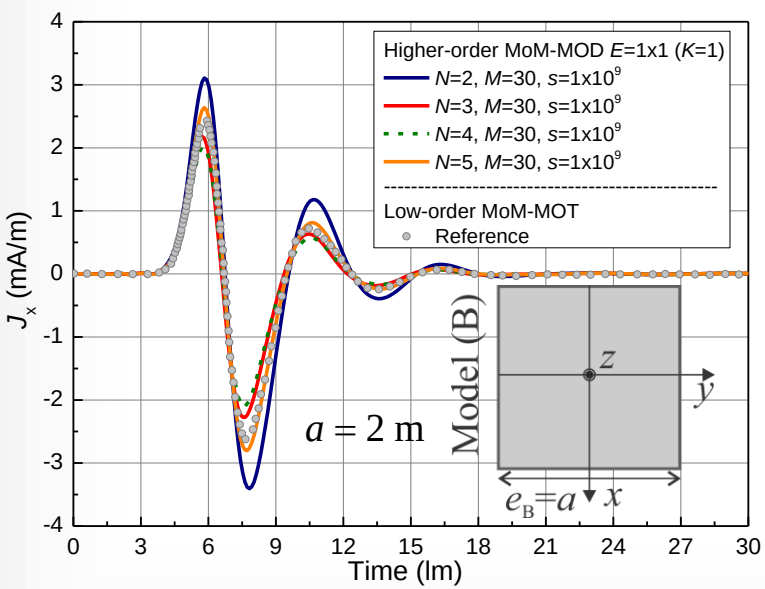
Post-processing: Current Coefficients

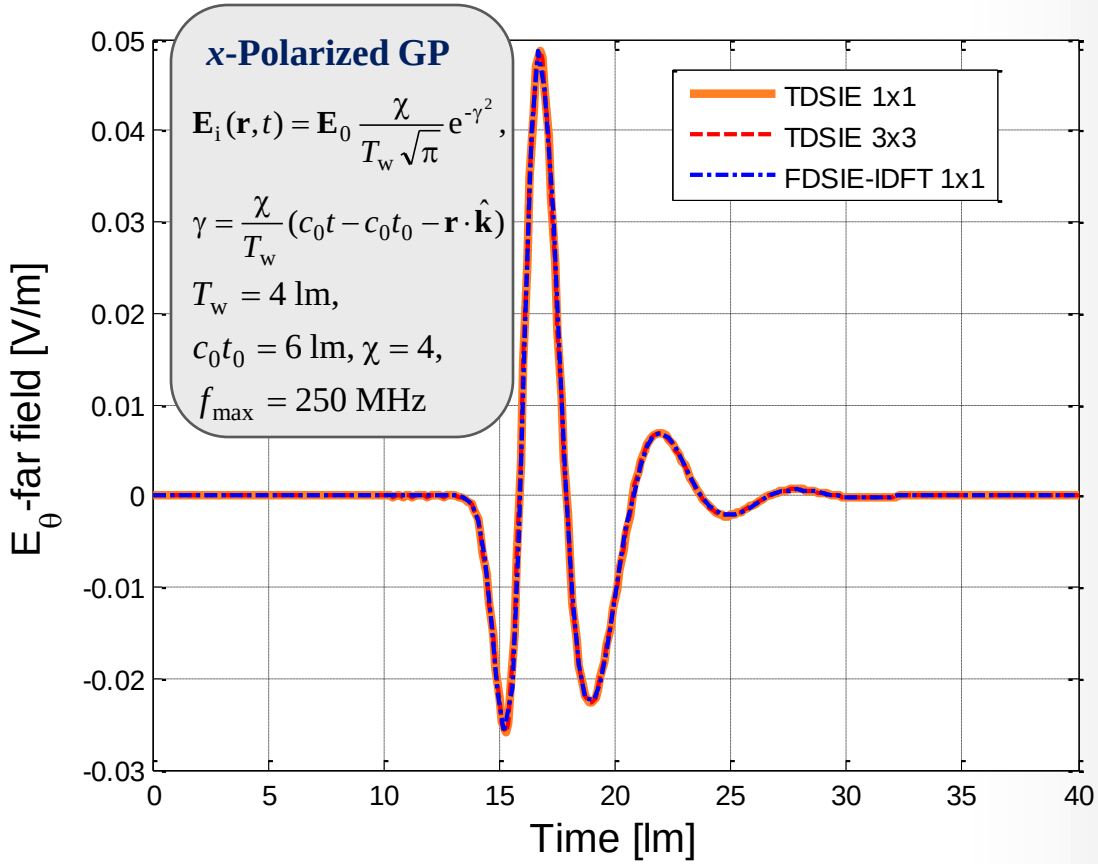
$$\mathbf{J}_{\mathrm{S}}(\mathbf{r},t) = \frac{\partial \mathbf{h}(\mathbf{r},t)}{\partial t} = \sum_{q=0}^{M} \frac{s}{2} h_{n,q} \left(\Psi_q(st) - \Psi_{q+2}(st)\right)$$

N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Spatially Large-Domain and Temporally Entire-Domain Electric-Field Integral Equation Method of Moments for 3-D Scattering Analysis in Time Domain," *IEEE Transactions on Antennas and Propagation*, accepted for publication.

PEC Plate, Temporal and Spatial Convergence







Reference: Low-Order Small-Domain TDEFIE-MOT

Reference: G. Manara, A. Monorchio, and R. Reggiannini, "A space-time discretization criterion for a stable time-marching solution of the electric field integral equation," *IEEE Transactions on Antennas and Propagation*, vol. 45, no. 3, pp. 527–532, March 1997.

PEC Cube, Non-resonant Region

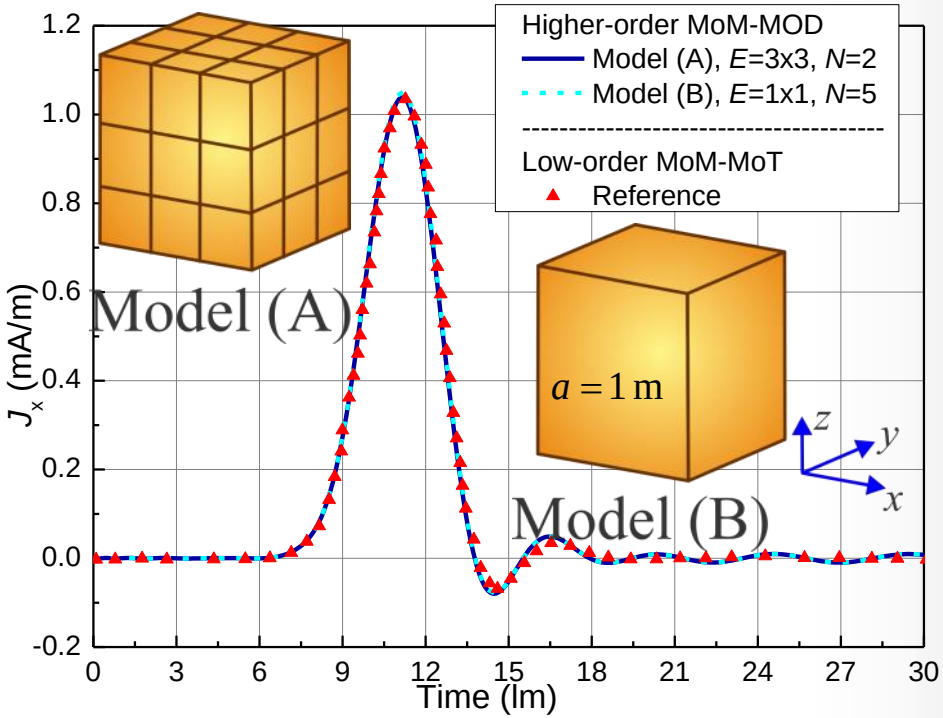
Transient Response of *x*-directed Surface Current Density

Higher-Order Large-Domain MoM-TDEFIE-MOD

x-Polarized Gaussian Pulse

$$\mathbf{E}_{\mathrm{i}}(\mathbf{r},t) = \mathbf{E}_0 \frac{\chi}{T_{\mathrm{w}}\sqrt{\pi}} \mathrm{e}^{-\gamma^2}, \quad \gamma = \frac{\chi}{T_{\mathrm{w}}}(c_0 t - c_0 t_0 - \mathbf{r}\cdot\hat{\mathbf{k}})$$

$$T_{\mathrm{w}} = 8\ \mathrm{lm},\ c_0 t_0 = 12\ \mathrm{lm},\ \chi = 4,\ f_{\max} = 125\ \mathrm{MHz}$$

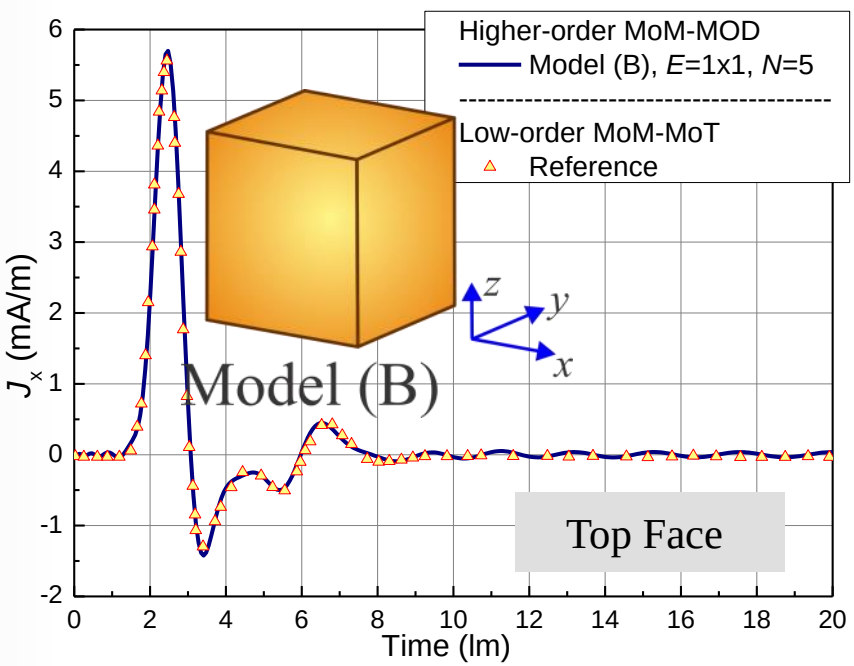


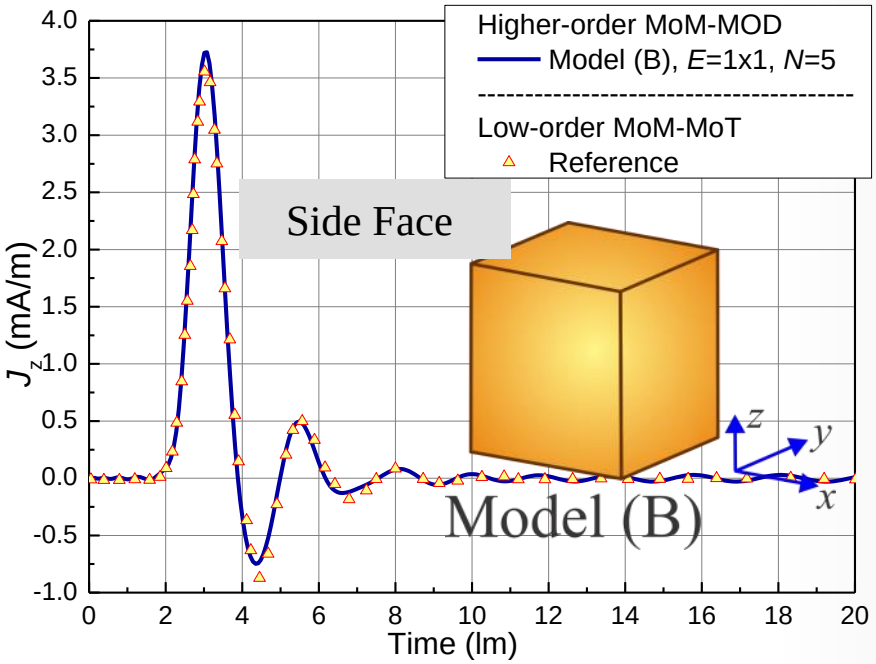
Reference: Low-Order Small-Domain Implicit TDCFIE-MOT

Reference: B. H. Jung and T. K. Sarkar, "Time-domain CFIE for the analysis of transient scattering from arbitrarily shaped 3D conducting objects", *Microwave and Optical Technology Letters*, vol. 34, no. 4, pp. 289-296, August 2002.

PEC Cube, Resonant Region

Gaussian Excitation: $T_w = 2\ \text{lm},\ c_0 t_0 = 3\ \text{lm},\ \chi = 4,\ f_{max} = 500\ \text{MHz}$





Reference: Low-Order Small-Domain: 832 triangular patches, 1,248 spatial unknowns

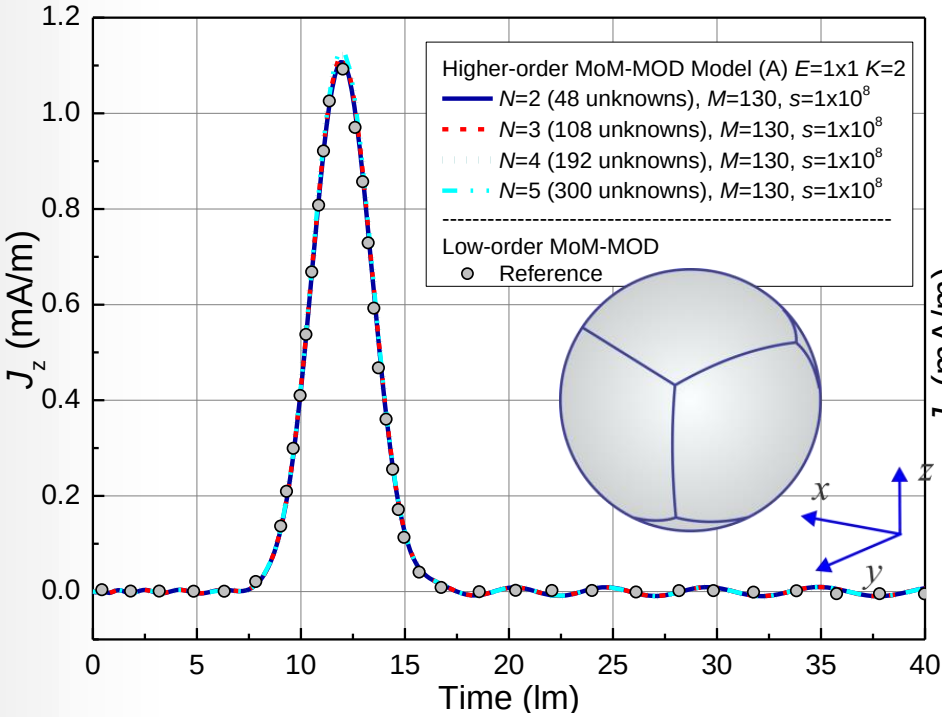
Higher-Order Large-Domain: 6 quadrilateral patches ($e \approx 1.7\ \lambda_0$), 300 unknowns, $N = 5$

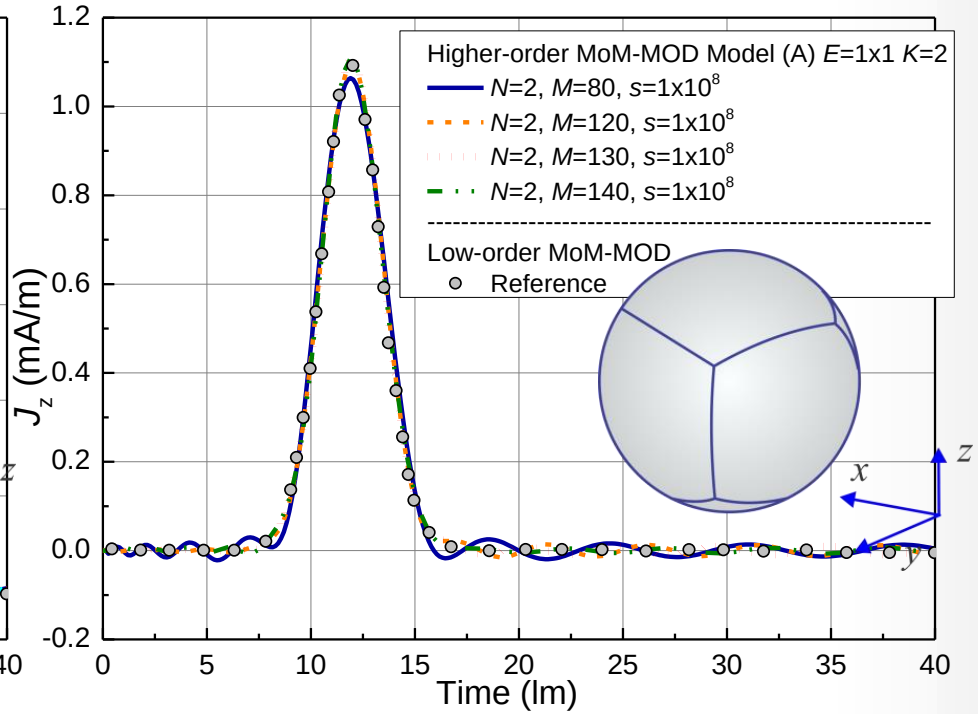
Reference: B. H. Jung and T. K. Sarkar, "Time-domain CFIE for the analysis of transient scattering from arbitrarily shaped 3D conducting objects", *Microwave and Optical Technology Letters*, vol. 34, no. 4, pp. 289-296, August 2002.

PEC Sphere, Non-resonant Region

Spatial and Temporal convergence

Non-resonant Region $T_w = 8\ \text{lm},\ c_0 t_0 = 12\ \text{lm},$
$\chi = 4,\ f_{max} = 125\ \text{MHz}$





Reference: Low-Order Small-Domain: 528 flat triangular patches, 792 spatial unknowns

Higher-Order Large-Domain: 6 quadrilateral patches ($e \approx 0.36\ \lambda_0$), 48 unknowns, $N = 2$

Reference: B. H. Jung, Y. S. Chung, and T. K. Sarkar, "Time-domain EFIE, MFIE, and CFIE formulations using Laguerre polynomials as temporal basis functions for the analysis of transient scattering from arbitrarily shaped conducting structures", *Progress in Electromagnetic Research*, vol. 39, pp. 1-45, 2003.

PEC Sphere, Resonant Region

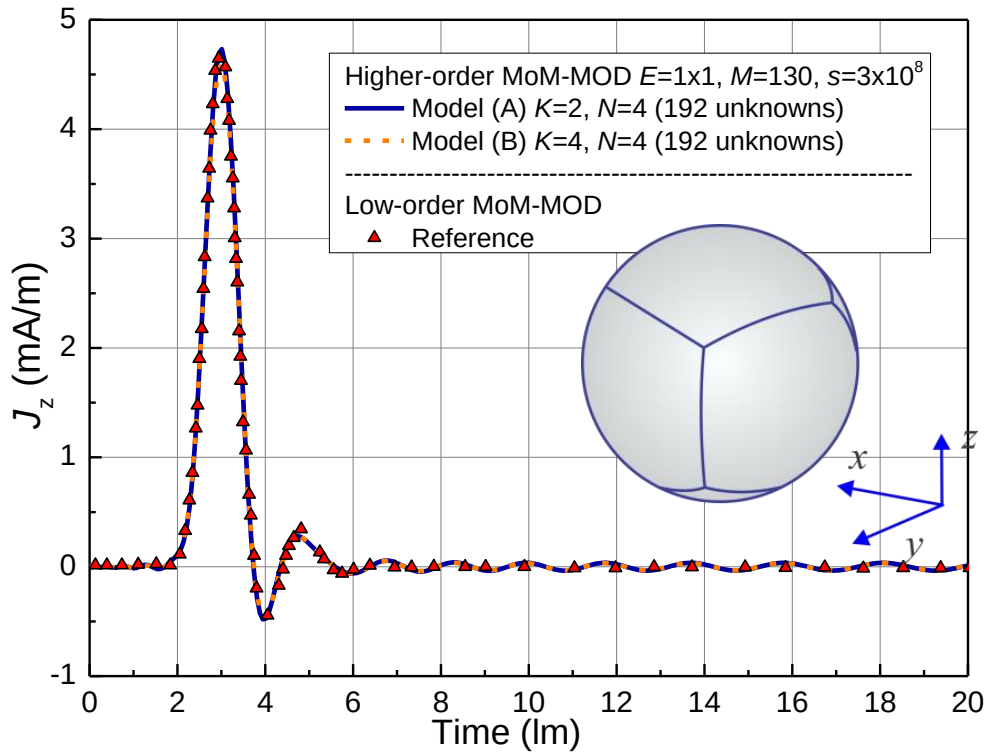
Convergence of the method in terms of the geometrical order *K*

First 3 Resonances: $f_1 = 262$ MHz, $f_2 = 369$ MHz, $f_3 = 429$ MHz

Resonant Region

$T_w = 2$ lm, $c_0 t_0 = 3$ lm,

$\chi = 4$, $f_{max} = 500$ MHz



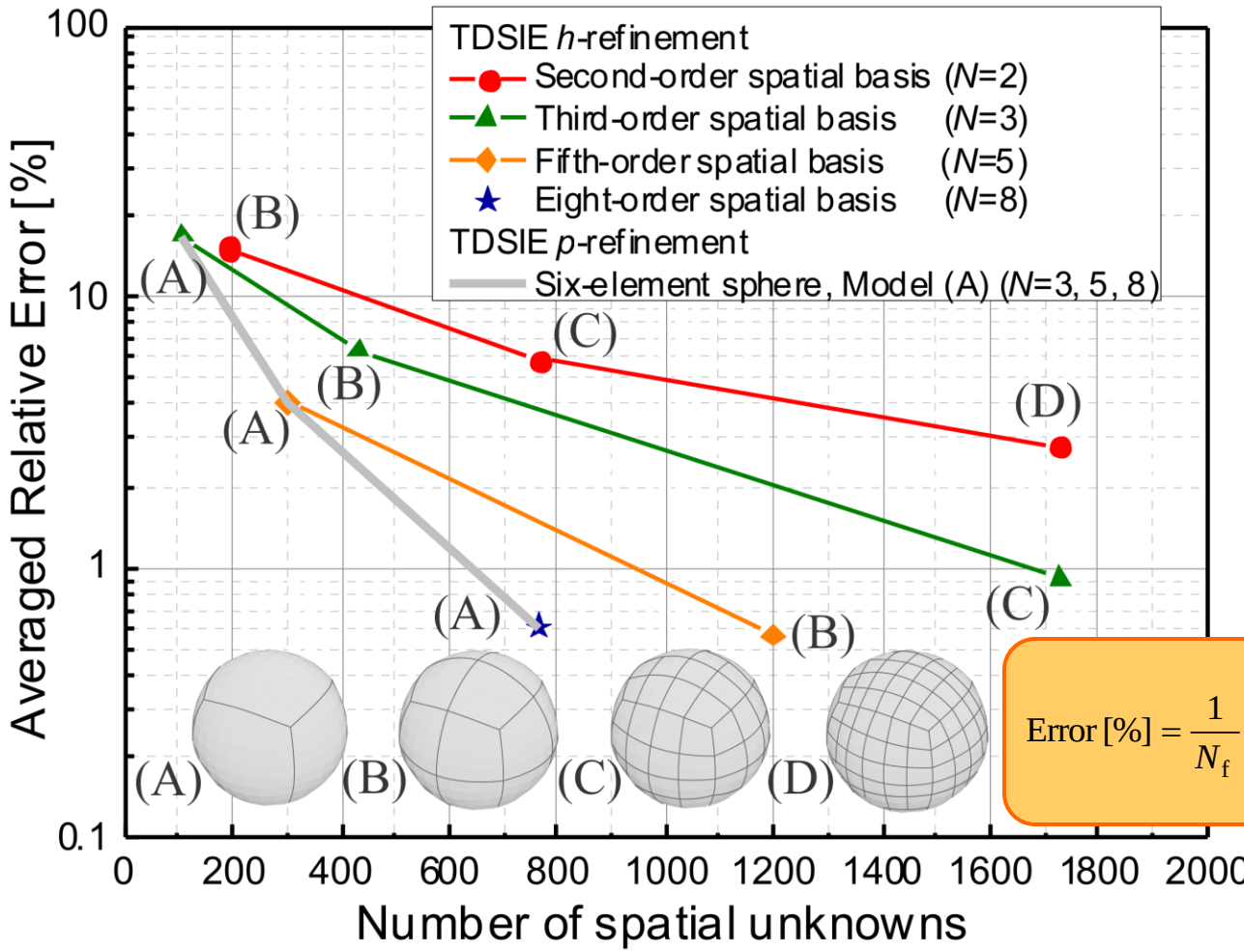
Reference: Low-Order Small-Domain: 528 flat triangular patches, 792 spatial unknowns

Higher-Order Large-Domain: 6 quadrilateral patches ($e \approx 1.34\ \lambda_0$), 192 unknowns, $N = 4$

Reference: B. H. Jung, Y. S. Chung, and T. K. Sarkar, "Time-domain EFIE, MFIE, and CFIE formulations using Laguerre polynomials as temporal basis functions for the analysis of transient scattering from arbitrarily shaped conducting structures", *Progress in Electromagnetic Research*, vol. 39, pp. 1-45, 2003.

TDSIE: *h*- and *p*-Convergence

Sphere , *K* = 4, Resonant Region: $T_w = 2\ \text{lm},\ c_0 t_0 = 3\ \text{lm},\ \chi = 4,\ f_{max} = 500\ \text{MHz}$



$$\text{Error}\,[\%] = \frac{1}{N_f}\sum_{i=1}^{N_f}\left|\frac{J_i^{SIE}(\theta,\phi)}{J_i^{Mie}(\theta,\phi)} - 1\right| \cdot 100$$

PEC NASA Almond

Transient Response of *x*-directed Surface Current Density, top face

x-Polarizesd Gaussian Wave Excitation

$T_w = 1\,\text{lm}, c_0 t_0 = 6\,\text{lm}, \chi = 1, f_{max} = 250\,\text{MHz}$

Low-Order Small-Domain:

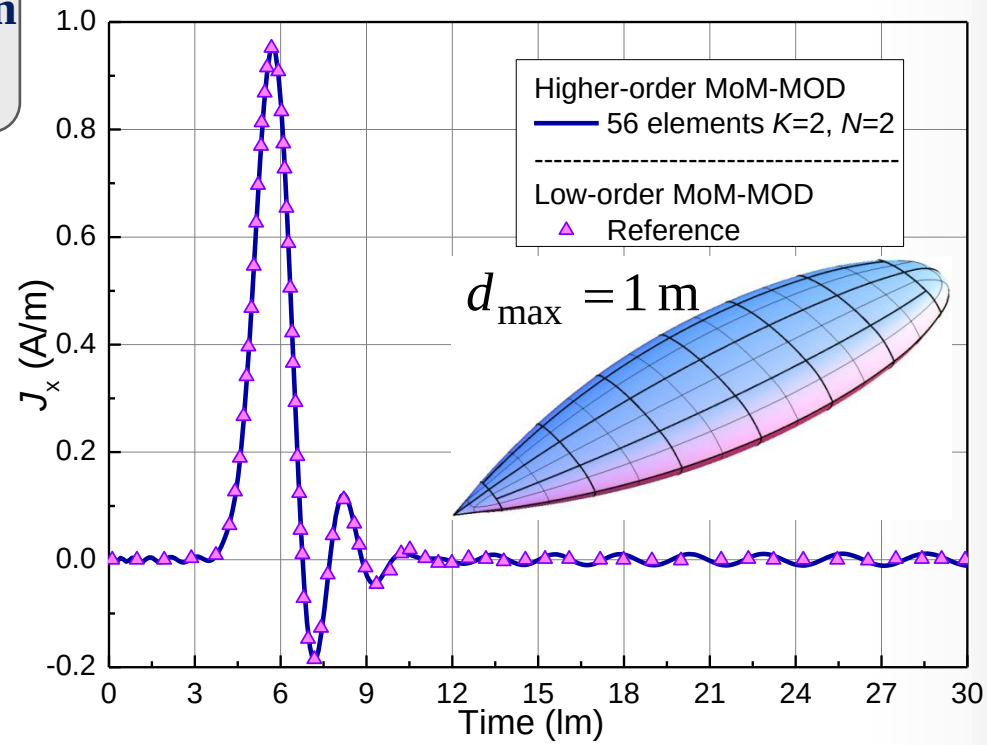
864 flat triangular patches,

1,296 spatial unknowns

Higher-Order Large-Domain:

56 quadrilateral patches K = 2,

448 unknowns, N = 2



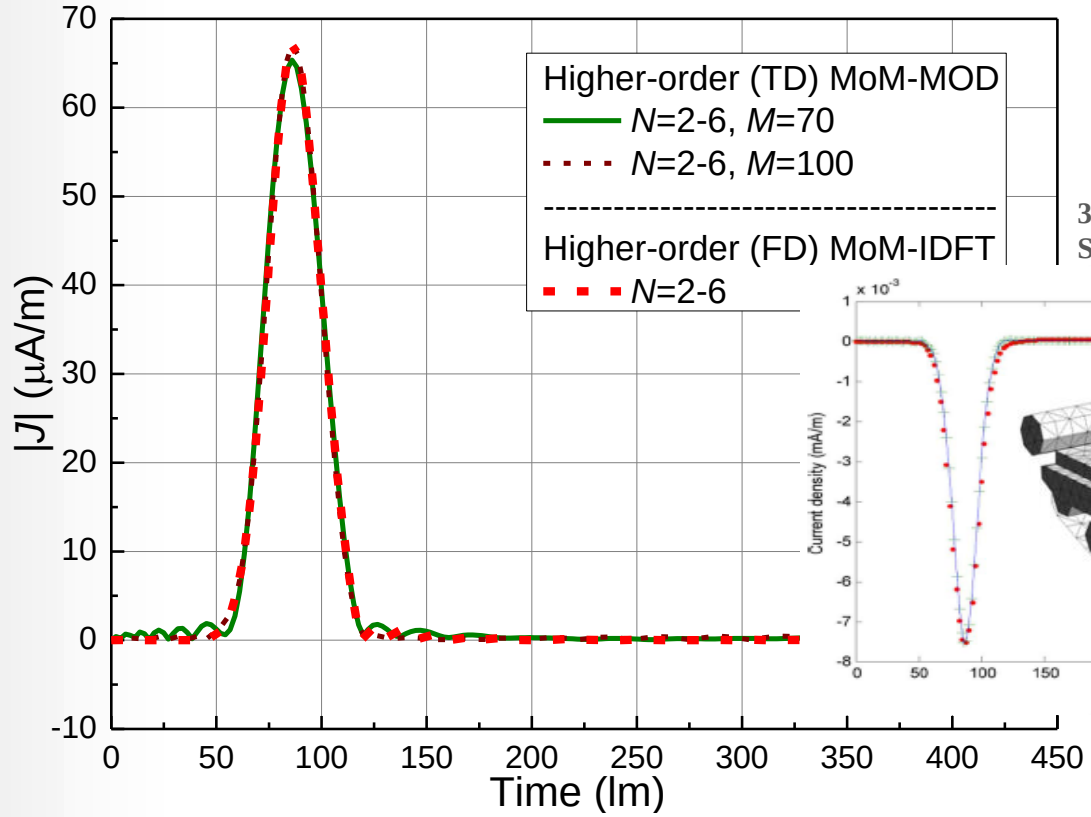
Reference: A. Geranmayeh, "Time Domain Boundary Integral Equations Analysis", Ph.D. Dissertation, Vom Fachbereich Elektrotechnik und Informationstechnik der Technischen Universitat Darmstadt, Germany 2011.

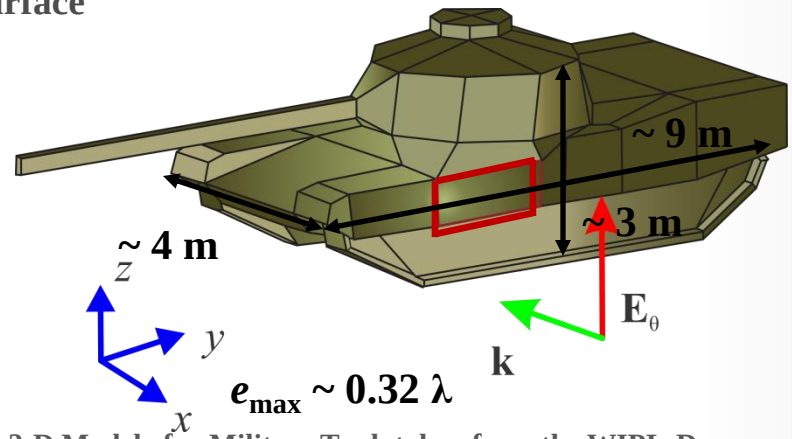
Military Tank (T-80 Series)

Higher-order MoM-TDEFIE-MOD: Transient Response of Surface Current Density at the center of the red squared patch

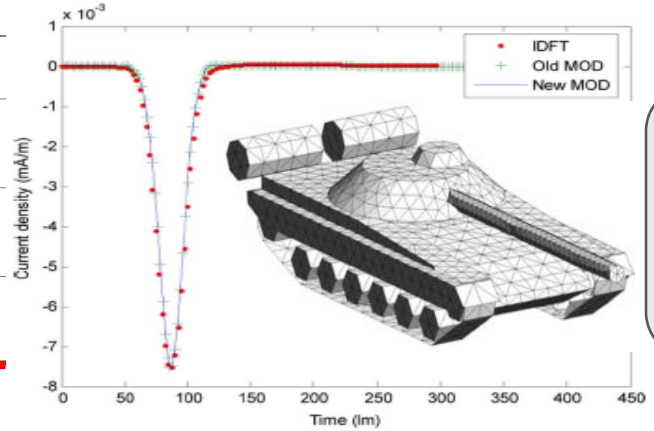
θ-Polarized Gaussian Plane Wave

$T_w = 70$ lm, $t_0 = 90$ lm,
$T_f = 200$ lm, $f_{max} = 15$ MHz





3-D Model of a Military Tank taken from the WIPL-D Simulation-ready EM Modeling Library, WIPL-D Pro V11, 2013



Referenced model

2, 737 triangular patched and around 4, 000 unknowns

Higher Order Solution

147 quadrilateral elements and around 1, 000 unknowns

Reference: Z. Mei, Y. Zhang, T. K. Sarkar, B. H. Jung, A. García-Lampérez, M. Salazar-Palma, "An improved marching-on-in-degree method using a new temporal basis", *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 12, pp. 4643-4650, December 2011.

TD-MoM: Conclusions

- **Novel large domain *p*-refined Galerkin-type MoM-TDSIE-MOD for transient 3-D modeling**
  - Conformal generalized curvilinear quadrilaterals
  - Spatial support: hierarchical divergence-conforming polynomial vector basis functions
  - Temporal support: associated Laguerre functions derived from the Laguerre polynomials
  - Unconditionally stable MOD iterative solution
- **Arbitrarily shaped 3-D EM metallic and dielectric structures**
- **MoM-TDEFIE-MOD: Preliminary results for PEC scatterers**
  - Canonical examples (benchmarks): Cube and Sphere (only 6 SIE elements), NASA Almond (56 SIE elements)
  - Real-world applications: Military Tank (147 SIE elements)
- **Current expansions of orders from 2 to 5 with less the 500 spatial unknowns for benchmarks, and approximately 1, 500 unknowns for the tank**
- **Higher order MoM-TDEFIE-MOD solution compared with**
  - Alternative small-domain low-order full-wave numerical solutions in TD: MoM-MOT, -MOD
- **Higher order MoM-TDEFIE-MOD: causal, stable, accurate, and converged solution**

Appendix A: Rules for Optimal Simulation Parameters in HO FEM and MoM

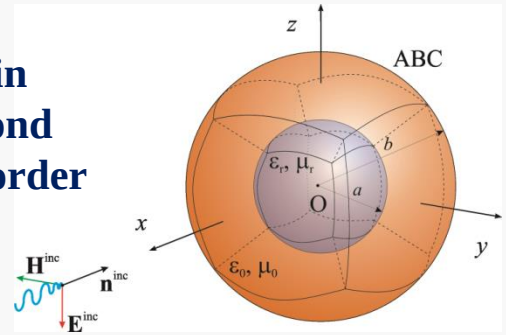
- The purpose of the project is to develop general guidelines for adoptions of **optimal higher order parameters** of computational electromagnetics (CEM) modeling using **MoM** and **FEM**
- The goal of this study, which is the first such study of higher order parameters, is:
  - to improve modeling flexibility of higher order elements, basis and testing functions, and integration procedures
  - to ease and facilitate the decisions to be made on how to actually use these parameters, by both CEM developers and practitioners
- **These rules result in considerable reduction of overall simulation complexity including modeling and computational time**

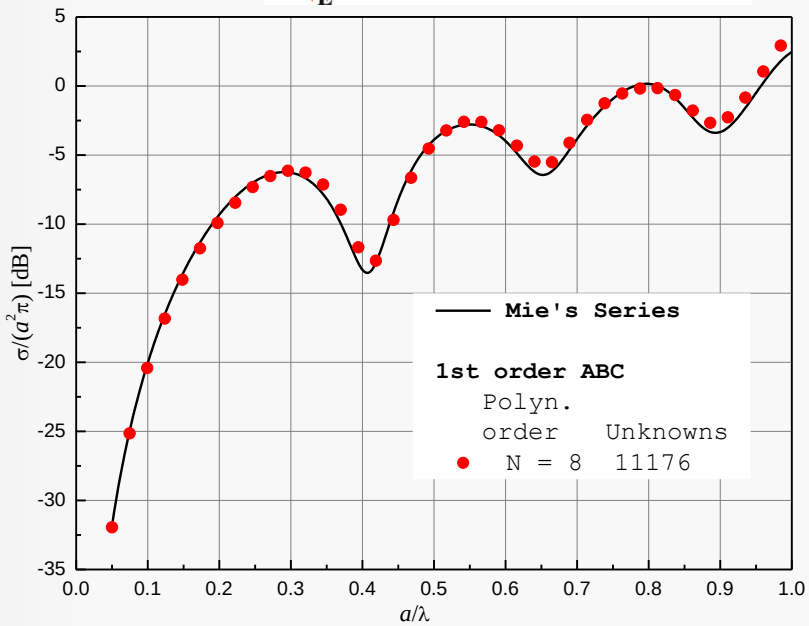
E. M. Klopf, N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Optimal Modeling Parameters for Higher Order MoM-SIE and FEM-MoM Electromagnetic Simulations", *IEEE Transactions on Antennas and Propagation*, vol.60, no.6, pp.2790-2801, June 2012.

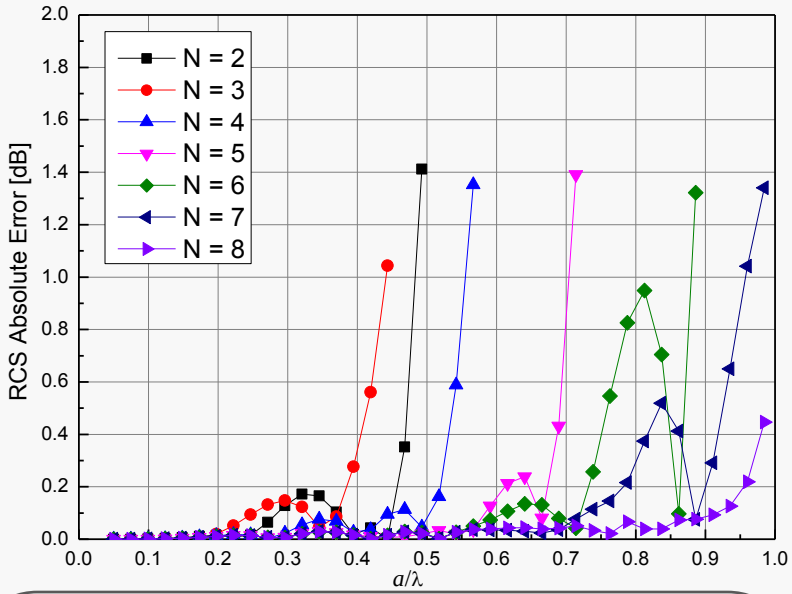
Appendix A: FEM-ABC Analysis of Dielectric Sphere

$a = 1\text{m},\ b = 2.5\text{ m},\ \varepsilon_r = 2.25$

Entire-domain model of second geometrical order







Optimal choice:

N = 2,3 for $a/\lambda < 0.4$,

N = 4 for $0.4 < a/\lambda < 0.5$,

N = 5 for $0.5 < a/\lambda < 0.6$,

N = 6 for $0.6 < a/\lambda < 0.7$,

N = 7 for $0.7 < a/\lambda < 0.8$,

N = 8 for $0.8 < a/\lambda < 1$, $NGL = N + 4$

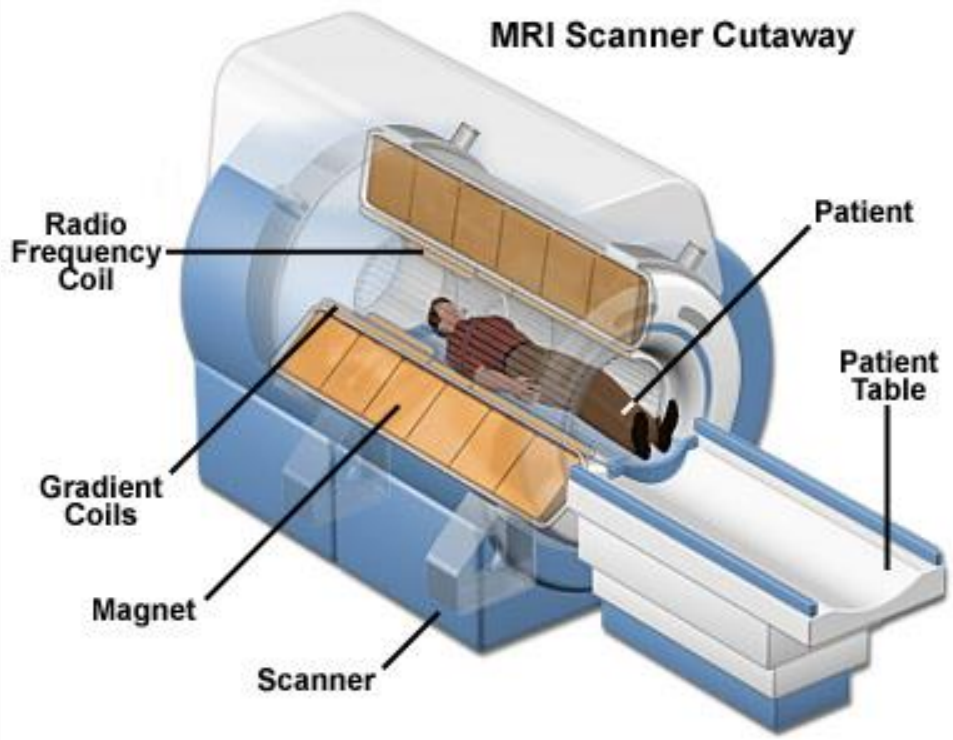
Accuracy of the results is limited by the accuracy of first order ABC!

N. J. Šekeljić, S. V. Savić, M. M. Ilić, and B. M. Notaroš, "Rules for Adoption of Expansion and Integration Orders in FEM Analysis Using Higher Order Hierarchical Bases on Generalized Hexahedral Elements," invited paper, Special Session "Adaptive FEM, Higher Order Bases, and Advanced FEM Formulations," 11th International Workshop on Finite Elements for Microwave Engineering – FEM2012, June 4-6, 2012, Estes Park, Colorado.

Appendix A: Conclusions

- MoM-SIE and FEM model should be *h*-refined if dimensions of elements become greater than $e = 2\lambda$
- The optimal (or nearly optimal) choice of orders N and NGL is $N = 6$ and $NGL = 8$, for both metallic and dielectric structures, with or without pronounced curvature
- It is generally optimal to use $NGL = N + 2$, for any N
- For surfaces with pronounced curvature, $K = 4$ should be adopted in order to enable efficient use of high orders N on electrically large elements
- Results of this study may contribute to considerable reduction of overall simulation time and lead to optimal or nearly optimal first-pass solutions

Appendix B: MRI Applications

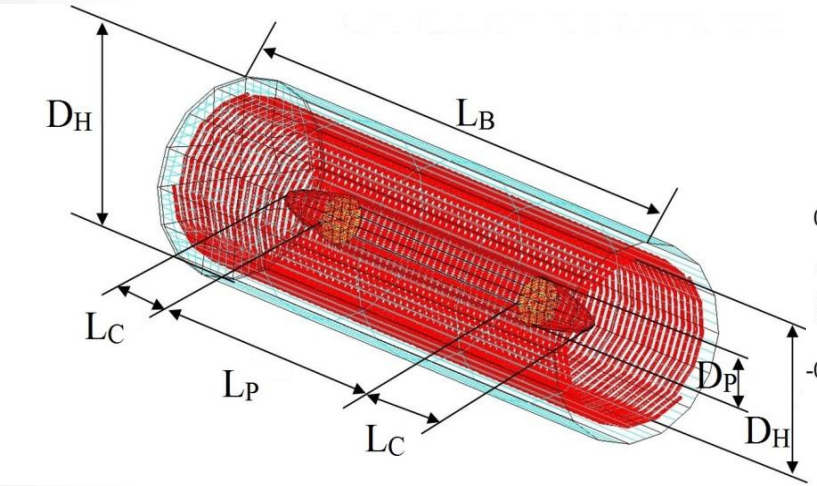


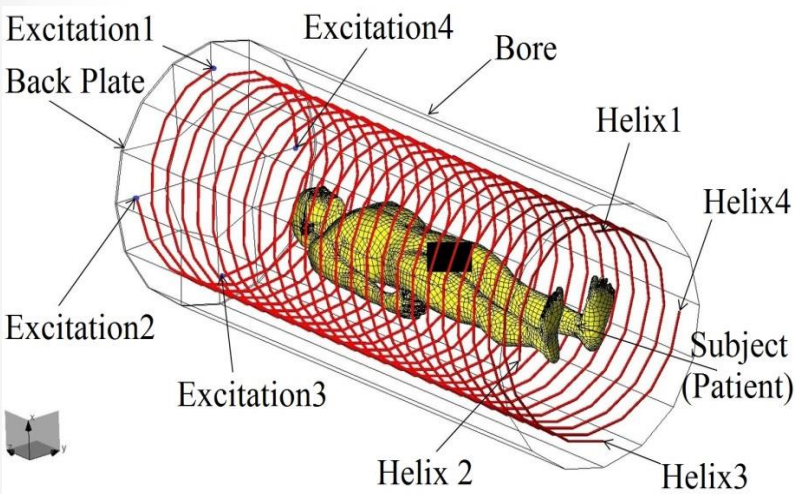
- B_0 – external polarizing static magnetic field
- f_0 – Larmor frequency
- B_1 – RF excitation magnetic field

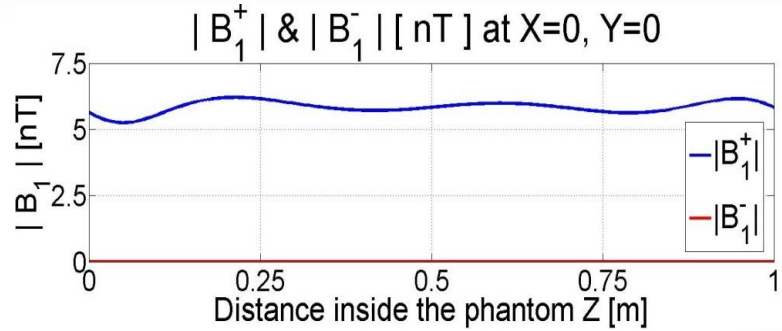
B_0 [T]	3	4	7	9.4	10.5	11	16.4	21.1
f_0 [MHz]	127.8	170	300	400	450	468	685	900

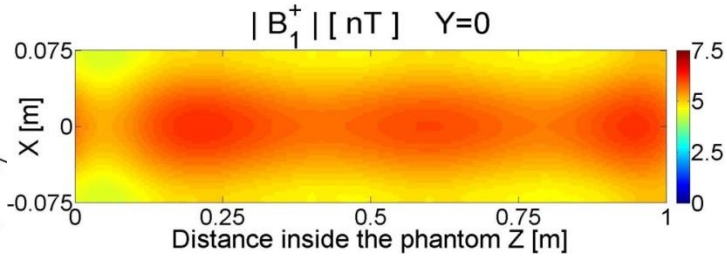
Image: http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/, National High Magnetic Field Laboratory

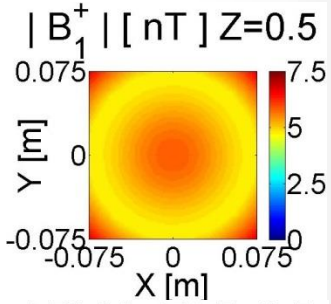
Appendix B: MRI Applications

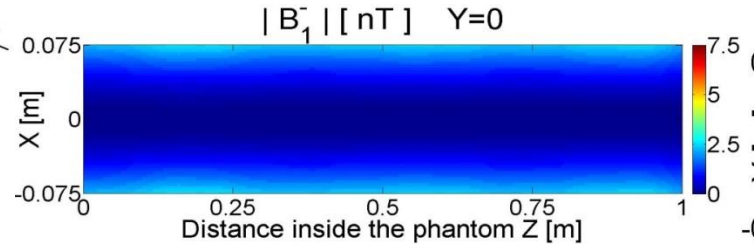


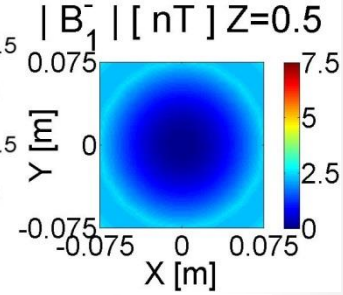


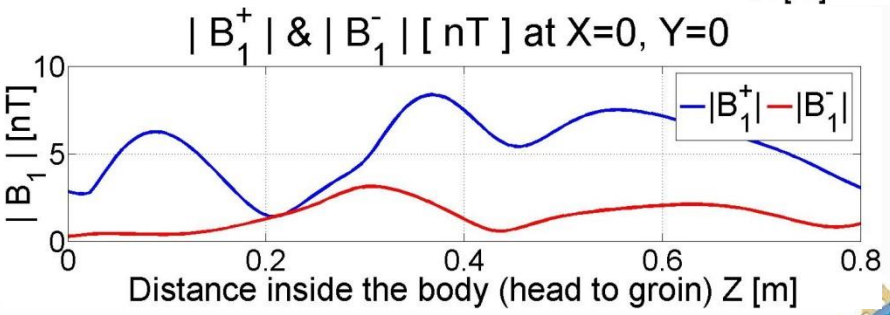












Branislav M. Notaros, Milan M. Ilic, Alexey A. Tonyushkin, Nada J. Sekeljic, and Pranav Athalye, "Quadrifilar Helical Antenna as a Whole-Body Traveling-Wave RF Coil for 3T and 7T MRI", Proceedings of 23rd ISMRM 2015, accepted.

Appendix C: Close Point Projection (CPP)

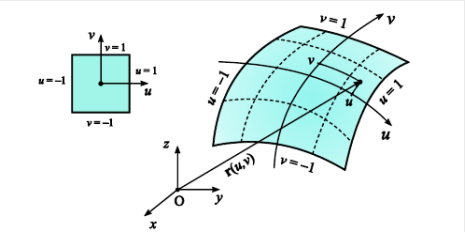
Extremal problem

$$F = ||\mathbf{r}_0 - \mathbf{r}(u,v)|| \mapsto \min \Leftrightarrow F = (\mathbf{r}_0 - \mathbf{r}(u,v)) \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) \to \min$$

$$\mathbf{r}(u,v) = \sum_{k=0}^{K_u} \sum_{l=0}^{K_v} \mathbf{r}_{kl} \, \Lambda_k^{K_u}(u) \Lambda_l^{K_v}(v) \qquad -1 \le u, v \le 1$$

$$\Lambda_k^{K_u}(u) = \prod_{\substack{j=0 \\ j \ne k}}^{K_u} \frac{u - u_j}{u_k - u_j}$$

Geometrical representation of the surface



Newton-Raphson method

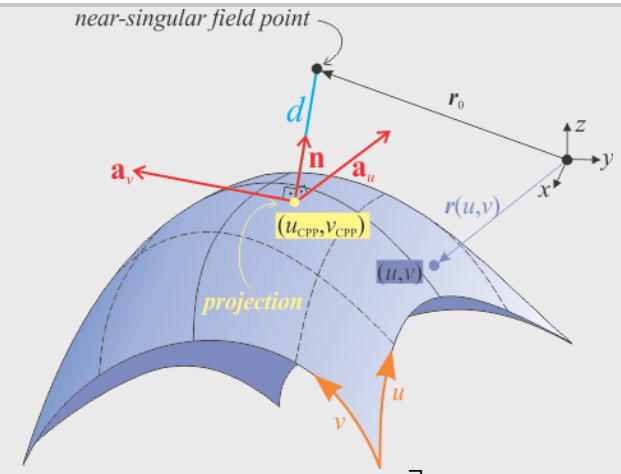
$$\begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix}_{(n)} = -(\Phi^2)^{-1}_{(n)} \Phi^1_{(n)}$$

$$u_{(n+1)} = u_{(n)} + \Delta u_{(n)} \quad v_{(n+1)} = v_{(n)} + \Delta v_{(n)}$$

$$\mathbf{a}_u = \frac{\partial \mathbf{r}(u,v)}{\partial u}$$

$$\mathbf{a}_{uu} = \frac{\partial^2 \mathbf{r}(u,v)}{\partial^2 u}$$

$$\mathbf{a}_{uv} = \frac{\partial}{\partial u}\left(\frac{\partial \mathbf{r}(u,v)}{\partial v}\right)$$



First fundamental

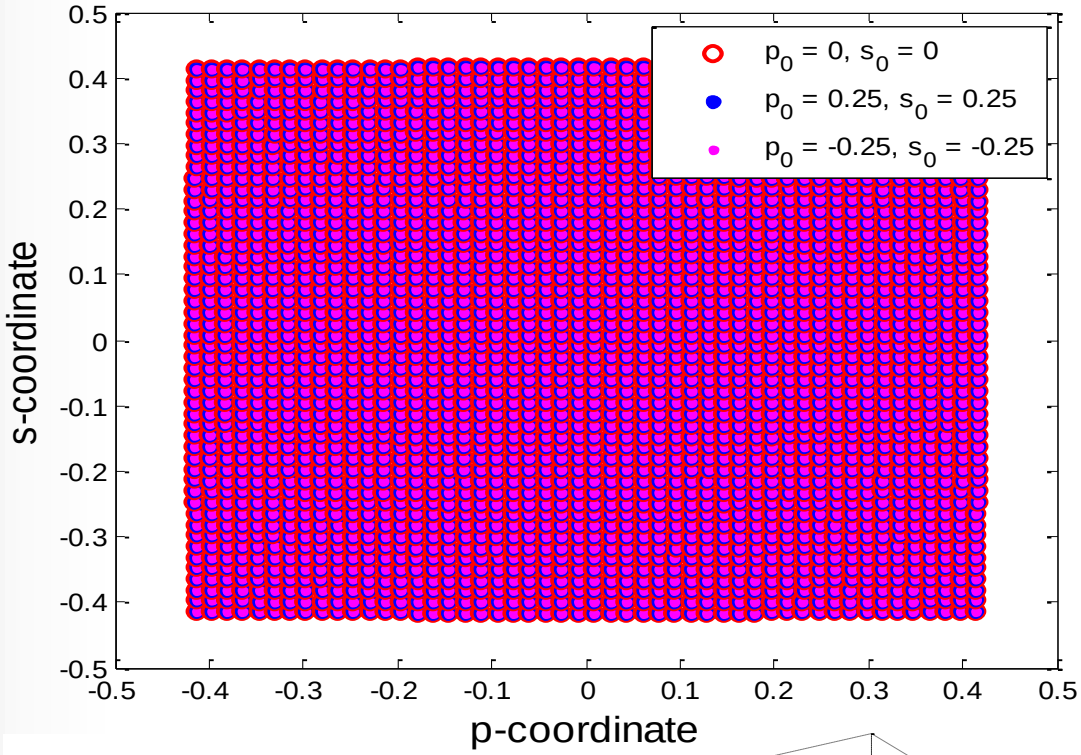
$$\Phi^{(1)} = \begin{bmatrix} \dfrac{\partial F}{\partial u} \\ \dfrac{\partial F}{\partial v} \end{bmatrix} = -\begin{bmatrix} \mathbf{a}_u \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) \\ \mathbf{a}_v \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) \end{bmatrix}$$

Second fundamental

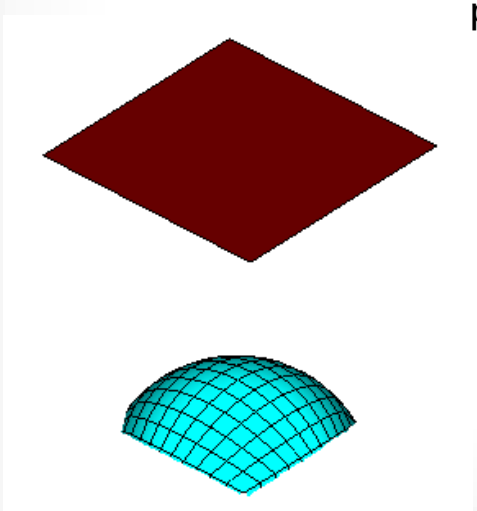
$$\Phi^{(2)} = \begin{bmatrix} \mathbf{a}_u \cdot \mathbf{a}_u - \mathbf{a}_{uu} \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) & \mathbf{a}_u \cdot \mathbf{a}_v - \mathbf{a}_{uv} \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) \\ \mathbf{a}_v \cdot \mathbf{a}_u - \mathbf{a}_{vu} \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) & \mathbf{a}_v \cdot \mathbf{a}_v - \mathbf{a}_{vv} \cdot (\mathbf{r}_0 - \mathbf{r}(u,v)) \end{bmatrix}$$

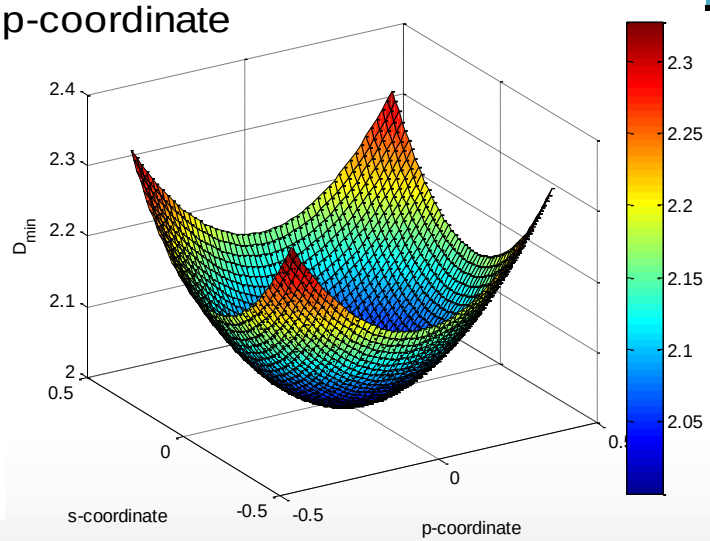
A. Konyukhov and K. Schweizerhof, "On the solvability of closest point projection procedures in contact analysis: Analysis and solution strategy for surfaces of arbitrary geometry", *Comput. Methods Appl. Mech. Engrg.* 197 (2008), 3045-3056.

Appendix C: Projection on a Spherical Patch (K = 2)



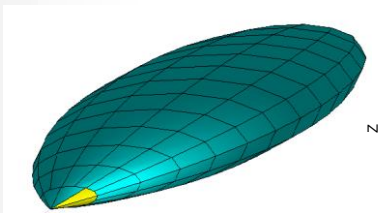
$N_{iteration}$	$u_{project}$	$v_{project}$
1	1.79555457895880	1.79555457895880
2	1.68449588484662	1.68449588484662
3	1.60329754566595	1.60329754566595
4	1.54745145376067	1.54745145376067
5	1.51453150603063	1.51453150603063
6	1.50182134291067	1.50182134291067
7	1.50003261339598	1.50003261339598
8	1.50000001066127	1.50000001066127
9	1.50000000000000	1.50000000000000
10	1.50000000000000	1.50000000000000

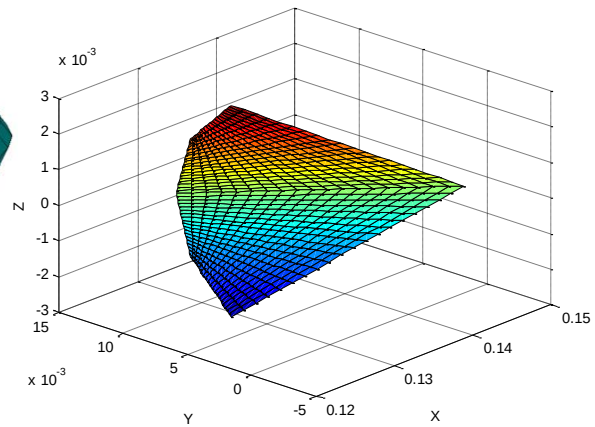




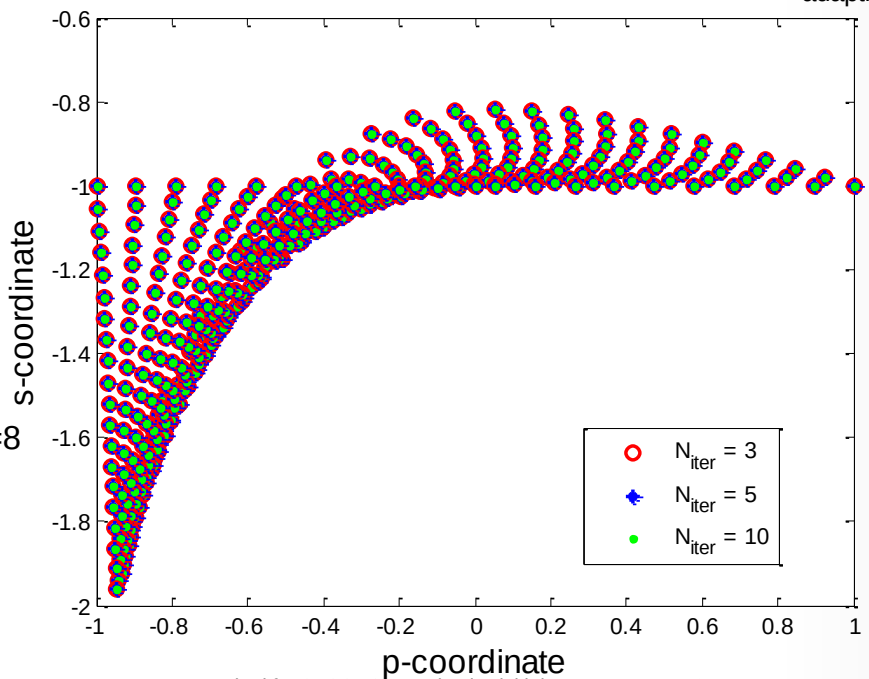
Projection falls outside of domain of *S*

Appendix C: Projection on the Patch of Irregular Shape (K = 1)

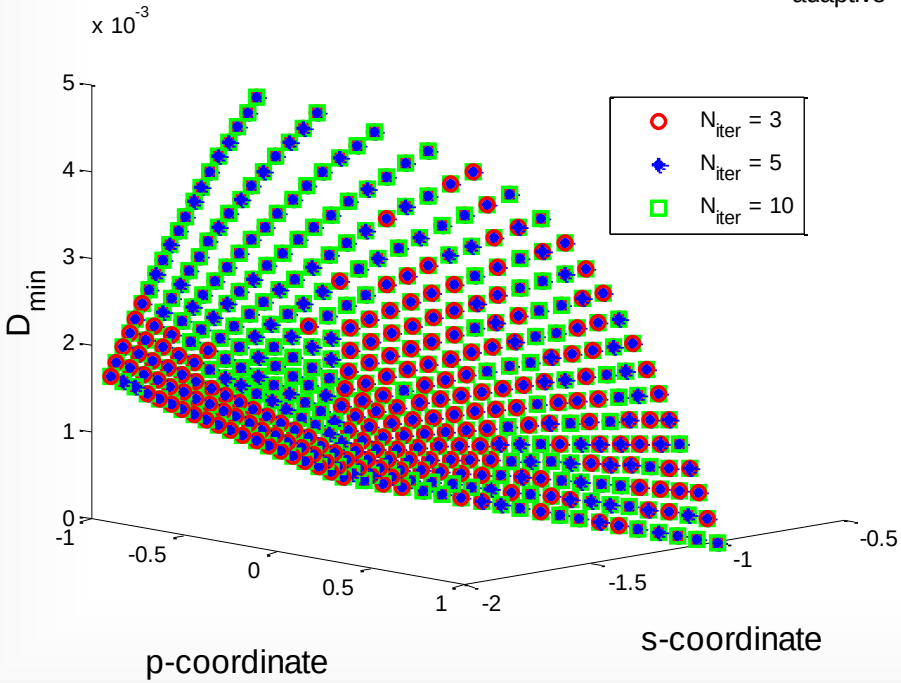




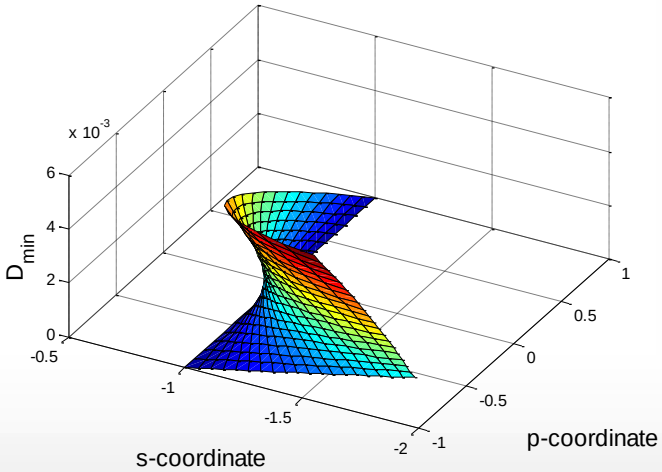
N-R Method for CLOSPOINT-Adaptive initial guess, $N_{adaptive}$=8



N-R Method for CLOSPOINT-Adaptive initial guess, $N_{adaptive}$=8



N-R Method for CLOSPOINT-Adaptive initial guess, $N_{adaptive}$=8, N_{iter}=5



Suggestions for Future Research

Coupled EFIE-MFIE Formulation in TD

Coupled EFIE/MFIE System of Equations

$$[\mathbf{E}(\mathbf{J}_S(\mathbf{r},t),\mathbf{M}_S(\mathbf{r},t),\varepsilon_1,\mu_1)]_{\text{tang}}+(\mathbf{E}_i(\mathbf{r},t))_{\text{tang}}=[\mathbf{E}(-\mathbf{J}_S(\mathbf{r},t),-\mathbf{M}_S(\mathbf{r},t),\varepsilon_2,\mu_2)]_{\text{tang}},$$

$$[\mathbf{H}(\mathbf{J}_S(\mathbf{r},t),\mathbf{M}_S(\mathbf{r},t),\varepsilon_1,\mu_1)]_{\text{tang}}+(\mathbf{H}_i(\mathbf{r},t))_{\text{tang}}=[\mathbf{H}(-\mathbf{J}_S(\mathbf{r},t),-\mathbf{M}_S(\mathbf{r},t),\varepsilon_2,\mu_2)]_{\text{tang}},$$

$$\mathbf{r}\in S,\quad (\forall)\, t\geq 0$$

Scattered Electric Field

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}-\nabla\Phi-\frac{1}{\varepsilon}\nabla\times\mathbf{F}$$

Scattered Magnetic Field

$$\mathbf{H}=-\frac{\partial\mathbf{F}}{\partial t}-\nabla U+\frac{1}{\mu}\nabla\times\mathbf{A}$$

Axillary Potentials

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu}{4\pi}\int_S\frac{\mathbf{J}_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S\qquad \mathbf{F}(\mathbf{r},t)=\frac{\varepsilon}{4\pi}\int_S\frac{\mathbf{M}_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S$$

$$\Phi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_S\frac{\rho_S(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S=-\frac{1}{4\pi\varepsilon}\int_S\int_0^{t-R/c}\frac{\nabla_s\cdot\mathbf{J}_S(\mathbf{r}',t')}{R}\,\mathrm{d}t'\mathrm{d}S$$

$$U(\mathbf{r},t)=\frac{1}{4\pi\mu}\int_S\frac{\rho_{mS}(\mathbf{r}',t-R/c)}{R}\,\mathrm{d}S=-\frac{1}{4\pi\mu}\int_S\int_0^{t-R/c}\frac{\nabla_s\cdot\mathbf{M}_S(\mathbf{r}',t')}{R}\,\mathrm{d}t'\mathrm{d}S$$

Conclusion

- Transient analysis of closed and open structures employing two different numerical approaches directly in time domain
- **Novel higher order, large-domain Galerkin-type TDFEM for vector wave equation modeling**
- **Novel higher order, large-domain Galerkin-type MoM-TDSIE for surface integral equation modeling**
- Excellent performances
  - Geometrical modeling accuracy and flexibility
  - Higher order modeling of transient field/current through electrically large elements
  - Accurate, stable, and efficient transient solution to real-world applications

Publications of the Candidate

- N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Spatially Large-Domain and Temporally Entire-Domain Electric-Field Integral Equation Method of Moments for 3-D Scattering Analysis in Time Domain," *IEEE Transactions on Antennas and Propagation*, Vol. 63, No. 6, June 2015.
- M. Thurai, V. N. Bringi, A. B. Manic, N. J. Sekeljic, and B. M. Notaros, "Investigating rain drop shapes, oscillation modes, and implications for radiowave propagation," *Radio Science*, Vol. 49, Issue 10, October 2014, pp. 921-932.
- N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Higher Order Time-Domain Finite Element Method for Microwave Device Modeling with Generalized Hexahedral Elements," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 61, No. 4, pp. 1425-1434, April 2013.
- E. M. Klopf, N. J. Šekeljić, M. M. Ilić, and B. M. Notaroš, "Optimal Modeling Parameters for Higher Order MoM-SIE and FEM-MoM Electromagnetic Simulations", *IEEE Transactions on Antennas and Propagation*, Vol.60, No.6, pp.2790-2801, June 2012.
- E. Chobanyan, N. J. Sekeljic, A. B. Manic, M. M. Ilic, V. N. Bringi, and B. M. Notaros, "Efficient and Accurate Computational Electromagnetics Approach to Precipitation Particle Scattering Analysis Based on Higher Order Method of Moments Integral Equation Modeling," *Journal of Atmospheric and Oceanic Technology*, under review.
- M. M. Ilic, I. Perovic, P. Athalye, N. Sekeljic, A. A. Tonyushkin, B. M. Notaros, "Full-Wave Frequency-Domain Electromagnetic Modelling of RF Fields in MRI Applications," *Proceedings of 2015 IEEE AP-S/URSI International Symposium*, accepted.
- M. M. Ilic, N. Sekeljic, P. Athalye, A. A. Tonyushkin, B. M. Notaros, "RF Excitation in 7 Tesla MRI Systems Using Monofilar Axial-Mode Helical Antenna," *Proceedings of 2015 IEEE AP-S/URSI International Symposium*, accepted.
- P. Athalye, N. Sekeljic, M. M. Ilic, A. A. Tonyushkin, B. M. Notaros, "Improving Traveling-Wave RF Fields inside Magnetic Resonance Imaging Bores by Incorporating Dielectric Loadings," *Proceedings of 2015 IEEE AP-S/URSI International Symposium*, accepted.
- Branislav M. Notaros, Milan M. Ilic, Alexey A. Tonyushkin, Nada J. Sekeljic, and Pranav Athalye, "Quadrifilar Helical Antenna as a Whole-Body Traveling-Wave RF Coil for 3T and 7T MRI", *Proceedings of 23th International Society for Magnetic Resonance Imaging (ISMRM) 2015*, accepted.
- N. J. Sekeljic, A. B. Manic, E. Chobanyan, M. Thurai, V. N. Bringi, and B. M. Notaros, "Electromagnetic scattering by oscillating rain drops of asymmetric shapes," *Proceedings of 2014 IEEE Antennas and Propagation Society International Symposium*, July 6-12, 2014, Memphis, Tennessee, pp. 1572-1573.
- N. J. Sekeljic, S. B. Manic, M. M. Ilic, and B. M. Notaros, "Efficient Models for Transient Analysis of Passive Microwave Devices Using FEM-TD with Time-Marching and FEM-FD with Discrete Fourier Transform," invited paper, Special Session "Time-Domain FEM," *12th International Workshop on Finite Elements for Microwave Engineering – FEM2014*, May 14-17, 2014, Chengdu, China.
- N. J. Sekeljic, M. M. Ilic, and B. M. Notaros, "p-Refined Large-Domain 3-D Curvilinear FEM Solutions of Arbitrarily Loaded and Shaped Waveguide Sections and Bends in the Time Domain," *Proc. 2014 USNC-URSI National Radio Science Meeting*, January 8-11, 2014, Boulder, Colorado.
- N. J. Sekeljic, A. Manic, M. M. Ilic, and B. M. Notaros, "Transient Analysis of 3D Waveguides Using Double- Higher-Order Time-Domain Finite Element Method," *Proceedings of 2013 IEEE Antennas and Propagation Society International Symposium*, July 7-12, 2013, Orlando, Florida.
- E. Chobanyan, N. J. Sekeljic, A. B. Manic, M. M. Ilic, and B. M. Notaros, "Atmospheric Particle Scattering Computation Using Higher Order MoM-SIE Method," *Proceedings of 2013 IEEE Antennas and Propagation Society International Symposium*, July 7-12, 2013, Orlando, Florida.
- N. J. Sekeljic, S. B. Manic, M. M. Ilic, and B. M. Notaros, "Direct and Indirect Time-Domain FEM Higher Order Solutions to 3-D Closed-Region Problems," *Proc. 2013 USNC-URSI National Radio Science Meeting*, January 9-12, 2013, Boulder, Colorado.
- N. J. Sekeljic, S. V. Savic, M. M. Ilic, and B. M. Notaros, "Rules for Adoption of Expansion and Integration Orders in FEM Analysis Using Higher Order Hierarchical Bases on Generalized Hexahedral Elements," invited paper, Special Session "Adaptive FEM, Higher Order Bases, and Advanced FEM Formulations," *11th International Workshop on Finite Elements for Microwave Engineering – FEM2012*, June 4-6, 2012, Estes Park, Colorado.
- B. M. Notaros, M. M. Ilic, S. V. Savic, N. J. Sekeljic, and A. Z. Ilic, "Accurate and Efficient Curvilinear Geometrical Modeling Using Interpolation Parametric Elements in Higher Order CEM Techniques," invited paper, Special Session "Higher Order Numerical Methods", *Proceedings of the 28th International Review of Progress in Applied Computational Electromagnetics – ACES 2012*, April 10-14, 2012, Columbus, Ohio, pp. 602-607.
- N. J. Sekeljic, E. Chobanyan, M. M. Ilic, and B. M. Notaros, "Rules for Adoption of Expansion and Integration Orders in Moment-Method Computation of Electromagnetic Scattering and Radiation," *Proc. 2012 USNC-URSI National Radio Science Meeting*, January 4-7, 2012, Boulder, Colorado.