AN ULTRA-HIGH RESOLUTION PULSED-WIRE MAGNET MEASUREMENT SYSTEM

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Overview

- Introduction and Background
- Pulsed-Wire Method Overview
- The CSU Undulator Specs
- Pulsed-Wire System Details
- Results
Introduction

- Synchrotron Radiation
- Undulator’s Function
- Characterization Techniques
- The CSU Undulator
- Description of Thesis
Synchrotron Radiation (SR)

- Occurs naturally in synchrotrons.
  - Relativistic electrons undergo transverse acceleration due to a magnetic field.
  - Produces electromagnetic radiation, called SR.

Undulators

Description

- Series of alternating polarity dipole magnets.
- Many bending magnets put together in a row.
- Enhances SR light.
  - Coherent emission.
  - More monochromatic.

Types

- Electro-magnet
- Pure Permanent Magnet
- Hybrid (magnet/pole combo)
- Super Conducting
Undulators

❖ Lorentz Force

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]
Undulators

❖ Lorentz Force

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]

The average velocity of the ultra-relativistic electron is slower than the speed of light.

The SR light out paces the electron by one optical period ($\lambda_{rn}$) for every undulator period ($\lambda_u$) of travel.

The 'slippage' of the electrons relative to the light must equal, $\lambda_{rn}$.

\[ \lambda_m = \frac{\lambda_u}{2n\gamma^2} \left(1 + \frac{K^2}{2}\right) \]

\[ K = \frac{eB\lambda_u}{2\pi m_e c} \]

\[ \gamma = \frac{E}{mc^2} \]
Undulator Errors

- Two types of errors.
  - Trajectory and phase.
- Reduces coherence and energy transfer between the electrons and light.
- Accurate undulator characterization is needed to be able to correct for any errors.
Trajectory Error

❖ Occurs when an electron passes a dipole with improper field strength.
❖ The electron gets an incorrect total kick or angle change from the ideal sinusoidal trajectory.
❖ The direction of the electron and emitted light is then incorrect.
❖ Reducing overlap between the light and the electron bunch.
Phase Error

- If a period in the undulator has a low field, the electron will have a higher average longitudinal velocity.
  - The electron becomes out of phase.
- The undulator resonance condition is not met if major phase errors are present.
- Errors reduce the coupling between the electrons and the EM wave.
Undulator Characterization

- **Traditional Hall probe/Gauss meter**
  - Accurate
  - Time Consuming

- **Pulsed-Wire Method**
  - Ultra-fast
  - Can be used where field is inaccessible to a Hall probe
The CSU Undulator

- Hall probe measurements impossible due to support brackets holding gap steady.
- Pulsed-Wire (PW) measurements were previously done at the University of Twente.
## CSU Undulator Specs

<table>
<thead>
<tr>
<th>Type</th>
<th>Hybrid: Sm$_1$Co$_5$ with Vanadium Permendur poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undulator Wavelength (Period) ($\lambda_u$)</td>
<td>25 mm</td>
</tr>
<tr>
<td># of periods (N)</td>
<td>50</td>
</tr>
<tr>
<td>Gap size</td>
<td>8 mm</td>
</tr>
<tr>
<td>Field Strength</td>
<td>0.61 T</td>
</tr>
</tbody>
</table>
## CSU Undulator Specs

### Undulator Design Parameters [mm]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half Gap</td>
<td>( h_s )</td>
<td>4.0</td>
</tr>
<tr>
<td>Half thickness of pole</td>
<td>( D_2 )</td>
<td>2.0</td>
</tr>
<tr>
<td>Half thickness of magnet</td>
<td>( h_2 )</td>
<td>4.25</td>
</tr>
<tr>
<td>Height of pole</td>
<td>( D_3 )</td>
<td>40.0</td>
</tr>
<tr>
<td>Height of magnet</td>
<td>( h_3 )</td>
<td>45.0</td>
</tr>
<tr>
<td>Half width of pole</td>
<td>( D_1 )</td>
<td>15.0</td>
</tr>
<tr>
<td>Half width of magnet</td>
<td>( h_1 )</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Description of Thesis

Created a Pulsed-Wire Method

- Built the physical system.
- Found both the mechanical and magnetic centers of the undulator.
- Applied algorithms to measure dispersive components of the PW signal and remove them.
- Compared corrected PW data of a reference magnetic field to Hall probe measurements for absolute scaling.
- Determined the local magnetic field profile of the magnetic center of the undulator.
Pulsed-Wire Method

- Simple History
- Basic Understanding
- Output
- Limitations
- Correction
PW History

- Has been used in a variety of specialized cases in the characterization of magnetic fields.
- The method’s accuracy was previously limited due to dispersive effects in the wire and the finite pulse width.
- Newly developed mathematical algorithms can correct for these limitations.
Basic Understanding

- A current pulse in a tensioned wire induces an acoustic wave (vibration) due to a magnetic field.
  - Lorenz force causes the wire to move in a direction corresponding to the orientation of the magnetic field.
  - The wave travels in both directions along wire and can be measured.

- Amplitude of wire vibrations are proportional to:
  - Amount of current in the wire.
  - Strength of magnetic field being applied.

- Using either short (µs) or long (ms) current pulses, the first or second field integrals can be deduced.
Basic Understanding

1\textsuperscript{st} and 2\textsuperscript{nd} magnetic field integrals.

Simulates both the transverse velocity and oscillation trajectory of a charged particle passing along the axis of the undulator.

\[ u_{s_0}(t) = \frac{I_c \delta t}{2T} \int_0^{c_0 t} B(\tilde{x}) d\tilde{x} \quad \leftrightarrow \quad v_x(z) = \frac{1}{\gamma m_e} \int_0^z qB_y(\tilde{z}) d\tilde{z} \]

\[ u_{s_0}(t) = \frac{I}{2T} \int_0^{c_0 t} \int_0^{\tilde{x}} B(\tilde{x}) d\tilde{x} d\tilde{\tilde{x}} \quad \leftrightarrow \quad x(z) = \frac{1}{\gamma m_e v_z} \int_0^z qB_y(\tilde{z}) d\tilde{z} d\tilde{\tilde{z}} \]
Wave Speed Determination

\[ \bar{u}_s^*(\omega) \bar{u}_{s\Delta z}(\omega) = |G(\omega)|^2 e^{i\kappa \Delta z} \]

\[ c = \frac{\omega \Delta z}{\phi} \]
Dispersion Correction

- From the Euler-Bernoulli equation for the bending of thin rods:

\[ c(\kappa) = c_0 \sqrt{1 + \frac{EI_w}{T} \kappa^2}. \]

\[ c_0 = \sqrt{T/\mu} \]

- Need to find \( c_0 \) and \( EI_w \) experimentally.
Dispersion Correction Algorithm

- Start with the dispersive wire displacement:

\[ u_s^{\text{short}}(t) = \frac{I\delta t}{2\mu} \int_{-\infty}^{+\infty} \frac{i}{\kappa c_0^2} \bar{B}(\kappa)e^{-i\omega t} d\omega \]

\[ u_s^{\text{long}}(t) = -\frac{I}{2\mu} \int_{-\infty}^{+\infty} \frac{1}{(\kappa c(\kappa))^2 \left(c + \kappa \frac{dc}{d\kappa}\right)} \bar{B}(\kappa)e^{-i\omega t} d\omega = \int_{-\infty}^{+\infty} H(\kappa)e^{-i\omega t} d\omega \]

- We can get a non-dispersive solution of \( u(t) \) by using the equation:

\[ u_{s0}(t) = \int_{-\infty}^{+\infty} F(\kappa)H(\kappa)e^{-i\omega t} d\omega = \int_{-\infty}^{+\infty} H_0(\kappa)e^{-i\omega t} d\omega \]
Dispersion Correction Algorithm

The function $F(\kappa)$ is a scaling function to solve for the non-dispersive solutions.

$$F^{\text{short}}(\kappa) = \frac{H_0(\kappa)}{H(\kappa)} = \left(\frac{c(\kappa)}{c_0}\right)\left(\frac{c(\kappa)+\kappa \frac{dc}{d\kappa}}{c_0}\right)\frac{i\omega(\kappa)\delta t}{e^{i\omega(\kappa)\delta t-1}}$$

$$F^{\text{long}}(\kappa) = \frac{H_0(\kappa)}{H(\kappa)} = \left(\frac{c(\kappa)}{c_0}\right)^2 \frac{c(\kappa)+\kappa \frac{dc}{d\kappa}}{c_0}$$

We can then solve directly for $H_0(\kappa)$ and thus, $B(x)$.
- 1st derivative for short, 2nd derivative for long.
Correction Algorithm Summary

1. Set evenly spaced $\omega_i$ over a large enough range to correctly capture $\bar{B}(\kappa(\omega))$.
2. For all $\omega_i$ numerically integrate

$$H(\kappa(\omega_i)) = G(\omega_i) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} u_s(\tau)e^{i\omega_i\tau} d\tau$$

to obtain $G(\omega_i)$.
3. Calculate unevenly spaced $\kappa$ values, $\kappa_i = \kappa(\omega_i)$, which are associated with $H(\kappa_i) = G(\omega_i)$.
4. Multiply $H(\kappa_i)$ by $F(\kappa_i)$ to obtain $H_0(\kappa_i)$.
5. For each time $t_i$ numerically integrate

$$u_{s0}(t_i) = c_0 \int_{-\infty}^{+\infty} H_0(\kappa)e^{-i\omega_0\kappa t_i} d\kappa$$

to determine the non-dispersive displacement solution $u_{s0}(t_i)$. 
Pulsed-Wire System Details

- Setup
- Procedures
- Final Results
- System Difficulties
- Conclusions
Setup

- Physical Design
- Pulse Generation
- Wire Positioning
- Wire Tension
- Vibration Detection
Setup: Design Specs

- Must be built such that reflections of the acoustic wave from the end wire mounts are not measured.
- The disturbance (vibration) travels in both directions at the same velocity, so the wire must be at least twice as long as the undulator.
- Total length = 3 m.
  - Addition of reference magnet increased length.
Setup: Pulse Generation

- The width of the pulses must be set to accurately extrapolate the first and second field integrals.
- \(1^{st}\) integral = short pulse.
  - 20 \(\mu\)s
- \(2^{nd}\) integral = long pulse.
  - 12 ms
Setup: Wire Positioning

- 2-Axis Translation Stage with 25 µm resolution.
- “V-Blocks” to hold wire steady during alignment and experiments.
Setup: Wire Tension

- **Weight**
  - Used 2.3 N and 0.85 N.

- **Higher tension reduces dispersive effects, increases wave speed, and decreases wire displacement.**
Setup: Wire Vibration Detection

- 635 nm fiber laser
- 40 µm Slit
- Amplified Si photo-detector
Procedure

- **Reference Magnet Base Measurement**
  - Hall probe and Gauss meter

- **Measurement and Data Acquisition**

- **Undulator Center Determination**
  - Mechanical
  - Magnetic
Hall Probe Data-Reference Magnet
Hall Probe Data-Reference Magnet

Hall Probe Data of Reference Magnet at Different Angles from the z-axis

Hall Probe Data for Different Heights above the Reference Magnet
Measurements and Data Acquisition

- Current pulse is introduced in the wire.
  - Oscilloscope starts measuring when the pulse turns “on.”
- Oscilloscope sees the wire vibration (displacement) as a voltage change.
  - Change proportional to the field and current amplitude.
- Data is transferred to a computer and processed through Matlab.
Procedures: Undulator Gap Center

- The PW needs to accurately simulate the electron beam.
  - Measurements must be done at the magnetic center of the undulator.

- Found the mechanical center and made fiducial marks to easily and accurately reposition the wire in the future.

- Determined the magnetic center of the undulator and documented its position from the mechanical center.
Mechanical Center

- Used mounted multi-meter probes to find horizontal center.
- Surveying scope was used to find vertical center.
- Fiducial marks were placed in different locations on the undulator.
Mechanical Center
Mechanical Center
Magnetic Center

- Curved poles for parabolic pole focusing assisted in determining the magnetic center.
  - Field strength increases the further you get from the magnetic center.
Magnetic Center

- RMS values of the field strength within the undulator at various locations within the gap.
Final Results

- Found dispersive wave speed.
- Corrected for dispersion in reference magnet signal.
- Comparison to Hall probe data to find absolute scaling needed for accurate magnetic field profile.
- Corrected 1st and 2nd field integral PW measurements for undulator.
- Found the magnetic field profile of the undulator.
Reference Magnet Measurements

\[ \Delta z = 30 \text{cm} \]
FFTs of Reference Magnet Signals

\[ \bar{u}_s^*(\omega) \bar{u}_{s\Delta z}(\omega) = |G(\omega)|^2 e^{i\kappa \Delta z} \]

\[ c = \frac{\omega \Delta z}{\phi} \]

\[ \Delta z = 30 cm \]
Dispersive Wave Speed

\[ c = \frac{\omega \Delta z}{\phi} \]
Wave Speed Parameters

\[ c(\kappa) = c_0 \sqrt{1 + \frac{EI_w}{T} \kappa^2} \]

\[ \kappa = \frac{\omega}{c} \]

<table>
<thead>
<tr>
<th>Table 3: Wave Speed Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension (N)</td>
</tr>
<tr>
<td>C0 (m/s)</td>
</tr>
<tr>
<td>EI_w (Nm²)</td>
</tr>
</tbody>
</table>
Dispersion Corrected: Short Pulse (1\textsuperscript{st} Integral)
Dispersion Corrected: Long Pulse (2nd Integral)

Wire displacement due to a long, 12ms, pulse, original (top), dispersion corrected (bottom)
1st Integral Compared to Derivative of 2nd Integral
Pulsed-Wire Measurements Compared to Hall Probe

\[ u_{s0}(t) = \frac{I_c \delta t}{2T} \int_0^{c_0 t} B(\tilde{x}) d\tilde{x} \]

\[ \frac{I_c \delta t}{2T} = 6.5 \times 10^{-4} \]

Absolute Scaling factor = \(5 \times 10^{-4}\)
$1^{st}$ Integral of the Undulator and Dipole
**2nd Integral of the Undulator**

Wire displacement, original (top), dispersion corrected (bottom)

Dispersive signal (V)

Distance from Detector (ms)

Corrected signal (V)
Local Magnetic Field of CSU’s Undulator Magnet

Magnetic Field of the CSU Undulator

Distance from Detector (m)

Magnetic Field (kGauss)
System Difficulties

- Large amount of noise was prominent.
  - Air
  - Poor table isolation from ground
  - Electrical

- Limitations
  - Oscilloscope resolution
Conclusions

- The results given are reproducible.
- The method can be used for many different types of magnets in the future for ultra-fast field profiling.
- Practical Applications: PW kit for KYMA
Future Work

- Find specific locations of field errors.
- Perform corrections on dipoles with errors.
  - Place ferromagnetic material (shims) at specific locations to correct errors.
    - Already documented where current shims are located.
- Shorter reference magnetic field.
  - Higher frequency components results in better calculations of the dispersive wave speed.
Thank you!

Experiment still setup for anyone interested to see!