

Efficient Macromodeling of Power Distribution Planes using Delay Extraction based Transmission Line Representation

Sourajeet Roy*, Anestis Dounavis

University of Western Ontario, London, Ontario, N6A 5B9, Canada

I. INTRODUCTION

With increasing clock speed along with decreasing signal rise times and supply voltages, transient currents caused due to simultaneous switching of large number of circuits can lead to voltage fluctuations in power distribution networks (PDN) [1]. Since PDN supply power to core logic and I/O circuits, such power distribution noise can severely limit the performance of the system [2]. Although the frequency response of PDN's can give sufficient insight into their performance, power supply noise is a transient phenomenon and needs to be characterized in the time domain when considering PDN's coupled with nonlinear drivers [3]. Thus, to accurately predict the electrical performance of these power planes over a large bandwidth, accurate broadband macromodeling of power planes is required.

Prominent techniques in macromodeling power plane structures are based on discretization of the plane into a grid of transmission lines [3]-[6]. Macromodeling the transmission line network using conventional lumped RLGC elements [4]-[6] can provide accurate results but require dense discretizations to implicitly model the delay in the transverse electromagnetic (TEM) wave from input to output ports of a plane structure. Hence, such macromodels suffer from large circuit matrices which are memory and CPU inefficient [3]. A methodology to improve the accuracy of conventional lumped was proposed in [5] based on HSPICE's W-element [7]. Other techniques to improve the memory and CPU costs in simulating PDN using lumped RLGC elements are based on transmission matrix method (TMM) [4] or model order reduction [6].

In the proposed work, the power distribution plane is discretized as a grid of transmission line segments where each line segment is represented as a cascade of resistances and lossless line segments based on the DEPACT macromodel [8]. Since the lossless line segments are exactly represented as distributed circuit elements using the MoC [9], a relatively coarser grid can be used to model the plane leading to a reduction of the circuit matrices compared to lumped RLGC macromodels. Moreover, the proposed macromodel is compared with the HSPICE's W-element and numerical

examples illustrate that the proposed model provides similar accuracy as the W-element but is superior in terms of CPU time required for simulation.

II. REVIEW OF THE DEPACT MACROMODEL

The solution of Telegraphers equations for transmission lines can be written as an exponential matrix function as

$$\begin{bmatrix} V(l, s) \\ -I(l, s) \end{bmatrix} = e^{\Phi} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} \quad (1)$$

where

$$\Phi = A + sB; A = \begin{bmatrix} 0 & -Rl \\ -Gl & 0 \end{bmatrix}; B = \begin{bmatrix} 0 & -sLl \\ -sCl & 0 \end{bmatrix} \quad (2)$$

's' is the Laplace transform variable; V and I represent the terminal voltage and current vectors of the transmission line, respectively; R , L , G and C are the per-unit-length resistance, inductance, conductance and capacitance matrices respectively and l is the length of the transmission line [8].

The basic idea of the DEPACT algorithm is to separate the delay terms from $e^{(A+sB)}$. However, since the matrices A and sB do not commute, (i.e. $e^{(A+sB)} \neq e^A e^{sB}$), a modified Lie product is used as

$$e^{A+sB} \approx \prod_{i=1}^n \Psi_i + \varepsilon_n \quad \text{and} \quad \Psi_i = e^{\frac{A}{n}} e^{\frac{sB}{n}} \quad (3)$$

where 'n' is the number of sections. The product of (3) converges to (1) when large number of sections is used. Equation (3) shows that the exponential function of (2) can be divided into subsections of $e^{A/n}$ and $e^{sB/n}$. The matrix $e^{sB/n}$ represents a lossless transmission line, which can be expressed in the time domain as delay algebraic equations [9] and $e^{A/n}$ represents the attenuation matrix which can be modeled as resistances [8]. This fact will be used to derive an efficient macromodel for PDN.

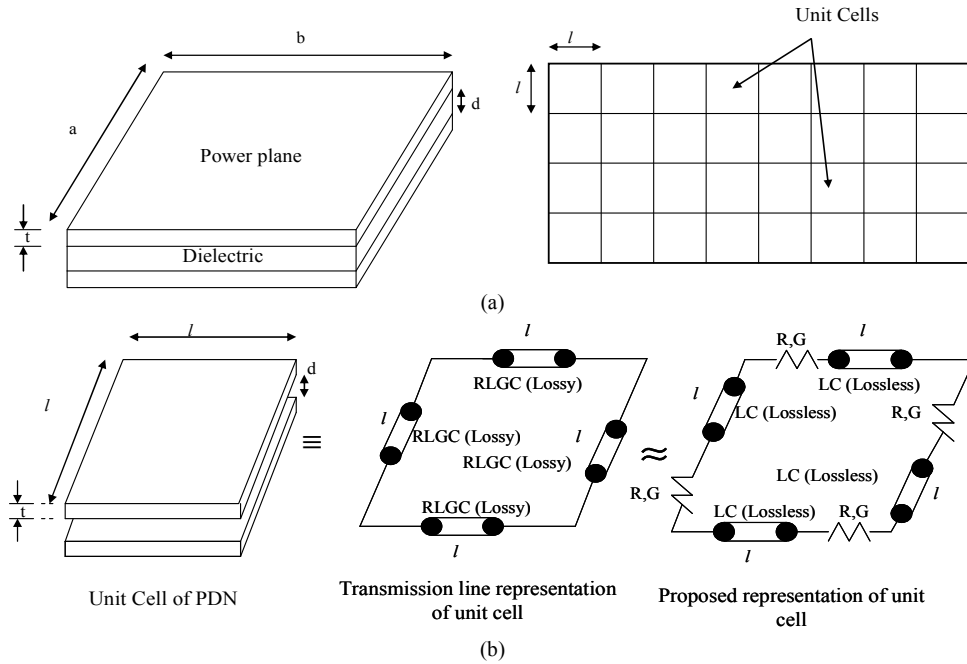


Fig. 1: Representation of a rectangular power plane using (a) grid of unit cells. (b) Representation of an unit cell using transmission line segments and the DEPACT macromodel

III. DEVELOPMENT OF PROPOSED MACROMODEL

In this section, the method of modeling PDNs using traditional transmission line grids is reviewed. Based on this discussion, the use of the DEPACT algorithm [8] to generate a more compact macromodel is formulated.

A. Modeling of the power plane

Without loss of generality, a single layer, rectangular power/ground plane pair is considered. The plane can be discretized into numerous unit cells shown in Fig. 1a. The equivalent circuit representing a unit cell can be obtained from the physical and electrical parameters of the plane [3], [4] provided the dielectric separation between the power and ground plane pairs is much smaller compared to the dimensions of the plane. Considering a square unit cell of dimensions (l) with a dielectric separation of (d) between planes, thickness of metal (t), metal conductivity (σ) and dielectric constant (ϵ_r), the equivalent parameters are

$$R = \frac{2}{\sigma t}, C = \epsilon_o \epsilon_r \frac{l^2}{d}, L = \mu_o d, G = \omega C \tan(\delta) \quad (4)$$

where ϵ_o and μ_o are the permittivity and the permeability of free space and ϵ_r is the relative permittivity of the dielectric. Equivalently, each cell can be represented by 4 transmission line segments with each segment having the per-unit-length parameters of $R_i = 2R/l$, $L_i = 2L/l$, $C_i = C/4l$ and $G_i = G/4l$ (Fig. 1b). Besides the circuit elements derived from the discretization of the plane, the decoupling capacitors

can be also included and are modeled as a series RLC circuit as discussed in [4].

B. Selection of Grid Size

In modeling of PDN using a grid of transmission line segments, two sources of modeling errors are identified. The first error is due to the discretization of the plane into transmission lines. To determine the order of discretization, traditional models for PDNs use the following rule to estimate the size of the unit cells [5].

$$\sqrt{LC} \leq T_r/k \quad (5)$$

where L and C represents the inductance and capacitance of the unit cell respectively as in (4); T_r is the rise time of the input signal and k is a chosen constant [3]-[5]. For accurate transmission line models, such as that using W-element, k is set to 5 thereby ensuring that the errors due to reflection at the far end of each segments is sufficiently reduced [5]. The second source of error is due to the macromodeling of each transmission line segment. If the line segment is represented using conventional lumped sections, such models may not explicitly capture the delay of the TEM wave propagation from the near to far end of the line and hence a larger k value has to be used in (5) [3], [4].

C. Representation of the line segments using DEPACT model

In order to overcome the errors due to the conventional lumped model, the DEPACT algorithm is used to model each line segment as a cascade of resistive elements and lossless

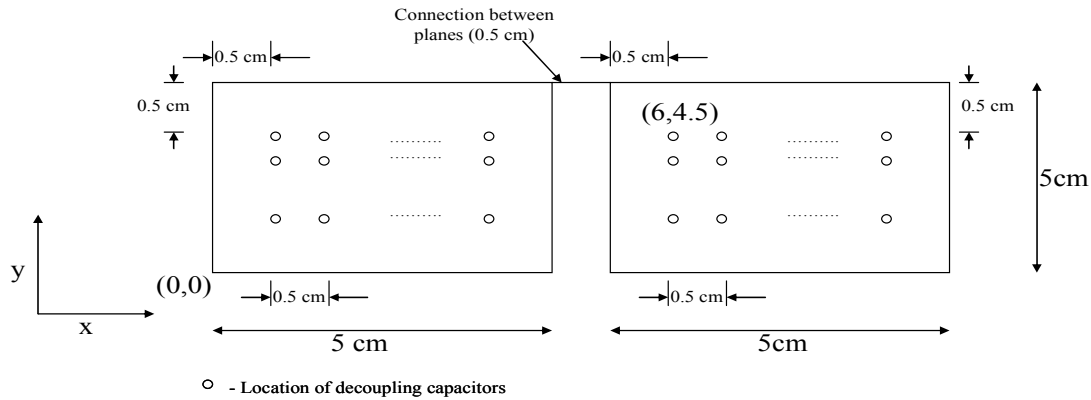
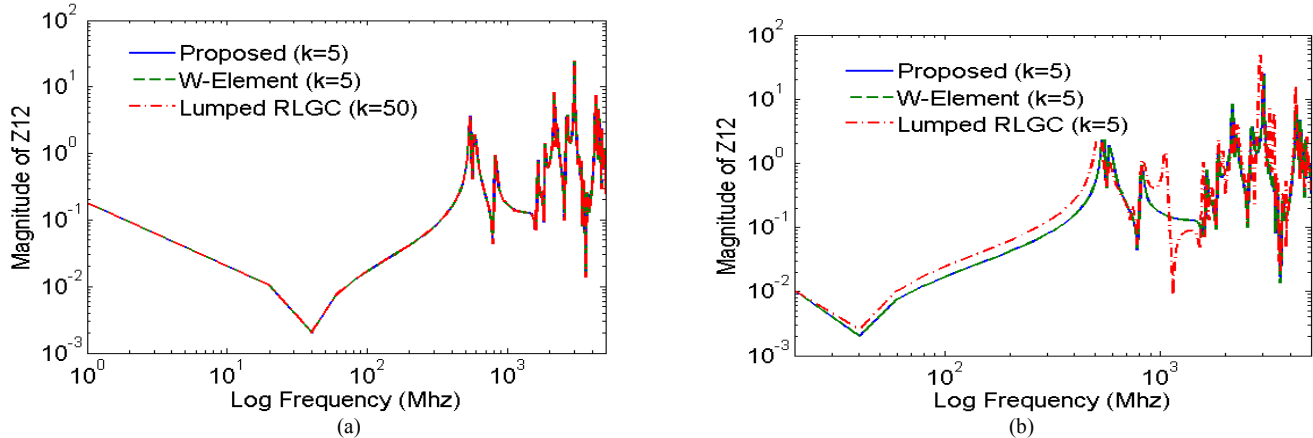


Fig. 2: Layout of the split plane example.

Fig. 3: Frequency response comparison of proposed model with existing models. (a) Accuracy of proposed macromodel compared against lumped RLCG segments and W element for $|Z_{12}|$. (b) Errors due to using lumped model of same discretization as proposed macromodel for $|Z_{12}|$.

line sections [8]. Since the lossless sections can be exactly represented as distributed circuit elements using a delay extraction technique, a coarser grid using $k=5$ (similar to discretization for W-element) is sufficiently accurate [5]. Furthermore using $k=5$, the length of each line segment is small enough to be represented using a resistive element and lossless section as shown in Fig. 1b where the MoC is used to represent the lossless section and matrix rational approximation (MRA) is used to represent the resistive line [8]. The resultant macromodel provides similar accuracy as the W-element and is computationally more efficient than both conventional lumped model and W-element model, as will be illustrated in the next section.

IV. NUMERICAL EXAMPLE

In this section a numerical example is provided to illustrate the validity of the proposed macromodel. A split power/ground plane pair of geometry given by Fig. 2 is considered. The top plane is split into two rectangular planes while the ground plane is continuous. The split planes are connected using a ferrite core modeled as a parallel RLC combination of $R=5\Omega$, $L=0.17$ nH and $C=0.15$ pF. The dimensions of each of the planes are 5 cm by 5 cm, thickness

12.7 micron and separated by a 25.4 micron thick FR4 dielectric with relative permittivity $\epsilon_r=4$. Decoupling capacitors are placed as shown in Fig. 2 and are represented as series RLC elements where $R_{cap}=100m\Omega$, $L_{cap}=2nH$ and $C_{cap}=10nF$. The input port is located at (0 cm, 0 cm) while the output port are located at (6cm, 4.5cm) as labeled in Fig 2.

Using $k=5$ in (6), the structure was divided into 100 unit cells of dimensions 0.5 cm by 0.5 cm. The resulting transmission line network is solved using the DEPACT model, the lumped model and the W-element model. Fig. 3a shows the comparative plot obtained using the proposed macromodel, the W-element model of same discretization ($k=5$) and the lumped model based on very fine discretization ($k=50$). It is noted that the discretization of $k=50$ for the lumped model provides very accurate results for comparison purposes. Both the proposed macromodel and the W-element model show good agreement with the lumped model ($k=50$) over the entire bandwidth (0-5 GHz). In Fig. 3b, the frequency response using a lumped model of similar discretization ($k=5$) is shown. For this example, the lumped model ($k=5$) cannot capture the high frequency resonant peaks beyond 20 MHz. The comparative analysis of the CPU run-times and accuracy using the proposed, lumped and W-element models is shown

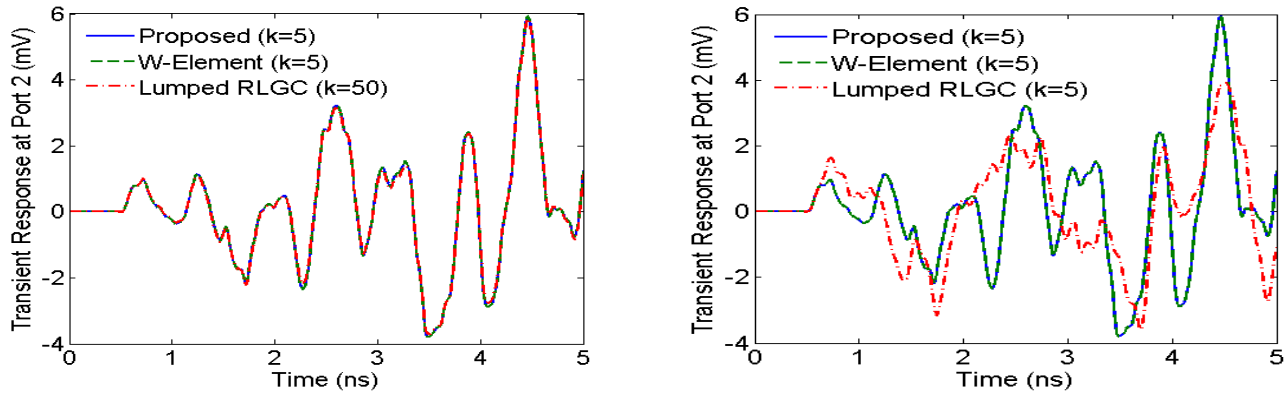


Fig. 4: Transient response using proposed macromodel compared with W-element model and lumped model of very high order of discretization

TABLE I

SIZE OF CIRCUIT MATRICES AND CPU COST OF PROPOSED MACROMODEL COMPARED WITH LUMPED MODEL AND W ELEMENT.

Model	Number of Cells	CPU time (seconds)	Average Error (%) over Bandwidth (0-5Ghz) w.r.t. Lumped RLGC (k=50)
Proposed (k=5)	200	4.68	0.22
W Element (k=5)	200	39.50	0.05
Lumped RLGC (k=5)	200	3.97	10.84
Lumped RLGC (k=10)	800	18.67	6.68
Lumped RLGC (k=15)	1800	29.44	0.99

in Table I. For this example for the same discretization, the proposed model is found to be 9 times faster than W-element while the lumped model gave inaccurate results. For the lumped model, to achieve an average error of less than 1% requires a denser discretization of $k=15$ which is at least 6 times more inefficient as the proposed model (Table I). For transient analysis, an input voltage source with $T_r = 0.1$ ns, pulse width of 0.8 ns, unit amplitude with a source resistance of 25Ω is considered. The load capacitance is 10pF. The transient responses obtained using the proposed model, the W-element and the lumped model are shown in Fig. 4. For this example, the proposed model ($k=5$), W-element model ($k=5$) and lumped model ($k=50$) show good agreement while for the lumped model ($k=5$) show significant errors (Fig 4).

VII. CONCLUSION

In this paper, an algorithm for a novel SPICE model for single layer power distribution planes is presented. The power plane is modeled as a grid of transmission line segments and a delay-extraction based representation is used to model the transmission line network. The proposed macromodel is compared with the HSPICE's W-element and numerical examples prove that the proposed model provides similar accuracy as the W-element but is superior in terms of CPU time required for simulation.

REFERENCES

- [1] R. R. Tummala, E. J. Ryemaszewski and A. G. Klopfenstein, *Microelectronics Packaging Handbook*, New York: Chapman & Hall, 1997.
- [2] E. E. Davidson, "Electrical design of high speed computer package," *IBM J. Res. Develop.*, vol. 26, no. 3, pp. 349-361, May 1982.
- [3] K. Lee and A. Barber, "Modeling and analysis of multichip module power supply planes," *IEEE Trans Comp. Packag. And Manufacturing Tech.*, vol. 18, no. 4, pp. 628-639, Nov. 1995.
- [4] J. H. Kim and M. Swaminathan, "Modeling of irregular shaped power distribution planes using transmission matrix method," *IEEE Trans Adv. Packag.*, vol. 24, no. 3, pp. 334-346, Aug. 2001.
- [5] L. D. Smith, R. Anderson and T. Roy, "Power plane SPICE models and simulated performance for materials and geometries," *IEEE Trans Adv. Packag.*, vol. 24, no. 3, pp. 277-287, Aug. 2001.
- [6] A. Tsuzaki *et al.*, "A fast transient simulation based on model order reduction and RLGC-MNA formulation," *18th European Conf. Circuit Theory and Design*, pp. 775-778, Aug. 2007.
- [7] *Star-HSPICE Manual*, Avant! Corporation, Fremont, CA, Dec. 1999.
- [8] N. M. Nakhla, A. Dounavis, R. Achar and M. S. Nakhla, "DEPACT: Delay extraction-based passive compact transmission-line macromodeling algorithm," *IEEE Trans. on Adv. Packaging*, vol. 28, no. 1, pp. 13-23, Feb. 2005.
- [9] C. R. Paul, *Analysis of Multiconductor Transmission Line*. New York, 2008.