Fast Electromagnetic Interference Analysis of Distributed Networks using Longitudinal Partitioning Based Waveform Relaxation

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Abstract — **In this paper, a waveform relaxation algorithm for the fast electromagnetic interference analysis of distributed transmission line networks is presented. The proposed work models lossy transmission lines as a cascade of lumped circuit elements and lossless line segments, where the incident field coupling with the network is represented as lumped sources connected to each lossless line segment. A longitudinal partitioning methodology which ensures that the resultant subcircuits are weakly coupled using delayed linear equations is presented, thereby leading to fast convergence of the waveform relaxation iterations. A numerical example is provided to demonstrate the validity of the proposed algorithm.**

Index Terms — **Electromagnetic interference, delay effects, partitioning algorithms, electromagnetic transients, transmission lines.**

I. INTRODUCTION

With the increased use of low powered devices, susceptibility of interconnects at the board and packaging levels to incident electromagnetic (EM) fields has become a major concern for signal integrity verification purposes [1]. However, accurate electromagnetic interference (EMI) analysis of large distributed networks using commercial circuit solvers with integrated circuit emphasis (like SPICE) require significant CPU time and memory making them computationally prohibitive for early design cycles.

The waveform relaxation (WR) algorithm, since its introduction in [2] has emerged as a powerful tool for addressing the increasing CPU costs of simulating large circuits. Presently, two approaches exist for application of waveform relaxation to transmission line networks. One such approach is the transverse partitioning (TP-WR) scheme [3] where multi-conductor transmission lines (MTLs) are partitioned into single lines by assuming weak capacitive and inductive coupling between the lines. Recently, TP-WR has been extended to perform fast EMI analysis for large MTL structures [4].

An alternative waveform relaxation algorithm is based on longitudinal partitioning of the network into cascaded subcircuits [5]. Longitudinal partitioning techniques typically lead to strong coupling between the subcircuits represented by the Dirichlet's transmission condition at the partitioning interface and consequently exhibit slow convergence [5], [6]. In [6], a longitudinally partitioning methodology based on the DEPACT segmentation model [7] was presented where the Dirtichlet's transmission condition at the partitions was replaced with a set of delayed linear equations which was found to significantly accelerate convergence. However, fast EMI analysis using longitudinal partitioning based WR algorithms are yet to be investigated.

This work extends the concepts of [6] to fast EMI analysis of transmission line networks. The distributed nature of the incident field coupling with the network is represented as lumped sources introduced into each DEPACT section. Longitudinal partitioning of the network is performed along the natural interfaces provided by the method of characteristics (MoC) within each DEPACT section. The coupling between subcircuits is represented by the delayed linear MoC equations which provide sufficient overlap between subcircuits for the WR iterations to converge rapidly.

II. REVIEW OF INCIDENT FIELD ANALYSIS USING DEPACT

Transmission lines for quasi transverse electromagnetic (TEM) mode of propagation are described by the Telegraphers equations. The solution of Telegraphers equations for a two conductor transmission line can be written, in the frequency domain as an exponential matrix function [7] as

$$\begin{bmatrix} V(l,s) \\ -I(l,s) \end{bmatrix} = e^{(\boldsymbol{A}+s\boldsymbol{B})} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} 0 & -(R(s)-s(L(s)-L_\infty)) \\ -(G(s)-s(C(s)-C_\infty)) & 0 \end{bmatrix} l; \qquad (1)$$
$$\boldsymbol{B} = \begin{bmatrix} 0 & -L_\infty \\ -C_\infty & 0 \end{bmatrix} l$$

where V and I represent the terminal voltage and current variables respectively of the transmission line, *R(s), L(s), G(s),* and *C(s)* represent the frequency dependent per-unit-length parameters; $L_\infty = L(s_\infty)$ and $C_\infty = C(s_\infty)$ are the p. u. l. inductive and capacitive parameters respectively at the maximum frequency of interest $s_\infty = j2\pi f_{\max}$ and l is the length of the transmission line.

In order to derive a time domain representation of (1), the DEPACT model approximates the exponential matrix of (1)

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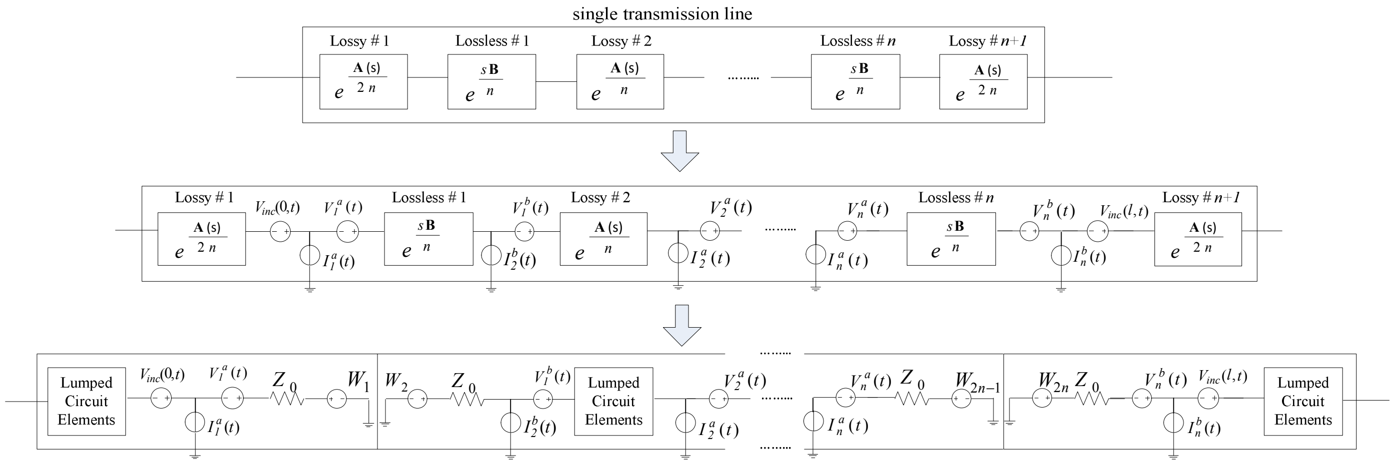

Fig. 1: SPICE equivalent circuit of a two conductor transmission line using DEPACT.

using a modified Lie product [7] as

$$e^{A+sB} \approx \prod_{i=1}^{n} \boldsymbol{\Psi}_i \quad \text{and} \quad \boldsymbol{\Psi}_i = e^{\frac{A}{2n}} e^{\frac{sB}{n}} e^{\frac{A}{2n}} \tag{2}$$

where *'n'* is the number of sections. Equation (2) provides a methodology of discretizing the transmission line into a cascade of *2n+1* alternating subsections with the individual stamps of $e^{\mathbf{A}/2n}$ and $e^{s\mathbf{B}/n}$ as illustrated in Fig. 1. The exponential matrix $e^{A/2n}$ represents the attenuation losses of the transmission line and is approximated by a low order rational function which can be represented as lumped RLC elements in SPICE [7]. On the other hand the matrix $e^{s\mathbf{B}/n}$ represents a lossless line sections which can be described by the MoC equations [1].

Considering the above transmission line to be exposed to an incident electric field $E(t)$, the EMI of $E(t)$ on the transmission line can be represented by the additional lumped sources connected to each i^{th} lossless section defined as

$$\begin{bmatrix} V_i^a(t) \\ I_i^a(t) \end{bmatrix} = E(t - k_z(i-1)\tau)\boldsymbol{\Psi}, \begin{bmatrix} V_i^b(t) \\ I_i^b(t) \end{bmatrix} = -E(t - k_z i\tau)\boldsymbol{\Psi}$$
$$\boldsymbol{\Psi} = \begin{bmatrix} k_z & -L_\infty \\ -C_\infty & k_z \end{bmatrix}^{-1} \begin{bmatrix} V_{F1} \\ 0 \end{bmatrix} \tag{3}$$
$$V_{inc}(0,t) = E(t)V_{F2}$$
$$V_{inc}(l,t) = E(t - n\tau)V_{F2}$$

where the voltages $\{V_{F1}, V_{F2}\}$ depend on the incident field and the geometry of the transmission line structure [4] and $\tau = l\sqrt{L_\infty C_\infty}/n$ is the delay of each lossless section respectively as shown in Fig. 1. The detailed derivation of (3) can be found in [4] and has been omitted for lack of space.

III. Development of the Proposed Algorithm

This section describes the proposed methodology to longitudinally partition the DEPACT model into subcircuits and to solve the subcircuits iteratively.

A. Proposed Longitudinal Partitioning

The i^{th} lossless line segment ($e^{s\mathbf{B}/n}$) in Fig. 1 can be exactly described in time domain by the method of characteristics equations as

$$\begin{aligned} V_{2i-1}(t) &= Z_0 I_{2i-1}(t) + W_{2i-1}(t) \\ V_{2i}(t) &= Z_0 I_{2i}(t) + W_{2i}(t) \\ W_{2i-1}(t) &= 2V_{2i}(t-\tau) - W_{2i}(t-\tau) \\ W_{2i}(t) &= 2V_{2i-1}(t-\tau) - W_{2i-1}(t-\tau); \qquad i = 1,2,...,n \end{aligned} \tag{4}$$

where V_{2i-2}, V_{2i-1} are the near and far end voltages respectively and I_{2i-2}, I_{2i-1} are the near and far end currents respectively and $Z_0 = \sqrt{L_\infty / C_\infty}$ the characteristic impedance of the i^{th} lossless line segment. The MoC equations of (4) can be realized by the simple circuit equivalent of Fig. 1. Partitioning the transmission line at the natural interfaces provided by MoC divides the network into many small, disjoint subcircuits which are by construction, weakly coupled leading to fast convergence of the WR iterations [6]. It is observed that the introduction of the lumped sources $\{V_i^a, I_i^a, V_i^b, V_i^b, V_{inc}\}$ does not in any way affect the partitioning methodology of [6].

B. Iterative Solution of Subcircuits

It is observed from Fig. 1 that the transmission line is partitioned into '*n+1*' subcircuits, where each subcircuit is coupled to the adjacent subcircuit through the delayed sources

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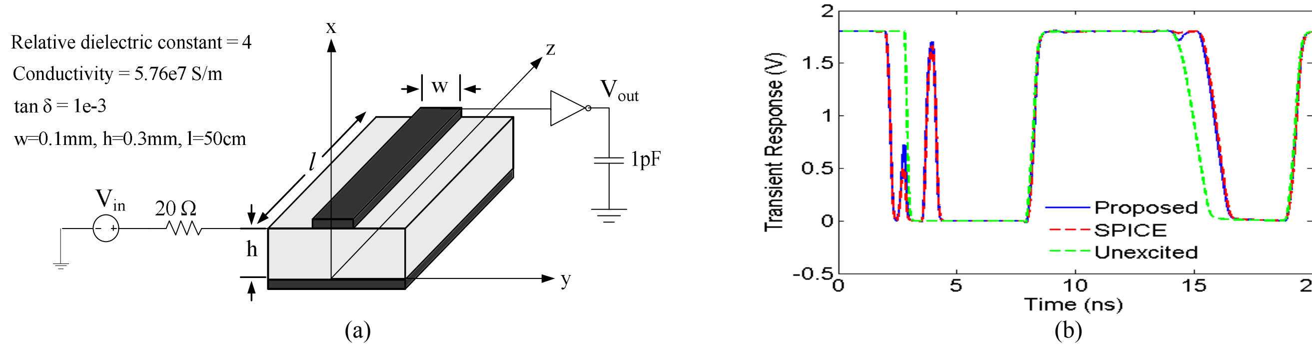

Fig. 2: Numerical example. (a) Microstrip structure. (b) Inverter output (V_{out}).

$W_{2i-1}(t)$ and $W_{2i}(t)$ defined using (4). Thus to begin the iterations, i.e. iteration count *k=0*, an initial guess of $\{W_1^{(0)}(t), W_2^{(0)}(t), ..., W_{2n}^{(0)}(t)\}$ are used as sources to excite the corresponding subcircuits. For each k^{th} iteration, all the subcircuits are solved individually either in sequence or in parallel. After each k^{th} iterations, the delayed sources is updated for the $k+1^{th}$ iteration using the MoC delayed linear equations of (4), expressed as

$$\begin{aligned} W_{2i-1}^{(k+1)}(t) &= 2V_{2i}^{(k)}(t-\tau) - W_{2i}^{(k)}(t-\tau) \\ W_{2i}^{(k+1)}(t) &= 2V_{2i-1}^{(k)}(t-\tau) - W_{2i-1}^{(k)}(t-\tau);\ i = 1,2,...,n \end{aligned} \tag{5}$$

where $W_{2i-1}^{(k)}(t)$, $W_{2i}^{(k)}(t)$, $V_{2i-1}^{(k)}(t)$ and $V_{2i}^{(k)}(t)$ are the known results obtained from the k^{th} iterations. Using the updated sources of (5) as the new sources for the $k+1^{th}$ iteration, the subcircuits are solved again. This iterative cycle continues till the absolute error between $W_i^{(k+1)}(t)$ and $W_{2i}^{(k)}(t)$ fall below a prescribed tolerance.

IV. Numerical Examples

In this section, a microstrip structure which is connected to nonlinear CMOS inverters with physical dimensions as shown in Fig. 2(a) is considered. The microstrip is excited with a trapezoidal voltage sources of rise time $T_r = 0.1ns$, pulse width $T_p = 5ns$ and amplitude of 1V. The structure is exposed to an incident field with Gaussian waveform described by $E(t) = E_0(\exp(t-t_0)^2)/T$ where $t_0 = 1ns$, T = 250ps, the peak amplitude $E_0 = 5kV/m$.

Modeling the microstrip required using 100 DEPACT sections. The structure was thereafter partitioned along the MoC interfaces into *N=101* subcircuits and solved in an iterative manner using the Gauss-Seidel (sequential) technique explained. The predefined error tolerance is set to $\eta = 1e-5$ and an initial guess of the relaxation sources set to the DC solution of zero. Convergence was achieved in 7 iterations and the results of the proposed WR algorithm was found to show good agreement with the full SPICE simulation using the DEPACT model of [7] as illustrated in Fig. 2(b).

TABLE I
Comparison of CPU cost

	Proposed	SPICE-DEPACT	Speedup
CPU time (sec)	39.88	264	6.62

The proposed WR algorithm exhibited a speed up more than 6 times over full SPICE simulation using [7].

V. Conclusion

This paper presents a longitudinal partitioning based waveform relaxation algorithm for the fast EMI analysis of transmission line networks. The proposed methodology exploits the DEPACT model to perform the longitudinal partitioning such that the resultant WR iterations converge efficiently.

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