Longitudinal Partitioning Based Waveform Relaxation Algorithm for Transient Analysis of Long Delay Transmission Lines

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Abstract — **In this paper a waveform relaxation algorithm based on longitudinal partitioning is presented to efficiently model large distributed networks. The proposed methodology represents lossy transmission lines as a cascade of lumped circuit elements and lossless line segments, where the lossless line segments are modeled using the method of characteristics. This allows the transmission line to be naturally partitioned into smaller, weakly coupled subcircuits, enabling the waveform relaxation algorithm to converge more efficiently compared to existing relaxation algorithms based on longitudinal partitioning using the conventional lumped model. A numerical example is provided to demonstrate the validity of the proposed algorithm.**

Index Terms — **Convergence, delay, longitudinal partitioning, transient simulation, signal integrity, transmission line, waveform relaxation.**

I. Introduction

The waveform relaxation algorithm, since its introduction in [1] has emerged as a powerful tool for addressing the increasing CPU costs of simulating large circuits [2]. Typically, waveform relaxation attempts to break a large circuit into smaller subcircuits which can be solved iteratively in sequence or in parallel. At each iteration, an exchange of voltage/current waveforms between the subcircuits is required to force the response to converge to the actual solution.

Presently, various approaches exist for applying the waveform relaxation to transmission line networks [3]-[6]. In [3], the waveform relaxation was applied to the generalized method of characteristics (MoC) which partitioned the line into two weakly coupled subcircuits. A waveform relaxation algorithm based on transverse partitioning of multi-conductor transmission lines (MTLs) has also been reported [4]. This algorithm proposed the partitioning of MTLs into individual transmission lines by exploiting the weak capacitive and inductive coupling between the lines. However, for tightly coupled MTLs, the above algorithm may require a large number of iterations to converge. An alternative waveform relaxation algorithm based on longitudinal partitioning of the line into lumped RLGC subcircuits has also been proposed in [5]. Generally, longitudinal partitioning of the transmission line using the conventional lumped RLGC model results in strong coupling between the subcircuits, leading to slow and inefficient convergence properties of the algorithm. To mitigate the above problem, in [5] the convergence was accelerated by exchanging additional voltage/current waveforms between the subcircuits followed by optimization routines. However, longitudinal partitioning of the line using lumped RLGC subcircuits is still not suitable for modeling long delay lines as the number of subcircuits required to implicitly model the delay of the line can quickly become exorbitant [7]. Hence, efficient waveform relaxation algorithms for simulation of long delay transmission lines based on longitudinal partitioning of the line is still an open problem. Other waveform relaxation algorithms are based on partitioning the linear transmission line network from the nonlinear terminations [6].

In the proposed work, a more suitable longitudinal partitioning scheme based on the DEPACT model [8] is proposed. According to the DEPACT model, the transmission line is discretized into cascaded lumped circuit elements and lossless line segments. Modeling the lossless segments using the MoC algorithm allows an elegant and simple partitioning of the line into smaller subcircuits, which by construction are relatively weakly coupled [3] compared to lumped RLGC subcircuits of [5]. Consequently, the proposed waveform relaxation algorithm can converge rapidly without exchange of additional waveforms or time consuming optimization routines. Moreover since a delay extraction based longitudinal partitioning is used, the number of subcircuits required to model the line is much smaller than that required using the conventional lumped model.

II. Review of DEPACT Model

Transmission lines for quasi-TEM mode of propagation are described by the Telegraphers equations. The solution of Telegraphers equations for a two conductor transmission line can be written as an exponential matrix function [8] as

$$\begin{bmatrix} V(l,s) \\ -I(l,s) \end{bmatrix} = e^{\boldsymbol{\Phi}} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} \tag{1}$$

$$\boldsymbol{\Phi} = \boldsymbol{A} + s\boldsymbol{B};\ \boldsymbol{A} = \begin{bmatrix} 0 & -Rl \\ -Gl & 0 \end{bmatrix};\ \boldsymbol{B} = \begin{bmatrix} 0 & -Ll \\ -Cl & 0 \end{bmatrix} \tag{2}$$

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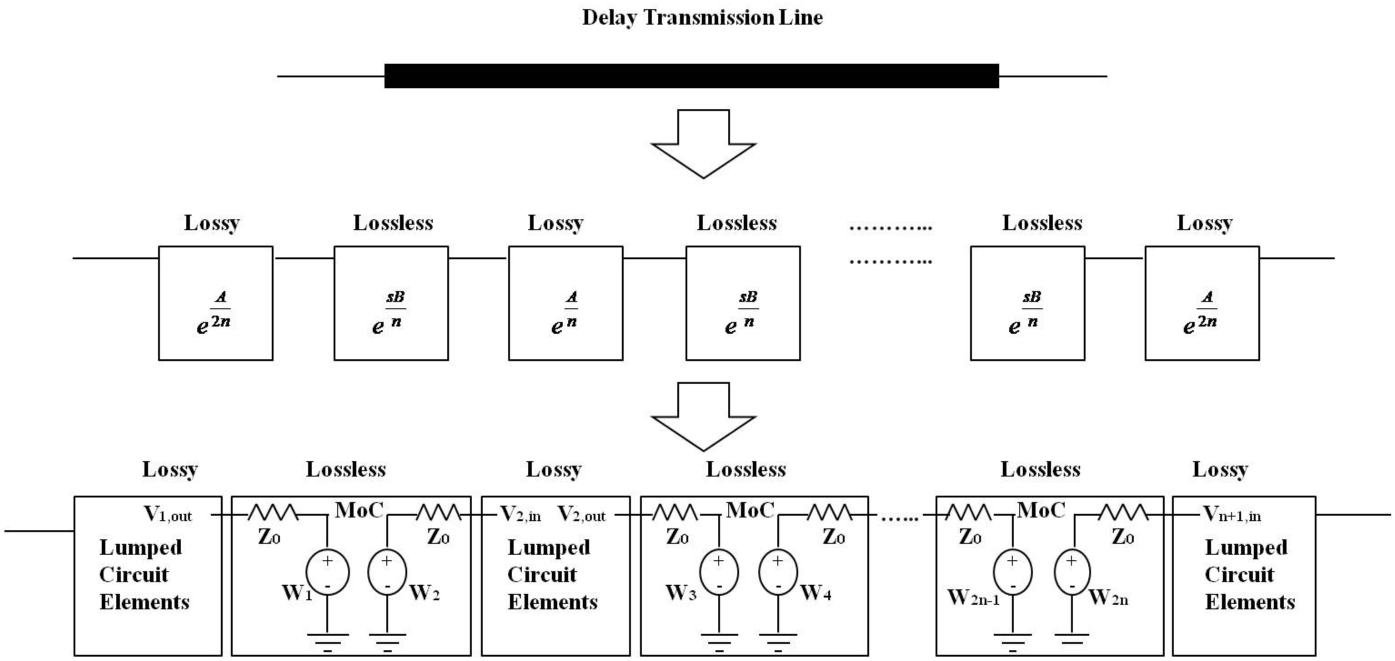

Fig. 1: SPICE equivalent circuit of a single transmission line using DEPACT.

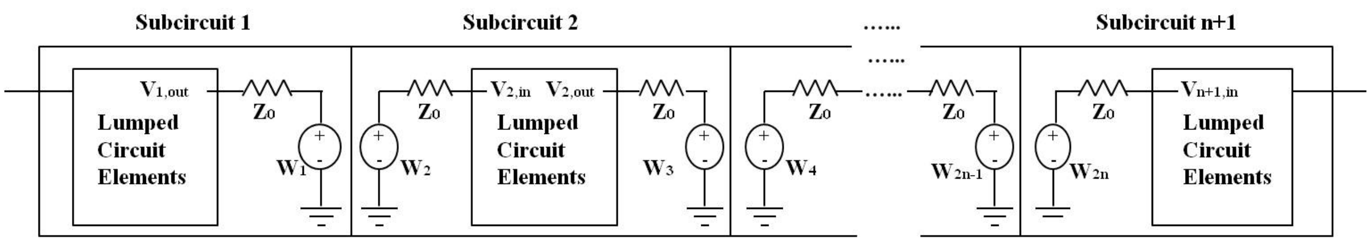

Fig. 2: Partitioning of DEPACT cells into subcircuits for waveform relaxation.

where V and I represent the terminal voltage and current variables respectively of the transmission line, R, L, G, and C represent the resistive, inductive, conductive and capacitive per-unit-length parameter respectively and l is the length of the transmission line.

To approximate $e^{(\mathbf{A}+s\mathbf{B})}$ in terms of a product of exponentials, a modified Lie product [8] is used as

$$e^{\mathbf{A}+s\mathbf{B}} \approx \prod_{i=1}^{n} \boldsymbol{\Psi}_i \quad \text{and} \quad \boldsymbol{\Psi}_i = e^{\frac{A}{2n}} e^{\frac{sB}{n}} e^{\frac{A}{2n}} \tag{3}$$

where *'n'* is the number of sections. Equation (3) shows that the exponential function of (1) can be divided into subsections of $e^{A/2n}$ and $e^{s\mathbf{B}/n}$ cascaded together. The matrix $e^{s\mathbf{B}/n}$ represents a lossless transmission line section and $e^{\mathbf{A}/2n}$ represents the the attenuation matrix. Hence, using (3), the transmission line can be discretized into a cascade of lumped circuit elements and lossless line sections as shown in Fig. 1 where the values of the lumped circuit block and the voltage sources $W_i(t)$ are described in [8], [9] and $Z_0 = \sqrt{L/C}$. The following section explains how the result of (3) is used to develop the waveform relaxation algorithm.

III. Development of the Proposed Algorithm

This section describes the partitioning of the DEPACT model into subcircuits and the proposed waveform relaxation algorithm.

A. Partitioning of the DEPACT model into subcircuits

Based on (3) of Section II, the single transmission line can be modeled as a cascade of lumped circuit elements and lossless line sections realized using MoC [8], [9] (Fig. 1). In the DEPACT model of Fig. 1, it is observed that there exists an inherent decoupling between the MoC subcircuits as reported in [3]. Exploiting this decoupling allows the DEPACT cells to be partitioned into '*n+1*' weakly coupled

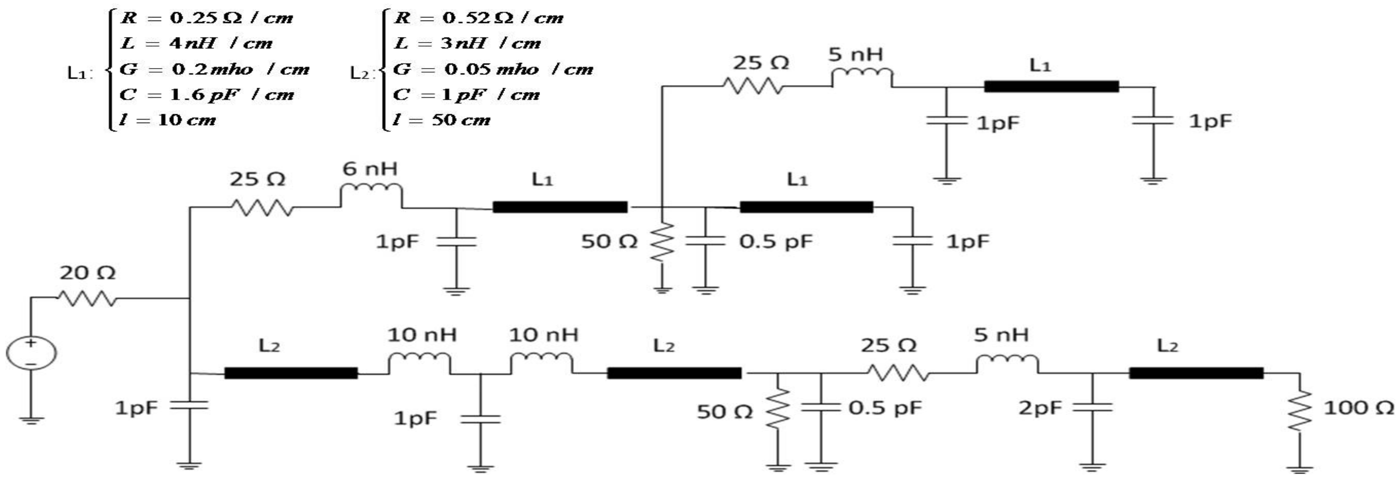

Fig. 3: Transmission line network for the numerical example.

subcircuits where n is the number of DEPACT cells in (3) as illustrated in Fig. 2.

This partitioning scheme differs from that used in [5] based on conventional lumped model where the far end voltage/current waveforms of one lumped subcircuit are the same as the near end voltage/current waveforms of the next subcircuit. Hence, typically for longitudinal partitioning of the line using conventional lumped model, the RLGC subcircuits are tightly coupled [5] and require exchange of additional waveforms followed by optimization routines to improve the convergence properties of the iterations. However, in the proposed partitioning scheme of Fig. 2 the subcircuits do not share their near/far end voltage/current waveforms. Thus, utilizing the natural partitioning provided by the MoC [3] makes the subcircuits more weakly coupled than that of [5]. Moreover, the subcircuits are coupled via the delayed sources $W_i(t)$ and $W_{i+1}(t)$ (Fig. 2). This introduces latency between the subcircuits and consequently a more relaxed exchange of information across the partitions [6]. The above reasons make the proposed partitioning scheme converge more efficiently that of [5] without additional waveform exchange and optimization routines. The following section explains the proposed waveform relaxation iteration technique using the subcircuits identified in Fig. 2.

B. Updating the waveform relaxation sources

It is observed from Fig. 2 that the transmission line is partitioned into '$n+1$' subcircuits, where each subcircuit is coupled to the adjacent subcircuit through the delayed sources $W_i(t)$ and $W_{i+1}(t)$. Thus to begin the iterations, i.e. iteration count $k=0$, an initial guess of $\{W_1^{(0)}(t), W_2^{(0)}(t), ..., W_{2n}^{(0)}(t)\}$ are used as sources to excite the corresponding subcircuits. For each the k iteration, all the subcircuits are solved individually either in sequence or in parallel for the time span $[0-T_{max}]$ where T_{max} is the maximum time point of interest. After each k iterations, the delayed sources need to be updated for the next iteration. This updating of the sources is done using the explicit relationship between the sources [8], expressed as

$$\begin{aligned} W_{2i-1}^{(k+1)}(t) &= 2V_{i+1,in}^{(k)}(t-\tau) - W_{2i}^{(k)}(t-\tau) \\ W_{2i}^{(k+1)}(t) &= 2V_{i,out}^{(k)}(t-\tau) - W_{2i-1}^{(k)}(t-\tau) \end{aligned} \quad (4)$$

where $W_{2i-1}^{(k)}(t)$, $W_{2i}^{(k)}(t)$, $V_{i+1,in}^{(k)}(t)$ and $V_{i,out}^{(k)}(t)$ are the results obtained from the k iterations and $\tau = l\sqrt{LC}/n$ is the delay of each section. Using the updated sources of (4) as the new sources for the $k+1$ iteration, the subcircuits are solved again. This cycle of iterations followed by updating of sources continues till the absolute error satisfies a predefined error tolerance expressed as

$$\varepsilon = \frac{1}{2n}\sum_{i=1}^{2n}\left|W_i^{(k+1)} - W_i^{(k)}\right| \le \alpha \quad (5)$$

where α is the predefined error tolerance. The convergence properties can be improved using the time windowing scheme [2], where the time span $[0-T_{max}]$ is further divided into 'm' uniform windows as $\{[0-T_1],[T_1-T_2],...,[T_{m-1}-T_m]\}$. The waveform iterations and updating the relaxation sources of (4) are repeated until (5) is satisfied for each window $[T_{j-1}-T_j]$ sequentially. In the following section, a numerical example is presented to illustrate the advantages of the proposed waveform relaxation algorithm.

IV. Numerical Examples

An example is presented in this section to demonstrate the validity of the proposed waveform relaxation algorithm. The subcircuits for each iteration was solved using SPICE and a customized C++ code was used to automatically extract the waveforms $\{W_i^{(k)}(t), V_{i,in}^{(k)}(t), V_{i,out}^{(k)}(t)\}$ for the i^{th} subcircuit and update the relaxation sources $\{W_i^{(k+1)}(t)\}$ of (4) without any external communication between the user and SPICE. The

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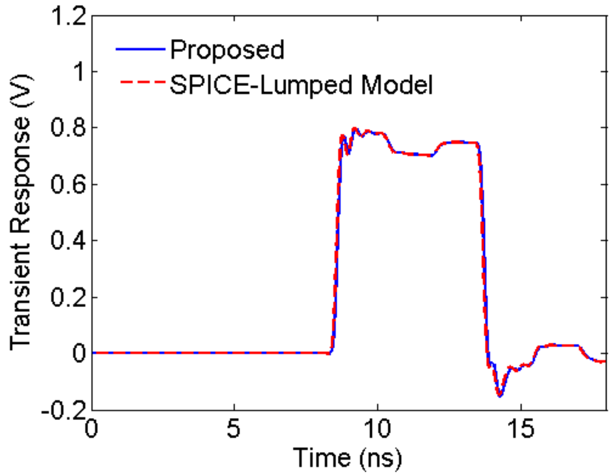

Fig. 4: Transient response using the proposed waveform relaxation algorithm and conventional lumped SPICE model.

invocation of SPICE and the C++ code for every subcircuit and iteration was automated using MATLAB 2010b.

To validate the proposed waveform relaxation algorithm, a transmission line network shown in Fig. 3 is considered. To match the impulse response of the considered example over 0-10 GHz, each 10 cm line required 15 sections and each 50 cm line required 55 sections. A voltage pulse of rise time 0.1 ns and pulse width 5 ns was considered as the input. To commence the waveform relaxation iterations, the initial guesses of $\{W_1^{(0)}(t), W_2^{(0)}(t),..., W_{2n}^{(0)}(t)\}$ was kept to zero and the subcircuits were solved sequentially. After every iteration, the waveform solution of $\{W_i^{(k)}(t), V_{i,in}^{(k)}(t), V_{i,out}^{(k)}(t)\}$ was used in (4) to update the $\{W_i^{(k+1)}(t)\}$ till a predefined error tolerance of $\varepsilon = 1e-4$ was achieved. In addition, the example was solved using [5] and conventional lumped SPICE model (both requires 2130 lumped segments). Fig. 4 shows the accuracy of the proposed waveform relaxation algorithm as compared to the conventional lumped SPICE model for the response of node P2.

It is observed that the proposed algorithm with 20 time windows requires 5 iterations to achieve an error of $\varepsilon = 1e-4$ while the work of [5] requires 10 iterations after optimization for same error tolerance and number of time windows. The CPU cost for simulation of the example using the proposed algorithm and conventional lumped SPICE model is provided in Table I. Although both the proposed algorithm and that of [5] use waveform relaxation to mitigate the CPU cost for transient analysis, the proposed algorithm is typically more efficient due to both the relatively smaller number of iterations required for convergence and also the smaller number of subcircuits required to model the network. It is expected that using a parallel processing paradigm (Gauss-Jacobi), the speedup of the proposed waveform relaxation over existing SPICE models will be even greater.

TABLE I
COMPARISON OF CPU COST FOR CONSIDERED EXAMPLE

	Proposed	Longitudinal discretization of [5]	SPICE Lumped
CPU time (sec)	6.31	48.60	123.66

V. CONCLUSION

This paper presents a waveform relaxation algorithm for the fast transient analysis of long delay transmission lines based on the longitudinal partitioning of the line. The proposed methodology represents the lossy transmission line as a cascade of lumped circuit elements and lossless line segments. Modeling the lossless segments using the method of characteristics (MoC) allows the line to be naturally partitioned into smaller, weakly coupled subcircuits, thereby allowing the iterations of the waveform relaxation algorithm converge efficiently.

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