Flow resistance estimation in mountain streams

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Abstract:

The quantification of flow resistance in complex natural channels continues to be a key source of uncertainty in basic research, engineering, and environmental management. This paper describes a meta-analysis of data compiled from a wide range of sources and flow conditions in mountain streams that compares the accuracy of equations developed by Jarrett (1984), Hey (1979), Bathurst (1985) and Bathurst (2002), with a new empirical model. The regression algorithm that was used to develop the model involved a four – fold cross validation procedure, with stratification of the compiled dataset by friction slope (S_f) values, to arrive at a dimensionless piecewise, nonlinear equation for Darcy–Weisbach friction factor. The new model is a function of S_f and relative submergence (ratio of hydraulic radius R to D_{84}) for streams <8% in slope, and relative submergence and ratio of D_{84} to D_{50} for streams with $S_f \geq 8\%$. Mean square error values, discrepancy ratios, and coefficients of determination indicate that the new model predicts friction factor better than the existing models that were tested, especially for mountain streams with large bed material and S_f values greater than 4%.

Introduction

An increasing demand for the limited water resources of mountain drainage basins has contributed to a need for more accurate and versatile models of the hydraulics of mountain streams. Central to the practice of hydraulic modeling is the evaluation of the roughness characteristics at the channel reach scale. The physical processes affecting flow resistance in mountain streams are very complex. Probably the most accurate and precise method for modeling all roughness effects, i.e. form drag around boulders, wake interference losses, skin friction, skimming flows, etc., would be to utilize a computational fluid dynamics (CFD) software package. However, the data requirements are such that the use of CFD is rarely practical in application to flows in natural mountain streams. Thus, an empirical approach remains the best option for modeling flow resistance in many situations.

Flow modeling in natural streams improves the prediction of flow conditions which may be expected to occur due to a range of design discharges. The discharge in an open channel, Q, is a function of the various hydrologic conditions upstream of the cross-section, i.e. rainfall, runoff, snowmelt, evaporation, flow diversion, flow augmentation, etc. Discharge may be expressed as a function of velocity and flow area by means of the simplified continuity equation, $Q = VA$, where V is the cross section averaged flow velocity and A is the area of the flow at a given cross section.

For a given discharge, flow velocity and depth vary between sites as a function of the roughness characteristics and the geometry of the channel. Channel roughness varies between sites due to a variety of channel and flow conditions that attenuate the current or increase turbulence. These include channel planform geometry, expansion or contraction of the channel cross section (Chow 1959), wave losses due to distortion of the water surface, localized hydraulic jumps in the flow over large bed roughness elements (Bathurst 1981), and form drag due to bed particles, bed forms, large woody debris, and live vegetation. Resistance due to form drag, wave losses and hydraulic jumps is largely a function of the size and disposition of bed and bank roughness elements relative to each other and the flow in the channel (Bathurst 1981).

At-a-station roughness has been observed to vary inversely with the depth of flow in the channel (e.g. Limerinos 1970, Hey 1979, Bathurst 1981, Jarrett 1984), thus it may not be prudent to assume that a measured roughness value will be constant for a reach over the range of design flows. It would be preferable to have a method to model changes in flow resistance at-a-station and between sites with changes in discharge, bed material, and channel geometry.

Much of the existing research in quantifying channel roughness has focused on quantifying the unitless Darcy – Weisbach (D-W) friction factor, f, as it is most suitable for the development of a dimensionally homogeneous empirical equation. Therefore, D-W f was used in this study. D-W f is expressed as in Equation (1), where R = hydraulic radius (L), g = acceleration due to gravity (L/T^2), S_f = friction slope (L/L), and V is as previously defined. Friction slope may by estimated by measurement of the slope of the water surface over a study reach (Jarrett 1984).

$$(8/f)^{0.5} = V/(gRS_f)^{0.5} \qquad (1)$$

D-W *f* may be converted to a Manning's roughness coefficient *n* and Chezy conveyance *C*, by means of Equation (2), where *R* and *g* are as defined above, expressed in SI units.

$$(8/f)^{0.5} = C/g^{1/2} = R^{1/6}/ng^{1/2} \qquad (2)$$

Flow resistance due to bedforms, grain roughness, channel planform and cross section geometry, width variability, local bank irregularities and vegetation effects may occur for any stream type. However, there is a difference in the relative importance of bedforms and grain roughness between low gradient streams and high gradient mountain streams. Flows in low gradient, sand bed streams are often affected by form drag caused by dunes and ripples on the channel bed. The highly mobile bedforms which are characteristic of sand bed streams are rarely present in coarse bed channels. Bedforms in mountain streams often occur as transverse gravel ribs or step-pool sequences. Transverse ribs may produce large amounts of form drag, resulting in the backwater pools observed in pool-riffle channel morphology (Hey 1979). While dunes and ripples may move rapidly through a reach of interest, channels with stable planform geometry tend to maintain pool-riffle geometry as well (Rosgen 1994). Thus one may avoid the complications of modeling the effects of channel bars in mountain streams by only considering flow over planar stream reaches (Hey 1979). In step-pool streams, large amounts of energy are lost due to spill resistance as the water flows over the largest particles and cascades into pool. Spill resistance in step-pool stream types account for large amounts of energy loss in steep headwater streams (MacFarlan and Wohl 2003).

Flow depths in mountain streams are generally of the same order of magnitude as the size of the larger bed particles over the range of inbank discharges. The relationship between flow depth and bed material size is often referred to as relative submergence, R/D_{84}, where D_{84} = diameter of the 84th percentile particle size measured on the intermediate axis by the Wolman (1955) method, and R is hydraulic radius. Relative submergence may also be defined as the ratio of hydraulic depth d to D_{84} (i.e. Bathurst 1981). For mountain streams with width to depth ratios greater than twenty, R and d are essentially the same (Jarrett 1984, Mussetter 1989).

Bathurst (1985) used relative submergence to categorize flow in mountain streams into three classes: large-, intermediate-, and small-scale roughness. Large-scale roughness corresponds to relative submergence values of less than 1.2; intermediate-scale roughness encompasses the range from 1.2 to 4; and small scale roughness is defined by relative submergence of greater than or equal to 4. Figure (1) presents an illustration of the flow and channel bed for conditions of large-, intermediate-, and small-scale roughness.

(Insert illustrations of roughness scales from Mussetter 1989, p. 76)

Figure 1. Conceptual illustration of roughness scales.

Flow resistance processes change with relative submergence conditions. Surface drag is the dominant characteristic affecting channel roughness in streams with small-scale roughness elements, i.e. high relative submergence values, and the velocity profile typically abides by the Prandtl – von Karman logarithmic velocity profile law (Keulegan 1938). Flows characterized by intermediate to large scale roughness conditions tend to have exaggerated surface velocities, resulting in an S – shaped velocity profile (Jarrett 1984). Intermediate scale roughness elements protrude into the flow and cause disruptions in the free surface, resulting in energy loss due to form drag and wave losses. Large scale roughness is characterized by form drag from the roughness elements, energy losses due to wave formation, disconnection of the free surface, and blockage, i.e. the forcing of flow over and between the largest bed particles (Bathurst 1981).

The proportion of the bed particles which contributes to form drag, blockage and the funneling of the flow into chutes and pools is determined by the degree to which large roughness elements protrude above the channel bed into the flow area. This characteristic of the channel boundary is largely a function of the grain size distribution (Bathurst 1981, Mussetter 1989). The gradation of the sediment in the channel boundary can be described by the geometric standard deviation $\sigma_g = D_{84}/D_g$ (e.g. Sturm 2001), where particle size data are collected by the methods described by Wolman (1955), and the geometric mean particle diameter D_g may be estimated by the median value, D_{50} (i.e. Bathurst 1981, Mussetter 1989).

Large amounts of energy losses occur where flow is funneled between large bed particles, causing localized hydraulic jumps (Herbick and Shulitz 1964). The amount of energy lost due to this phenomenon is a function of relative submergence, the disposition of the particles, and the Froude number (Bathurst 1981). Unfortunately, there is little utility in using Froude number, F_r, in a regression for $(8/f)^{0.5}$, as $F_r = (S_f)^{0.5}(8/f)^{0.5} = V/(gd)^{0.5}$, and thus flow depth and velocity would need to be known *a priori*. However, not accounting for changes in F_r should not introduce error greater 5 – 10% for streams with gradients less than 5% (Rosso et al. 1990, quoted in Grant 1997).

The most widely used empirical equations for predicting roughness in gravel bed streams are simply functions of relative submergence (e.g. Limerinos 1970, Hey 1979, Bathurst 1985, Bathurst 2002) and thus do not account for variations in Froude number or the disposition of the bed roughness elements in the channel. Mussetter (1989) developed an equation for D-W *f* using friction slope, D_{84}/D_{50}, and R/D_{84} using data from steep mountain streams, and tested its efficacy versus the equations developed by Jarrett (1984), Hey (1979), Bathurst (1979), and Bathurst (1985). His relationship performed better streams steeper than 4%, but there was little improvement for flatter streams.

The objectives of this study were to investigate the effectiveness of existing roughness prediction equations for various conditions and to improve the estimation of flow resistance in mountain streams by using data compiled from a wider range of flow conditions in natural streams than were used to fit Mussetter's equation. It was hypothesized that prediction of D-W *f* may be improved by incorporating the three channel parameters used in Mussetter's equation, S_f, D_{84}/D_{50}, and R/D_{84} into a piecewise regression relationship.

Model Development and Testing

A total of 357 observations of mountain streams at 102 sites were used to develop a new model and to evaluate the roughness prediction capabilities of several existing equations. The data for this study were compiled from: Barnes (1967), Judd and Peterson (1969), Emmett (1972), Bathurst (1978), Zevenbergen (1983), Jarrett (1984), Bathurst (1985), and Mussetter (1989). At a minimum, each observation used in the present study included measured or calculated values of R, A, D_{84}, $D_{50,}$ S_f, V, Q, F_r and top width w. The data were collected from natural streams in the western United States and the United Kingdom. All of the data were originally collected for the purpose of investigating the roughness characteristics or hydraulic geometry of steep, coarse bed streams. At least two observations were made at each station for two different discharges for approximately 90% of the sites, thus the compiled data describe changes in at-a-station flow resistance with corresponding changes in discharge.

Although step-pool streams are likely to occur in the observed S_f range, classic step-pool geometry parameters such as step height and length were not available. Resistance effects due to large woody debris (LWD) are not considered in this study, although form drag and spill resistance due to LWD may account for the majority of energy loss in streams where LWD is present in the flow (Curran and Wohl 2003). Explicit reports related to geomorphogy, magnitude of channel forming discharge, flow regime and bed forms were not included in all of the studies. It was assumed, where not expressed in the original studies, that the channel reaches were of relatively uniform width and free of significant resistance-causing vegetation and LWD.

Relative submergence was most often reported in the literature as the ratio of R to D_{84}, but was sometimes presented as d/D_{84}. Measurements of w, A, R and/or d were missing from some of the original sources. Therefore, in order to calculate relative submergence by using a single ratio while guarding the largest amount of data possible, it was assumed that R and hydraulic d were equal for channels with width to depth ratios greater than 20. Observations from Bathurst (1985) and Emmett (1972) with width to depth ratios less than 20 were omitted from the data set because observations of R were not reported.

Multiple regression analyses were used to develop a dimensionless nonlinear regression equation for $(8/f)^{0.5}$ using R/D_{84} , S_f and D_{84}/D_{50}. The best regression equation was computed as a function of the Mallow's Cp score using the best subsets procedure (Mallow 1973). This method is well suited to arrive at relationship including only the most important predictive variables. A four-fold cross validation procedure was used to test the efficacy of the new equation, identify an optimal threshold value of S_f, and detect over-fitting. The data were randomly partitioned into four groups of approximately 90 flow observations, with no data from any one particular site present in more than one group, so as to ensure that the four groups were independent. The multiple regression analysis procedure was performed using data from three groups. The mean square error generated by using the empirical relationships derived from the regression algorithm was then calculated for the fourth group test set. This process was repeated four times for each model type, such that each flow observation was used once and only once to train and test the new model.

In order to verify that no regression assumptions were violated by methods used in this study, plots of the distributions of the regression residuals and observed versus predicted values were made to validate normality of the regression residuals and to check for homoscedasticity in the final regression equation. Correlation matrices were calculated for the independent variables used in the regressions for $(8/f)^{0.5}$. Plots and matrices are included in the Appendix. Energy slope, relative submergence and gradation coefficient were found to be collinear, but the correlation coefficients between the regression parameters did not exceed 0.66 (i.e. maximum $R^2 = 0.44$). Plots of the independent variables versus the observed roughness were made to verify that empirical coefficients generated by the regression procedure made physical sense.

In consideration of the wide range of slopes (and thus stream-types) present in the dataset ($0.002 < S_f < 0.168$), the data in each training set were stratified by slope to so as to allow for the fitting of a piecewise nonlinear regression equation. The model development procedure was also conducted using no stratification of the data, as done in Mussetter's original work, the difference being that the training set in the present study included many more data points. The threshold values and the corresponding sample sizes used in the regressions are presented in Table 1.

Table 1. Regression threshold values and corresponding sample sizes

Energy Slope (m/m)	Number of Observations Less than Value	Number of Observations Greater than Value
No Split	357	N/A
0.008	41	316
0.01	64	293
0.02	166	191
0.03	239	118
0.04	269	88
0.05	278	79
0.06	299	58
0.07	307	50
0.08	312	45

The mean of the four mean square error (MSE) values calculated from the four test sets in the cross validation procedure was calculated for each of the new models created by the stratification method. The MSE values generated by using the equations developed by Hey (1979), Bathurst (1985), Jarrett (1984), and Bathurst (2002), listed below, were calculated for comparison with the results of the new model. All of the equations are dimensionless except for Jarrett, where *R* is expressed in feet.

$$(8/f)^{0.5} = 3.17 + 5.75 \log(R/D_{84}) \qquad \text{(Hey, 1979)} \qquad (3)$$

$$(8/f)^{0.5} = 4 + 5.62 \log(R/D_{84}) \qquad \text{(Bathurst, 1985)} \qquad (4)$$

$$(8/f)^{0.5} = 0.671\, R^{0.33}\, S_f^{-0.38} \qquad \text{(Jarrett, 1984)} \qquad (5)$$

$$S_f < 8\%: (8/f)^{0.5} = 3.84(R/D_{84})^{0.547} \qquad (6a)$$

$$S_f \geq 8\%: (8/f)^{0.5} = 3.1(R/D_{84})^{0.93} \qquad \text{(Bathurst, 2002)} \qquad (6b)$$

Equations (3) through (6) were developed using flow observations with S_f less than 4%, but the available dataset included many observations of higher slopes. Once the best-performing new empirical relationship was identified, all of the observations were partitioned into two groups: $S_f < 4\%$ and $S_f \geq 4\%$, so that the new model could be compared to the existing equations within the range of flows for which they were developed. Mean square errors, correlation coefficients, and discrepancy ratios (ratio of predicted to observed values) were calculated for each model type, and plots of predicted versus observed values were made to compare the results. Mean error and mean square error values were also calculated based on relative submergence category.

Results

The model development procedure and cross validation analyses yielded a simple dimensionless empirical relationship which predicts Darcy – Weisbach *f* in mountain streams better than the models developed by Jarrett, Bathurst and Hey. The identification of an optimal threshold value for the piecewise regression was difficult due to the similarity of results between stratification at the S_f = 4% and 8% thresholds. Ultimately, the equation based on the S_f = 8% threshold proved to be the better candidate, and is presented below as Equation 7.

$$S_f < 8\%: \quad (8/f)^{0.5} = 0.65\, S_f^{-0.37} (R/D_{84})^{0.33} \qquad \text{(7a)}$$

$$S_f \geq 8\%: \quad (8/f)^{0.5} = 4.76\, (R/D_{84})^{0.66} (D_{84}/D_{50})^{-1.27} \qquad \text{(7b)}$$

The MSE values for each test set from the cross validation procedure are presented in Table (2). The equations that were created by stratifying the data at the S_f = 8% and 4% thresholds had the lowest average MSE values. The MSE values generated by the cross validation algorithm for the equations developed Hey (1979), Jarrett (1984), Bathurst (1985) and Bathurst (2002) are shown for comparison in Table (3).

Table 2. Cross validation results for tested threshold values

	Mean Square Error by Threshold Value									
Cross Valid. #	**No Split**	**0.8%**	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**
1	1.61	3.39	1.48	1.36	1.29	1.04	1.18	1.05	1.08	1.13
2	2.13	5.61	5.30	2.77	2.13	1.89	1.89	2.14	2.93	1.89
3	1.31	1.07	1.43	0.99	1.00	1.12	0.98	1.01	1.06	1.00
4	5.23	2.64	2.63	2.58	2.27	2.07	2.09	2.08	2.11	2.11
Mean	2.57	3.18	2.71	1.93	1.67	1.53	1.54	1.57	1.80	1.53

Table 3. Cross validation results for Eq. 7 and existing models tested

	Mean Square Error				
Cross Validation	**Eq. 7**	**Jarrett**	**Hey**	**Bathurst (1985)**	**Bathurst (2002)**
1	1.13	0.89	2.00	3.39	4.24
2	1.89	1.59	3.94	5.81	16.73
3	1.00	1.02	1.58	2.91	3.20
4	2.11	3.26	2.78	3.65	8.39
Mean	1.53	1.69	2.58	3.94	8.14

The coefficients of determination (R^2) for the two new equations and the four existing relationships are presented in Table (4), which shows that the S_f = 8% threshold model, Equation 7, fits the higher gradient flow observations better than the S_f = 4% model.

Table 4. Correlation coefficients for models tested

	Coefficients of Determination (R^2)					
S_f Range	**4%**	**Eq. 7**	**Jarrett**	**Hey**	**Bathurst (1985)**	**Bathurst (2002)**
All Data	0.71	0.71	0.64	0.50	0.50	0.35
0.002 - 0.039	0.58	0.58	0.51	0.37	0.37	0.21
0.040 - 0.168	0.62	0.68	0.22	0.50	0.50	0.50

Table (5) presents the MSE values generated by application of Equation 7 and the other existing equations for the two friction slope classes. The MSE and mean percent error values for the various equations, stratified by relative submergence class, are presented in Table (6) and (7).

Table 5. Mean square error values, stratified by friction slope range

			Mean Square Error			
S_f Range	**N**	**Eq. 7**	**Jarrett**	**Hey**	**Bathurst (1985)**	**Bathurst (2002)**
0.002 - 0.039	268	1.7	2.0	2.7	3.8	9.6
0.040 - 0.168	89	0.2	0.5	2.0	4.2	2.4

Table 6. Mean square error values stratified by relative submergence category

			Mean Square Error			
Rel. Sub. Range	**n**	**Eq. 7**	**Jarrett**	**Hey**	**Bathurst (1985)**	**Bathurst (2002)**
All Data	357	1.36	1.66	2.51	3.87	7.80
< 1.2	153	0.61	0.94	1.23	2.25	1.08
1.2 - 4	179	1.53	2.00	2.67	4.25	5.44
> 4	25	4.82	3.59	9.22	11.09	65.83

Table 7. Mean percent error values by relative submergence category (%)

			Mean Percent Error (%)			
Rel. Sub. Range	**n**	**Eq. 7**	**Jarrett**	**Hey**	**Bathurst (1985)**	**Bathurst (2002)**
All Data	357	-7	-31	-14	-68	-67
< 1.2	153	-11	-71	10	-83	-66
1.2 - 4	181	-1	0	-31	-56	-56
> 4	24	-20	-16	-42	-57	-159

The resistance values predicted by Equation 7 and the existing equations developed by Jarrett, Hey, and Bathurst were plotted versus the observed resistance values for all flow observations. Figures (1) through (5) present observed versus predicted values for $S_f <$ 4%, i.e. the range of applicability for the existing equations. Figures (6) through (10) present observed versus predicted values for $S_f \geq 4\%$. Lines were added to the plots show the range of ± 33% error and the line of perfect agreement.

Discrepancy ratios were calculated for each model type, and the results are presented as histograms relating the frequency of the discrepancy ratios, expressed in percent of all observations, occurring in a given ratio class. Comparisons of the distributions of the discrepancy ratios for the two S_f categories are presented in Figures (11) through (18). Discrepancy ratio values greater than one indicate that the model overpredicted $(8/f)^{0.5}$, and values less than one represent underprediction of $(8/f)^{0.5}$.

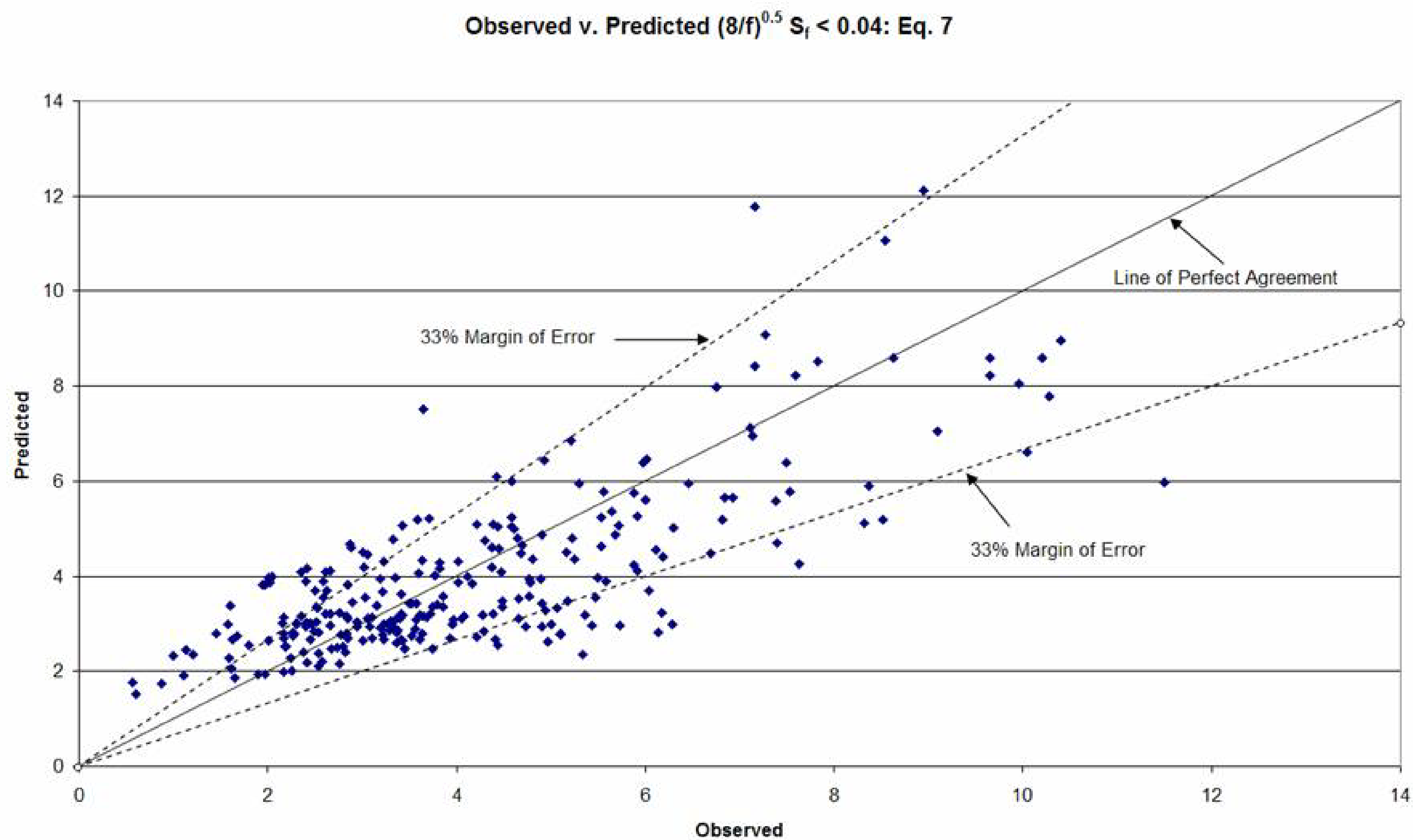

Figure 1. Observed versus predicted values of (8/f)0.5 for Sf < 0.04: Eq. 7

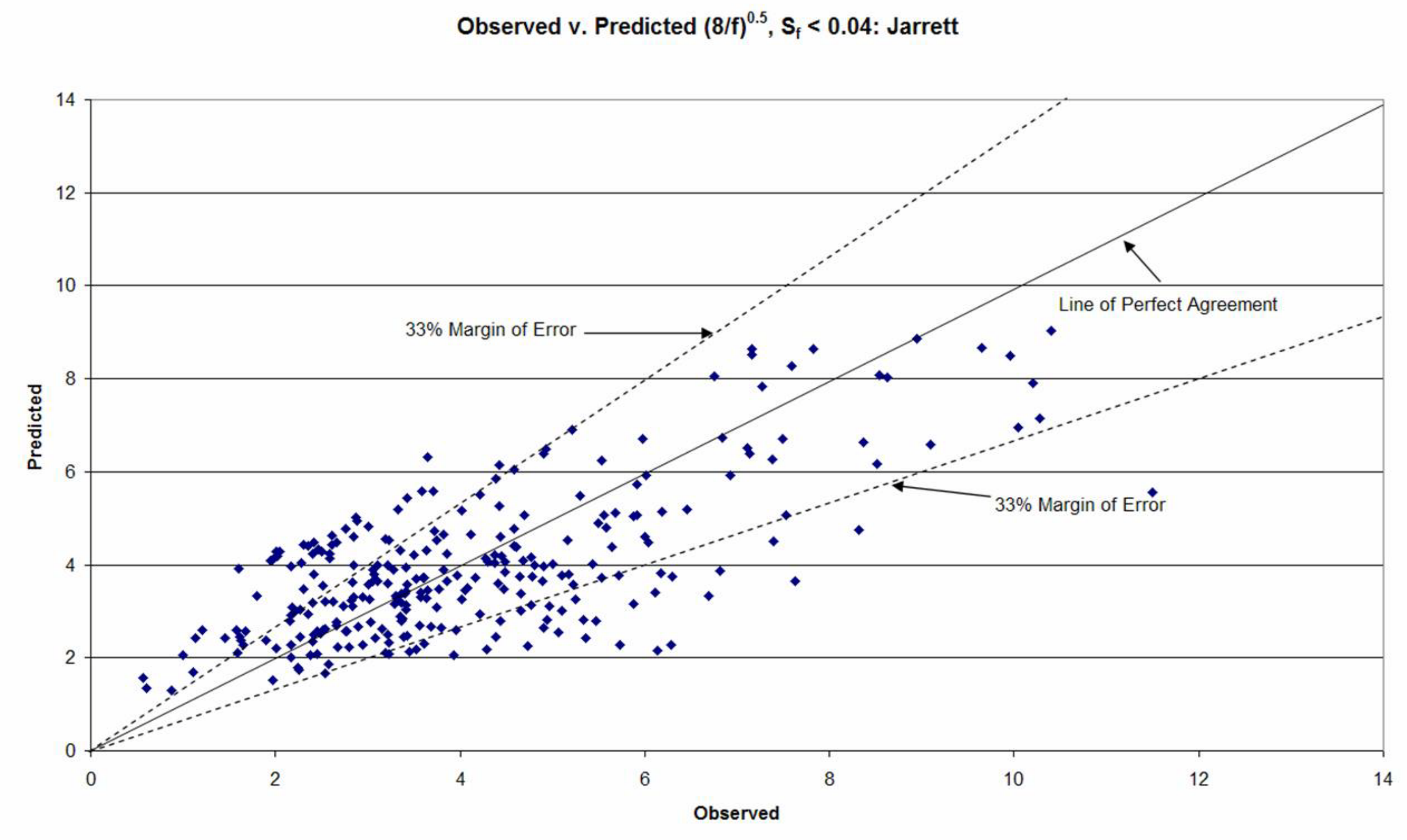

Figure 2. Observed versus predicted values of $(8/f)^{0.5}$, $S_f < 0.04$: Jarrett

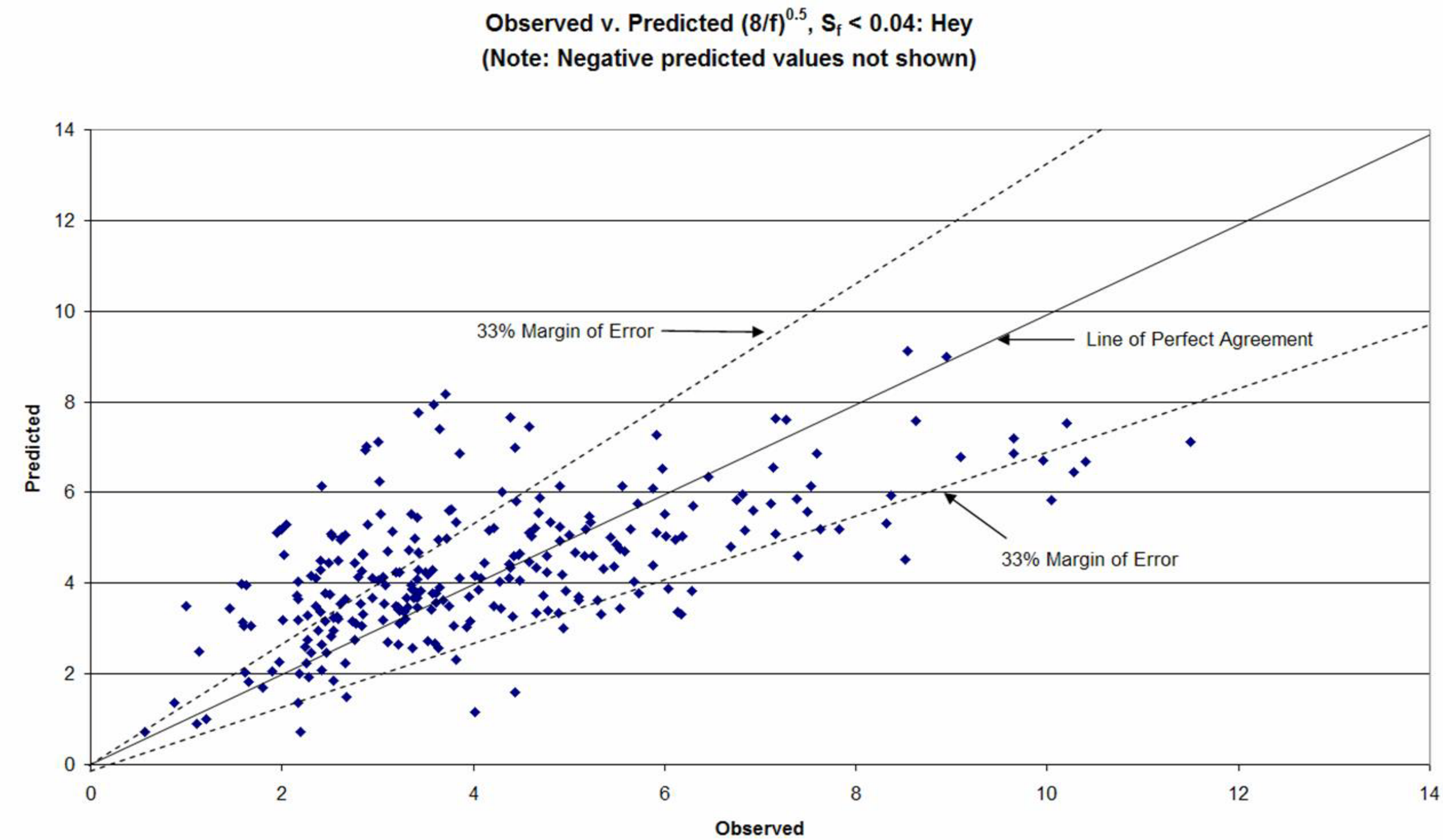

Figure 3. Observed versus predicted values of $(8/f)^{0.5}$, $S_f < 0.04$: Hey

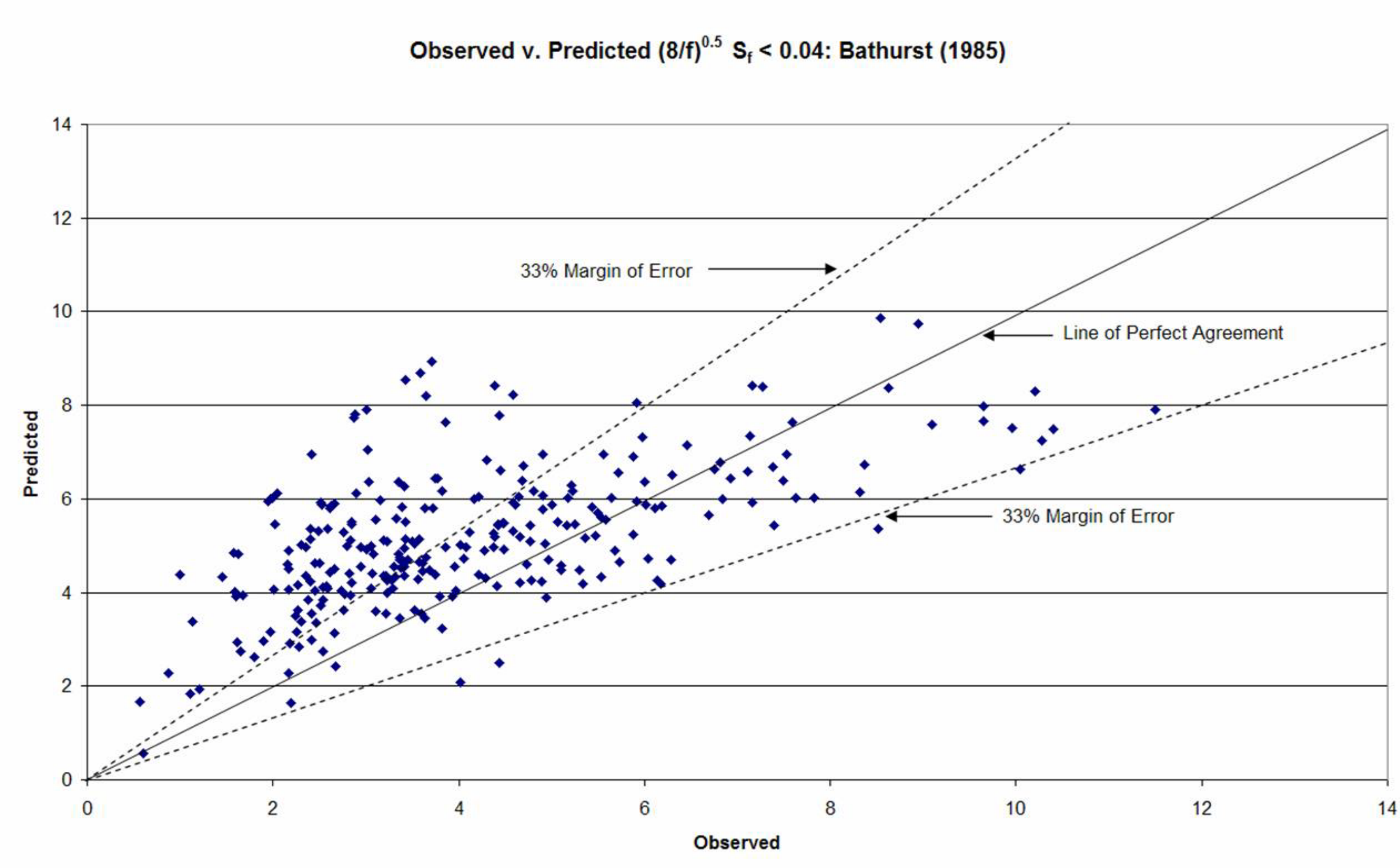

Figure 4. Observed versus predicted values of $(8/f)^{0.5}$, $S_f < 0.04$: Bathurst (1985)

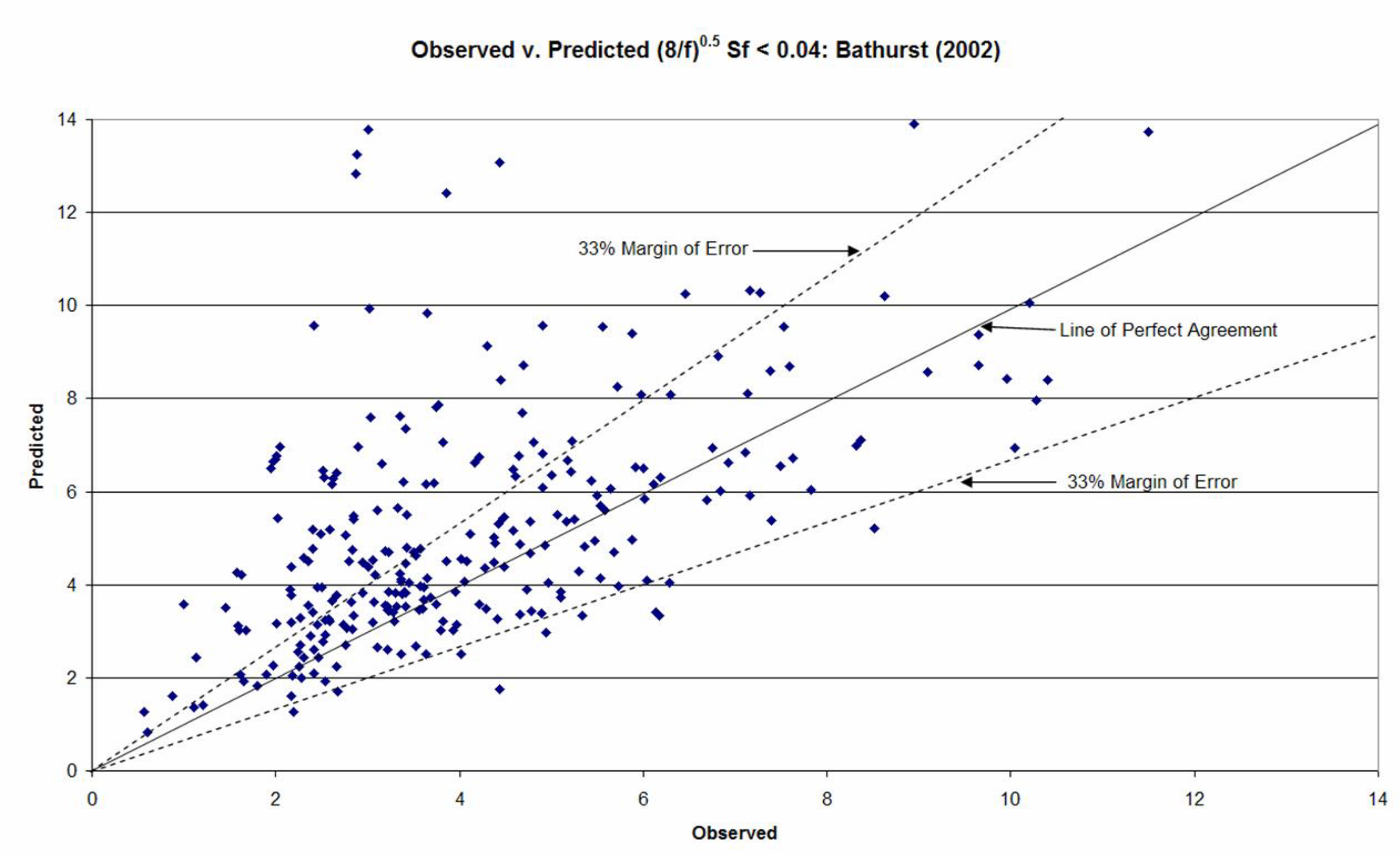

Figure 5. Observed versus predicted $(8/f)^{0.5}$, $S_f < 0.04$: Bathurst (2002)

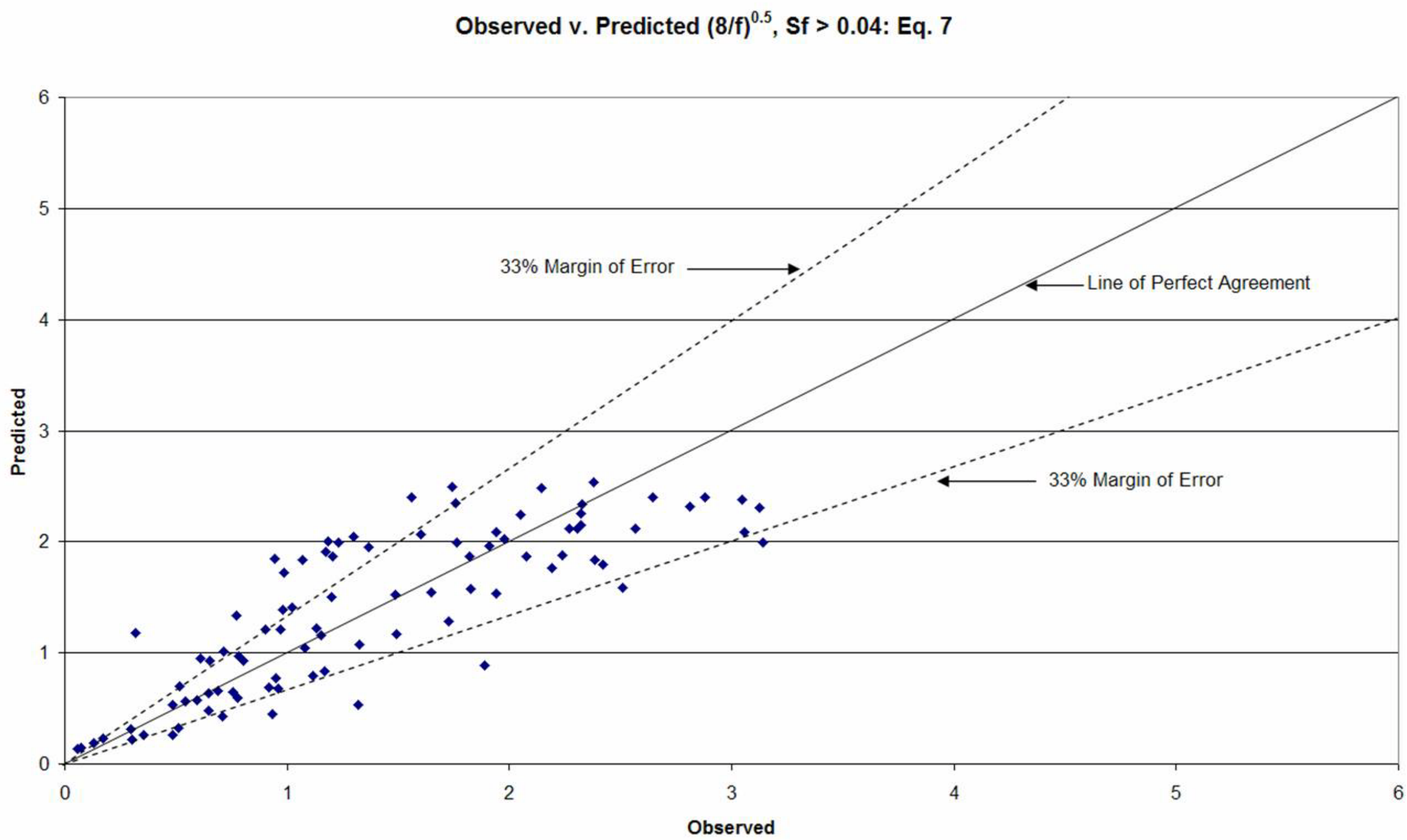

Figure 6. Observed versus predicted $(8/f)^{0.5}$, $S_f > 0.04$

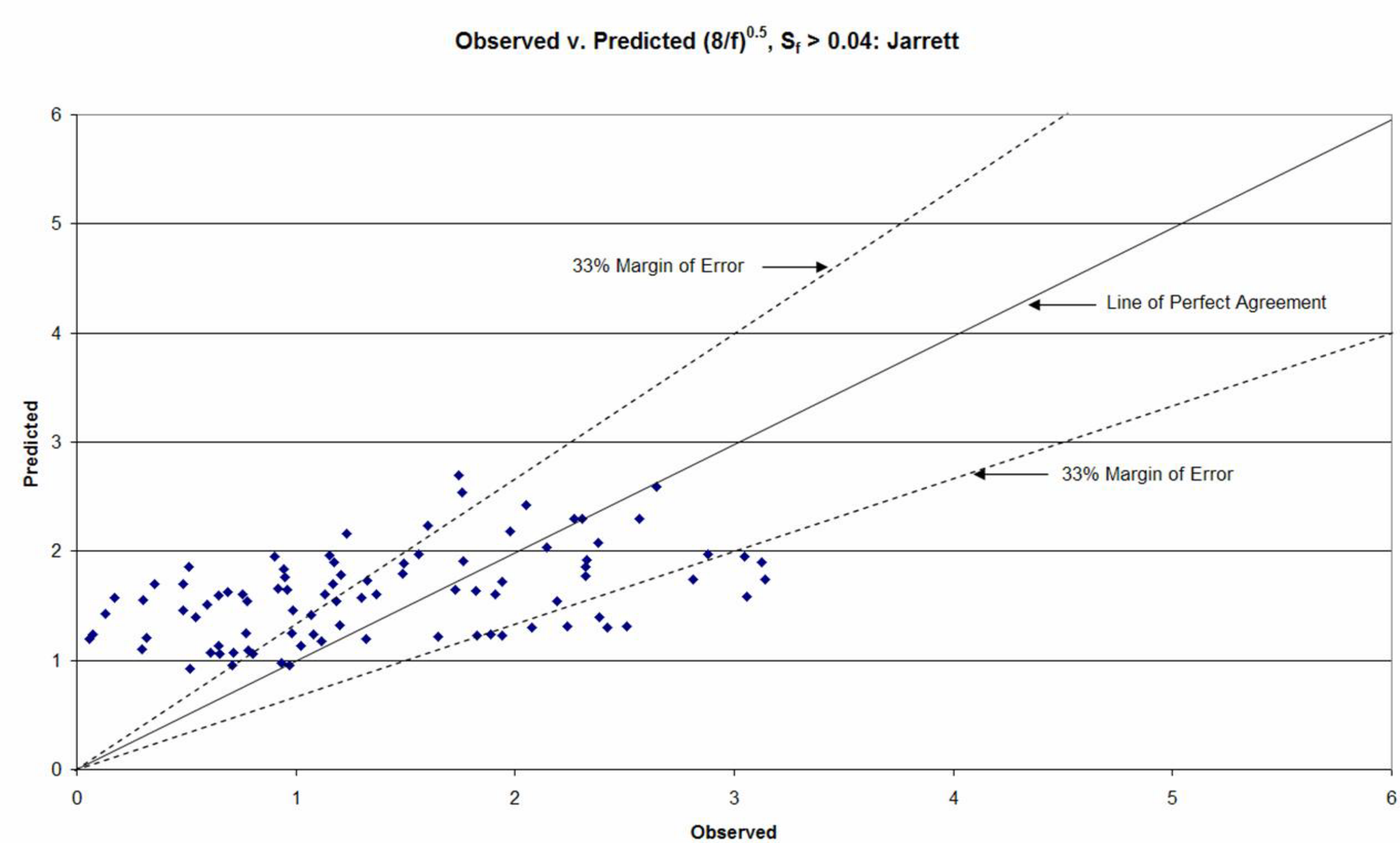

Figure 7. Observed versus predicted $(8/f)^{0.5}$, $S_f > 0.04$: Jarrett

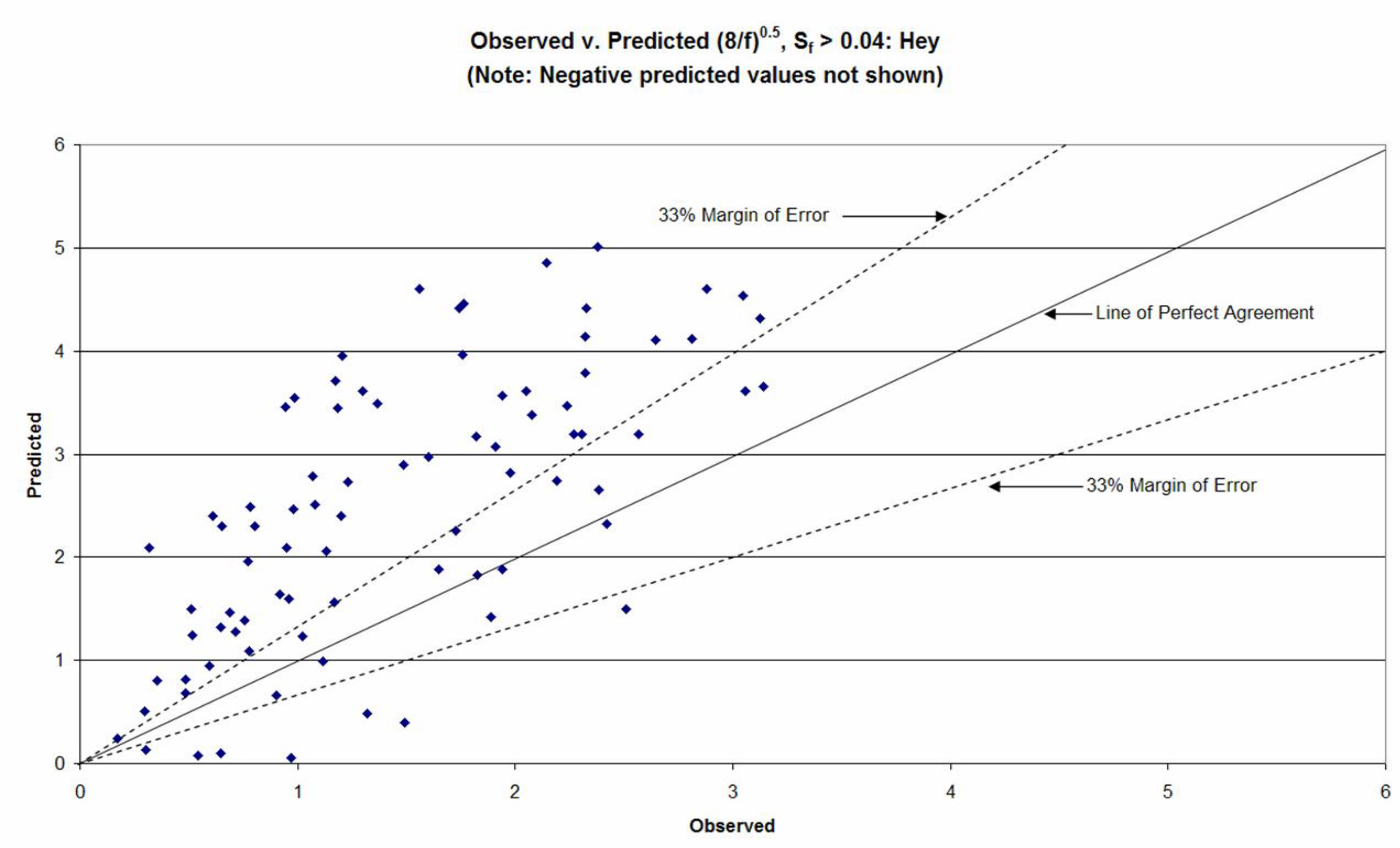

Figure 8. Observed versus predicted $(8/f)^{0.5}$, S_f > 0.04: Hey

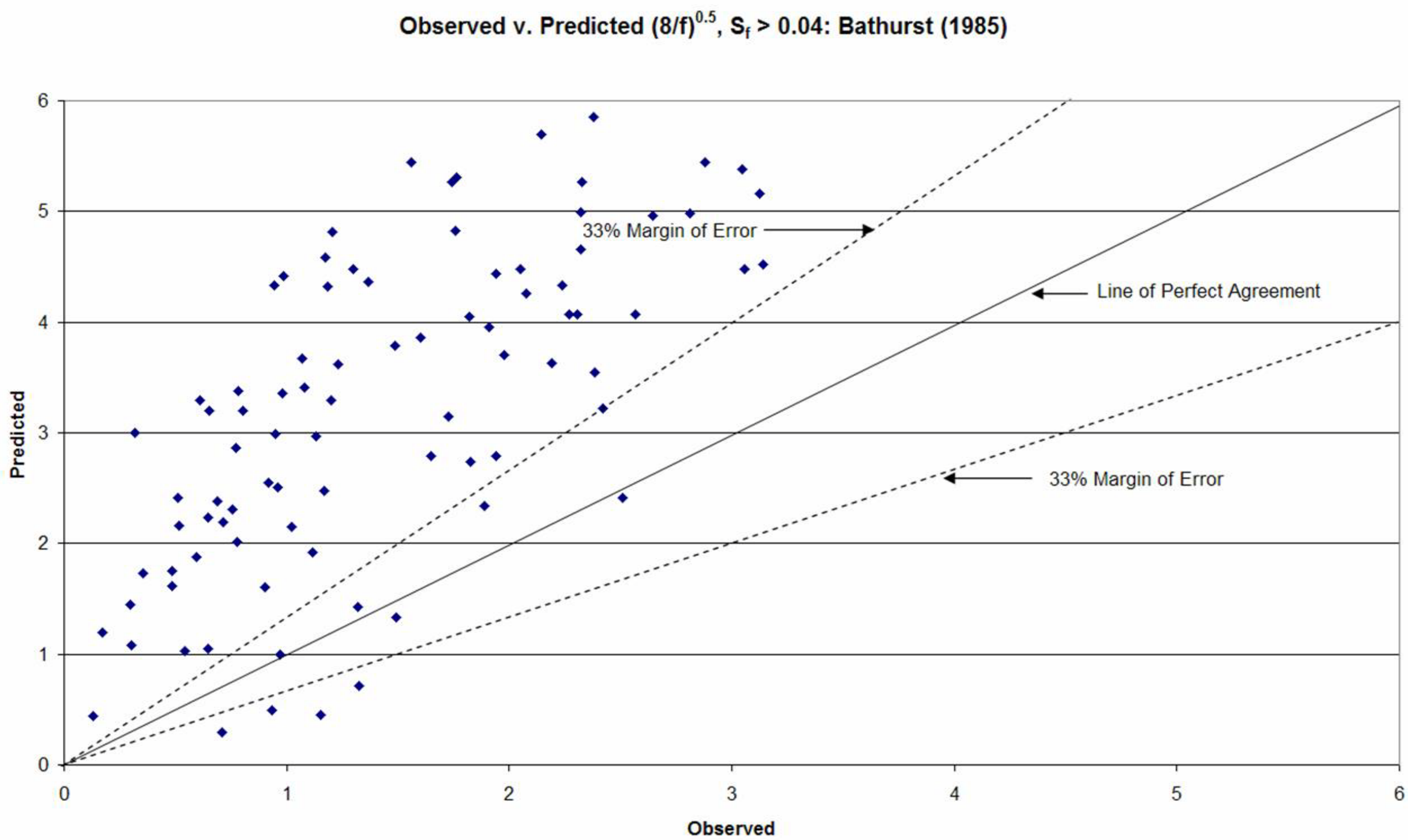

Figure 9. Observed versus predicted $(8/f)^{0.5}$, $S_f > 0.04$: Bathurst (1985)

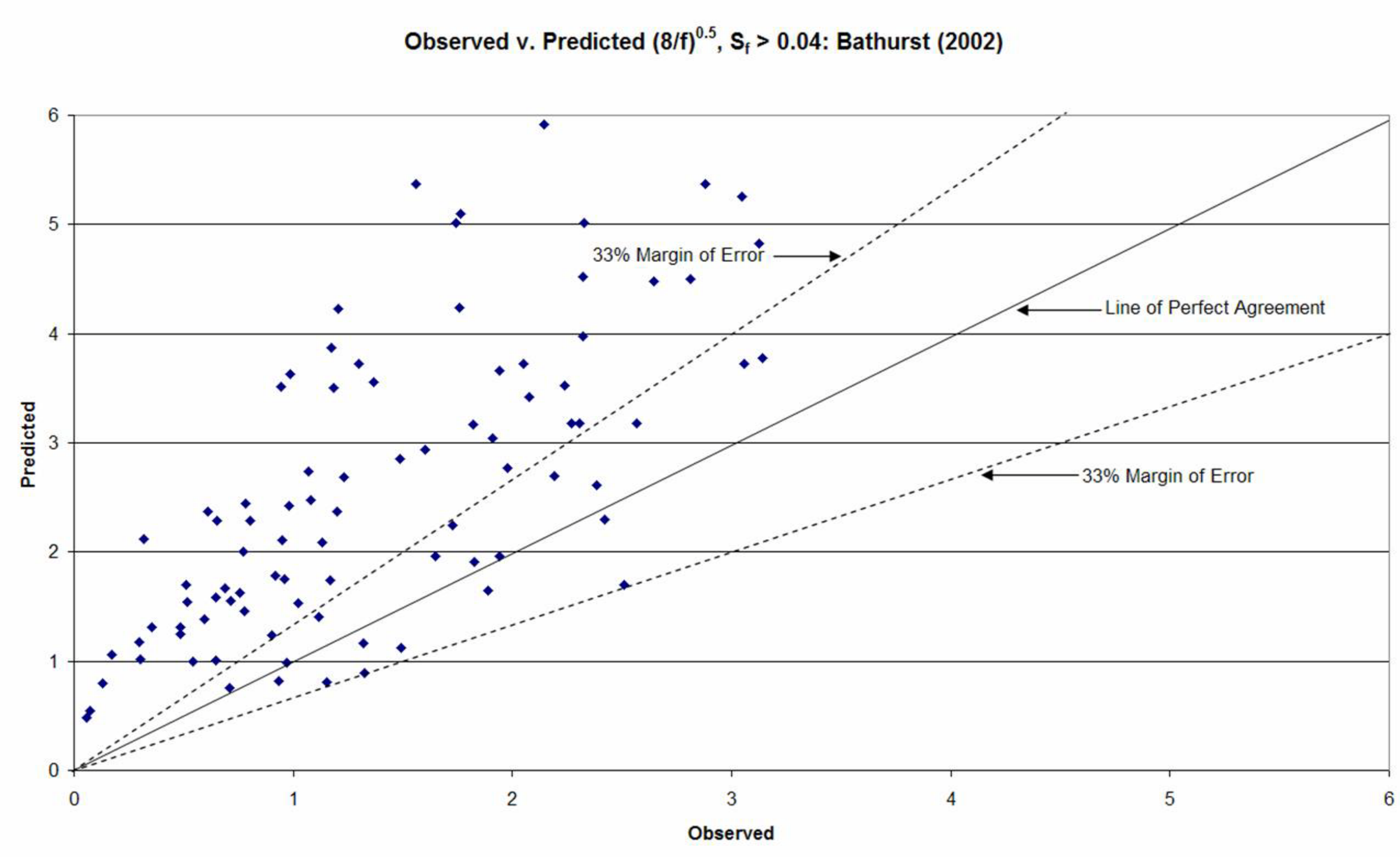

Figure 10. Observed versus predicted $(8/f)^{0.5}$, $S_f > 0.04$: Bathurst (2002)

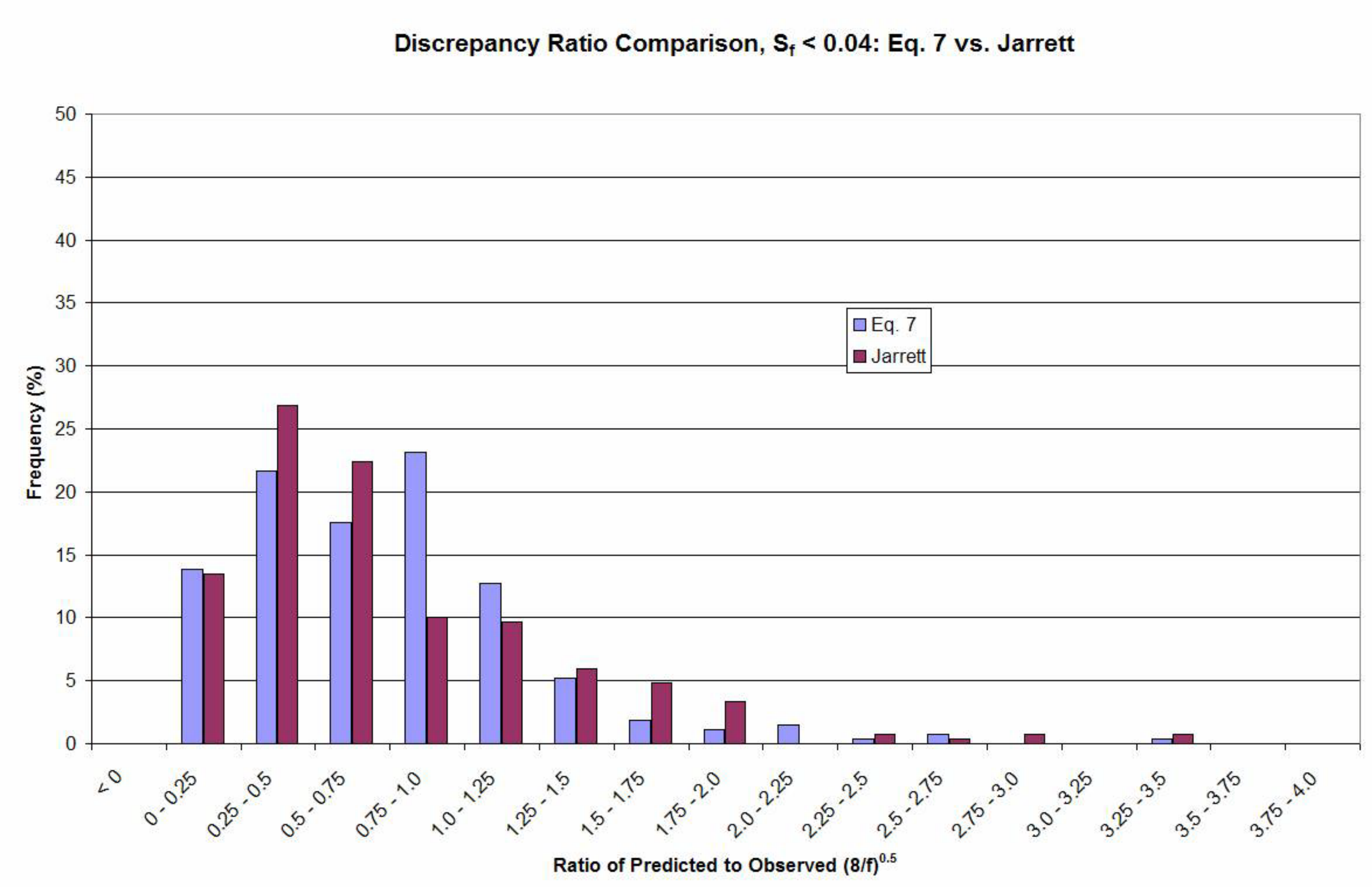

Figure 11. Comparison of the discrepancy ratios of Eq. 7 and Jarrett, S_f < 0.04

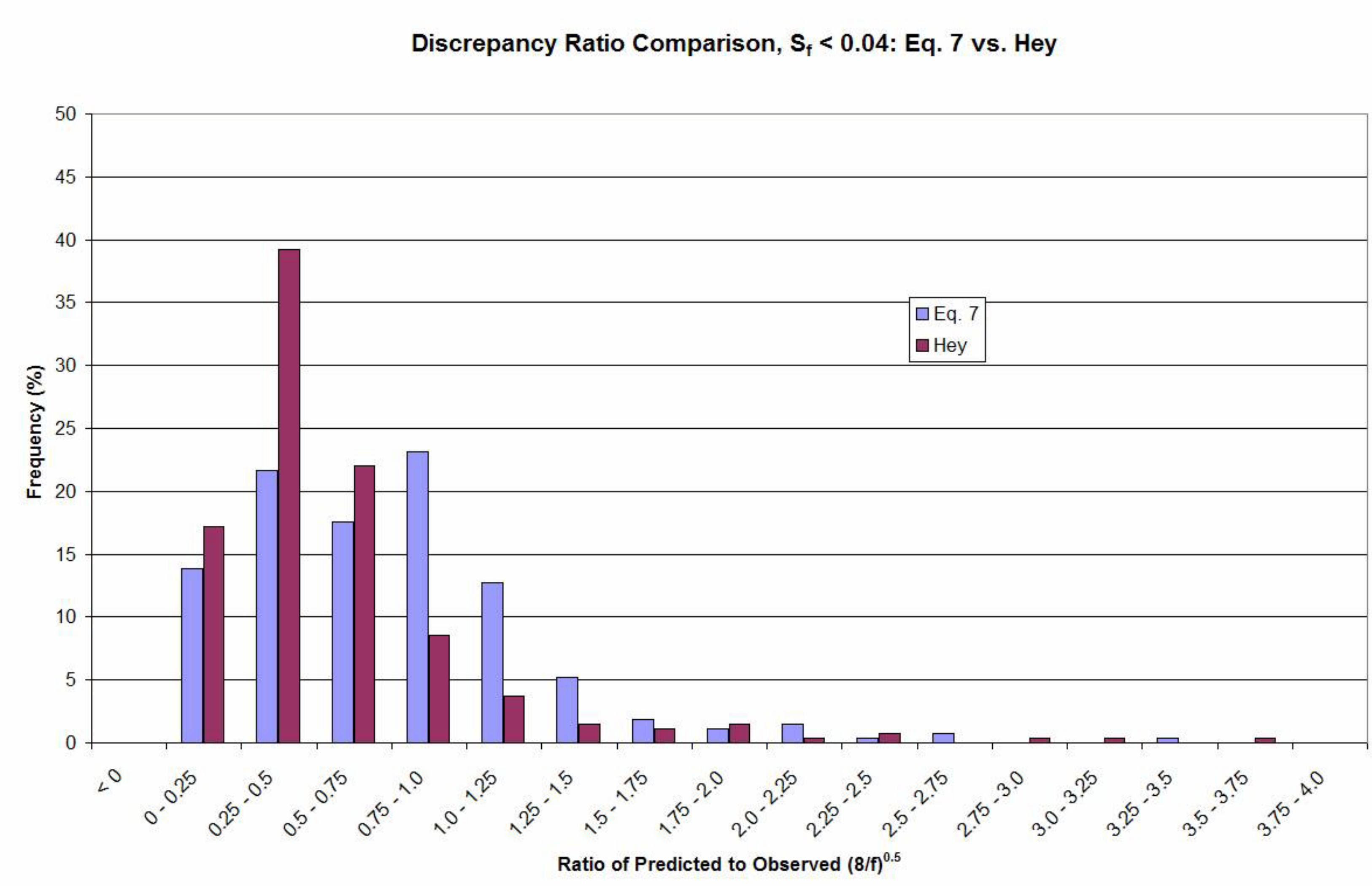

Figure 12. Comparison of the discrepancy ratios of Eq. 7 and Hey, S_f < 0.04

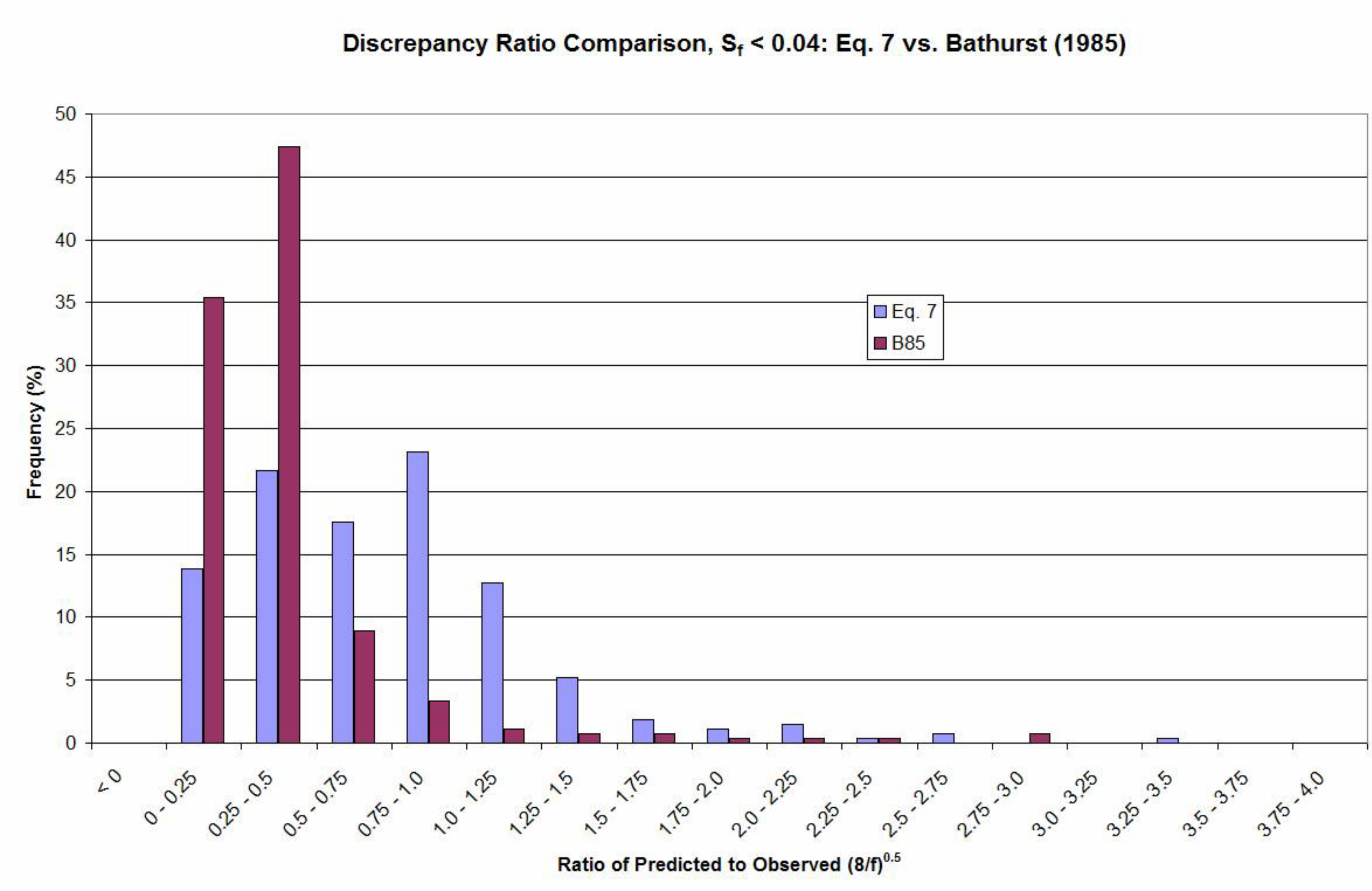

Figure 13. Comparison of the discrepancy ratios of Eq. 7 and Bathurst (1985)

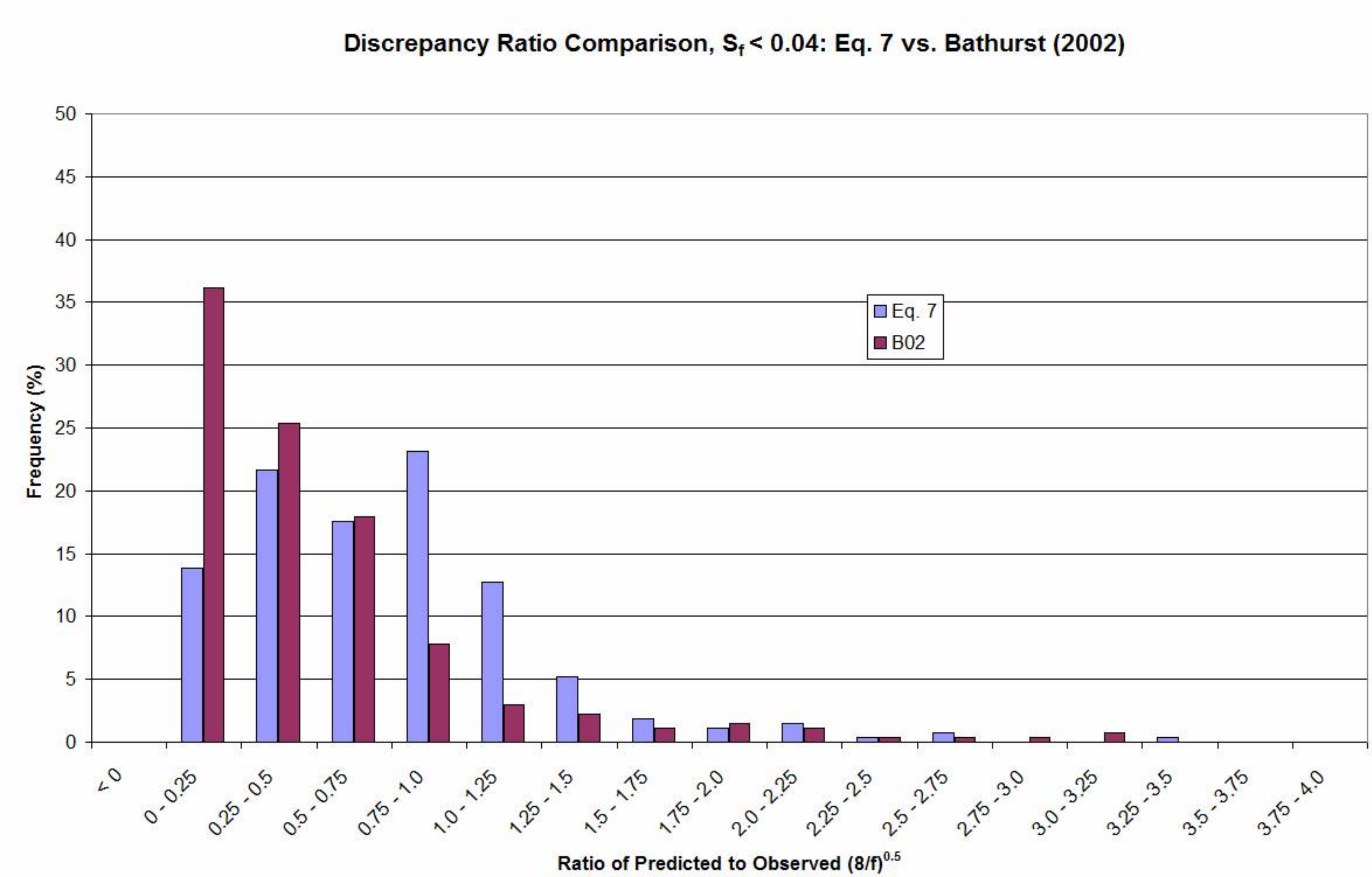

Figure 14. Comparison of the discrepancy ratios of Eq. 7 and Bathurst (2002)

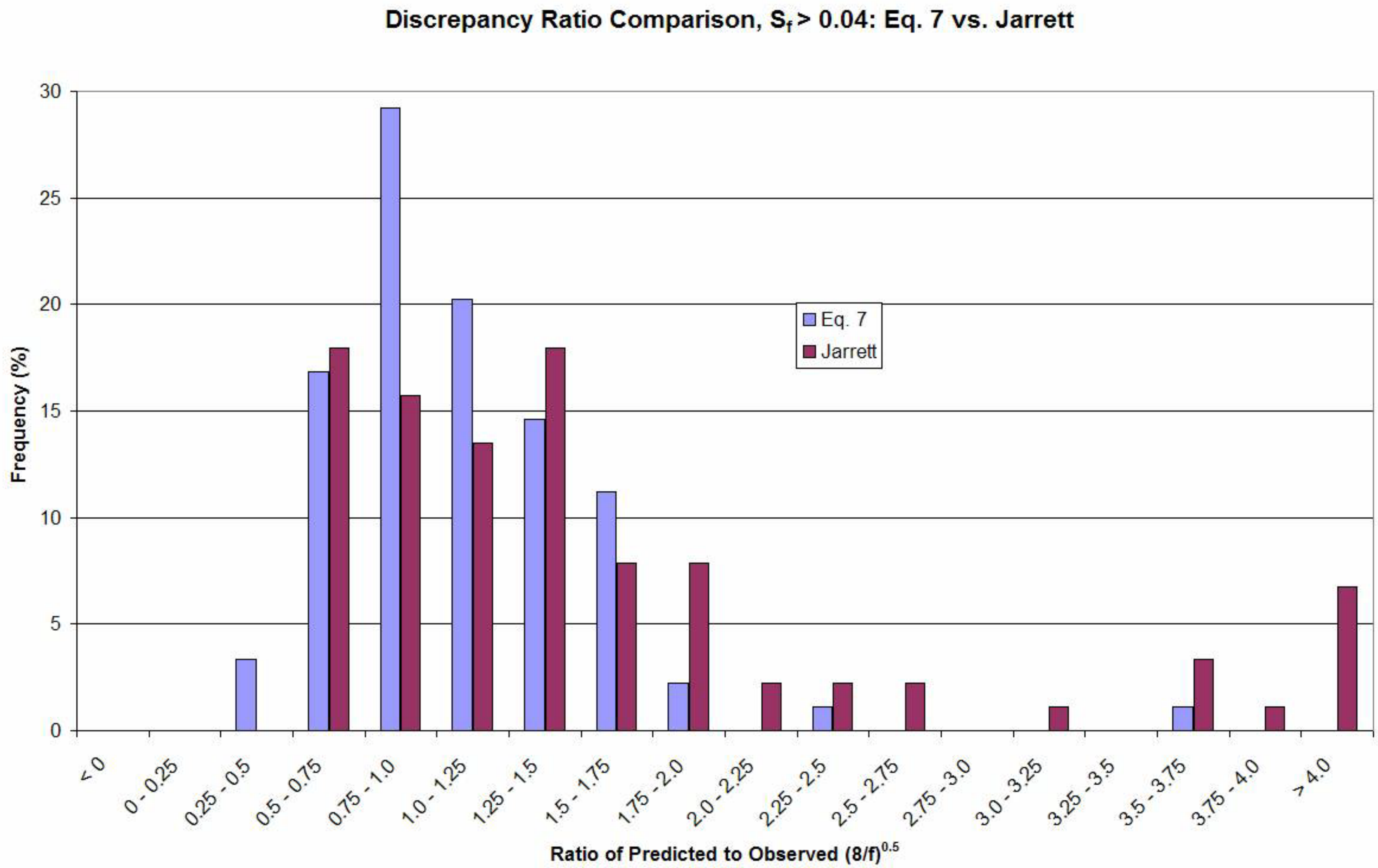

Figure 15. Comparison of the discrepancy ratios of Eq. 7 and Jarrett, S_f > 0.04

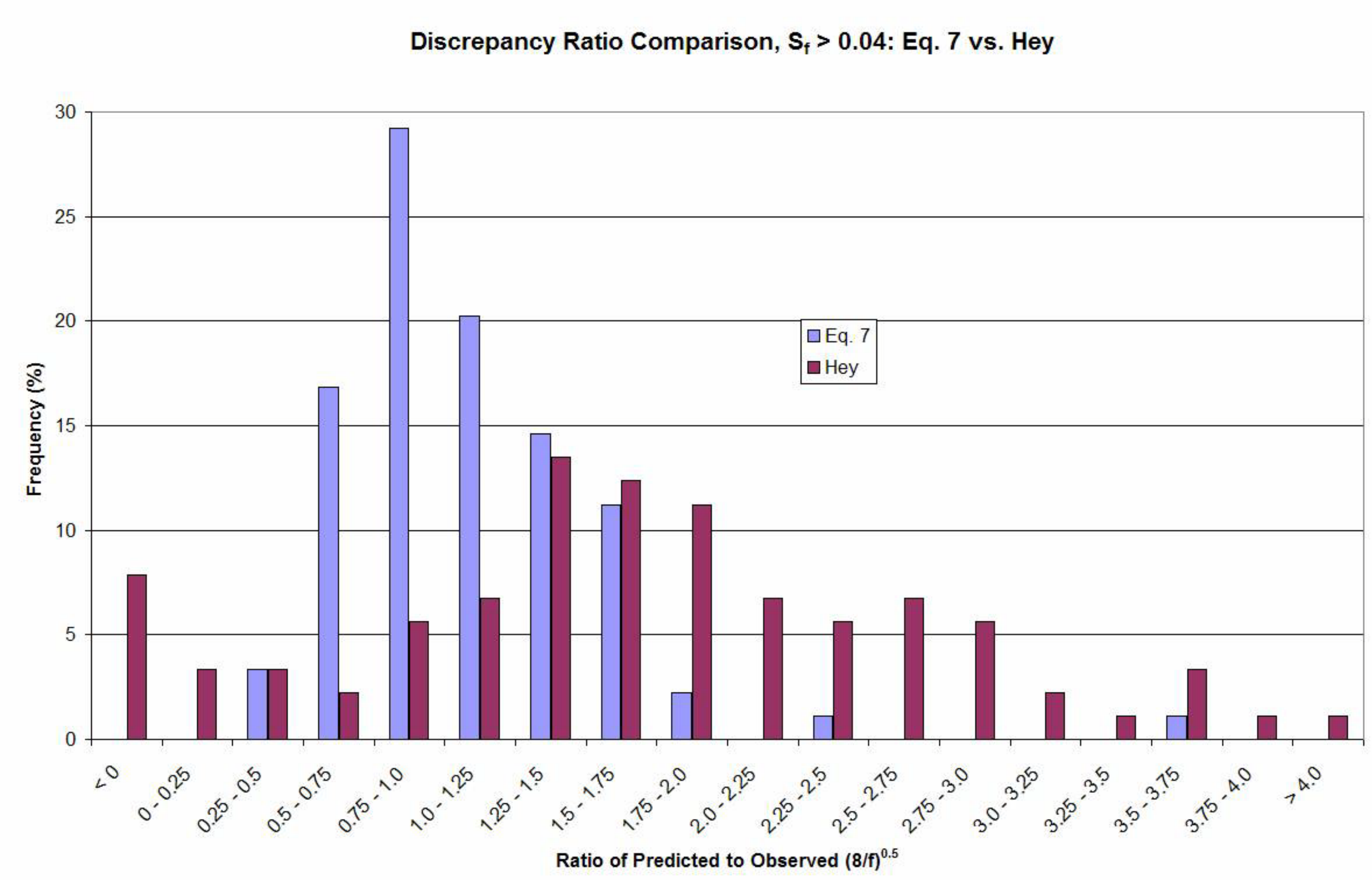

Figure 16. Comparison of the discrepancy ratios of Eq. 7 and Hey, $S_f > 0.04$

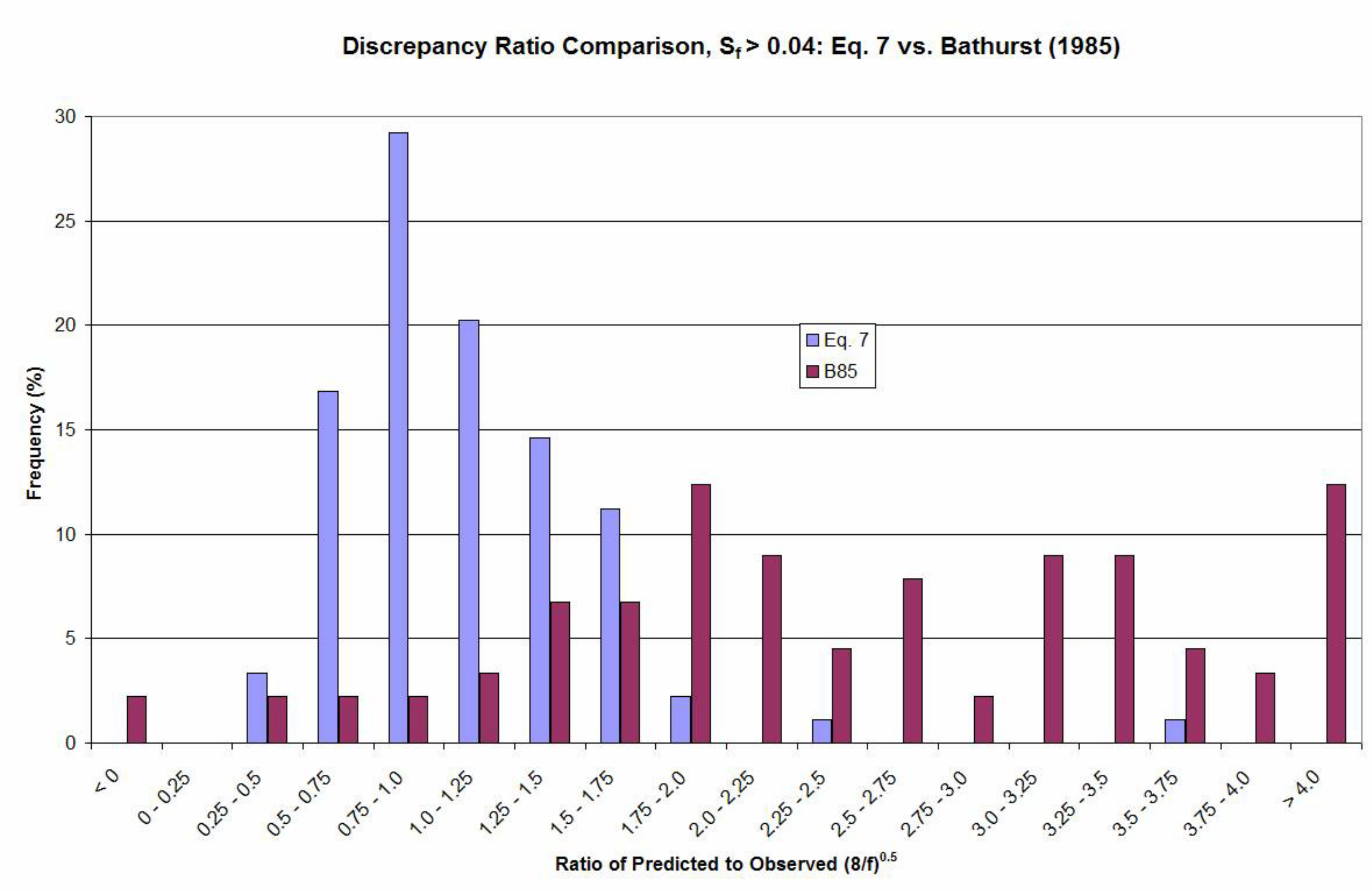

Figure 17. Comparison of the discrepancy ratios of Eq. 7 and Bathurst (1985), S_f > 0.04

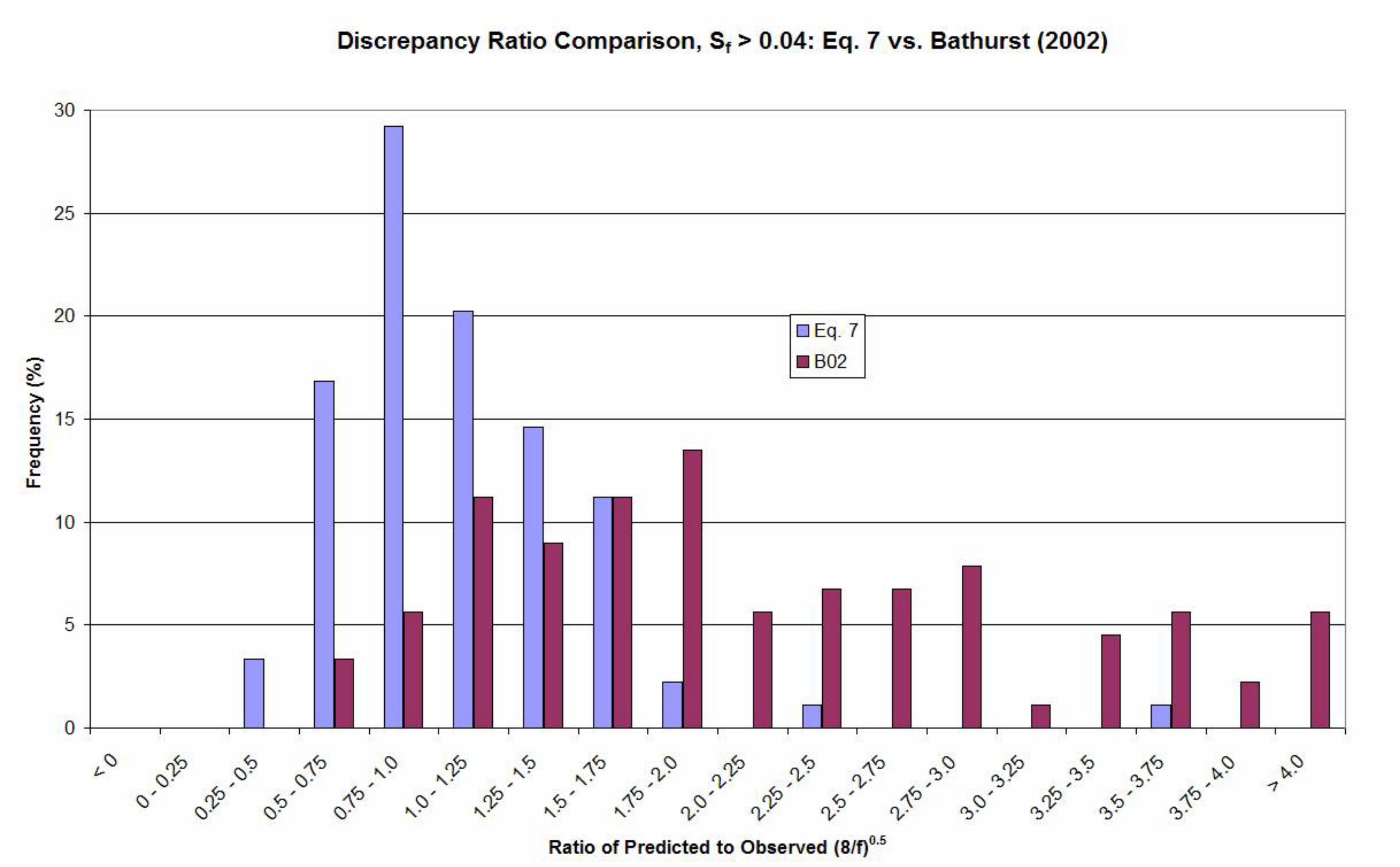

Figure 18. Comparison of the discrepancy ratios of Eq. 7 and Bathurst (2002), Sf > 0.04

Discussion

The use of S_f, D_{84}/D_{50} and R/D_{84} in a piecewise nonlinear regression improved the prediction of channel roughness, especially for flows with S_f greater than 4%. Stratification of the dataset at the S_f = 8% threshold improved the predictive capacity of the model without over fitting. This is apparently due to a change in the relative contribution of the three variables to the overall channel roughness with variation of S_f.

The empirical analyses of this study were performed in terms of the dimensionless expression $(8/f)^{0.5}$. The observed f values spanned a range of four orders of magnitude, while the range of observed $(8/f)^{0.5}$ values covered two orders of magnitude. For ease of interpretation, Equations (7a) and (7b) are rearranged and presented below in terms of f.

$$S_f < 0.08: \qquad f = 18.9\, S_f^{0.74}\, (R/D_{84})^{-0.66} \qquad (7a)$$

$$S_f \geq 0.08: \qquad f = 0.353\, (R/D_{84})^{-1.32}\, (D_{84}/D_{50})^{2.54} \qquad (7b)$$

Equation (7a) predicts that roughness is inversely proportional to relative submergence, which is logical as the amount of energy loss due to distortion or discontinuity of the free surface of the flow is directly related to the relative submergence value. The proportional relationship between S_f and f is likely due to energy loss due to the formation of localized hydraulic jumps in the vicinity of large bed particles, as the propensity for the flow to approach critical depth, and thus for hydraulic jumps to form, is a function of the channel gradient in natural streams. There are also correlations between S_f, D_{84}, D_{50}, and R/D_{84}. Thus, the S_f may account for multiple features of the channel geometry and the flow itself.

At any given site, D_{84}/D_{50} was always constant, and only small amounts of variation in S_f were observed, while R/D_{84} changed in some instances by an order of magnitude. The combination of relative submergence with the two other parameters of interest therefore accounts for changes in flow resistance both at–a–station and between sites.

Observed resistance values were consistently very high above the S_f = 8% threshold. This is likely due to the predominance of step-pool bedforms in this range, where energy losses due to hydraulic jumps and spill resistance are highest (Wohl and Thompson 2000). Grant (1997) has observed that incipient-motion flows with gradients steeper than approximately 4% are generally all tending toward critical velocity, thus localized hydraulic jumps are prevalent in this range. Additionally, the effects of D_{84}/D_{50} were only significant for S_f greater 4%. This most likely describes, in a general way, the geometry of the "steps", and thus the degree to which the flow is forced over and between large particles, resulting directly in localized hydraulic jumps and spill resistance.

The friction factor becomes more responsive to changes in R/D_{84} at steeper slopes. This may be due to the aforementioned tendency of the flow to locally exceed critical velocity in passing over large bed particles. In the highest slope range, R/D_{84} may account for not only energy lost due to deformation and discontinuity of the free surface of the flow, but also the energy lost due to hydraulic jumps.

The mean square error values, plots of observed versus predicted values, and discrepancy ratio data support the assertion that Equation (7) is a better tool for predicting flow resistance than the other methods that were tested, especially for mountain streams with S_f greater than 4%. It was anticipated that the new equation would perform better than the others at slopes greater 4%, as this is beyond the range of conditions for which the others were developed. Equation (7a) and Jarrett's equation are very similar in terms of form and results for predicting flow resistance in channels with $S_f < 0.04$. Both are functions of friction slope, and have almost exactly the same intercept and exponent values for S_f and hydraulic radius. However the results of this study show that relative submergence is a better predictor of flow resistance than hydraulic radius alone over the range of flow conditions encountered in this study. It is evident in comparison with the results from the Hey and Bathurst equations, which are simply functions of relative submergence, that it is necessary to account for other factors in order to adequately model flow resistance, and that an improvement has been made by incorporating the gradation coefficient and friction slope in the empirical relationship.

There is still a significant amount of scatter around the lines of perfect agreement for Equation 7 for both slope ranges. This is likely due to a combination of factors. Firstly, there are limitations inherent in using an empirical equation, as opposed to distributed model such as CFD, in that empirical relationships are based on reach average hydraulic parameters, and cannot represent exactly the flow field around each significant resistance generating bed particle, turbulent eddy formation, and channel irregularity. Secondly, there is no parameter in Equation 7 to account for energy losses due to changes in channel width or bed height through the reach of interest. Also, the data contain some unknown degree of measurement error. It becomes increasingly difficult to accurately measure velocity at very low discharges. The resolution of a cross-section survey may limit the accuracy of flow area calculations, especially at low flows. Water surface slope measurements are often difficult to collect as the water surface height may fluctuate due to wave action. Finally, the longitudinal water surface profile is rarely planar, especially in step-pool stream types, which introduces yet another source of uncertainty due to reach averaging and potential for measurement error.

Equation 7 is best applied to flow conditions where relative submergence is less than 4. As shown by Table (6), the MSE values for all of models tested in the study are highest for relative submergence conditions greater than 4. The mean percent error values presented in Table (7) suggest that all of the equations tested tend to underpredict roughness when the relative submergence value of 4 is exceeded. It is hypothesized that measurement error, bedload transport, and vegetation effects may be greatest for this roughness category. Review of the data indicated that the observations in this range were performed on coarse bed material streams at the high discharge, not finer bed material streams at average discharges. It is difficult and potentially dangerous to collect flow velocity data at high flows in mountain streams. In the case where the observer is collecting velocity and depth data safely removed from the flow atop a bridge or other structure, the structure itself may influence flow parameters. Also, as average boundary shear stress exceeds the critical shear stress necessary for particle entrainment, bedload transport ensues, thereby adding an additional complication in the relationship between the flow and channel boundary.

Another possible explanation of underprediction of roughness is the effect of riparian vegetation and large woody debris. Neither Equation (7) nor the other models tested can account for these effects, which may significantly retard the flow and increase flow depth. The results of using Equation (7) to predict flow conditions will be best for straight channel reaches free of vegetation effects and relatively uniform cross sections geometry throughout the reach; it may be used for water surface slopes ranging from 0.2% to 17%, relative submergence from 0.14 to 11, and D_{84}/D_{50} values ranging from 1.4 to 6.

In reality, natural mountain streams are rarely straight, free of vegetation and of uniform cross section geometry. Future research could be directed towards improving flow modeling throughout other types of channel reaches, for example the backwater pools in the riffle-pool type channel morphology, which are of great importance for aquatic habitat, especially at low flows. It may be possible to integrate the results of the present study with methods for evaluating resistance due to large woody debris, bed forms and irregularities in channel. In that way, a more complete method for modeling flow in mountain streams could be developed.

Applications

Equation (7) can be used for a variety of engineering applications concerning the hydraulics of mountain streams. The results of this and many existing research endeavors have shown that channel roughness values are generally not constant over the range of design flows. The design of stable channels and culverts, two common requirements for sustainable development of mountainous areas, rely on the accurate estimation of channel roughness in order to predict flow velocity and depth over the range of commonly encountered conditions in open channels. Equation (7) could also be used to arrive at a rough estimate of geomorphic bankfull discharge in steep, ungaged mountain streams by means of a Wolman pebble count and surveying of the channel cross section.

Equation (7) may have applications in habitat conservation programs. An increasing amount of water resources are being allocated for use as instream flows, which are essentially minimum discharge levels appropriated to preserve the fish and wildlife habitat. Instream discharge determination is generally approached via the application of a one – dimensional flow model, for example R2CROSS or PHABSIM, which calculates the amount of flow required to wet a riffle-type stream reach to maintain a minimum value of aquatic habitat during low flow conditions. As is the case with many one – dimensional flow models, there is no algorithm in the R2CROSS program to account of changes in at-a-station roughness with variation in discharge. Thus the user must assume that roughness value observed during flow measurements will remain constant. Equation (7) could be used in a one - dimensional flow model such as R2CROSS, and may improve the ability of such programs to prescribe appropriate instream flows.

Conclusion

Whenever flow in an open channel is a consideration for engineering works, research or environmental conservation, one is often faced with the challenge of estimating flow resistance so as to apply a flow equation such as Darcy – Weisbach or Manning's equation to predict flow velocities and depths for a range of design discharges. This study investigated the existing theory related to the processes which drive flow resistance, and evaluated the ability of various existing methods to predict flow resistance in mountain streams. A new empirical relationship was created using S_f, R/D_{84}, and the ratio of D_{84} to D_{50}. Stratification of the data compiled for this study by S_f allowed for the fitting of a dimensionally homogeneous, piecewise nonlinear regression equation. The new model performs arguably better than the existing methods developed by Hey, Jarrett and Bathurst, and it performs best for low relative submergence conditions and streams with water surface slopes greater than 4%. The relationship developed by the present study should be applied with caution and supplemented with field data whenever possible, as it will most likely underpredict flow resistance in reaches with variable cross-section geometry, gravel bars, or large woody debris.

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