STOCHASTIC ANALYSIS OF GROUNDWATER LEVEL TIME SERIES IN THE WESTERN UNITED STATES

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Acknowledgment is also made to the District Chiefs of the U. S. Geological Survey in the twenty-two states west of the Mississippi River who provided information on the wells studied, and in a number of cases supplied the basic data in a convenient format or on magnetic tape for machine processing.

ABSTRACT

The primary objective of this investigation was to determine the stochastic structure of monthly groundwater levels in the western United States. The secondary objective was to study the regional variation of the time dependence in the form of the stochastic components of autocorrelation coefficients for the groundwater level time series.

For investigation of the stochastic structure, it was first necessary to remove the deterministic components. The trend and jump components were investigated first, and they were removed if found to be significant. Hydrologic time series exhibit periodicity in the mean, standard deviation, and autocorrelation coefficients, and it was also necessary to identify and remove these components. Upon removal of the periodic mean and periodic standard deviation, the resulting stochastic series was examined for dependence using Markov models of lag one, two and three, including the periodicity in the autocorrelation coefficients. When the dependence effects were removed, the resulting process was tested for independence. Thus the original nonstationary process is reduced to a second-order stationary and independent stochastic component process. The final independent stochastic residuals were analyzed to find their probability distribution functions.

A total of 84 wells in the 22 states west of the Mississippi River were selected for analysis. For each well the parameters necessary for the stochastic model were evaluated. Most wells could be described by a first, second, or third order Markov model, but for 29 wells higher-order dependence models were indicated.

Analysis of the regional variation of the first autocorrelation coefficient in the stationary stochastic component shows a much greater dependency in monthly groundwater level series than in either monthly precipitation or streamflow time series. The second and third autocorrelation coefficients of groundwater series also displayed high values, and in fact the mean third autocorrelation coefficient for the groundwater series is greater than the mean first autocorrelation coefficient for the streamflow series.

The regional correlation coefficients, r_i, between the pairs of stochastic components of groundwater level time series depend on the distance between wells. However, the absolute values of r_i oscillate widely, probably due in large part to sampling errors, and no particular conclusion can be drawn in their pattern.

FOREWORD

Starting from the basic fact that functions of random variables are also random variables, the fluctuations of groundwater properties in general and of their levels in particular, must also be stochastic processes since all variables affecting groundwater levels are stochastic processes. Such variables are: precipitation, infiltration, evaporation, transpiration, deep water percolation, groundwater outflow, aquifer geometry, permeability, etc. Because the time series of all inputs to the saturated part of an aquifer are periodic-stochastic processes, the groundwater outputs and state variables (such as groundwater level, water storage, velocities, etc.) are also periodic-stochastic processes. The annual period is the major periodic component. The lunar cycle may also be present in groundwater processes of aquifers close to seas or tidal estuaries.

The hydrologic process becomes more complex as water moves from one earth environment to another. The water in groundwater aquifers has already passed through several environments: from evaporation from oceans and seas into the atmosphere; to deposited rainfall, accumulated and melted snow; to water movements over the earth's surface or in the rivers; to infiltration and movement through the unsaturated zone to reach the saturated zone of the groundwater aquifer. Therefore, the groundwater state variables and their outflows should be relatively complex hydrologic processes.

Two fundamental characteristics of groundwater, in general, are: high delays with attenuations in water outflow, and man's influence on groundwater states and outflow. Because of the large storage capacity between the low and high groundwater levels, in comparison with the total annual inflow and outflow, the groundwater response functions of the unit-time and unit-amount of infiltrated water in the form of groundwater unit hydrographs are usually very flat and time-delayed. Exceptions to this are highly pervious formations (karstified limestones and dolomites, some lava beds, some basalts and sandstone, etc.). Because groundwater aquifers vary widely in their physical properties (transmissivity, storage and type of flow), their response functions also vary over a wide range from a relatively rapid to a very slow reaction to inputs. The response functions of classical alluvial groundwater aquifers are highly time-delayed and attenuated unit hydrographs. This exceptional flow attenuation makes the groundwater outflow and state variables also highly time-dependent stochastic processes. It is likely that they are among the most attenuated hydrologic periodic-stochastic processes in nature, with much smoother periodicity in the basic parameters (mean, standard deviation, autocorrelation coefficients, etc.) and a highly dependent second-order stationary stochastic component.

The fundamental characteristic of most groundwater aquifers is the presence of various types of nonhomogeneity in their processes of outflow and state varibles. This nonhomogeneity may be of both natural and man-made origins. Any natural disruption affects aquifers, but they are particularly influenced by human activities. Even land use and cultivation, affecting infiltration, evaporation, transpiration, overland flow, and movement through the unsaturated zone, may produce nonhomogeneity in these processes in the form of trends and/or jumps in basic parameters. Pumping or artificially recharging groundwater aquifers, irrigating land over the aquifers, or affecting them by changes in the interconnected streamflow regime, further accentuate nonhomogeneity in groundwater time series. Therefore, it is very difficult to find completely virgin time series of aquifer variables except in uninhabitated areas.

This study by Dr. Albert G. Law of groundwater level time series shows all the complexity, high sequential dependence, and versatile nonhomogeneities of these hydrologic processes. This study departs from the classical investigations of groundwater aquifers which most often stress their deterministic properties, such as hydraulics of porous media flow or the input-response-output approach of the presently fashionable applied system analysis for groundwater management, and similar techniques. Dr. Law follows techniques of structural decomposition of series, which are used for other time series, such as precipitation, runoff, water use, sediment transport, and water quality variables. As expected, Dr. Law's investigations reinforce the significance of the above emphasized characteristics of groundwater time series, namely the high sequential dependence and the large degrees of nonhomogeneity in the form of trends.

A particular attitude by many investigators can often be discerned as related to stochastic groundwater time processes. Namely, the high smoothness of series of outflow and state variables is often misinterpreted as only the periodic or almost-periodic processes with a negligible stochastic component. Regardless of the smoothness of time series, the stochastic component usually represents a sufficiently high part of the total variance of the process that it cannot be neglected. The smoothness of a series does not imply nonexistence or insignificance of stochasticity in hydrologic series in general, or in groundwater time series in particular.

The nonhomogeneity in historical samples of groundwater time series poses a particularly difficult problem in the current pressure for using more and more groundwater aquifers for additional storage and regulation of the artificially recharged water flow. Namely, without the removal of the nonhomogeneity in the historical samples and a reliable extrapolation into the future of the expected nonhomogeneity, the planning of these additional uses of groundwater aquifers would be subject to high errors and misjudgments.

Investigations by Dr. Albert G. Law in this paper give some light and open some possibilities for a better treatment in practical applications both of the high dependence in the second-order stationary stochastic component of groundwater time series, and of the various types of nonhomogeneity in historical samples of these series. This paper, with the structural analysis of groundwater time series, should be viewed as a contribution toward a better understanding, analysis and description of groundwater state time series, even though only the water level time series of groundwater wells in the western United States have been used as examples in the investigation.

The association of Dr. A. G. Law with the two NSF sponsored research projects at Colorado State University, GK-11564 (Large Continental Droughts) and GR-11444 (Stochastic Processes in Water Resources), during his sabbatical year spent at this University, has been mutually fruitful and is appreciated.

Vujica Yevjevich, Professor-in-Charge
Hydrology and Water Resources Program

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CHAPTER I
INTRODUCTION

1.1. General

The water contained beneath the surface of the earth serves as one of the major sources of supply for agricultural, municipal and industrial uses. It has been estimated (E. A. Ackerman & G. O. Löf, 1959) that the volume of usable groundwater is three and one-half times the volume of lake and reservoir storage in the United States, and about thirty-five times the annual volume of runoff discharged to the oceans. However, no reasonable approach would consider using all this stored water as one would not consider using all water stored in a large lake. In some areas groundwater is the only dependable source of supply, while in other regions it is chosen because of its ready availability, lack of pollution, and for its relatively uniform temperature throughout the year. In some cases it is more attractive, as the aquifers serve not only for storage but also as transmission systems with evaporation losses greatly reduced in comparison with the evaporation from surface storage and conveyance systems. In many situations surface water and groundwater are used conjunctively and will be used so more and more in the future.

Replenishment of groundwater is dependent upon precipitation and streamflow and is greatly influenced by the other hydrologic processes such as evaporation, overland flow, infiltration, transpiration and flow in the unsaturated zone. The occurrence and movement of groundwater in the saturated zone is controlled by the geologic environment. Due to these factors the groundwater subsystem has highly damped system state processes in comparison with the precipitation or streamflow subsystems. Thus it would appear that groundwater in its natural state should not experience the excesses and deficiencies that a surface water system usually exhibits. One may reasonably assume then that management of groundwater resources may be especially attractive in aleviating the effects of droughts, particularly large-scale continental droughts. This concept provided the impetus and justification for initiation of this investigation.

Various subsystems of the hydrologic system have been analyzed to determine their stochastic properties. Much of the work has been directed toward streamflow studies, as may be observed in the works of Julian (1961), Brittan (1961), Thomas and Fiering (1962), Beard (1965), and Fiering (1967). At Colorado State University, Yevjevich (1963) initiated studies on streamflow, followed by studies of monthly precipitation and streamflow by Roesner and Yevjevich (1966).

More recently, J. D. Salas-LaCruz and V. Yevjevich (1972) have investigated water use time series of urban water supply, irrigation and hydropower generation. The present project is aimed at complementing these studies. It is anticipated that in the near future this work will be extended to bivariate processes of precipitation and groundwater levels, as well as streamflow and groundwater levels; a further step will be the study of all these variables as multivariate hydrologic processes. This groundwater study provides a base for these future investigations.

Analysis of time series yields valuable information on the past occurrences of a hydrologic process that may provide guidance as to the future behavior of the phenomenon. The stochastic analysis of a time series will decompose the process into its deterministic components (trends and periodicities) and stochastic component so that the process may be described by a set of mathematical equations making a general mathematical model. Once the phenomenon is described mathematically, other possible realizations of the stochastic process may be generated for use in planning water resource developments, such as drought severity attenuation. Generated series, however, must include the expected future trends if the procedure is to be valid.

1.2. Main Objectives of the Study

The main objectives of this study are:

(1) To investigate the degree of damping of the groundwater subsystem relative to the precipitation and surface water subsystems. Part of this aim must await the investigation of bivariate or multivariate processes.

(2) To detect and remove the trend and/or jump components from the groundwater time series.

(3) To detect, describe, and remove from the time series the periodic components in parameters.

(4) To investigate the structure of the stochastic component so that the time dependence may be appropriately described mathematically.

(5) To remove the dependence within the dependent stationary stochastic series so as to arrive at a time series which is a second-order stationary but independent stochastic process.

(6) To investigate the probability distribution function of the independent stochastic component.

(7) To express mathematically the structure of the groundwater time series which may be useful in generating time series of groundwater levels for use in water resource planning and management, especially with relation to droughts.

(8) To study the spatial variation of the autocorrelation coefficients of groundwater time series in the western United States.

(9) To investigate the regional correlation between the dependent stochastic components of groundwater time series.

CHAPTER II
TIME SERIES OF GROUNDWATER LEVELS

2.1. General

A time series is a chronological sequence of observations of a specified variable. In some cases the observations are continuous over time, and in other cases the observations are made only at discrete time intervals, such as once per day, week, or month, or summed or averaged over these intervals. In the case of groundwater level time series, the depth from the land surface datum to the water table of an unconfined (or water table) aquifer or to the piezometric surface of a confined (or artesian) aquifer is measured either continuously or sampled discretely in time. Such a variable is not a cumulative one in the way that precipitation or streamflow may be accumulated to provide a total for each discrete interval or period. However, a groundwater level time series may also be conceived of as the cumulative of the net groundwater recharge-discharge. Where groundwater recharge exceeds discharge for a time period, the water table or piezometric surface will be higher at the end of that time period than at the end of the preceding period; the corresponding depth-to-water measurement will show a decrease. The reverse will be true if the groundwater discharge exceeds the recharge.

A groundwater level time series reflects all natural hydrologic processes as well as the influences of man upon the aquifer, upon other hydrologic processes which affect the aquifer, and upon other hydrologic units to which the aquifer is connected. The geologic environment plays a prominent role in the water level fluctuations; an aquifer with a high capability for transmitting water will behave differently than an aquifer with a low transmission capability. Aquifers also have varying capacities for water storage. A low storage coefficient will result in a greater cone of depression in response to a given pumping rate, or in a greater recharge cone in the case of water inputs into the aquifer by recharge wells.

The groundwater level time series resulting from the above-mentioned factors will have some deterministic components as trends, jumps, and periodicities, with the dependent and/or independent stochastic component superimposed. Because of differing influential factors, each well will exhibit these components to a different degree, or in statistical terms, the part of the explained variance of the time series due to these components will vary from well to well. The purpose of time series analysis is to decompose the series into its component parts and to determine the relative importance of each component. When this decomposition is described mathematically, the series may then be reconstituted in such a way that the fundamental properties of the series are preserved.

Traditionally, groundwater resource planning has been treated in a deterministic manner relating cause and effect. After description of the hydrologic system by a mathematical model, solutions of the model are obtained in response to stresses placed on the system. The stresses are in the form of pumpage or artificial recharge. Methods commonly employed in obtaining solutions involve numerical methods, electric analog models, or analog or digital computer models. Implicit in this approach is the assumption of a known groundwater surface at the time the stress is applied to the system.

The analysis of a groundwater time series as a stochastic process provides information as to future states of the system. For a natural hydrogeologic system, i.e., one not affected by pumpage or artificial recharge, time series analysis allows the description of any natural trends or periodicities as well as the description of the dependent and independent stochastic components. Should this aquifer then be pumped or recharged artifically, the deterministic responses to these stresses as outlined above could be superimposed on generated sequences of the time series. However, for a time series of a stressed hydrogeologic system, i.e., an aquifer subjected to pumping or artificial recharge, analysis of the series allows the description of any trends or periodicities whether natural or induced by man's activities or a combination thereof. The dependent and independent stochastic components are also described but again reflect the combined influences of the natural system and man's activities. Possible future states of the system could then be generated, and any changes in stress could be determined as set forth previously and superimposed on the system. Thus stochastic analyses of groundwater systems should be of value in the planning and management of groundwater resources.

2.2. The Natural Hydrologic System and Groundwater Subsystem

The relationship between the inputs and outputs of a hydrologic system, with special emphasis on the groundwater subsystem, is illustrated in Fig. 2.1.

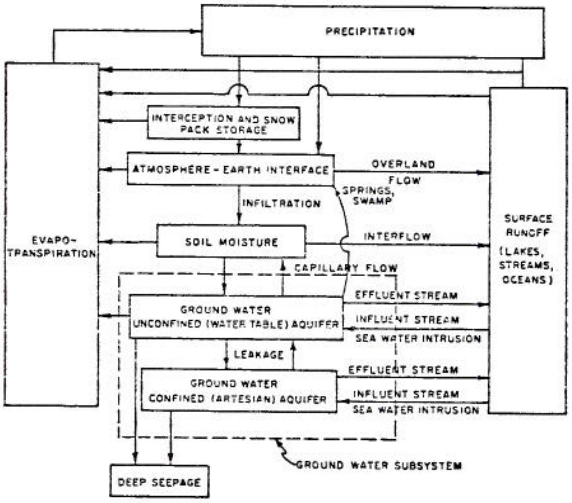

Fig. 2.1 The natural hydrologic system and groundwater subsystem.

The primary source of recharge for aquifers is from precipitation, which must pass through the surface retention and snow pack storage and the atmosphere-earth interface storage sybsystems, infiltration, the soil moisture storage subsystem, and perhaps one or more other aquifers. Groundwaters may also be recharged through influent conditions in surface lakes and streams. Each of these storages acts as a filter operating on the input processes in such a way as to cause the output processes to have somewhat different but related properties; the output processes and groundwater state processes (levels, storage) are "smoother" and are more "damped" than the input processes for most groundwater aquifers. Since water in these storages is released gradually, the output from storage is dependent upon the water in storage. Thus a carry-over or dependence factor is introduced by each storage phase in the process. For a comprehensive discussion of carry-over, the reader is referred to V. Yevjevich (1963).

Natural discharge of aquifers determines the degree to which storage within the aquifer system is utilized. As may be noted in Fig. 2.1, such discharge may be in the form of flow to effluent lakes and streams, evapotranspiration to the atmosphere, groundwater discharge to the earth's surface (springs and swamps), leakage to other aquifers, and loss of water by seepage to deep groundwater zones. Not all of these processes are in operation in any one aquifer, or at all times within a given aquifer. The geologic controls as well as the general climatic conditions govern which processes are dominant in a given aquifer at a given time.

The subsystems of the hydrologic system affect the groundwater time series in various ways. Because of the effects of the various storages, the groundwater series will lag behind the precipitation series. Water in these storages is released gradually at a rate dependent on the amount of water in storage. Moisture in storage and water use by vegetation will have a pronounced effect on this relationship, as precipitation during a period of low soil moisture may all go to moisture replenishment, whereas precipitation occurring when the soil moisture content is high will probably result in recharge to the groundwater. When precipitation is highly seasonal, clear periodicities may appear in the groundwater time series. Snow accumulation and melt are especially important in this respect as they introduce a strong annual periodicity. Vegetation also plays a major role in that water use by plants is heavy during the growing season, thereby removing water that would otherwise go to groundwater recharge. This process may introduce an annual periodicity into the groundwater time series.

The surface water - groundwater interrelationship may also be important if the aquifer is hydraulically connected to a stream, lake, or ocean. The degree to which an aquifer is influenced by a surface water source depends on how well the surface water and aquifer are connected (a region of low permeability may impede the flow of water). The proximity of a point in the aquifer to the surface water is also a factor in the degree of influence. Also, an influent stream condition serves to recharge the aquifer, and the characteristics of the streamflow may be transferred to the aquifer although in dampened form. For example, a stream greatly affected by high seasonal differences, such as snowmelt runoff, would have an annual periodicity; groundwater in an aquifer influenced by this stream might also exhibit an annual periodicity but not as pronounced. Influent conditions in a stream below a well-regulated reservoir would provide a somewhat uniform recharge to the groundwater. On the other hand, an effluent surface water source is a recipient of discharge from an aquifer; such a condition may have only a minor effect on the groundwater time series. Certain surface water conditions may have a noticeable effect on groundwater conditions. Flooding may change an effluent stream into an influent stream. Groundwater in aquifers adjacent to coasts may fluctuate in response to tidal action.

The various forms of recharge thus serve as inputs to the groundwater subsystem and discharge as outputs, while the system itself is controlled by environmental geologic factors. Since the geologic environment is essentially unchanging, in a reasonable period of time of water resource development and use, groundwater level fluctuations are in response to changing inputs and outputs.

2.3. The Groundwater Subsystem as Affected by Man's Activities

Superimposed on the natural hydrologic cycle are the works of man. For the groundwater subsystem, this results in artificial discharge and artificial recharge.

Artificial discharge usually refers to pumping, but may also take the form of wells to allow drainage through a lower confining strata to a formation below. Pumping from aquifers is usually for the purpose of supplying water for domestic, municipal, agricultural, or industrial purposes, although pumpage for control of groundwater during excavation may be encountered sometimes.

Artificial recharge is the introduction of water into the ground by wells, pits, excavations, or by spraying. This recharge may be for alleviating salt water intrusion, to dispose of wastewater. to recharge a declining water table or piezometric surface, or to use the aquifer as a storage reservoir. Often wells adjacent to effluent streams are pumped to reverse the hydraulic gradient and thereby artificially recharge the aquifer by induced infiltration from the stream. Construction of a dam on a stream may also create an artificial recharge situation by raising the water level in the reservoir above the groundwater table thus effecting an influent reservoir condition.

Irrigation deserves special mention in connection with artificial recharge. If the water is derived from a surface water source, the irrigation may be considered to be artificial surface recharge. Some of this artificial recharge may reach the water table, depending on the rate of application, soil characteristics and soil moisture conditions. Should the irrigation water be obtained from a water table aquifer with the water table close to the surface, there will first be a discharge from the aquifer, and as the artificial recharge is applied some water may return to the water table, completing a recycling process. If the irrigation water is obtained from a confined aquifer, then upon application some water may reach the water table; whether it would eventually be recycled to the confined aquifer from which it was pumped would depend upon leakage conditions between the aquifers. The manner in which artificial discharge or recharge affects the time series of groundwater level fluctuations depends upon the circumstances. Heavy pumping from one or more wells in an aquifer may result in a long term trend being introduced into all the groundwater level time series of that aquifer. Pumping may also introduce periodicities into the time series if the pumping has seasonal variations, such as heavy summer pumping and no pumping in winter with a resulting annual cycle. Quite likely, a combination of both trends and periodicities is artificially introduced into the series in many areas. If the pattern of pumpage is irregular, however, the effect on the time series may be to reduce the dependency within the series. Recharge also introduces trends and periodicities within the series; however, because of filtering

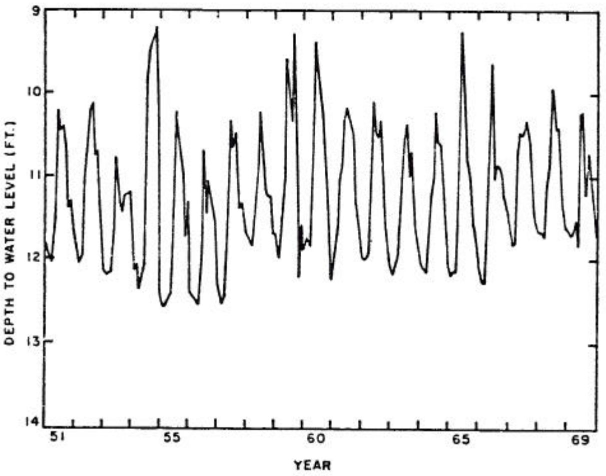

Fig. 2.2 Well hydrograph of Wyoming well 14-66-31bda (WY-021-2). Trendless times series of water table well not affected by pumpage, but slightly affected due to lawn irrigation.

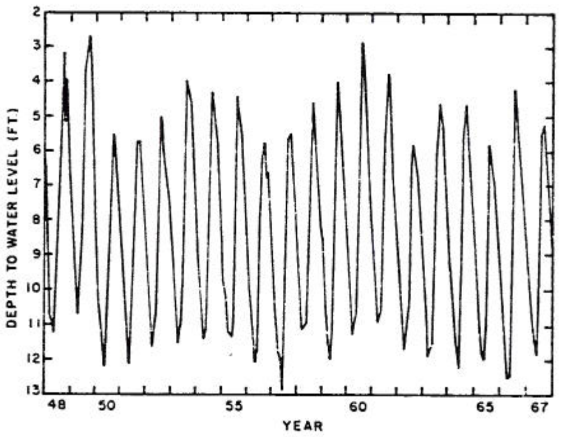

Fig. 2.3 Well hydrograph of Montana well D-2-23-29bd (MT-095-1). Trendless time series of water table well affected by local pumpage.

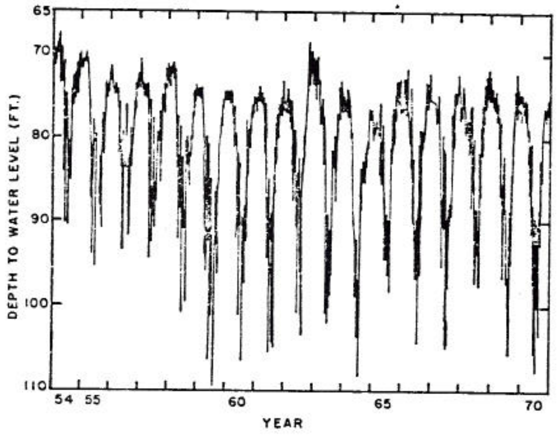

Fig. 2.4 Well hydrograph of Minnesota well 117-21-16cca (MN-053-4). Trendless time series of artesian well with periodicity due to pumping.

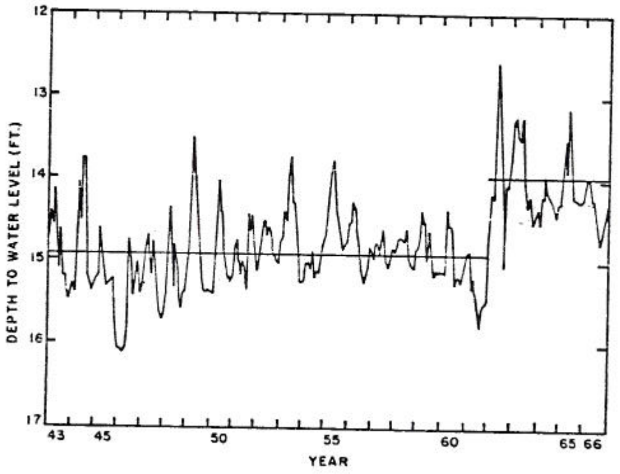

Fig. 2.5 Well hydrograph of Wyoming well 54-65-12add (WY-011-1). The effect of reservoir construction (1961) on adjacent groundwater levels: aquifer connected to Belle Fourche River below Keyhold Reservoir, Wyoming.

and storage processes at the land surface and in the soil moisture zone, the dependency within the series will probably be increased.

2.4. Groundwater Time Series in the Western United States

To illustrate some of the features discussed in the previous sections, some characteristic well hydrographs for selected wells in the 22 states west of the Mississippi are presented in Figs. 2.2 through 2.7. Each well is identified by the U. S. Geological Survey identification number. However, for the present study a code is assigned to each well for internal identification purposes (shown in parentheses in the figures). The groundwater levels are measured in feet below the land surface datum.

Figures 2.2 through 2.7 show graphically that groundwater levels may be characterized by complex deterministic-stochastic processes, with components such as trends, jumps, periodicities, and stochastic dependence. For example, Figs. 2.2, 2.3, and 2.4 do not show trends; however, periodicities and superimposed stochasticity are present in the series. Figure 2.2 illustrates a series of a water table well unaffected by pumpage, but slightly affected by recharge due to lawn irrigation. Figures 2.3 and 2.4

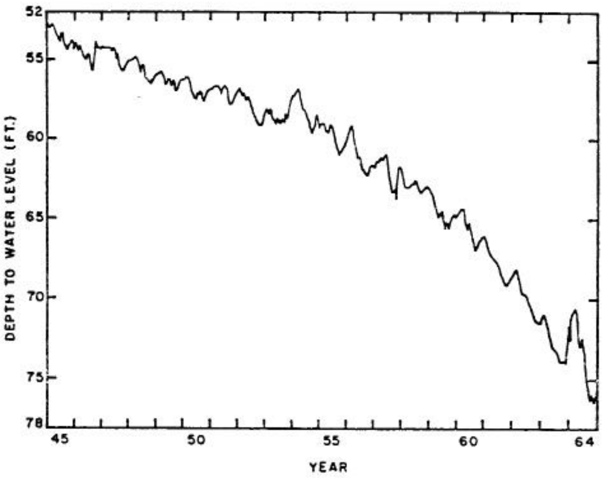

Fig. 2.6 Well hydrograph of Kansas well 28-38W-6ad (KS-067-1). Nonlinear decreasing trend of water table well caused by local pumping.

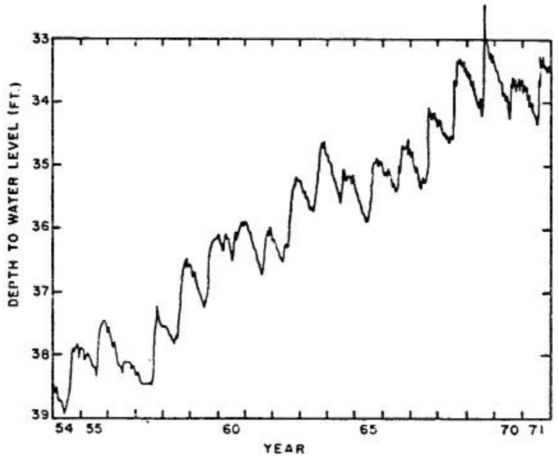

Fig. 2.7 Well hydrograph of Nebraska well 29N-46W-10aa1 (NB-061-1). Time series with linear increasing trend probably due to irrigation from surface water source with local pumpage superimposed.

show groundwater hydrographs affected by local pumping. Figures 2.2, 2.3 and 2.4 show the different amplitudes and phases of periodic parameters, with a high degree of stochasticity present in the series. For example, while Fig. 2.2 shows a midyear high peak, Fig. 2.4 shows a midyear low; on the other hand, Fig. 2.3 presents a low in the first four months and a peak in the last four months of the year. It is also interesting to observe the differences in the random fluctuations of the well hydrographs. For example, while Fig. 2.3 is very regular, Fig. 2.4 is very irregular with high random fluctuations around the periodic mean of the series. On the other hand, Fig. 2.2 shows a somewhat moderate regularity.

Figures 2.5, 2.6 and 2.7 give examples of nonhomogeneous groundwater level series created by the continuous development and use of water in the groundwater basin. Figure 2.5 illustrates an upward jump in the last five years of the time series. In this case, the jump appears to have been caused by putting Keyhole Reservoir on the Belle Fourche River into operation, raising the water level in the connected aquifer. Figure 2.6 shows a decreasing nonlinear trend caused by the continuous increase of local pumping, and Fig. 2.7 shows an increasing linear trend created by irrigation seepage superimposed on a periodic pattern (induced by the local pumping) and a stochastic fluctuation.

CHAPTER III
RESEARCH DATA ASSEMBLY AND METHOD OF ANALYSIS

3.1. Type and Sources of Data

In order to select wells for analysis, the Catalog of Information on Water Data of the Office of Water Data Coordination was reviewed in conjunction with the "groundwater level series" of the Water Supply Papers of the U. S. Geological Survey. The most recent in this series, Water Supply Paper 2010, provides a reference to the other papers in the series, and it may be used to locate published data on groundwater levels in all states. The original criteria used in selecting wells for analysis were: (1) depth to water readings taken at least once per month; (2) at least 15 years of record available, and (3) the well represented the natural hydrology as much as possible.

A total of 276 wells in the 22 states west of the Mississippi River was selected for possible analysis, and for each well a data summary sheet was prepared listing all pertinent data, as indicated in Table 3.1.

TABLE 3.1

DATA SOUGHT FOR EACH GROUNDWATER WELL SELECTED

A. Well location
   1. State
   2. County
   3. Office of Water Data Coordination number
   4. Station number and name (U.S.G.S.)
   5. Latitude
   6. Longitude

B. Basic data regarding well
   1. Period of record
   2. Data published?
   3. Data on cards or tape?
   4. Frequency of measurement
   5. Well affected by pumpage?
   6. Water table or artesian?

C. Additional well information
   1. Well diameter
   2. Well depth
   3. Screened depths
   4. Land surface datum, above mean sea level
   5. Measuring point
   6. Highest water level
   7. Lowest water level

D. Aquifer description
   1. Groundwater basin (after McGuinness)
   2. Formation
   3. Description of formation
   4. Recharge to formation
   5. Hydraulic characteristics: T, S, specific capacity
   6. Aquifer hydraulically connected to a stream?

E. Related hydrologic information
   1. Surface water basin
   2. Stream gages
   3. Precipitation stations
   4. Evaporation stations

F. Comments and discussions

For ease in well identification, each well was assigned a study well number, consisting of the state abbreviation, county number (obtained from the Catalog of Information on Water Data), and a within-the-county number, e.g., AZ-009-1. Data for all wells was assembled using the data on tape supplied by the U. S. Geological Survey, or else copying the data from the groundwater level series of the Water Supply Papers. This data was reviewed for frequency of measurement, completeness, and suitability for analysis.

The data for some wells could not be used because a portion of the record was missing or the frequency of observation was erratic. After reviewing all the information on each of the wells, 145 wells were chosen for further analysis and the data placed on punched cards. For uniformity, the data was keypunched according to the U.S.G.S. format, as described by S. M. Lang and A. R. Leonard (1967).

Well hydrographs of the 145 wells were plotted by computer processing. Visual examination was used to make the final selection: 84 wells were chosen for analysis by the method described in section 3.2. Some of these wells required the removal of jumps in the annual means, and other wells required the removal of linear or quadratic trends in the annual means. For some wells only part of the record could be used, resulting in a period of record of less than 15 years.

Many of the wells selected were not free of the effect of pumping as originally intended. Because of the extensive use of groundwater in the western states it is difficult to locate many wells free from the effects of pumping. Another factor is that the well measurement program is usually set up to use the existing wells, and these wells are usually being pumped or are in regions where pumping is common. The degree of pumping is also important, but this information is usually only available in qualitative form, such as "minor local pumping" or "pumping well."

Whether or not an aquifer is hydraulically connected to a stream is also important. This information is not available in the groundwater series of the Water Supply Papers, but many District Offices of the U.S.G.S. were able to supply this information for wells in their states although again this is often in a qualitative form, such as "aquifer believed to be poorly connected to stream."

The location of the wells selected for analysis is shown in Fig. 3.1. Table 3.2 provides basic information on the 84 wells selected for analysis. The study well number used for identification in this report is cross referenced in Table 3.2 to the U.S.G.S. Well Number. Other information presented is the type of aquifer, i.e. water table or artesian, the aquifer formation, and indications as to pumping condition and hydraulic connection to stream as nearly as could be determined.

3.2. Method of Analysis

The method for analyzing the stochastic structure of groundwater level time series is based on a general mathematical method developed by J. D. Salas-LaCruz and

V. Yevjevich (1972) for identifying and estimating the deterministic and stochastic components of water use time series. This method first detects, estimates, and removes any trend and/or jump present in the original series. Subsequently, the periodicities in the mean, standard deviation and autocorrelation coefficients are identified and mathematically described. The periodicities in the mean and standard deviation are removed first, and then the periodicities in the autocorrelation coefficients are removed by using either the first, second or third-order Markov model for describing the dependence structure of the stochastic component. This yields a residual which is a second-order stationary, independent stochastic component. Finally, the probability distribution function of this independent stochastic component is derived by fitting either the normal or the lognormal-3 probability density function to the sample frequency curve.

The method outlined above, and subsequently briefly described, may be used for the analysis of daily, weekly, monthly (or their multiples) and annual values. In general, groundwater level time series may exhibit trends in the mean and in the standard deviation. They may be either increasing or decreasing. They also show within-the-year periodicity in the mean, standard deviation and autocorrelation coefficients and a time dependent stochastic component. Annual values of groundwater levels may or may not exhibit trends and a time dependence in the stochastic component.

3.2.1. Deterministic Components

Trends in the mean and standard deviation. Consider the $x_{p,\tau}$ series as the original nonstationary stochastic process with $\tau = 1, 2, \ldots, \omega$ where ω is the basic periodicity of descrete series equal to 365, 52, or 12, respectively, for daily, weekly and monthly value series; and $p = 1, 2, \ldots, n$, where n is the number of years of record. Assume that $x_{p,\tau}$ has trends, periodicities, and a dependent stochastic component. Let $Tm_{p,\tau}$ and $Ts_{p,\tau}$ be the trends in the mean and standard deviation of the process $x_{p,\tau}$.

A new process $y_{p,\tau}$ is generated by removing the trend in the mean $Tm_{p,\tau}$ from $x_{p,\tau}$ by

$$y_{p,\tau} = x_{p,\tau} - Tm_{p,\tau} \tag{3.1}$$

Because the process $y_{p,\tau}$ may still have a trend in the standard deviation, $Ts_{p,\tau}$, a new process $z_{p,\tau}$ is obtained by

$$z_{p,\tau} = \frac{y_{p,\tau}}{Ts_{p,\tau}} \tag{3.2}$$

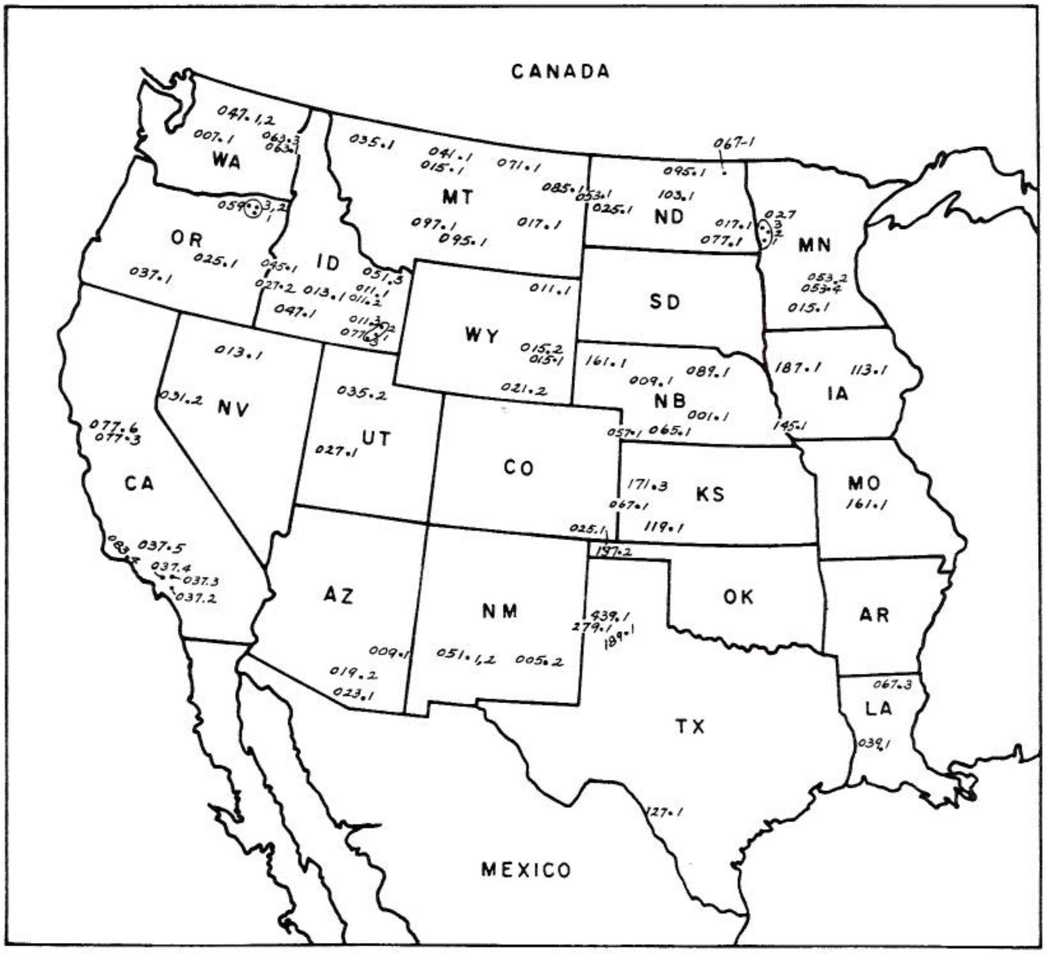

Fig. 3.1 Areal distribution of groundwater wells selected for analysis.

TABLE 3.2

WELL DESCRIPTIONS

Study Well No.	USGS Well No.	Type of Aquifer	Aquifer formation	No. of years	Pumping Condition*	Hydraulic Connection to stream
AZ-009-1	(D-6-28)31aab	WT	Gila conglomerate	20	NP	yes
AZ-019-2	(D-17-14)18cab	WT	sand and gravel	31	MP	no
AZ-023-1	(D-24-17)11cda	WT	alluvial fill	16	P	yes
CA-037-3	3S/12W-8L3S	Art.	Gaspur formation	23	PP	no
CA-037-4	3S/14W-21B1S	Art.	sand and gravel	35	P	no
CA-037-5	4S/13W-14L1S	Art.	Gaspur formation	37	PP	no
CA-059-2	4S/11W-19K1S	Art.	Pleistocene deposits	34	PP	no
CA-077-3	3N/7E-10L4M	WT	Victor formation	32	NP	maybe
CA-077-6	4N/7E-30E4M	WT	Victor formation	20	NP	maybe
CA-083-4	4N/29W-14A3S	WT	Santa Barbara form.	23	NP	maybe
ID-011-1	2N-31E-25dcc1	WT	Snake River group	21	NP	no
ID-011-2	1S-30E-15bca	WT	Snake River group	18	NP	no
ID-011-3	5S-31E-27aba1	WT	Snake River group	18	NP	no
ID-013-1	2S-20E-1acc2	WT	sand and gravel	15	NP	no
ID-027-2	2N-1W-7bbc1	WT	Idaho group	17	MP	no
ID-045-1	7N-2W-35abb1	WT	alluvial sand	12	NP	maybe
ID-047-1	8S-14E-16cbb1	WT	Snake River group	18	NP	no
ID-051-3	5N-34E-9bda1	WT	Snake River group	20	MP	no
ID-077-1	6S-33E-20abc1	WT	gravel	14	NP	maybe
ID-077-2	5S-33E-35ccd1	WT	gravel	11	P	maybe
ID-077-3	6S-32E-27add1	WT	sand	14	MP	no
IA-113-1	84-7-13E2	Art.	glacial drift	30	NP	no
IA-145-1	68-38-7N1	Art.	glacial drift	33	NP	no
IA-187-1	87-28-29N1	Art.	glacial drift	28	NP	no
KS-067-1	28-38W-6ad	WT	Pleistocene deposits	20	LP	no
KS-119-1	30-27W-23abb	WT	Ogallala formation	31	RP	no
KS-171-3	20-33W-9bbb	WT	Ogallala formation	39	LP	no
LA-039-1	EV-229	Art.	Chicot formation	22	NP	yes
LA-067-3	MO-15	WT	terrace deposits	25	LP,RP	yes
MN-015-1	108-30-9add	WT	glacial drift	27	NP	no
MN-027-1	137-45-30cdb1	WT	glacial drift	21	P	no
MN-027-2	139-45-1ccd2	WT	glacial drift	18	P	no
MN-027-3	139-47-5cdc	WT	glacial sand	17	P	no
MN-053-2	B29-23-30bda1	Art.	Jordan sandstone	18	RP	yes
MN-053-4	117-21-16cca	Art.	Prairie due Chein	12	RP	no
MO-161-1	37-10-13k1	WT	Gasconade dolomite	23	NP	no
MT-015-1	A-29-13-21aa2	Art.	Pleistocene deposits	21	NP	no
MT-017-1	A-7-47-13dd	WT	Ft. Union formation	21	NP	no
MT-035-1	B-32-11-3dd	WT	Pleistocene deposits	21	NP	no
MT-041-1	A-32-15-17dd	Art.	Pleistocene deposits	21	MLP	yes
MT-071-1	A-31-34-8ca	WT	alluvium	19	NP	yes
MT-085-1	A-28-57-28dd	WT	alluvium	21	NP	no
MT-095-1	D-2-23-29bd	WT	alluvium	20	NP	no
MT-097-1	A-4-14-14ba	WT	alluvium	15	LP	yes
NB-001-1	7N-10W-23ab1	WT	sand and gravel	14	LP,RP	no
NB-009-1	22N-24W-33ca1	WT	fine sand	29	NP	yes
NB-057-1	1N-40W-29bb1	WT	silt and clay	25	MP	yes
NB-065-1	4N-22W-29ad1	WT	sand	14	MP	yes
NB 089-1	27N-9W-34da1	WT	sand and gravel	27	NP	yes
NB-161-1	29N-46W-10aa1	WT	sand	18	MP	no
NV-013-1	42/39-25c1	WT	alluvium	19	MLP	no
NV-031-2	18/20-7dcb1	Art.	alluvium	11	LP,RP	no
NM-005-2	12-25-23-113	Art.	Los Andres limestone	22	NP	no
NM-051-1	6.(Harry Dakos)	Art.	Magdalena group	29	NP	yes
NM-051-2	6A.(Harry Dakos)	WT	alluvium	24	NP	yes
ND-017-1	139-48-6ccd1	Art.	glacial drift	25	NP	maybe
ND-025-1	145-92-25ad1	WT	Ft. Union formation	28	P	no
ND-053-1	150-100-12cc1	WT	Ft. Union formation	28	NP	no
ND-067-1	161-56-22bb1	WT	glacial L. Agassiz	29	P	yes
ND-077-1	133-52-34	WT	glacial L. Agassiz	33	PP	no
ND-095-1	160-66-28ba1	WT	glacial drift	32	NP	no
ND-103-1	150-70-20dad2	WT	glacial drift	20	MLP	no
OK-025-1	3N-7E-9bbb1	WT	Ogallala	9	NP	no
OK-139-2	1N-12E-35bdd1	WT	Ogallala	12	NP	no
OR-025-1	23/31-33E1	WT	alluvium	23	MP	no
OR-037-1	27/15-4G1	WT	Basaltic agglomerate	16	MP	no
OR-059-1	5N-35-1C1	WT	gravel	34	P	no
OR-059-2	6N-35-14L1	WT	alluvium	34	NP	no
OR-059-5	6N-35-28H1	WT	gravel	33	P	no
TX-127-1	HZ-77-33-301	WT	Carrizo sand	14	LP,RP	no
TX-189-1	KY-11-51-503	WT	Ogallala formation	20	LP,RP	no
TX-279-1	RU-10-53-602	WT	Ogallala formation	19	LP,RP	no
TX-437-1	XT-11-42-501	WT	Ogallala formation	17	LP,RP	no
UT-027-1	(C-21-5)21aba1	Art.	alluvium	31	NP	no
UT-035-2	(D-1-1)16caa1	WT	alluvium	21	NP	no
WA-007-1	23/19-4E2	WT	sand and gravel	25	LP,RP	no
WA-047-1	34/26-26Q1	WT	stream gravel	27	P	yes
WA-047-2	34/26-35R1	WT	gravel	23	P	no
WA-063-1	25-42-14L1	WT	outwash gravel	28	P	no
WA-063-3	20-43-19A1	WT	fluvioglacial gravel	31	P	no
WY-011-1	54-65-12add	WT	alluvium	24	MP	yes
WY-015-1	26-64-28bbb1	WT	alluvium	21	P	yes
WY-015-2	26-64-29ada	WT	alluvium	20	MP	yes
WY-021-2	14-66-31bda	WT	Ogallala	19	NP	no

*Legend for Pumping Condition

NP - not affected by pumping
MP - may be affected by pumping
P - affected by pumping
MLP - minor local pumping
LP - local pumping
RP - regional pumping

The process $z_{p,\tau}$ now has both trends in the mean and in the standard deviation removed, while maintaining the periodic and stochastic components of the original process $x_{p,\tau}$. Rarely would one expect significant trends in the other parameters, autocorrelation coefficients or higher-order coefficients.

The trends $Tm_{p,\tau}$ and $Ts_{p,\tau}$ may be in general approximated by the polynomial equations of the type

$$Tm_{p,\tau} = A_m + B_m t + C_m t^2 + D_m t^3 + \ . \ . \ . \qquad (3.3)$$

and

$$Ts_{p,\tau} = A_s + B_s t + C_s t^2 + D_s t^3 + \ . \ . \ . \qquad (3.4)$$

in which $t=(p-1)\,\omega + \tau$, and A, B, C and D are coefficients of the polynomial regressions to be estimated from data.

In many cases the linear term of Eqs. 3.3 and 3.4 is sufficient; however, higher-order terms may be necessary when the regression of Tm or Ts on t is far from linear. The regression constants A, B, C and D of Eqs. 3.3 and 3.4 may be estimated either by the least squares procedure or by the multiple-linear-regression method. The least squares procedure may be used with weighted (say, by the trend variance for the trend in the mean) or unweighted deviations. The unweighted least squares method was used in this study.

Periodic components in the mean, standard deviation and autocorrelation coefficients. Following a procedure outlined by V. Yevjevich (1972), the process $z_{p,\tau}$ of Eq. 3.2 may be further represented by the nonparametric equation

$$z_{p,\tau} = m_\tau + s_\tau \varepsilon_{p,\tau} \qquad (3.5)$$

or by the parametric equation

$$z_{p,\tau} = \mu_\tau + \sigma_\tau \varepsilon_{p,\tau}, \qquad (3.6)$$

in which m_τ and μ_τ are the estimated and periodic mean fitted, by a periodic function, s_τ and σ_τ are the estimated and periodic standard deviation fitted, by another periodic function, and $\varepsilon_{p,\tau}$ is in general a nonstationary stochastic component which depends on the periodic autocorrelation coefficients $r_{k,\tau}$ (computed) or $\rho_{k,\tau}$ (fitted), or periodic higher-order coefficients (skewness, kurtosis), and a model for describing the dependence structure.

The sample estimates of the periodic mean, m_τ, periodic standard deviation, s_τ, and periodic autocorrelation coefficients, $r_{k,\tau}$, are computed by

$$m_\tau = \frac{1}{n} \sum_{p=1}^{n} z_{p,\tau}, \qquad (3.7)$$

$$s_\tau = \left[\frac{1}{(n-1)} \sum_{p=1}^{n} (z_{p,\tau} - m_\tau)^2\right]^{\frac{1}{2}}, \qquad (3.8)$$

and

$$r_{k,\tau} = \frac{\frac{1}{n^*}\sum_{p=1}^{n^*} \varepsilon_{p,\tau}\,\varepsilon_{p^*,\tau^*} - \frac{1}{nn^*}\sum_{p=1}^{n} \varepsilon_{p,\tau}\sum_{p=1}^{n^*} \varepsilon_{p^*,\tau^*}}{\left[\frac{1}{n}\sum_{p=1}^{n} \varepsilon^2_{p,\tau} - \left(\frac{1}{n}\sum_{p=1}^{n} \varepsilon_{p,\tau}\right)^2\right]^{\frac{1}{2}} \left[\frac{1}{n^*}\sum_{p=1}^{n^*} \varepsilon^2_{p^*,\tau^*} - \left(\frac{1}{n^*}\sum_{p=1}^{n^*} \varepsilon_{p^*,\tau^*}\right)^2\right]^{\frac{1}{2}}},$$

(3.9)

with $k<\omega$; $n^*=n$, $\tau^*=\tau+k$ and $p^*=p$ if $\tau+k \le \omega$ and $n^*=n-1$, $\tau^*=\tau+k-\omega$ and $p^*=p+1$ if $\tau+k > \omega$.

By Fourier analysis, the periodicities in the mean, standard deviation and autocorrelation coefficients may be fitted mathematically by

$$\nu_\tau = \bar{\nu}_\tau + \sum_{j=1}^{m} (A_j \cos 2\pi f_j \tau + B_j \sin 2\pi f_j \tau) \qquad (3.10)$$

in which ν_τ may represent μ_τ, σ_τ, or $\rho_{k,\tau}$; $\bar{\nu}_\tau$ is the mean of ν_τ; m is the number of significant harmonics; A_j and B_j are the Fourier coefficients; and f_j is the frequency of the j-th harmonic. The estimation from the sample of the periodic function ν_τ (in form of μ_τ, σ_τ, or $\rho_{k,\tau}$) is given elsewhere (V. Yevjevich, 1972).

3.2.2. Stochastic Components

The stochastic process, $\varepsilon_{p,\tau}$ may be obtained by the nonparametric approach of Eq. 3.5 or by the parametric approach of Eq. 3.6. The $\varepsilon_{p,\tau}$ process, whether stationary or not, usually shows a high time dependence structure.

J. D. Salas-LaCruz and V. Yevjevich (1972) expressed the m-th order nonstationary Markov model by

$$\epsilon_{p,\tau} = \sum_{j=1}^{m} \alpha_{j,\tau-j}\ \epsilon_{p,\tau-j} + [1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{i,\tau-i}\ \alpha_{j,\tau-j}\ \rho_{|i-j|,\tau-k}]^{1/2}\ \xi_{p,\tau}\ ,$$

(3.11)

in which k=i if i<j and k=j if i>j with $\alpha_{j,\tau-j}$ the autoregression coefficient at the position $\tau-j$ which is dependent on the autocorrelation coefficients $\rho_{k,\tau-j}$. In this case $\rho_{k,\tau-j}$ represent the population autocorrelation coefficients, to be replaced either by the estimates $r_{k,\tau-j}$, or by fitted periodic function of Eq. 3.10. The relation between $\alpha_{j,\tau-j}$ and $\rho_{k,\tau-j}$ may be found in the above reference for m=1, 2, and 3. One of the first three linear models, m=1, m=2 and m=3 of Eq. 3.11, usually are good approximations. When one of these models is satisfied, then the process $\xi_{p,\tau}$ becomes approximately a second-order stationary independent component. In order to choose the appropriate model of Eq. 3.11, the correlogram $r_\xi(k)$ of $\xi_{p,\tau}$ may be computed and tested for $E[r_\xi(k)]=\rho_\xi(k)=0$ for $k\neq 0$, at the given level of significance. Once the order of the model is determined, Eq. 3.11 is used to obtain the independent series, $\xi_{p,\tau}$.

The independent stochastic process, $\xi_{p,\tau}$ should be further investigated for its probability distribution function. For this purpose, one symmetric distribution, the normal, and one asymmetric distribution, the three-parameter lognormal function, were used in this study, although other distributions may also be used. The estimation of parameters and fitting criteria may be found elsewhere (R. D. Markovic, 1965).

The above description of the decomposition and mathematical description of different components of the original series $x_{p,\tau}$ yields a deterministic-stochastic model (J. D. Salas-LaCruz and V. Yevjevich, 1972) in the form

$$x_{p,\tau} = Tm_{p,\tau} + Ts_{p,\tau}\left\{\mu_\tau + \sigma_\tau\left[\sum_{j=1}^{m} \alpha_{j,\tau-j}\,\epsilon_{p,\tau-j} + \left(1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{i,\tau-i}\,\alpha_{j,\tau-j}\,\rho_{|i-j|,k}\right)^{1/2} \xi_{p,\tau}\right]\right\} \tag{3.12}$$

with k=i if i<j and k=j if i>j, with $Tm_{p,\tau}$ and $Ts_{p,\tau}$ the trends in the mean and standard deviation, respectively; μ_τ and σ_τ the within-the-year periodic mean and standard deviation; $\alpha_{j,\tau-j}$ the within-the-year periodic autoregression coefficients dependent on the periodic autocorrelation coefficient $\rho_{j,\tau-j}$; $\epsilon_{p,\tau-j}$ a nonstationary and dependent stochastic process; and $\xi_{p,\tau}$ the second-order stationary independent stochastic process.

Groundwater level time series may be described by the above deterministic-stochastic model, Eq. 3.12. This model makes it feasible to obtain other sequences of groundwater levels by generating samples of the independent component, $\xi_{p,\tau}$, conforming to its inferred probability distribution function, and by combining these samples with the other parameters and transformatives as shown by Eq. 3.12. These new generated series may then be used in water resources planning to indicate possible future groundwater levels. Implicit in this approach is the assumption that the causal forces that generated the observed series will not be changed, except for the trends which likely will be different for future samples than they were for the observed series.

CHAPTER IV
DATA PROCESSING AND ANALYSIS OF GROUNDWATER LEVEL TIME SERIES

4.1. General

The frequency of the depth-to-water measurements for the 84 selected wells was either monthly, weekly, or continuously. Data from wells on which continuous recorders were installed were published as of 12 noon for each day, or more often for just six days of the month: 5, 10, 15, 20, 25 and EOM, i.e., the end-of-month reading. In order to choose the time interval for use in this study, the longest available daily record, 11 years of daily water level measurements, was analyzed. This well was TX-463-1 (U.S.G.S. well number YP-69-50-302). Table 4.1 shows the results of this analysis. For the time series of daily data the lag-one autocovariance was practically the same as the variance, and the same condition existed for six readings per month, i.e., readings on day 5, 10, 15, 20, 25 of each month plus the end-of-month. Use of 5 readings plus the EOM reading introduced a discrepancy in that the last time interval was 3, 4, 5, or 6 days rather than 5 days as for all other readings. For this reason, the variance and autocovariance were determined for a series with a constant 5-day interval, again with no appreciable difference. Finally, a series with only the EOM readings was used. In this case the autocovariance was less than the variance, but only by a small amount. Based on these results, the decision was made to use the EOM-values for the determination of the stochastic structure of the time series. This seems reasonable in view of the highly-damped groundwater system, and will also be consistent in future studies of groundwater series when dealing conjunctively with monthly precipitation and monthly streamflow series. In other words, the results of using only one reading per month, EOM, gives approximately the same results as in using several readings per month. Only in those cases in which the barometric pressure affects the level fluctuations, this approximation may not be acceptable.

TABLE 4.1

AUTOCOVARIANCE STRUCTURE OF WELL TX-463-1 FOR DIFFERENT FREQUENCY OF MEASUREMENT, BASED ON 11 YEARS OF DAILY DATA

Frequency of Measurement	mean	variance	lag-one autocovariance
daily	54.402	505.339	505.329
5,10,15,20,25,EOM*	54.392	505.215	505.176
5 day intervals	54.386	505.232	505.187
EOM*	54.288	505.260	502.433

*EOM end-of-month

All data were placed on magnetic tape at Colorado State University for processing. Preliminary analysis was done on the CDC 6400 computer there. Final processing was accomplished on the IBM 370/155 system at Clemson University.

For the decomposition of the original observed time series of groundwater levels, four main programs written for the Hydrology and Water Resources Program of Colorado State University were used. The following sections present some of the computational details and the analysis of the results obtained in the present investigation.

4.2. Analysis of Trends and Jumps

Many of the wells exhibited a long-term linear or quadratic trend in the annual mean. The annual mean water level was determined by averaging the water levels at the end of each of 12 months. A regression line was fitted to this time series of mean annual groundwater levels and the t-test was applied to see if the slope coefficient, B, was significantly different from zero. For those wells in which a trend was found, the F-test was used to determine whether the trend was linear or quadratic. Table 4.2 expresses the results. For each well with a trend, the mean and variance of the annual series is given along with the regression coefficients A, B, and C in the equation

$$Tm_t = A + Bt + Ct^2. \qquad (4.1)$$

Where no value is given for C in Table 4.2, the trend in the annual mean could be approximated satisfactorily with a linear trend. Twenty-one wells required removal of a linear trend, while a quadratic trend was removed from 28 wells. Figure 4.1 shows the observed monthly groundwater level hydrograph with the fitted linear trend in the annual mean for the well NB-161-1. The trend was removed by assuming a horizontal line through the mean of the last year and then for each month of each year subtracting the difference between the trend line and horizontal line. Thus, for planning purposes, new series could be generated for future years starting with the mean value for the last year of observation in this study. If the expected trend in the future is different than the trend in the past, this should be incorporated into the generated series.

Wells selected for analysis in this study did not exhibit an annual trend in the standard deviation so no corrections were necessary for this statistic.

By a visual inspection, it was hypothesized that eight of the wells contained a jump in the annual mean. For these wells, identified in Table 4.2 as "jump," the series of annual means was divided into two subseries, and the t-test was applied to determine if the difference of the means of the two subseries was different from zero. The jump was removed from these eight wells by subtracting the difference in the subseries means from the first subseries. No series was encountered for which both the jump and the trend in the mean were significant.

All wells in Table 4.2 for which no information is given did not require any removal of a trend or jump, and the original $x_{p,\tau}$ series is satisfactory for further analysis.

4.3. Analysis of Periodicities in the Mean, Standard Deviation and Autocorrelation Coefficients.

In 51 of the 84 wells, only the 12-month cycle was significant in the monthly means. The subharmonics of the annual cycle were not very much in evidence in that the 6-month component was found only in 33 wells, while the 4-, 3-, 2.4-, and 2-month subharmonics were

TABLE 4.2

MEAN, VARIANCE, AND REGRESSION COEFFICIENTS (IF APPLICABLE) FOR TREND IN ANNUAL MEAN

Study Well No.	mean $\overline{T}_m$	variance s^2_{Tm}	Regression Coefficients		
			A	B	C
AZ-009-1					
AZ-019-2	71.276	98.769	49.591	1.980	-0.030
AZ-023-1					
CA-037-3	60.815	334.966	14.423	6.307	-0.156
CA-037-4	101.517	400.467	58.256	3.411	-0.043
CA-037-5	38.543	154.996	35.529	-1.416	0.063
CA-059-2	35.195	291.437	2.148	3.040	-0.050
CA-077-3	52.571	203.114	27.717	1.506	
CA-077-6	jump				
CA-083-4	68.438	164.056	67.902	2.938	-0.185
ID-011-1	584.478	0.881	581.892	0.494	-0.018
ID-011-2	712.572	0.683	711.590	0.103	
ID-011-3					
ID-013-1					
ID-027-2	9.539	0.363	9.764	-0.198	0.015
ID-045-1	76.376	4.238	78.166	-0.197	
ID-047-1	38.291	0.189	37.317	0.177	-0.006
ID-051-3	256.935	0.743	254.895	0.433	-0.018
ID-077-1	34.869	0.287	33.447	0.394	-0.021
ID-077-2					
ID-077-3	35.889	1.238	37.040	-0.153	
IA-113-1					
IA-145-1					
IA-187-1	5.015	1.172	5.770	-0.052	
KS-067-1	61.199	35.046	54.727	-0.073	0.051
KS-119-1	jump				
KS-171-3	70.120	110.818	55.066	0.445	0.012
LA-039-1	73.044	59.701	60.366	1.103	
LA-067-3	82.713	38.256	78.050	-0.453	0.046
MN-015-1	6.966	3.403	5.566	0.100	
MN-027-1	jump				
MN-027-2	jump				
MN-027-3	25.657	4.251	19.319	1.565	-0.074
MN-053-2	111.736	22.350	100.176	2.024	-0.066
MN-053-4					
MO-161-1	jump				
MT-015-1	jump				
MT-017-1	38.606	3.058	40.144	0.100	-0.017
MT-035-1	1.886	0.174	1.217	0.061	
MT-041-1	41.801	1.884	45.339	-0.566	0.017
MT-071-1					
MT-085-1	23.536	2.393	24.567	0.106	-0.014
MT-095-1					
MT-097-1					
NB-001-1	107.075	16.081	99.930	0.595	
NB-009-1					
NB-057-1	13.243	1.642	11.101	0.165	
NB-065-1					
NB-089-1					
NB-161-1	35.857	2.413	38.650	-0.294	
NV-013-1					
NV-031-2	11.625	1.102	13.188	0.284	
NM-005-2	56.659	453.379	19.071	3.269	
NM-051-1	- 0.316	0.236	- 0.788	0.032	
NM-051-2					
ND-017-1	jump				
ND-025-1	7.876	1.485	7.008	0.325	-0.014
ND-053-1	113.307	0.410	114.359	-0.073	
ND-067-1					
ND-077-1					
ND-095-1	14.837	3.803	19.864	-0.542	0.011
ND-103-1					
OK-025-1					
OK-139-2	189.527	0.280	189.446	-0.206	0.025
OR-025-1					
OR-037-1					
OR-059-1	23.836	3.874	25.558	-0.098	
OR-059-2	9.113	1.543	7.390	0.098	
OR-059-5	11.861	0.766	11.245	-0.040	0.003
TX-127-1	138.869	63.502	128.975	0.235	0.112
TX-189-1	77.960	251.882	42.649	4.643	-0.094
TX-279-1	40.138	55.979	26.536	1.360	
TX-437-1	99.888	471.516	57.601	5.211	-0.044
UT-027-1	29.864	471.887	11.923	-0.808	0.092
UT-035-2					
WA-007-1	17.187	0.636	16.527	0.051	
WA-047-1					
WA-047-2	27.631	3.113	24.798	0.236	
WA-063-1					
WA-063-3	137.573	1.350	137.673	-0.188	0.008
WY-011-1	jump				
WY-015-1					
WY-015-2					
WY-021-2					

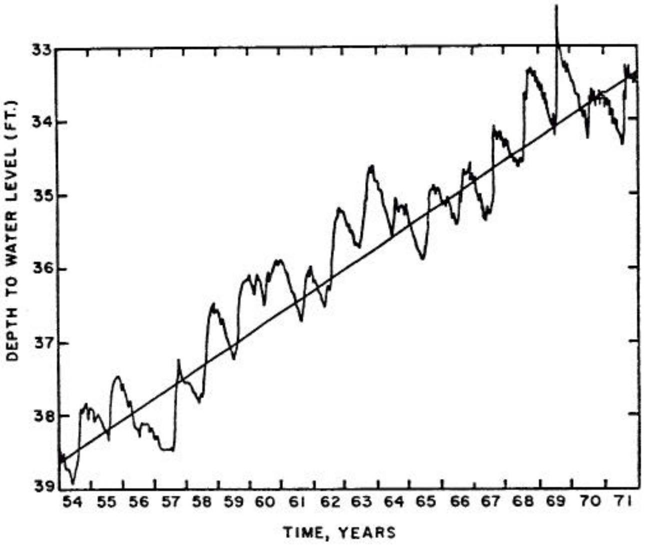

Fig. 4.1 Observed monthly groundwater levels time series $x_{p,\tau}$ and the fitted linear trend in the mean Tm_t for well NB-161-1.

significant only in 8, 1, 3 and 0 wells, respectively. Generally speaking, the wells in the northern states, unaffected by pumping but affected by snow and below freezing temperatures, had lower water levels in the winter months when recharge was low due to the snow layer, and higher water levels occuring during the spring and summer. Deeper wells appeared to have the high and low water levels a month or so after the shallower wells, but study of the bivariate processes of precipitation and groundwater levels should be conducted to confirm this conclusion. Wells affected by pumpage, especially for irrigation, usually showed lower levels in July and August. Figures 4.2 and 4.5 show the computed and fitted periodicities in the mean for two wells: well NB-161-1 is affected by local pumping, while well WY-021-2 is not affected by pumping. They show a close agreement of the fitted periodic functions to the estimated values, m_τ.

In dealing with the periodicity of the standard deviation of water levels, the 6-month and 4-month subharmonics of the annual cycle were more in evidence than for the periodic mean, being found significant in 50 and 32 cases, respectively. The 3- and 2.4- month subharmonics were also stronger than for the mean, being significant in 11 and 13 wells, respectively. The 2-month subharmonic was not found significant for any wells. No discernible pattern could be found for the variations in the standard deviations with the variations of the mean. Sometime, high deviations were found when the mean monthly water level was high, but often the opposite situation was found. Similar results were noted by wells in northern locations vs. southern locations, wells not affected by pumpage and those that were so affected, and also when the type of use was considered. The computed and fitted periodicities in the standard deviation are shown in Figs. 4.2 and 4.5 for wells NB-161-1 and WY-021-2, respectively. The agreement between the fitted periodic function (σ_τ) and the estimates (s_τ) is good, though the subharmonics (4-, 3-, 2.4- month) may be due to sampling errors.

The mean, variance and significant harmonics of the periodic mean and standard deviation are given in Table 4.3. The Fourier coefficients for the significant harmonics of the periodicity in the mean and in the standard deviation are given in Tables 4.4 and 4.5, respectively.

After the periodic components in the mean and standard deviation have been removed by application of Eq. 3.6, the stochastic process, $\varepsilon_{p,\tau}$, was further analyzed. The computations have shown that in all cases the dependent stochastic component, $\varepsilon_{p,\tau}$, has periodic autocorrelation coefficients. For each well the significant harmonics of the first, second and third autocorrelation coefficients were determined as presented in Table 4.6. The autocorrelation coefficients for lags 1, 2, and 3 for almost all wells showed one or more significant subharmonics of the annual cycle, with the 6-, 4-, and 3-month subharmonics being significant for a number of wells. Figures 4.3 and 4.6 show the computed and fitted periodic first, second and third autocorrelation coefficients of the variable $\varepsilon_{p,\tau}$ for the wells NB-161-1 and WY-021-2, respectively.

To determine the dependence model, a hypothesis is first made that a certain order of Markov dependence model will describe the structure of the series $\varepsilon_{p,\tau}$. Equation 3.11 gives the independent stochastic component $\xi_{p,\tau}$. The next step is to determine if, in fact, this $\xi_{p,\tau}$ process is independent. The procedure selected for use here is the correlogram analysis of the residuals $\xi_{p,\tau}$. If the process is found to be independent, then the assumed Markov dependence model is considered satisfactory.

For the wells investigated in this study, the model(s) that may be used to represent the dependence within the stochastic component is shown in Table 4.7. In most cases more than one model will satisfactorily represent the dependence, and for 23 wells either the first-, second-, or third-order Markov model may be used. However, in 29 wells none of the three models used in this study were satisfactory. Because of the nature of the groundwater system it would appear that higher-order Markov models will be necessary to describe the stochastic dependence in these cases.

To complete the description of the stochastic component, a distribution function was fitted to the independent stochastic process, $\xi_{p,\tau}$. Using the chi-square criterion, either the normal distribution or the three-parameter lognormal distribution was fitted to its empirical frequency distribution. The results are shown in Table 4.7 where the distribution is indicated along with the two-parameters (for the normal) or three parameters (for the lognormal) estimated by the method of maximum likelihood. Figures 4.4 and 4.7 show the empirical and fitted normal density and cumulative distribution functions of the independent stochastic component $\xi_{p,\tau}$ for the wells NB-161-1 and WY-021-2, respectively. The mean and the standard deviation of the normal distribution are not exactly 0 and 1, but are close to them. The reason for this departure is the use of the fitted periodic functions (μ_τ, σ_τ, and $\rho_{k,\tau}$) instead of the use of the estimated values (m_τ, s_τ, $r_{k,\tau}$).

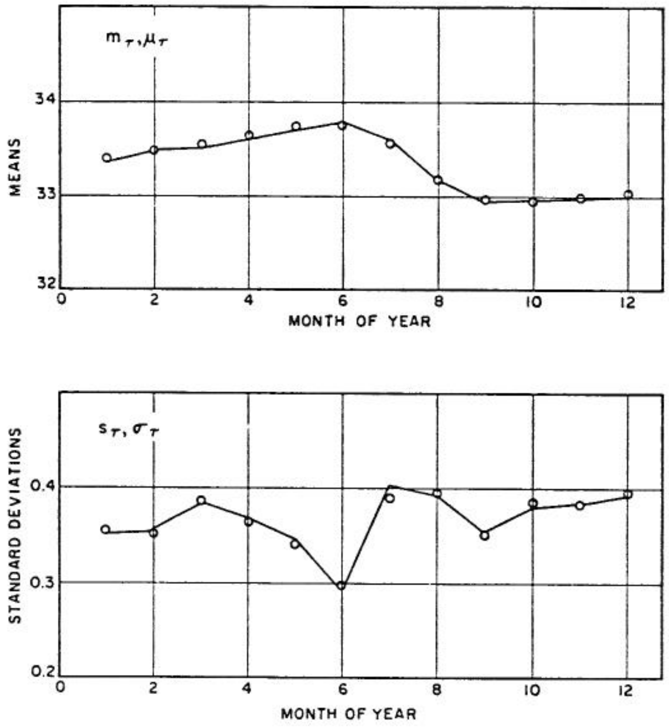

Fig. 4.2 Computed and fitted periodicities in the mean and standard deviation for well NB-161-1.

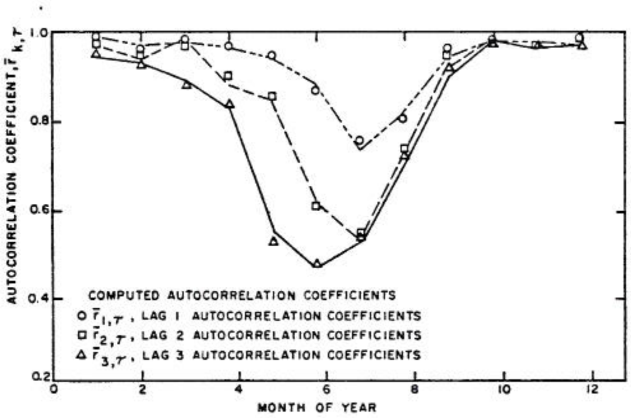

Fig. 4.3 Computed and fitted periodicities in the autocorrelation coefficients for well NB-161-1.

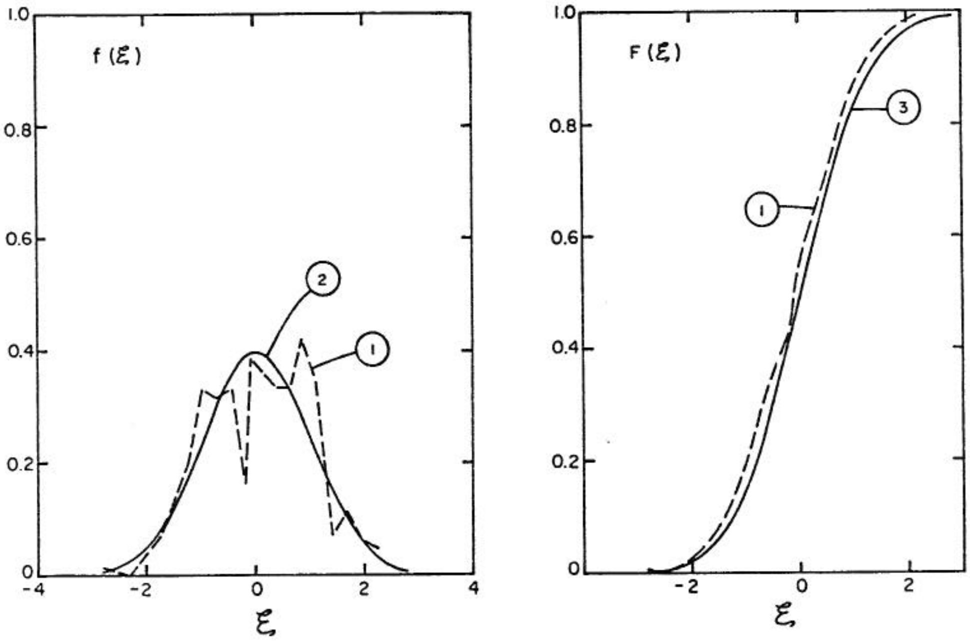

Fig. 4.4 Empirical (1) and fitted normal density (2) and cumulative distribution (3) functions of the independent stochastic component $\xi_{p,\tau}$ for well NB-161-1.

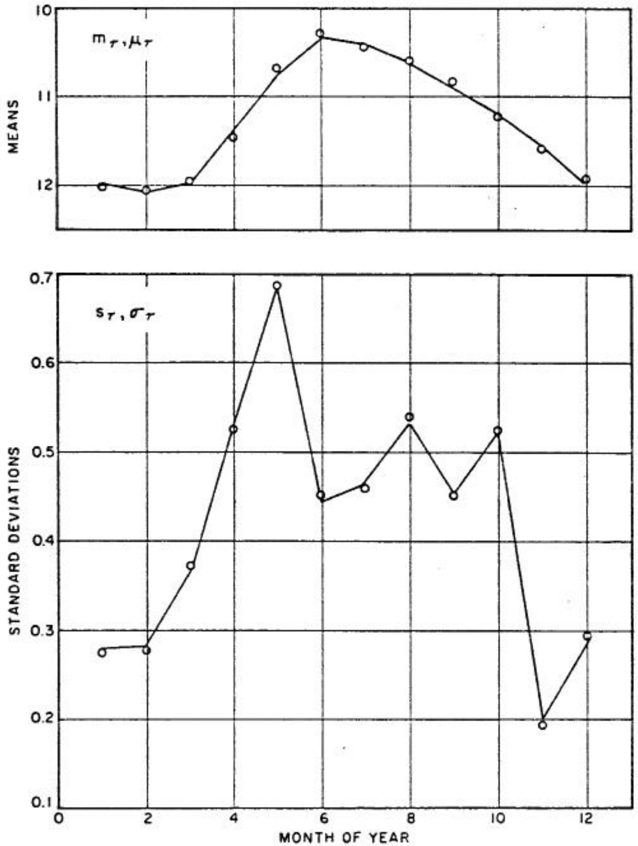

Fig. 4.5 Computed and fitted periodicities in the mean and standard deviation for well WY-021-2.

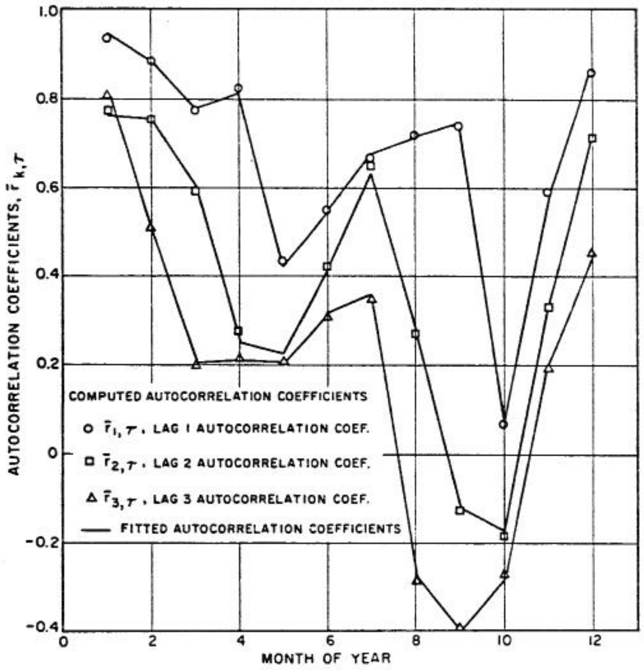

Fig. 4.6 Computed and fitted periodicities in the autocorrelation coefficients for well WY-021-2.

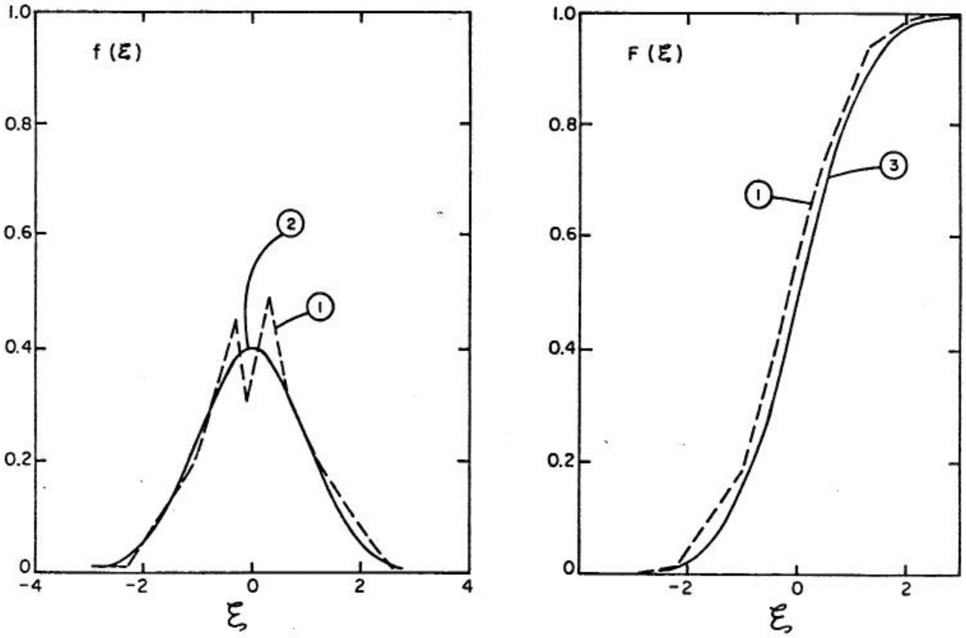

Fig. 4.7 Empirical (1) and fitted normal density (2) and cumulative distribution (3) functions of the independent stochastic component $\xi_{p,\tau}$ for well WY-021-2.

TABLE 4.3

MEAN, VARIANCE, AND SIGNIFICANT HARMONICS OF PERIODICITY IN MEAN AND STANDARD DEVIATION

Study Well No.	in mean			in standard deviation		
	$\bar{m}_\tau$	$s^2_{m_\tau}$	signif. harm.	$\bar{s}_\tau$	$s^2_{s_\tau}$	signif. harm.
AZ-009-1	32.059	2.399	12	4.436	0.045	12,6,4
AZ-019-2	82.340	0.588	12	3.820	0.087	12,6,4
AZ-023-1	5.598	0.393	12,6	0.901	0.051	12,4
CA-037-3	77.013	7.094	12	3.510	0.136	12,6
CA-037-4	125.445	7.206	12	5.607	0.175	12,6,4,2.4
CA-037-5	69.327	0.919	12	2.493	0.025	12,4
CA-059-2	47.617	52.804	12	11.276	0.906	12,6
CA-077-3	75.919	6.140	12	4.649	2.495	12,4
CA-077-6	47.260	6.688	12	2.445	0.069	12,4
CA-083-4	37.723	0.177	12,6,2.4	3.918	0.058	12,6,4
ID-011-1	584.354	0.096	12	0.504	0.002	12,4
ID-011-2	713.449	0.176	12	0.690	0.004	12
ID-011-3	18.730	16.795	12	1.156	0.039	12,6,4
ID-013-1	144.722	10.684	12	2.950	0.780	12,6
ID-027-2	10.683	2.021	12	0.639	0.011	12,4,3
ID-045-1	74.002	4.952	12	1.721	0.032	12
ID-047-1	38.500	0.365	12	0.350	0.001	12,6,4,3
ID-051-3	256.549	1.589	12	0.676	0.005	12,6,4
ID-077-1	34.791	0.227	12,6	0.458	0.024	12,6
ID-077-2	24.487	0.102	12,6	0.492	0.012	12,6
ID-077-3	34.892	0.813	12	1.158	0.128	12
IA-113-1	5.838	1.385	12	2.660	0.337	12
IA-145-1	12.333	1.777	12	3.154	0.197	12,6,3,2.4
IA-187-1	4.311	0.835	12,6	1.654	0.161	12,6
KS-067-1	73.472	0.417	12	0.822	0.007	12,6,4
KS-119-1	21.203	3.785	12,4	3.758	2.918	12,6,4,3
KS-171-3	90.224	0.412	12	1.977	0.022	12,4
LA-039-1	84.621	77.028	12,6	4.963	4.669	12
LA-067-3	96.305	0.131	12	1.591	0.004	12,6,4
MN-015-1	8.265	1.139	12	2.681	0.371	12,6
MN-027-1	9.415	0.197	12,6	0.654	0.022	12,6
MN-027-2	18.250	0.162	12,6	0.658	0.040	12,4
MN-027-3	24.601	0.059	12	0.787	0.010	12
MN-053-2	115.388	68.823	12	4.825	1.503	12,6,2.4
MN-053-4	82.805	53.833	12,6	3.969	6.816	12
MO-161-1	10.025	0.732	12	1.437	0.038	12,6,4
MT-015-1	19.787	1.779	12	1.956	0.376	12,6,4
MT-017-1	34.876	0.847	12	0.607	0.050	12,6,2.4
MT-035-1	2.494	0.409	12,6	0.511	0.012	12,6,4,2.4
MT-041-1	40.982	0.006	12,6,4	0.568	0.003	12,4
MT-071-1	5.149	0.506	12	0.487	0.016	12,6
MT-085-1	20.637	0.019	12,6	0.926	0.003	12,4,2.4
MT-095-1	8.372	5.271	12	0.765	0.080	12,4
MT-097-1	11.288	6.347	12,6	1.286	0.437	12,6,3
NB-001-1	113.417	0.066	12	0.732	0.017	12,6
NB-009-1	3.952	0.293	12	0.507	0.014	12,2.4
NB-057-1	15.220	0.068	12	0.631	0.015	12,6
NB-065-1	10.689	1.015	12	1.269	0.069	12
NB-089-1	7.843	0.278	12	0.872	0.058	12,3
NB-161-1	33.358	0.087	12	0.366	0.001	12,4,2.4
NV-013-1	7.825	1.034	12	1.754	0.077	12,6
NV-031-2	10.347	2.053	12	0.828	0.036	12,6
NM-005-2	90.979	711.678	12	13.118	11.616	12,6,2.4
NM-051-1	0.125	0.039	12	0.507	0.004	12,6,3
NM-051-2	1.745	0.045	12,6	0.570	0.021	12,3,2.4
ND-017-1	50.284	1.187	12	3.388	2.580	12,6,4,3
ND-025-1	5.169	0.277	12,6	0.964	0.085	12
ND-053-1	112.328	0.001	12,6,3,2.4	0.294	0.001	12,6,4,3
ND-067-1	7.948	0.676	12,6	1.376	0.045	12,6
ND-077-1	5.778	1.420	12,6	1.564	0.064	12,6
ND-095-1	13.740	0.002	12,6,4	0.620	0.001	12,6
ND-103-1	3.563	0.626	12,6	1.762	0.062	12,6
OK-025-1	32.893	0.003	12,4,2.4	0.537	0.023	12
OK-139-2	190.675	0.009	12,6,4	0.224	0.006	12,6
OR-025-1	5.179	3.143	12	1.032	0.238	12
OR-037-1	36.953	0.280	12,6	0.941	0.409	12,6,4
OR-059-1	22.214	2.750	12,6	2.725	0.156	12,3
OR-059-2	10.737	0.127	12,4	1.390	0.336	12,6,4
OR-059-5	13.638	1.516	12,6	1.057	0.075	12,6
TX-127-1	154.202	0.743	12,6	1.663	0.397	12
TX-189-1	98.086	0.880	12,6	2.555	0.002	12,6,4,2.4
TX-279-1	52.380	0.118	12,6	0.761	0.003	12
TX-437-1	133.457	2.061	12,6	1.786	0.053	12,6
UT-027-1	75.165	38.386	12	8.694	0.880	12
UT-035-2	58.034	0.197	12,6	2.199	0.062	12,6,4
WA-007-1	17.796	0.424	12	1.144	0.062	12,6,2.4
WA-047-1	13.362	1.537	12,6,4	1.389	0.560	12,6
WA-047-2	30.228	0.712	12,6,4	1.274	0.218	12,6,3
WA-063-1	95.104	6.963	12,6	1.850	0.421	12,6
WA-063-3	139.979	0.207	12	1.019	0.052	12
WY-011-1	14.073	0.045	12	0.377	0.008	12
WY-015-1	15.725	0.242	12	0.404	0.008	12,6
WY-015-2	17.950	0.043	12,6	0.371	0.007	12
WY-021-2	11.253	0.403	12	0.421	0.018	12,6,4,2.4

TABLE 4.4

FOURIER COEFFICIENTS FOR PERIODICITY IN THE MEAN

Study Well No.	A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4
AZ-009-1	0.340	-2.149						
AZ-019-2	-1.038	0.130						
AZ-023-1	-0.567	-0.399	0.281	-0.344				
CA-037-3	-2.162	-3.055						
CA-037-4	-2.416	-2.898						
CA-037-5	-0.428	-1.274						
CA-059-2	-7.086	-7.278						
CA-077-3	-3.094	-1.343						
CA-077-6	-2.646	-2.355						
CA-083-4	-0.212	-0.315	-0.228	0.280	-0.226	-0.002		
ID-011-1	-0.379	0.214						
ID-011-2	-0.401	0.430						
ID-011-3	1.838	5.359						
ID-013-1	1.128	4.320						
ID-027-2	0.964	1.742						
ID-045-1	-2.506	1.881						
ID-047-1	-0.003	0.844						
ID-051-3	-1.274	-1.234						
ID-077-1	-0.526	-0.116	0.184	0.336				
ID-077-2	-0.203	0.338	0.130	0.128				
ID-077-3	0.486	-1.147						
IA-113-1	1.214	-1.014						
IA-145-1	1.404	-1.166						
IA-187-1	1.056	-0.503	0.039	0.491				
KS-067-1	-0.043	-0.843						
KS-119-1	-1.957	-1.299	0.998	-0.365				
KS-171-3	0.085	-0.840						
LA-039-1	-9.753	-6.051	4.435	-0.410				
LA-067-3	0.266	-0.379						
MN-015-1	1.412	-0.229						
MN-027-1	0.557	-0.089	-0.156	0.203				
MN-027-2	0.361	-0.276	0.152	0.237				
MN-027-3	-0.176	-0.271						
MN-053-2	-9.319	-6.089						
MN-053-4	-8.952	-3.480	2.154	2.919				
MO-161-1	0.383	-1.121						
MT-015-1	-1.334	-1.208						
MT-017-1	-0.276	1.259						
MT-035-1	0.150	-0.839	0.149	0.249				
MT-041-1	0.073	0.013	-0.047	0.029	-0.050	0.010		
MT-071-1	0.550	0.832						
MT-085-1	0.094	0.137	-0.063	0.052				
MT-095-1	-0.269	3.141						
MT-097-1	2.933	1.387	-1.316	0.354				
NB-001-1	-0.004	-0.349						
NB-009-1	-0.082	-0.718						
NB-057-1	0.096	-0.347						
NB-065-1	0.552	1.227						
NB-089-1	0.115	-0.689						
NB-161-1	-0.285	0.280						
NV-013-1	1.169	-0.689						
NV-031-2	-0.405	1.943						
NM-005-2	-34.886	-1.370						
NM-051-1	0.180	-0.206						
NM-051-2	0.238	-0.104	0.050	-0.082				
ND-017-1	-0.492	-1.342						
ND-025-1	0.618	-0.068	-0.272	0.282				
ND-053-1	0.009	0.010	-0.002	0.013	-0.014	-0.017	-0.009	0.008
ND-067-1	0.992	-0.067	-0.440	0.374				
ND-077-1	1.283	-0.215	-0.348	0.969				
ND-095-1	0.036	-0.019	-0.016	0.031	-0.026	0.013		
ND-103-1	0.985	-0.005	-0.263	0.450				
OK-025-1	-0.051	0.014	0.027	0.013	-0.006	0.029		
OK-139-2	-0.009	-0.104	-0.012	-0.053	0.042	-0.033		
OR-025-1	2.045	-1.246						
OR-037-1	-0.632	-0.110	0.334	0.126				
OR-059-1	-0.530	-1.736	-0.116	1.417				
OR-059-2	-0.136	0.392	-0.105	-0.234				
OR-059-5	1.146	1.033	-0.286	0.731				
TX-127-1	0.770	-0.625	-0.017	-0.516				
TX-189-1	0.262	-1.081	0.207	-0.455				
TX-279-1	0.103	-0.348	0.092	-0.199				
TX-437-1	0.609	-1.557	0.533	-0.685				
UT-027-1	-7.110	-4.659						
UT-035-2	0.384	0.392	-0.064	0.255				
WA-007-1	0.419	-0.779						
WA-047-1	0.690	-0.848	-0.793	0.905	0.251	-0.536		
WA-047-2	0.442	-0.454	-0.853	0.207	-0.136	0.172		
WA-063-1	1.751	-2.802	-0.994	1.259				
WA-063-3	0.252	-0.567						
WY-011-1	0.189	-0.223						
WY-015-1	0.575	0.349						
WY-015-2	0.258	0.055	-0.098	0.029				
WY-021-2	0.733	0.485						

TABLE 4.5

FOURIER COEFFICIENTS FOR PERIODICITY IN THE STANDARD DEVIATION

Study Well No.	A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4
AZ-009-1	0.078	0.147	0.178	-0.035	-0.019	0.108		
AZ-019-2	-0.181	0.123	0.123	0.229	-0.115	0.194		
AZ-023-1	0.020	-0.265	0.113	-0.068				
CA-037-3	0.086	0.271	-0.007	0.375				
CA-037-4	-0.187	-0.110	-0.183	-0.315	0.167	-0.280	0.196	-0.013
CA-037-5	0.139	0.001	0.116	-0.082				
CA-059-2	-0.926	0.815	0.262	-0.339				
CA-077-3	-1.077	1.494	0.742	-0.384				
CA-077-6	-0.232	0.144	-0.134	0.125				
CA-083-4	0.004	-0.184	-0.149	0.076	0.170	-0.026		
ID-011-1	0.040	0.032	-0.031	0.014				
ID-011-2	-0.020	-0.074						
ID-011-3	-0.116	-0.109	-0.098	-0.081	-0.018	0.157		
ID-013-1	-0.119	-1.061	-0.537	0.057				
ID-027-2	-0.024	-0.069	-0.041	0.096	-0.052	-0.042		
ID-045-1	0.116	-0.178						
ID-047-1	0.014	-0.009	-0.026	-0.019	-0.006	0.013	0.015	0.005
ID-051-3	-0.024	-0.054	-0.051	0.025	0.048	0.020		
ID-077-1	-0.145	-0.110	0.112	0.023				
ID-077-2	-0.063	-0.111	0.048	0.067				
ID-077-3	0.213	-0.443						
IA-113-1	0.780	0.132						
IA-145-1	-0.255	0.352	-0.103	-0.256	0.223	0.020	-0.211	-0.054
IA-187-1	0.418	0.271	-0.235	-0.021				
KS-067-1	0.067	0.033	-0.028	-0.060	-0.030	-0.027		
KS-119-1	-1.769	-0.438	-0.221	0.796	0.921	-0.407	-0.817	0.272
IS-171-3	-0.092	-0.147	0.073	0.005				
LA-039-1	-2.474	0.349						
LA-067-3	-0.014	0.052	0.010	-0.052	0.036	-0.005		
MN-015-1	0.632	0.461	-0.201	-0.274				
MN-027-1	-0.071	-0.053	0.156	-0.082				
MN-027-2	-0.219	0.065	0.096	-0.085				
MN-027-3	0.093	0.092						
MN-053-2	-1.296	-0.418	0.610	-0.409	-0.109	-0.576		
MN-053-4	-3.320	-0.519						
MO-161-1	0.179	-0.073	-0.147	0.074	-0.068	0.073		
MT-015-1	-0.618	-0.332	-0.335	-0.071	0.214	0.245		
MT-017-1	0.212	0.098	0.052	0.104	0.091	-0.073		
MT-035-1	0.076	-0.017	-0.039	0.095	0.010	-0.057		
MT-041-1	0.065	-0.019	0.026	0.009				
MT-071-1	-0.120	0.035	0.112	0.003				
MT-085-1	-0.049	0.023	-0.002	-0.037	-0.012	-0.021		
MT-095-1	-0.276	-0.219	-0.109	0.076				
MT-097-1	-0.652	-0.355	0.051	-0.326	-0.342	-0.071		
NB-001-1	-0.053	0.150	-0.024	0.071				
NB-009-1	-0.142	0.006	0.001	0.062				
NB-057-1	0.050	-0.117	-0.098	-0.016				
NB-065-1	-0.179	-0.299						
NB-089-1	-0.200	0.172	0.112	-0.048				
NB-161-1	0.015	-0.010	0.020	-0.019	0.013	0.010		
NV-013-1	-0.277	0.100	0.146	-0.134				
NV-031-2	-0.047	-0.213	-0.103	0.072				
NM-005-2	-2.100	-1.361	1.378	2.909	1.211	-1.587		
NM-051-1	-0.049	0.004	0.051	-0.015	-0.018	-0.030		
NM-051-2	0.001	-0.096	-0.080	-0.107	0.035	-0.103		
ND-017-1	-0.838	-1.184	0.724	0.090	-1.117	-0.527	0.356	0.829
ND-025-1	-0.341	-0.184						
ND-053-1	0.024	0.025	-0.022	0.009	-0.022	-0.011	0.001	-0.021
ND-067-1	-0.197	0.112	0.017	-0.163				
ND-077-1	-0.273	-0.108	-0.091	-0.130				
ND-095-1	0.004	-0.043	0.009	0.013				
ND-103-1	0.047	0.170	-0.272	0.033				
OK-025-1	0.159	-0.124						
OK-139-2	0.081	0.030	0.006	0.052				
OR-025-1	-0.032	0.634						
OR-037-1	-0.563	-0.200	0.366	0.376	-0.150	-0.314		
OR-059-1	0.263	0.398	-0.055	0.224				
OR-059-2	0.119	0.525	-0.124	0.464	-0.257	-0.115		
OR-059-5	0.125	0.059	-0.186	0.280				
TX-127-1	0.681	-0.298						
TX-189-1	0.015	-0.041	-0.021	-0.002	0.009	0.020	0.012	-0.015
TX-279-1	0.053	-0.049						
TX-437-1	-0.186	0.146	-0.193	-0.050				
UT-027-1	-0.644	-1.061						
UT-035-2	-0.167	-0.195	-0.025	-0.164	0.104	0.078		
WA-007-1	0.114	0.240	-0.071	0.131	-0.007	0.120		
WA-047-1	-0.877	0.424	0.353	-0.013				
WA-047-2	-0.180	-0.144	0.550	-0.127	0.186	-0.145		
WA-063-1	-0.490	0.621	0.414	0.044				
WA-063-3	-0.287	-0.097						
WY-011-1	-0.075	0.098						
WY-015-1	-0.019	-0.069	-0.069	-0.092				
WY-015-2	-0.064	0.088						
WY-021-2	-0.146	-0.024	-0.033	-0.055	0.018	0.064	0.050	0.050

TABLE 4.6

MEAN AND SIGNIFICANT HARMONICS OF FIRST, SECOND, AND THIRD AUTOCORRELATION COEFFICIENTS, $r_{k,\tau}$

Study Well No.	1st autocorrelation coef.		2nd autocorrelation coef.		3rd autocorrelation coef	
	$\bar{r}_{1,\tau}$	signif. harm.	$\bar{r}_{2,\tau}$	signif. harm.	$\bar{r}_{3,\tau}$	signif. harm.
AZ-009-1	0.931	12,6,4,3,2.4	0.850	12,6,4,3	0.776	12,6,4
AZ-019-2	0.946	12,6,4	0.850	12,6,4	0.744	12,6,4
AZ-023-1	0.716	12,6,4,3	0.539	12,6,4	0.472	12,6,4
CA-037-3	0.893	12,6,4,2.4	0.779	12,6	0.649	12,6
CA-037-4	0.928	12,3,2	0.865	12,4	0.827	12
CA-037-5	0.953	12,6,2.4	0.896	12,6,4	0.841	12,6,4
CA-059-2	0.957	12,2.4	0.918	12,6,4	0.888	12,6,4
CA-077-3	0.807	12,6,3,2.4	0.584	12,6	0.389	12,6,4
CA-077-6	0.829	12,6,4,3,2.4	0.719	12,4,3	0.599	12
CA-083-4	0.925	12,4	0.880	12,6,4,3,2.4,2	0.844	12,6,3
ID-011-1	0.957	12,6,4,3,2.4	0.910	12,6,4,3	0.861	12,3
ID-011-2	0.966	12,6,4,3	0.925	12,6,4,3,2.4	0.884	12,6,4,3
ID-011-3	0.768	12,3,2.4	0.549	12,3	0.427	12
ID-013-1	0.718	12,3	0.495	12,6	0.385	12,6,4
ID-027-2	0.770	12,3	0.615	12,6,2.4	0.495	12,6
ID-045-1	0.867	12,6,4	0.799	12,6,2.4	0.738	12,6
ID-047-1	0.878	12,4,3	0.761	12,4	0.618	12,6,4
ID-051-3	0.938	12,6,4,3	0.858	12,6,4	0.765	12,6,4
ID-077-1	0.763	12,2.4	0.571	12,3	0.460	12,6,3
ID-077-2	0.936	12,3,2.4	0.891	12,6,3	0.841	12,6,2.4
ID-077-3	0.909	12	0.809	12,4,3,2.4	0.721	12,4,3
IA-113-1	0.880	12,6	0.789	12,6	0.687	12,6
IA-145-1	0.772	12,3,2	0.566	12,6,2.4	0.436	12,6,3,2.4
IA-187-1	0.824	12	0.694	12	0.585	12
KS-067-1	0.883	12,6,4,3,2.4	0.730	12,6,4	0.601	12,6,4
KS-119-1	0.661	12,3,2.4	0.415	12,6,4,2.4	0.314	12,6,4
KS-171-3	0.943	12,4,3	0.894	12,6,4,2.4	0.872	12,6,4,3
LA-039-1	0.789	12,6	0.668	12,6	0.516	12,6
LA-067-3	0.975	12,6,3,2.4	0.951	12,6,4	0.925	12,6,4
MN-015-1	0.885	12,6,4,2.4	0.744	12	0.601	12
MN-027-1	0.676	12,4,3,2.4,2	0.433	12,4,3	0.289	12,4
MN-027-2	0.484	12,3	0.261	12,6	0.260	12,6
MN-027-3	0.878	12,6,4,3	0.807	12,6,4,3	0.741	12,6,3
MN-053-2	0.304	12,4,2.4	0.149	12,4,2.4	0.190	12,6,4
MN-053-4	0.222	12,4,2.4	0.182	12,6,3,2.4	0.150	12,6,4,3,2.4
MO-161-1	0.711	12,6	0.611	12,3	0.594	12,6,4,2.4
MT-015-1	0.597	12,6,4,2.4	0.322	12,6,4,3	0.122	12,6,4,2.4
MT-017-1	0.613	12,6,4	0.271	12,6,2.4	0.116	12,4,3,2.4
MT-035-1	0.482	12,6,2.4	0.232	12,2.4	0.146	12,2
MT-041-1	0.931	12,6,4,3,2.4	0.874	12,6,3,2.4	0.819	12,6,4,3,2.4
MT-071-1	0.802	12,6,3,2.4	0.564	12,6	0.489	12,6,4
MT-085-1	0.946	12,6,4,3	0.919	12,6,2.4,2	0.898	12,6,4,3,2.4
MT-095-1	0.886	12,4,2.4	0.759	12,6,4,3	0.650	12,6,4,2.4
MT-097-1	0.313	12,6,4,3	-0.018	12,6,4	0.003	12,4,3,2.4
NB-001-1	0.959	12,6,4,3,2.4	0.927	12,6	0.898	12,6
NB-009-1	0.631	12,2.4	0.504	12,3,2.4	0.405	12,6,3
NB-057-1	0.918	12,6,2.4	0.811	12,6	0.720	12,6
NB-065-1	0.880	12,6,4	0.731	12,6	0.639	12,6,3
NB-089-1	0.772	12,6,4	0.630	12,6,4,2	0.561	12,4
NB-161-1	0.932	12,6	0.868	12,6	0.809	12,6
NV-013-1	0.935	12,4,3,2.4	0.859	12,6,4	0.790	12,6,4
NV-031-2	0.868	12,6,3	0.767	12,6,3	0.681	12,6,4
NM-005-2	0.427	12,6,2.4	0.262	12,6,4,2.4	0.077	12,6,4
NM-051-1	0.910	12,6,4	0.816	12,6,4,2.4	0.732	12,6,4,3
NM-051-2	0.480	12,6,4,3,2.4	0.468	12,6,3	0.379	12,3
ND-017-1	0.716	12,6,3	0.579	12,6	0.523	12,6,4,3
ND-025-1	0.743	12,6	0.573	12,6	0.426	12,6
ND-053-1	0.809	12,6,3	0.805	12,2.4	0.801	12,4,3,2.4
ND-067-1	0.933	12,6	0.862	12,6	0.806	12,6
ND-077-1	0.823	12,4	0.731	12,4	0.681	12,4
ND-095-1	0.976	12,6,4,2.4	0.946	12,6,4,2.4	0.925	12,6,4,3
ND-103-1	0.865	12,6,3,2.4	0.739	12,6,3	0.648	12,6,4
OK-025-1	0.861	12,6,4,3	0.708	12,6	0.585	12,6,2.4
OK-139-2	0.541	12,6,4,2.4	0.492	12,6,4,3	0.371	12,6,4,2.4
OR-025-1	0.850	12,6,3	0.721	12,6,3	0.629	12,6
OR-037-1	0.675	12,4,3	0.549	12	0.519	12,6
OR-059-1	0.712	12,6,3	0.464	12,6,4,3,2.4	0.340	12,6,3,2.4
OR-059-2	0.618	12,6,4,3,2.4	0.222	12,6,4,3,2.4	0.113	12,6,2.4
OR-059-5	0.667	12,6,3	0.294	12,6	0.083	12,6
TX-127-1	0.719	12,6,4,2.4	0.609	12,6,4,3,2	0.469	12,3
TX-189-1	0.985	12,6,4,2.4	0.963	12,6,4,2.4	0.942	12,6,4
TX-279-1	0.973	12,6,2.4,2	0.936	12,6,4	0.911	12,6,4
TX-437-1	0.931	12,6	0.810	12,6,4	0.697	12,6,4
UT-027-1	0.961	12,6,4,3,2.4	0.912	12,6,4	0.872	12,6,4
UT-035-2	0.964	12,4,3	0.916	12,4,3	0.865	12,4
WA-077-1	0.729	12,4,3,2.4	0.565	12,6,3	0.388	12,6,4
WA-047-1	0.656	12,6,3,2.4	0.293	12,6,3	0.092	12,6,4
WA-047-2	0.708	12,6,4,2	0.444	12,6,4,3	0.384	12,6,3
WA-063-1	0.783	12,6	0.573	12,6	0.437	12,6
WA-063-3	0.714	12,6,3,2.4	0.565	12,6,3	0.426	12,4,3,2.4
WY-011-1	0.821	12,6,4,2.4	0.600	12,6,3,2.4	0.425	12,6,4,3
WY-015-1	0.673	12,3,2.4	0.359	12,6,4,3,2.4	0.209	12,6
WY-015-2	0.637	12,6,4,3	0.310	12,6,4,2.4	0.225	12,6,3
WY-021-2	0.669	12,6,4,2.4	0.388	12,6	0.188	12,6

TABLE 4.7

MARKOV DEPENDENCE MODEL AND DISTRIBUTION FUNCTION OF INDEPENDENT STOCHASTIC COMPONENT, $\xi_{p,\tau}$

Study Well No.	Markov model*	Distribution	Parameters of distribution**		
			A	B	C
AZ-009-1	1, (3)	normal	0.031	1.026	
AZ-019-2	1	normal	-0.009	0.971	
AZ-023-1					
CA-037-3	1, (3)	normal	-0.018	0.967	
CA-037-4					
CA-037-5					
CA-059-2	1, (2,3)	log normal 3	1.849	0.154	-6.440
CA-077-3					
CA-077-6	2, (1,3)	log normal 3	2.160	0.118	-8.732
CA-083-4	2	log normal 3	2.316	0.097	-10.177
ID-011-1	2, (1,3)	normal	-0.021	1.011	
ID-011-2					
ID-011-3	1, (3)	normal	0.004	1.000	
ID-013-1	1, (2,3)	normal	0.002	1.017	
ID-027-2					
ID-045-1					
ID-047-1					
ID-051-3	1, (3)	log normal 3	2.232	0.111	-9.398
ID-077-1	1, (3)	log normal 3	2.194	0.112	-9.029
ID-077-2	1, (2,3)	normal	0.041	1.062	
ID-077-3	1, (2)	normal	0.0	1.064	
IA-113-1	2, (1,3)	normal	-0.013	0.995	
IA-145-1	1, (2,3)	normal	0.002	0.997	
IA-187-1					
KS-067-1	1, (2,3)	normal	0.007	1.029	
KS-119-1					
KS-171-3					
LA-039-1	2	normal	-0.029	1.260	
LA-067-3	1, (2,3)	log normal 3	2.087	0.128	-8.119
MN-015-1	1, (2,3)	normal	0.029	1.000	
MN-027-1	1, (2,3)	normal	0.002	0.985	
MN-027-2	1, (2,3)	normal	0.000	1.002	
MN-027-3					
MN-053-2					
MN-053-4	1, (3)	normal	0.004	0.994	
MO-161-1	2	normal	0.009	1.001	
MT-015-1					
MT-017-1	2, (1,3)	normal	0.002	0.950	
MT-035-1					
MT-041-1					
MT-071-1					
MT-085-1	1, (3)	normal	-0.025	0.999	
MT-095-1					
MT-097-1	1, (3)	log normal 3	2.506	0.086	-12.287
NB-001-1	1	normal	-0.134	1.466	
NB-009-1	2	normal	-0.001	1.003	
NB-057-1	1, (2)	normal	0.012	0.985	
NB-065-1					
NB-089-1	3	normal	-0.003	1.002	
NB-161-1	2, (1,3)	normal	0.030	1.001	
NV-013-1	2	normal	-0.009	1.095	
NV-031-2	1, (2,3)	normal	-0.008	1.013	
NM-005-2	1, (2)	normal	-0.017	0.998	
NM-051-1	1, (2,3)	normal	-0.003	1.002	
NM-051-2					
ND-017-1	1, (3)	log normal 3	1.906	0.144	-6.797
ND-025-1					
ND-053-1					
ND-067-1	1, (2,3)	normal	0.004	1.009	
ND-077-1	2, (1,3)	normal	-0.001	1.006	
ND-095-1					
ND-103-1					
OK-025-1	1, (2)	normal	-0.096	1.178	
OK-139-2	1, (2,3)	log normal 3	2.026	0.126	-7.622
OR-025-1	3, (1)	normal	-0.001	0.992	
OR-037-1	1	normal	-0.059	1.129	
OR-059-1					
OR-059-2	1, (3)	normal	-0.001	1.022	
OR-059-5	1, (2)	log normal 3	1.949	0.142	-7.094
TX-127-1					
TX-189-1					
TX-279-1	1, (2)	normal	-0.056	1.053	
TX-437-1	3	normal	-0.052	1.108	
UT-027-1					
UT-035-2	1, (2)	normal	-0.012	1.000	
WA-007-1	1, (3)	normal	-0.004	0.999	
WA-047-1	1, (2,3)	normal	0.001	0.999	
WA-047-2	1, (3)	normal	-0.001	0.989	
WA-063-1	3, (1,2)	normal	0.005	1.007	
WA-063-3					
WY-011-1	1, (2,3)	normal	0.010	0.990	
WY-015-1	3, (1,2)	log normal 3	2.2689	0.108	-9.732
WY-015-2	3	log normal 3	2.439	0.091	-11.524
WY-021-2	2	normal	-0.001	1.000	

*Best model as well as other acceptable models in parentheses.

**For normal distribution A and B represent the mean and standard deviation of $\xi_{p,\tau}$; for lognormal-3 distribution A, B and C represent the mean and standard deviation of $\ell_n(\xi-\xi_0)$ and the location parameter, respectively.

CHAPTER V

REGIONAL ANALYSIS OF DEPENDENCE IN GROUNDWATER LEVEL TIME SERIES

One of the objectives of this investigation was to look at the spatial variation of the dependence of the stochastic component in the western United States for comparison with those of monthly precipitation and streamflow series (see L. A. Roesner and V. Yevjevich, 1966). The areal distribution of the first, second and third autocorrelation coefficients are shown in Figs. 5.1, 5.2 and 5.3, respectively.

The first autocorrelation coefficient, r_1, of the stochastic component, $\varepsilon_{p,\tau}$, of the monthly groundwater level series for the 84 wells in the western U. S. has a mean of 0.79. This may be compared with a mean of 0.053 for the stochastic components of monthly precipitation for 219 stations west of the Mississippi River. For stochastic components of monthly streamflow time series, the mean first autocorrelation coefficient was 0.54 for 36 stations in the Washington-Idaho-Oregon area, and 0.38 for 48 stations in the Missouri-Kansas area. Comparison of the r_1-values for the stochastic components of precipitation, streamflow and groundwater level series demonstrates that the groundwater system has a greater storage and carry-over effect than the streamflow system. Comparison of the distributions of r_1 for the three series indicate that whereas the precipitation and streamflow coefficients were approximately normally distributed, the r_1-values for the groundwater series is non-normal and highly skewed to the right, as illustrated in the upper diagram of Fig. 5.4. This should be expected because the skewness of the sampling distribution of r_1 increases as the $|\rho_1| = |E(r_1)| \doteq |\bar{r}_1|$ increases.

Looking at the second and third autocorrelation coefficients, one finds the frequency distribution has a reduced skewness, because $|\bar{r}_2|$ and $|\bar{r}_3|$ are much smaller than $|\bar{r}_1|$. The mean value of r_2 is 0.65, and the distribution is skewed to the right but flatter. For r_3 the distribution is less skewed because the value of $\bar{r}_3$ is 0.56. All these $\bar{r}_k$ values illustrate the high dependence in groundwater time series.

Returning to the areal distribution of the r_1, r_2, and r_3 values of the groundwater series, no apparent patterns could be found. In the beginning of the study it was thought that wells in water table aquifers would have different dependence characteristics than wells in artesian aquifers. No such effect could be discerned. Knowledge of the hydraulic characteristics of all the aquifers, i.e., the coefficient of transmissivity, T, and the coefficient of storage, S, might be of benefit here, but estimates of these coefficients could only be obtained for 15 wells. The Ogallala formation is a major source of groundwater, and the data available on the aquifer is presented in Table 5.1 along with the autocorrelation coefficients of $\varepsilon_{p,\tau}$ for the series of eight wells being investigated. The values of T and S shown here are the range of values for wells in the vicinity of the indicated well. In comparing OK-025-1 and OK-139-2 one observes that OK-025-1 has higher values of r_1, r_2 and r_3 even though the T and S values are estimated to be in the same range. The depth in OK-139-2 is greater than OK-025-1, but comparing the two shallow wells, OK-025-1 and WY-021-2, one again notes a variation in values of r. The value of S for WY-021-2 does look suspiciously low for a water table aquifer, and the value of T has a much lower value at the lower end of the range. The two Kansas wells also have different ranges of the autocorrelation coefficients. The three Texas wells have similar r values and somewhat the same range of T values, although the depths to groundwater vary considerably.

Four other wells could also be compared as they are in the Snake River group, as shown in Table 5.2. Although the hydraulic parameters of the aquifer, T and S, are not available, there is some uniformity in that the two deep wells have higher values of r than the shallow wells.

In summary, while the first autocorrelation coefficients of stochastic components of monthly groundwater levels are much higher than those for precipitation or streamflow series in the western U. S., there does not appear to be a pattern to these variations. This is most likely due to the multiplicity of factors affecting water levels, including geologic conditions, pumping conditions, and recharge, as well as the relative small number of wells available for this analysis.

Inter-station correlation of the dependent stochastic component, $\varepsilon_{p,\tau}$, of the water levels in wells in the Ogallala formation, as well as the wells in the Snake River group, was investigated to determine the degree of association in areas that might be considered to be hydrogeologically homogeneous. As may be observed in Tables 5.3 and 5.4, there was no general pattern of correlation in either group, although several wells in each group exhibited some degree of correlation. In general, the inter-station correlation coefficient for precipitation and streamflow stations decreases with an increase in distance between stations. One would expect the same general relationship between wells in the same aquifer. It would appear again that the multiplicity of factors affecting water levels in the regions obscured such a selection for the inter-station correlation.

A localized study of the geology and hydrology of the Ogallala formation and Snake River group would be expected to yield information that would enable better interpretation of the results of the regional variation of the autocorrelation coefficients and of the inter-station correlation coefficient.

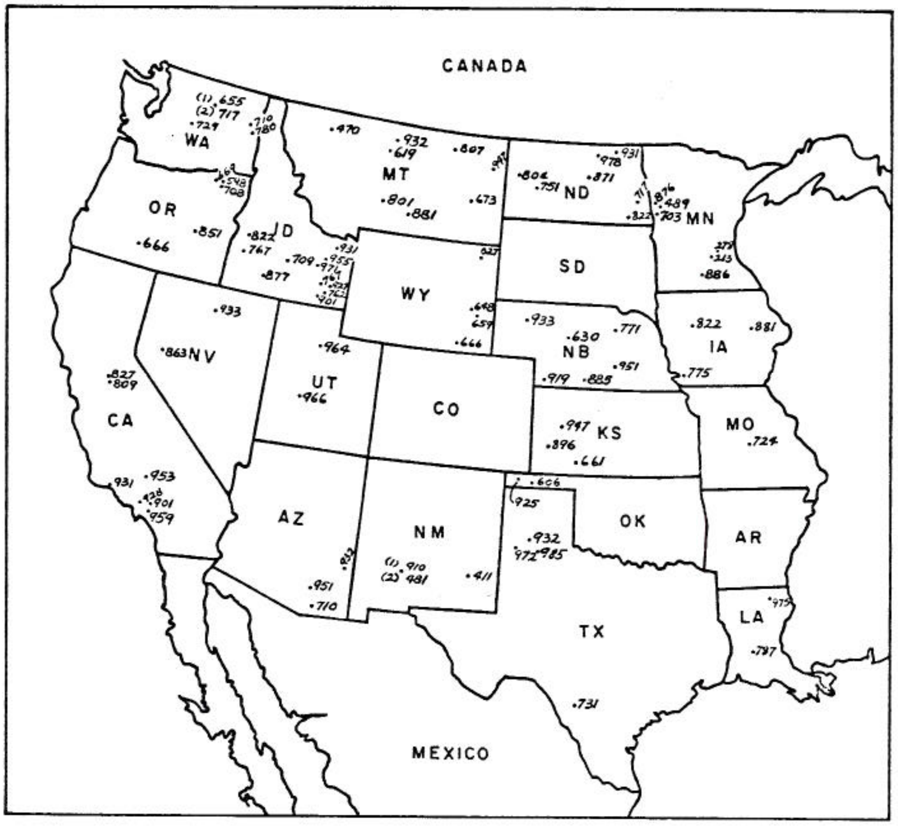

Fig. 5.1 Areal distribution of first autocorrelation coefficient, r_1.

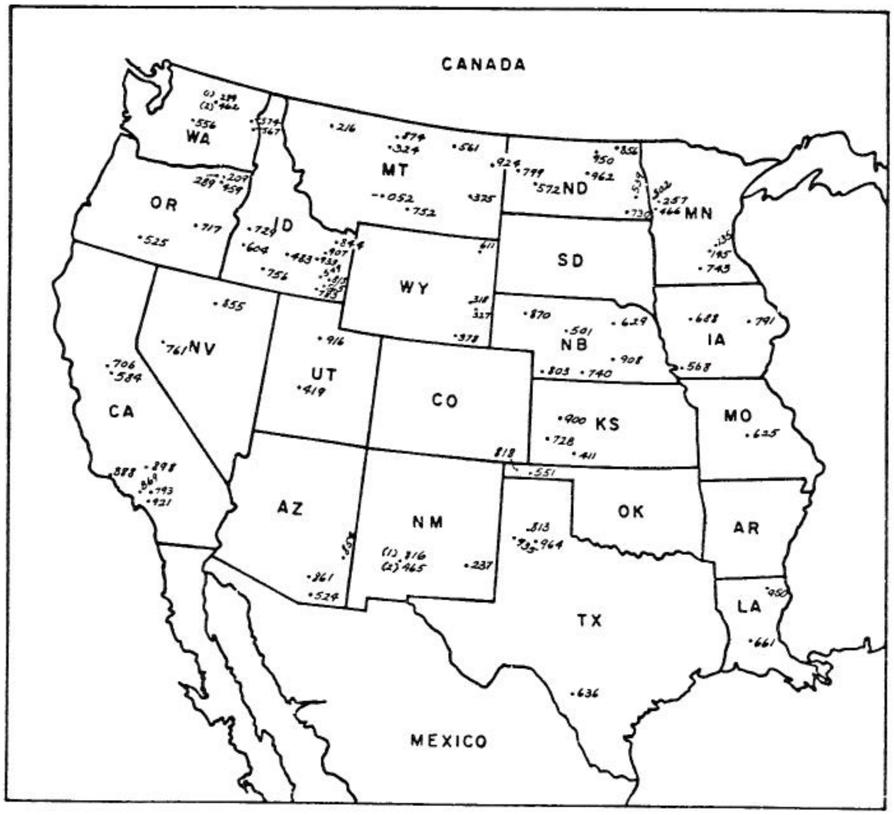

Fig. 5.2 Areal distribution of second autocorrelation coefficient, r_2.

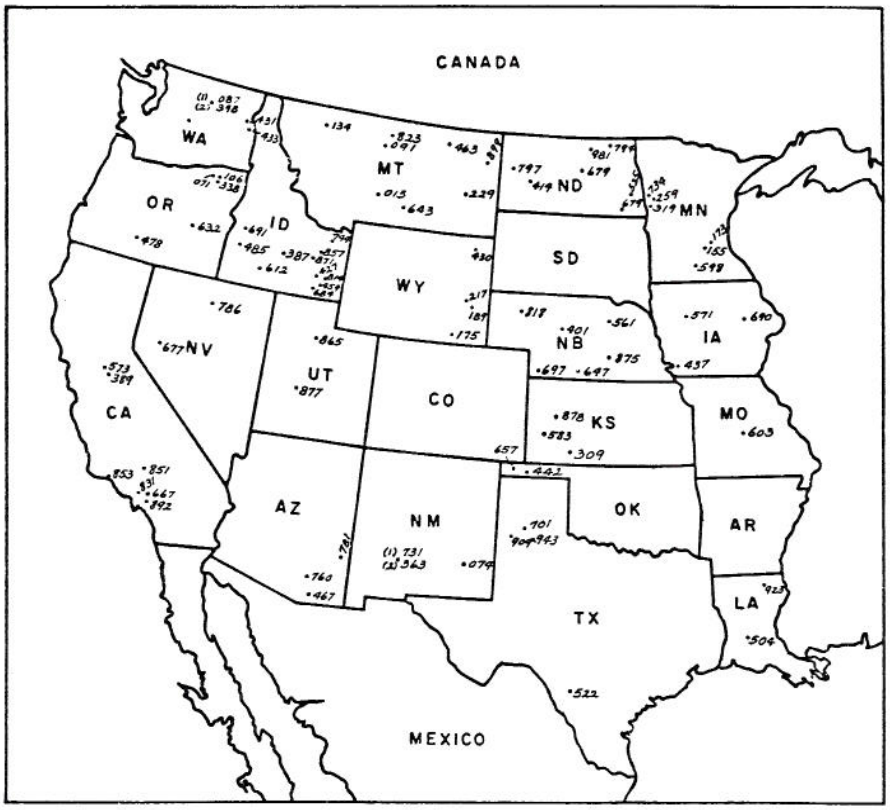

Fig. 5.3 Areal distribution of third autocorrelation coefficient, r_3.

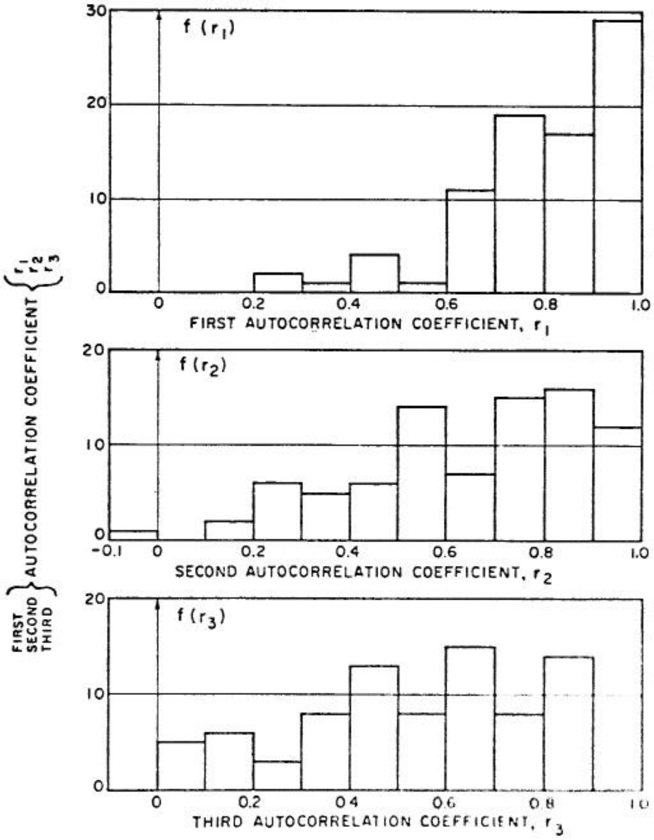

Fig. 5.4 Frequency distribution of autocorrelation coefficients of monthly groundwater levels in the western U. S.

TABLE 5.1

WELLS IN THE OGALLALA FORMATION (WATER TABLE CONDITIONS)

Study Well No.	Pumping Conditions*	T(gpd/ft)	S	Mean depth to water (ft)	Screened depth (ft)	r_1	r_2	r_3
OK-025-1	NP	20,000-35,000	0.1-0.15	32.89	41-61	0.925	0.818	0.657
OK-139-2	NP	20,000-35,000	0.1-0.15	190.68		0.606	0.551	0.442
WY-021-2	NP	3,800-39,200	6.74×10^{-5}-1.4×10^{-4}	11.25		0.666	0.378	0.173
KS-119-1	RP			21.20		0.661	0.411	0.309
KS-171-3	LP			90.22		0.947	0.900	0.878
TX-189-1	LP,RP	33,000;51,000		98.09		0.985	0.964	0.943
TX-279-1	LP,RP	25,000-76,000		52.38	99-202	0.972	0.935	0.909
TX-437-1	LP,RP	29,000-56,000		133.46	open@120'	0.932	0.813	0.701

TABLE 5.2

WELLS IN THE SNAKE RIVER GROUP (WATER TABLE CONDITIONS)

Study Well No.	Pumping Conditions*	T	S	Mean depth to water (ft)	Screened depth	r_1	r_2	r_3
ID-011-1	NP	--	--	584.35	below water level	0.955	0.907	0.857
ID-011-3	NP	--	--	18.73		0.767	0.549	0.427
ID-047-1	NP	--	--	38.50		0.877	0.756	0.612
ID-051-3	NP	--	--	256.55	below water level	0.931	0.844	0.744

*NP not affected by pumping
MP may be affected by pumping
LP local pumping
RP regional pumping

TABLE 5.3

INTER-STATION CORRELATION COEFFICIENTS, WELLS IN SNAKE RIVER GROUP

	ID-011-1	ID-011-3	ID-047-1	ID-051-3
ID-011-1	1.000	0.039	0.430	0.556
ID-011-3		1.000	0.409	-0.266
ID-047-1			1.000	-0.038
ID-051-3				1.000

TABLE 5.4

INTER-STATION CORRELATION COEFFICIENTS, WELLS IN OGALLALA FORMATION

	OK-025-1	OK-139-2	WY-021-2	KS-119-1	KS-171-3	TX-189-1	TX-279-1	TX-437-1
OK-025-1	1.000	0.028	0.193	-0.291	0.649	-0.685	0.387	-0.291
OK-139-2		1.000	-0.063	-0.284	-0.024	0.191	-0.455	-0.044
WY-021-2			1.000	-0.137	0.171	-0.359	0.407	-0.048
KS-119-1				1.000	-0.062	0.060	0.175	0.163
KS-171-3					1.000	-0.637	0.275	-0.296
TX-189-1						1.000	-0.636	0.621
TX-279-1							1.000	-0.186
TX-437-1								1.000

CHAPTER VI
CONCLUSIONS

Results of the analysis of the stochastic structure of monthly groundwater level time series as presented in the preceding chapters may be summarized in the following conclusions:

(1) Due to man-made interferences on groundwater basins many groundwater level time series exhibit nonhomogeneities in the form of trends and jumps. In order to study the stochastic structure of these types of series these nonhomogeneities must be properly detected, described and removed.

(2) The groundwater level series show periodicities in the mean and the standard deviation which may be described by Fourier series with its 12-month cycle and its subharmonics.

(3) Periodicity is also present in the first three autocorrelation coefficients of all the series studied, and may be described by Fourier series.

(4) Dependence within the stochastic component may be described by Markov models of lag one, two, or three for many time series, but in other series higher-order models are necessary.

(5) The probability distributions of the independent stochastic components may be sufficiently well described either by fitting a normal or a three-parameter lognormal probability function, as the case may be.

(6) The mean value of the first autocorrelation coefficient of the stochastic components for series of the 84 wells in the western United States is 0.79. This compares with a corresponding value of 0.053 for monthly precipitation series; for streamflow series the corresponding mean value was 0.54 for the northwest region and 0.38 for the midwest region.

(7) The mean values of the second and third autocorrelation coefficients of the groundwater level series are also high, being 0.65 and 0.56, respectively.

(8) The distribution of the first autocorrelation coefficient is highly skewed to the right, as expected, while the second and third autocorrelation coefficients appear less skewed, also as expected.

(9) There was no clear regional pattern for the dependence of the series with regard to confined or unconfined aquifers, pumping or nonpumping wells, or aquifers connected or not connected to a surface water source.

(10) No patterns of inter-station correlation could be discerned in wells pumping from the same geologic formations, the Ogallala and the Snake River group.

(11) Groundwater subsystems have highly damped effects on state and outflow variables in comparison with discharges of surface water systems.

(12) Study has shown that the model of Eq. 3.12, together with trend and periodic functions, and simple-normal and lognormal functions for the independent stochastic components, may be rightfully used for many groundwater level time series.

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The first autocorrelation coefficient in the stationary stochastic component is much greater in monthly groundwater level series than in either monthly precipitation or streamflow time series. The second and third autocorrelation coefficients of groundwater series also are high. The regional correlation coefficients between the pairs of stochastic groundwater level time series depend on the distance between wells. The absolute values of the regional correlation coefficients oscillate widely.

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