ECE 562

Week 11 Lecture 2

Fall 2008

Week 11 Lecture 2 Summary

Slides	Topic
3-9	RLC circuits and resonance
10-13	Resonant converter principles
14-17	Trends in power electronics
18-336	Switching efficiency
37-44	Zero current and zero voltage switching
45-63	Problem 19.1
64-73	Analysis of resonant converters
74-86	Conversion ratio and filter networks

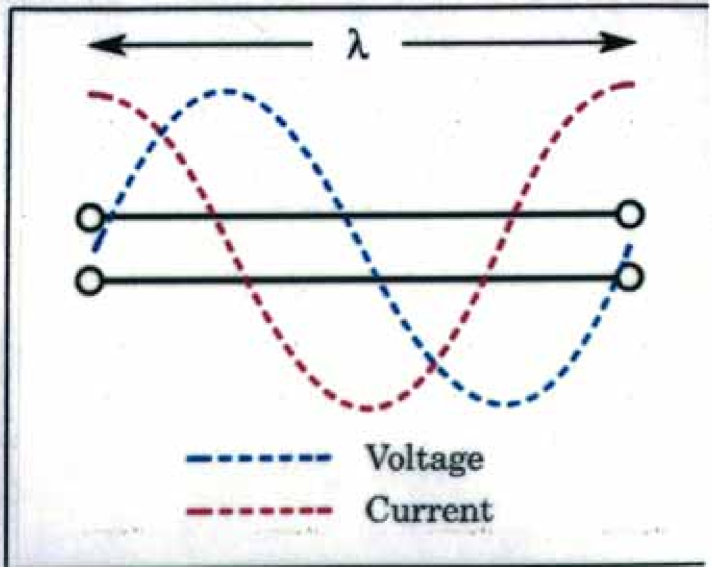
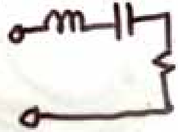


Figure 4 · Voltage and current on a one-wavelength long transmission line.

5 to 20 kHz	-AUDIBLE NOISE -SLOW BIPOLAR SWITCHES -LARGE L's AND C's
20 to 100 kHz	-ABOVE AUDIBLE RANGE -FAST BIPOLAR TRANSISTOR -MAGNETICS BECOME IMPORTANT -SMALL SIZES
100 to 500 kHz	-POWER MOSFET SWITCHES -LOSSES IN L's AND C's -DIODE RECOVERY TIME -RFI AND EMI -PACKAGING PROBLEMS

Fig. 2 - Power Supply Switching Frequencies

Series RLC:

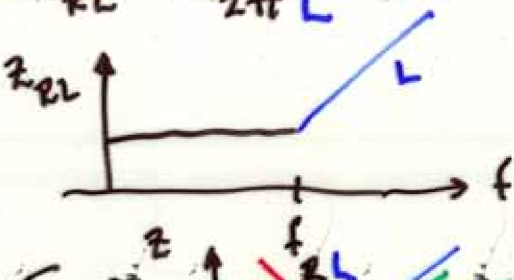


$$f_{RC} = \frac{1}{2\pi RC}$$

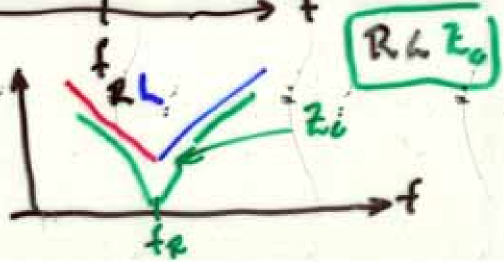
Z_C series with R

$$f_{RL} = \frac{R}{2\pi L}$$

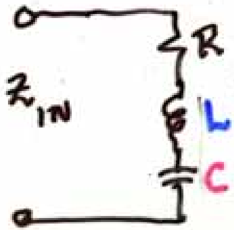
Z_L series with R

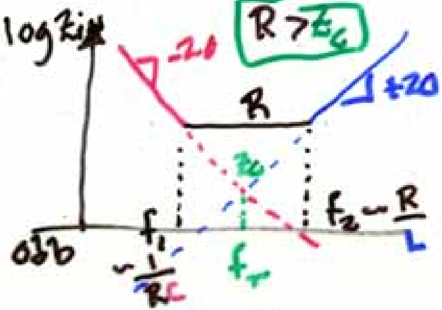


Finally R-L-C



Series RLC





$$f_1 = \frac{1}{2\pi RC} \qquad f_2 = \frac{R}{2\pi L}$$

$$f_R = ? \qquad \left|\frac{1}{\omega C}\right| = |\omega L| \;@\; \omega_R$$

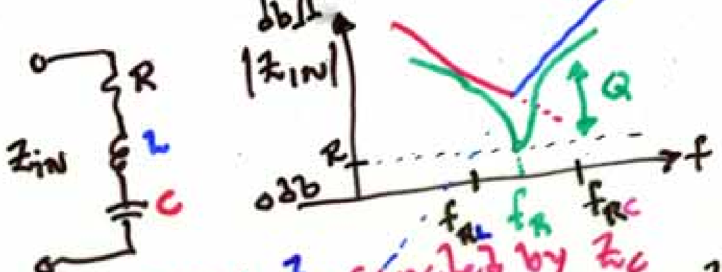
$$\omega_R = \frac{1}{\sqrt{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_C = Z_L \;@\; \omega_R = \sqrt{\frac{L}{C}} \equiv Z_c$$

Above case for $R > Z_c$

Case: R L $Z_C \equiv \sqrt{\frac{L}{C}}$



Z_L cancled by Z_C

Resonance:

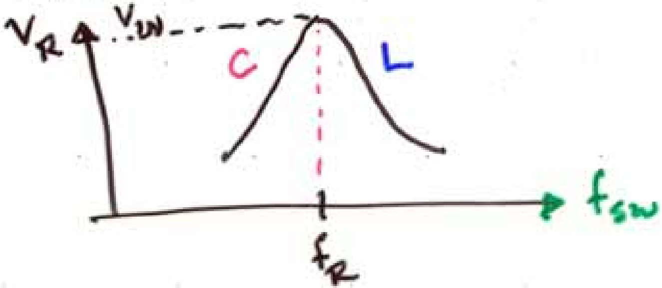
Q = Deviation from asymptote @ f_r

$= \frac{Z_C}{R}$

$Q(dB) = Z_C(dB_\Omega) - R(dB_\Omega)$

If Z_{in} "Q's" down then $i_{in}(t)$?

Resonant Buck



$f_{SW} = f_R$ $\quad V_R = V_{IN}$

$f_{SW} \neq f_R$ $\quad \underbrace{V_R < V_{IN}}_{\text{Buck}}$

$f < f_R$: Capacitive

ZVS for FET

$f > f_R$: Inductive :

ZCS for FET

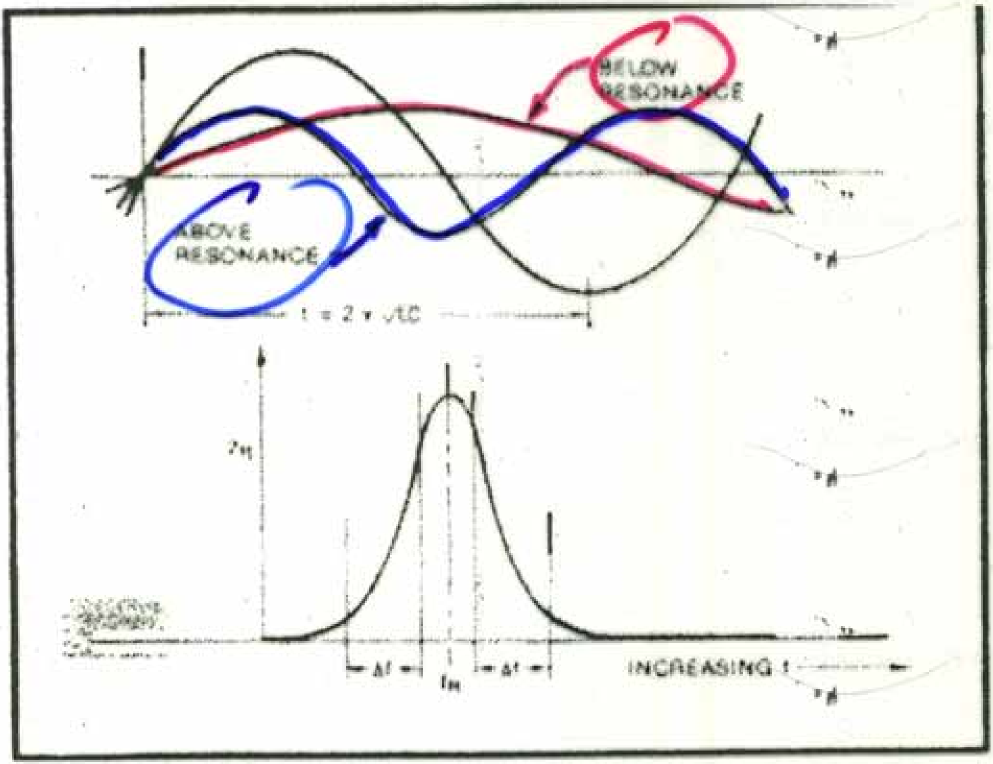


Fig. 11 - Variable Frequency Continuous Resonance

Introduction to Resonant Conversion

Resonant power converters contain resonant L-C networks whose voltage and current waveforms vary sinusoidally during one or more subintervals of each switching period. These sinusoidal variations are large in magnitude, and the small ripple approximation does not apply.

No small ripple approx.

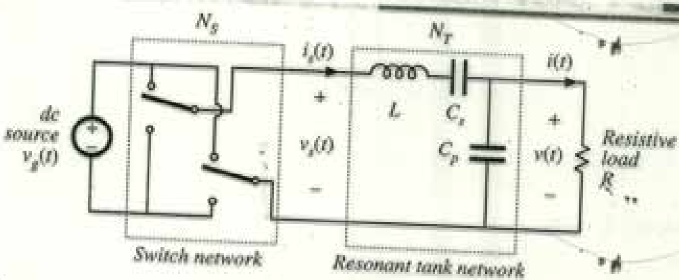
Some types of resonant converters:

- Dc-to-high-frequency-ac inverters AE ½, 2, 13, 60, 160 MHz
- Resonant dc-dc converters AE Pinnacle, Pinnacle Plus
- Resonant inverters or rectifiers producing line-frequency ac 461

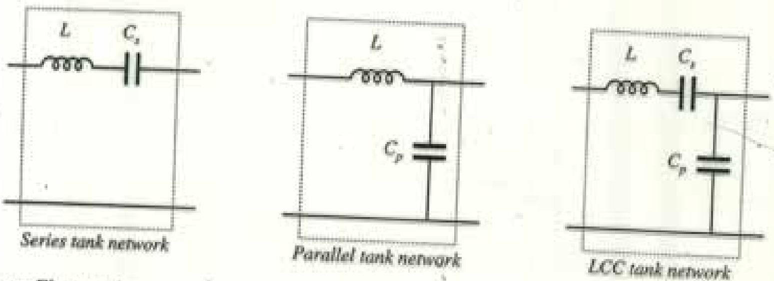
cycloconverters f < 60 Hz

A basic class of resonant inverters

Basic circuit



Several resonant tank networks



1. SERIES OR PARALLEL LOADED?
2. FIXED OR VARIABLE FREQUENCY?
3. CONTINUOUS / DISCONTINUOUS RESONANCE?
4. ZERO CURRENT OR VOLTAGE SWITCHING?
5. HALF OR FULL CYCLE CONDUCTION?

Fig. 8 - Classifying Resonant Converters

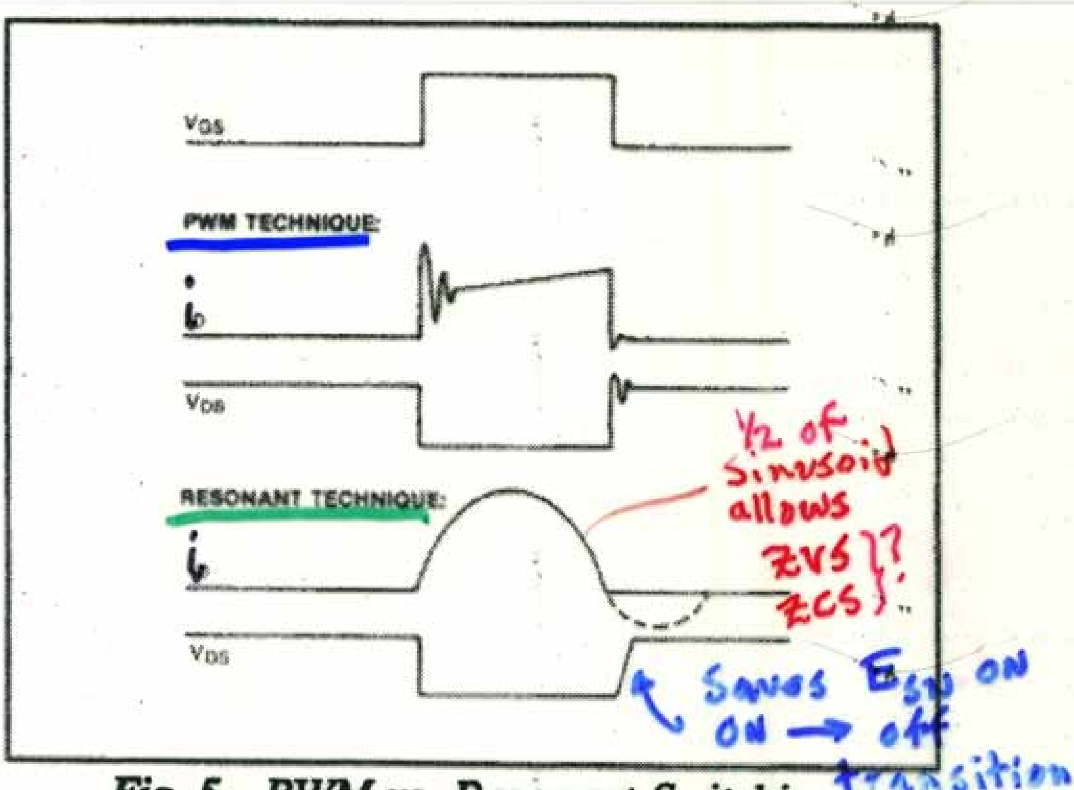


Fig. 5 - PWM vs. Resonant Switching

Trends in the power electronics field

- Improved packaging of semiconductors, to reduce size of control circuitry and power switches. Use of application-specific ICs (ASICs)
- Increased switching frequencies, to reduce size of magnetics
- System integration, leading to distributed power supplies having high density

Much R&D effort has been devoted to high-density power supplies, including resonant converters and soft switching techniques

Introduction to Resonant Conversion

Resonant power converters contain resonant L-C networks whose voltage and current waveforms vary sinusoidally during one or more subintervals of each switching period. These sinusoidal variations are large in magnitude, and the small ripple approximation does not apply.

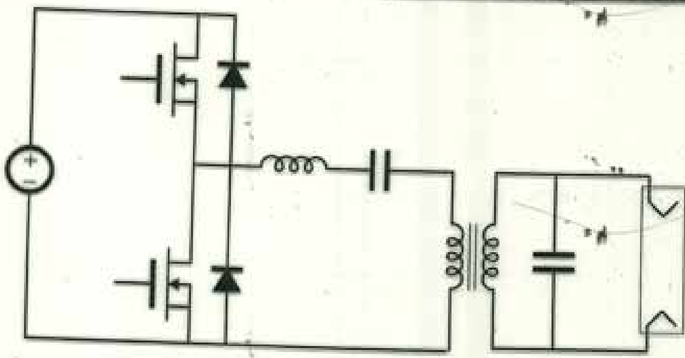
Some types of resonant converters:

- Dc-to-high-frequency-ac inverters DC - IF / RF
- Resonant dc-dc converters - DC - DC
- Resonant inverters or rectifiers producing line-frequency ac

DC → 60 Hz

An electronic ballast Rapid Start Instant Start

- Must produce controllable high-frequency (50 kHz) ac to drive gas discharge lamp
- DC input is typically produced by a low-harmonic rectifier
- Similar to resonant dc-dc converter, but output-side rectifier is omitted

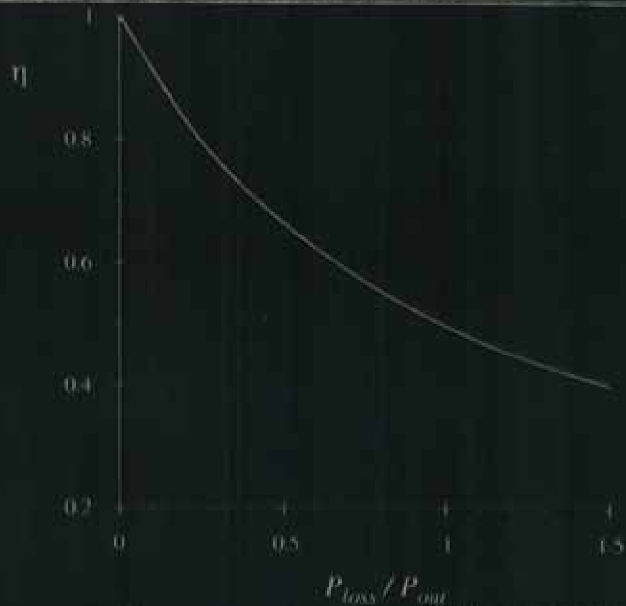


Half-bridge, driving LCC tank circuit and gas discharge lamp

High efficiency is essential

$$\eta = \frac{P_{out}}{P_{in}}$$

- High efficiency leads to low power loss within converter
- Small size and reliable operation is then feasible
- Efficiency is a good measure of converter performance



Switching loss

(Section 4.3 of Power Electronics 1 text)

- Energy is lost during the semiconductor switching transitions, via several mechanisms:
  - Transistor switching times
  - Diode stored charge
  - Energy stored in device capacitances and parasitic inductances
- Semiconductor devices are *charge controlled*
- Time required to insert or remove the controlling charge determines switching times

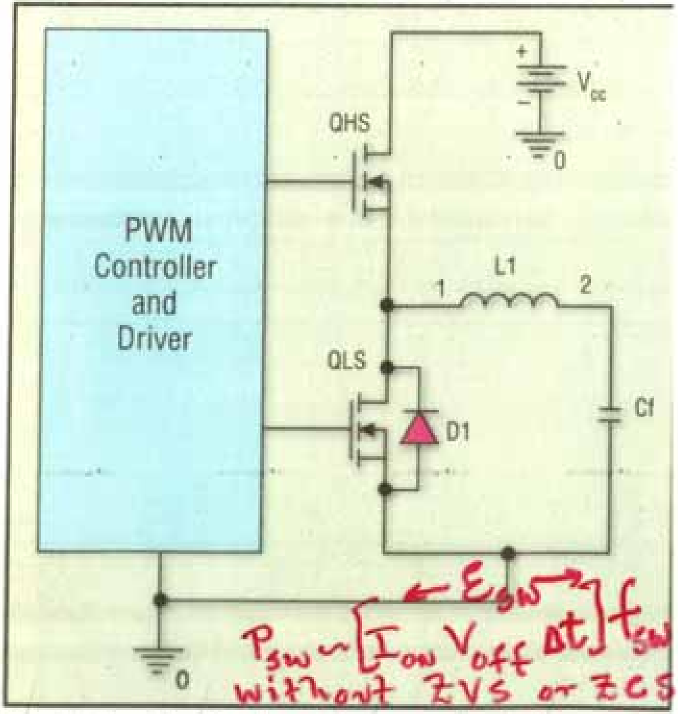


Fig. 1. *The classical synchronous buck converter uses two switching MOSFETs: the high-side (control) device, QHS, and the low-side synchronous rectifier, QLS.*

4.3.4. Efficiency vs. switching frequency

Add up all of the energies lost during the switching transitions of one switching period:

$$W_{tot} = W_{on} + W_{off} + W_D + W_C + W_L + \ldots$$

Average switching power loss is

$$P_{sw} = W_{tot} f_{sw}$$

Total converter loss can be expressed as

$$P_{loss} = P_{cond} + P_{fixed} + W_{tot} f_{sw}$$

Key

where

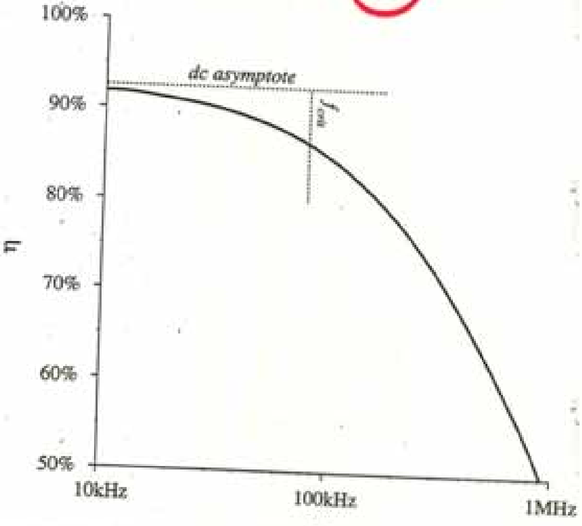
P_{fixed} = fixed losses (independent of load and f_{sw})
P_{cond} = conduction losses

$$\left[\mathcal{E}(\text{off to on}) + \mathcal{E}(\text{on to off})\right]$$

$W_T \;\underline{\quad\Delta\quad}\; f_{sw}$

Efficiency vs. switching frequency

$$P_{loss} = P_{cond} + P_{fixed} + W_{tot} f_{sw}$$



Switching losses are equal to the other converter losses at the critical frequency

$$f_{crit} = \frac{P_{cond} + P_{fixed}}{W_{tot}}$$

This can be taken as a rough upper limit on the switching frequency of a practical converter. For $f_{sw} > f_{crit}$, the efficiency decreases rapidly with frequency.

Unless $W_{total} \to 0$

Resonant conversion: disadvantages

Can optimize performance at one operating point, but not with wide range of input voltage and load power variations

Significant currents may circulate through the tank elements, even when the load is disconnected, leading to poor efficiency at light load

Quasi-sinusoidal waveforms exhibit higher peak values than equivalent rectangular waveforms

33% ↑

Fourier Analysis

These considerations lead to increased conduction losses, which can offset the reduction in switching loss

Resonant converters are usually controlled by variation of switching frequency. In some schemes, the range of switching frequencies can be very large

Need V-f Chips

Complexity of analysis

Introduction to Resonant Conversion

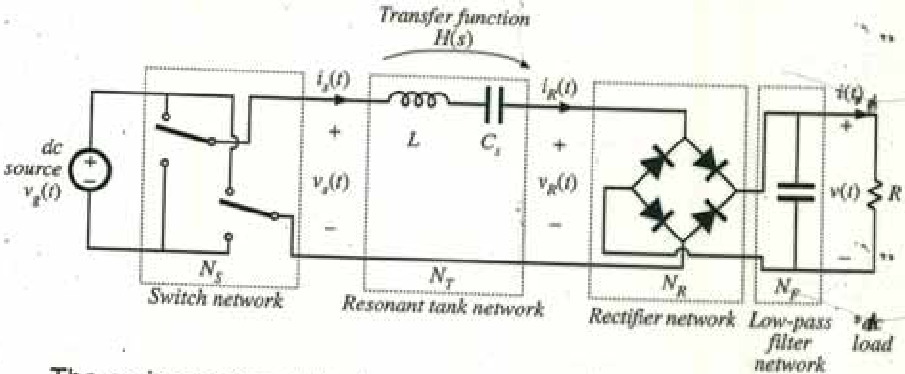
Resonant power converters contain resonant L-C networks whose voltage and current waveforms vary sinusoidally during one or more subintervals of each switching period. These sinusoidal variations are large in magnitude, and the small ripple approximation does not apply.

Some types of resonant converters:

- Dc-to-high-frequency-ac inverters 1/2 → 162 MHz
- Resonant dc-dc converters Pinnicle
- Resonant inverters or rectifiers producing line-frequency ac 360 KW

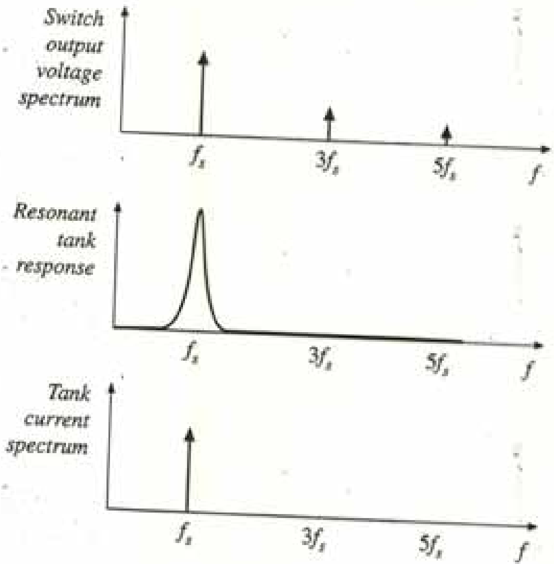
Derivation of a resonant dc-dc converter

Rectify and filter the output of a dc-high-frequency-ac inverter



The series resonant dc-dc converter

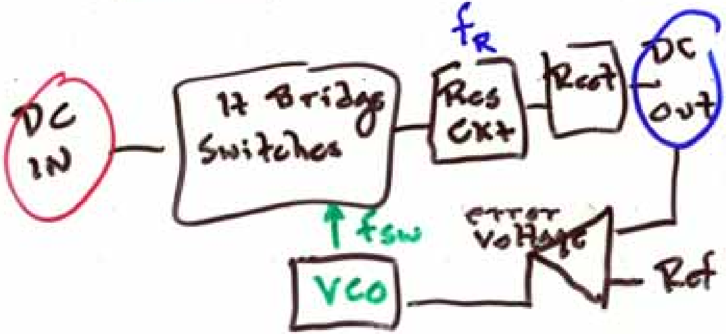
Tank network responds only to fundamental component of switched waveforms



Tank current and output voltage are essentially sinusoids at the switching frequency f_s.

Output can be controlled by variation of switching frequency, closer to or away from the tank resonant frequency

Control Loop for Resonant Converter



f_{sw} varied until

error voltage $\rightarrow 0$

AEROJET
LOCKHEED MARTIN
Glenn Research Center
Colorado Power Electronics, Inc.

Three-Phase Resonant Converter PPU 10KW Breadboard

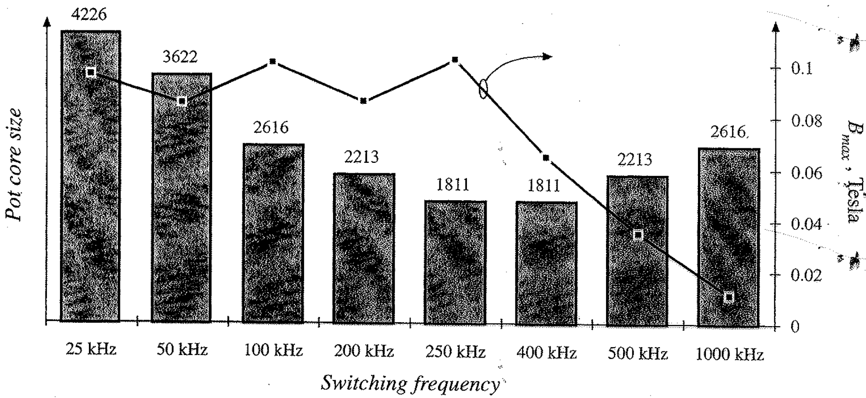
Magnetics development for sinusoids is unique

Two magnetic designs are being developed . The candidate with the best combination of efficiency and specific mass will be chosen.

Critical attention is needed for larger magnetic structures due to eddy current losses in the ferrite. The larger structures have an increased volts per turn which causes loss factors not prevalent in lower power magnetic structures.

Effect of switching frequency on transformer size

for this P-material Cuk converter example



- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases

$$\text{SW Loss in Joules} \sim \left[\tfrac{1}{2} \underbrace{V_{off}\, I_{ON}\, \Delta t}_{\text{energy to switch}}\right]$$

Switching loss

(Section 4.3 of Power Electronics 1 text)

- Energy is lost during the semiconductor switching transitions, via several mechanisms:
  - Transistor switching times
  - Diode stored charge
  - Energy stored in device capacitances and parasitic inductances
- Semiconductor devices are *charge controlled*
- Time required to insert or remove the controlling charge determines switching times

$$P_{loss} = \mathcal{E}_{sw} f_{sw}$$

4.3.4. Efficiency vs. switching frequency

Add up all of the energies lost during the switching transitions of one switching period:

$$W_{tot} = W_{on} + W_{off} + W_D + W_C + W_L + \ldots$$

What is missing?

Average switching power loss is

$$P_{sw} = W_{tot}\, f_{sw}$$

Total converter loss can be expressed as

$$P_{loss} = P_{cond} + P_{fixed} + W_{tot}\, f_{sw}$$

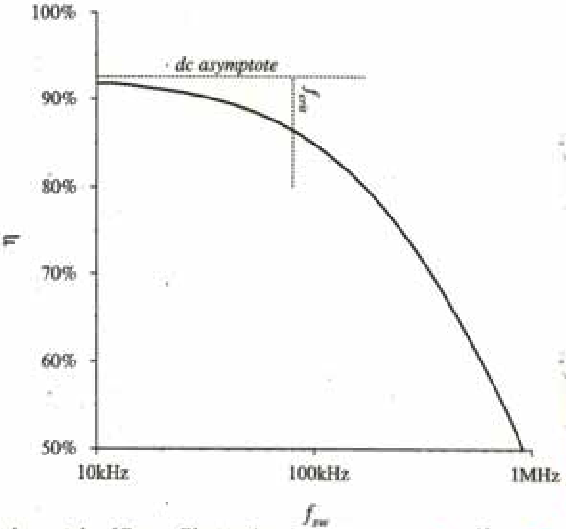
where P_{fixed} = fixed losses (independent of load and f_{sw})
P_{cond} = conduction losses

Efficiency vs. switching frequency

GSU student days
$f_{sw} \doteq 25\,kHz$

Now: ? 2.5 MHz

$$P_{loss} = P_{cond} + P_{fixed} + W_{tot}\, f_{sw}$$



Switching losses are equal to the other converter losses at the critical frequency

$$f_{crit} = \frac{P_{cond} + P_{fixed}}{W_{tot}}$$

This can be taken as a rough upper limit on the switching frequency of a practical converter. For $f_{sw} > f_{crit}$, the efficiency decreases rapidly with frequency.

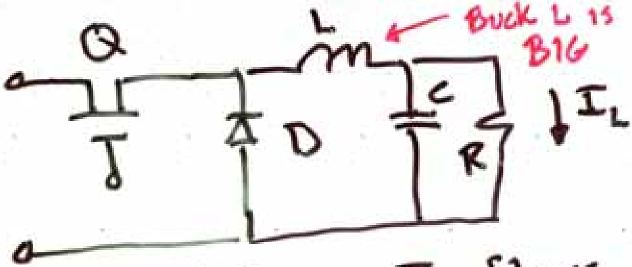
ZVS

$f_{sw}\uparrow$ requires ZCS

$\therefore\ [I_{on} V_{off}\, \Delta t] = E_{sw} \downarrow$

Applications of resonant and soft-switching converters

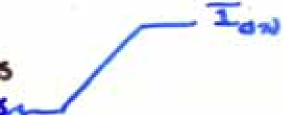
- Electronic ballasts for gas-discharge lamps
  - Produce high-frequency ac
- High-frequency high-density dc–dc converters
  - Reduce switching loss and improve efficiency
- High-voltage and other specialized converters
  - Transformer nonidealities lead to ringing waveforms
- Converters using IGBTs
  - Mitigate switching loss caused by current tailing
- Converters using piezoelectric transformers
  - Converter is designed to excite one mode of piezo
- Low-harmonic rectifiers
  - Mitigate switching loss caused by diode stored charge



① if $\underline{Q}$ off D on, I_L flows through D

② i_D will continue to flow due to Q_{rr} for Δt_{rr} after $\underline{Q \text{ on}}$

③ When Q goes from $\underline{\text{off to on}}$ large $\mathcal{E}_{\substack{off\\on}}$ occurs since V_{DS} large as current increases



force i_Q to be zero as

Q turns ON $V_{ds} \to 0$

} ZCT switching

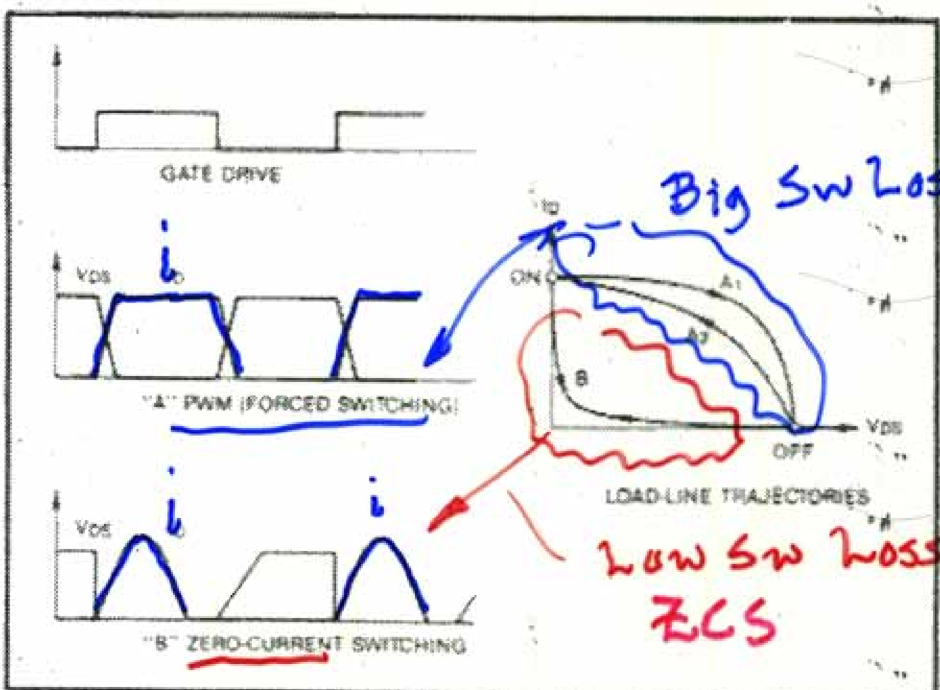


Fig. 6 - Switching Stress and Switching Loss

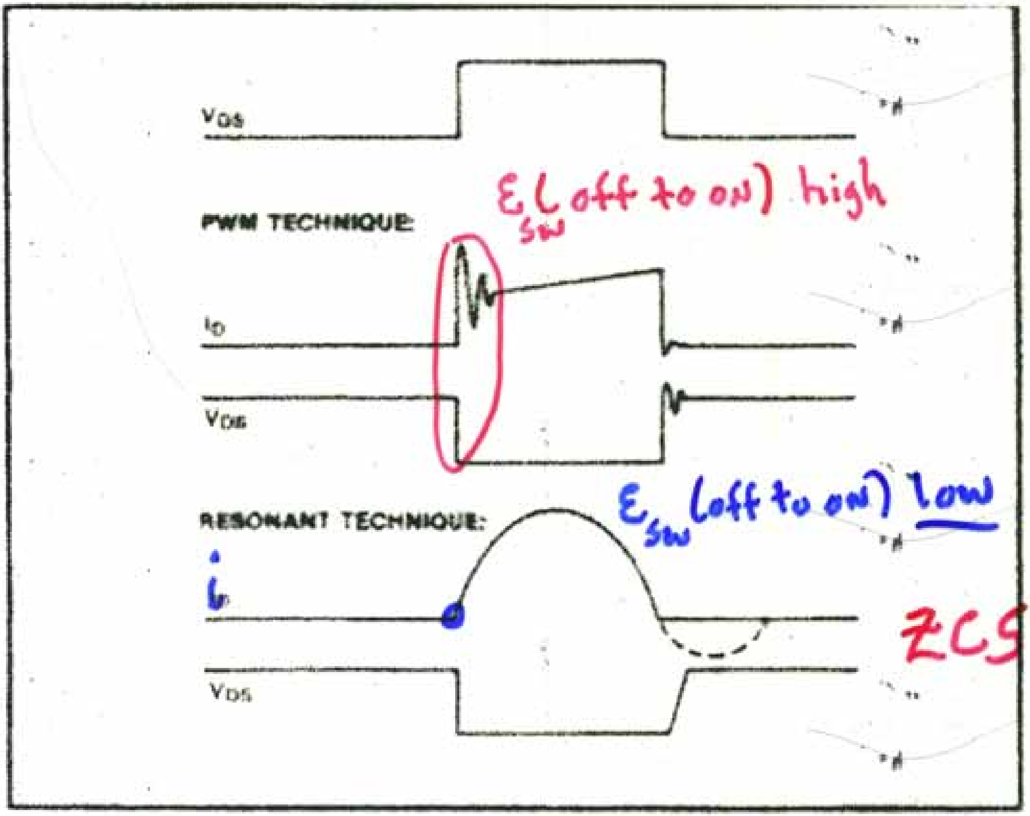


Fig. 5 - PWM vs. Resonant Switching

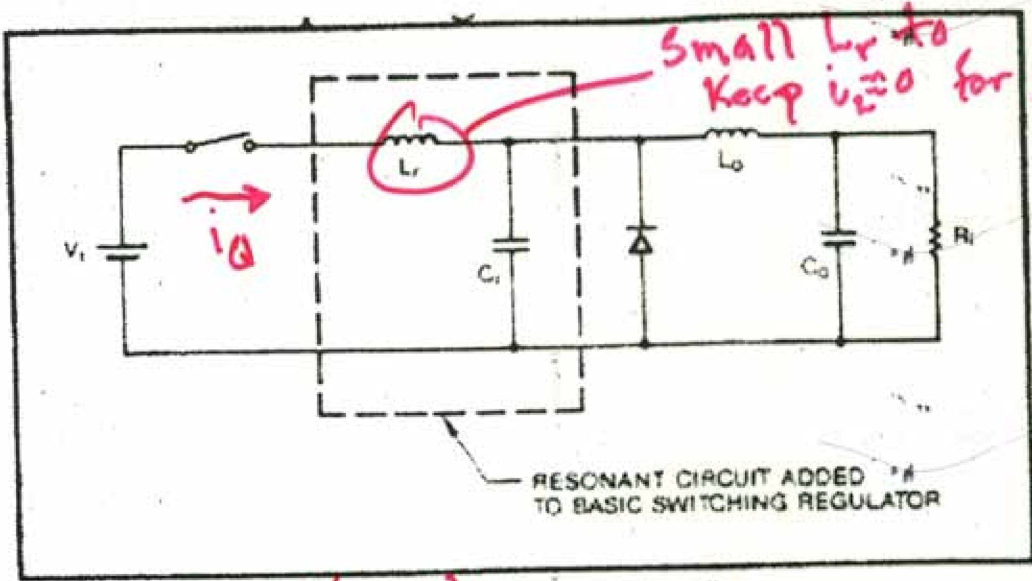


Fig. 3 - Basic Resonant Converter

(Buck)

L_r, C_r allow reduction of Sw loss

force ZCT

How? L_r Keeps $i_Q \cong 0$ while V_{DS}

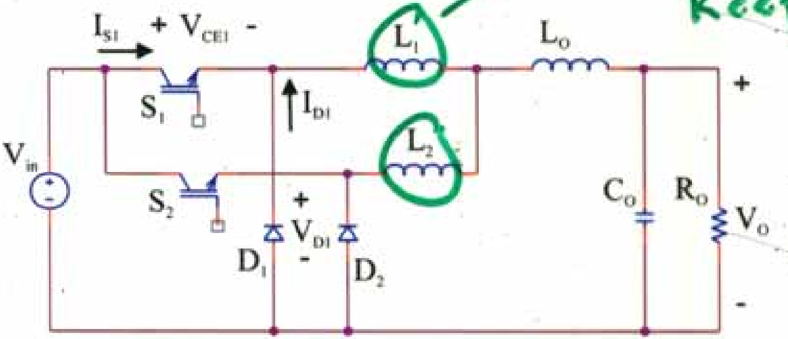
V_t

0

t

Interleaved ZCT Buck

lags i_{L_1} wrt v_{L_1}
Keep $i_Q = 0$



- Two small inductors (L_1, L_2) added to achieve zero-current-transition turn on of the switches S_1 and S_2
- Out-of-phase operation of S_1 and S_2, as in two-phase converters
- Significant reduction of losses associated with diode reverse recovery
- Simple operation, no resonant circuit

ZCT Only for turn-on

Comparison of Losses

Powerex: CM150DU-24NFH		Hard switched	New circuit
Turn-on loss	*(mJ)*	*4.0*	*0.4*
Turn off loss	(mJ)	3.8	3.8
Conduction loss (one sw. period)	(mJ)	6.1	6.1
Loss per switch (32 kHz)	*(W)*	*444.2*	*164.5*
Diode recovery loss	*(mJ)*	*12.0*	*2.0*
Diode conduction loss (one sw. per.)	(mJ)	3.5	3.5
Diode loss (32 kHz)	*(W)*	*496.6*	*88.3*
Aux switch, eq (10)* in [2]	(mJ)		
Total loss (32 kHz)	*(W)*	*940.8*	*505.6*

*m=0.6, To=3 µs, k=2, Vc=10 V

10x

No advantage

6x

2x better overall

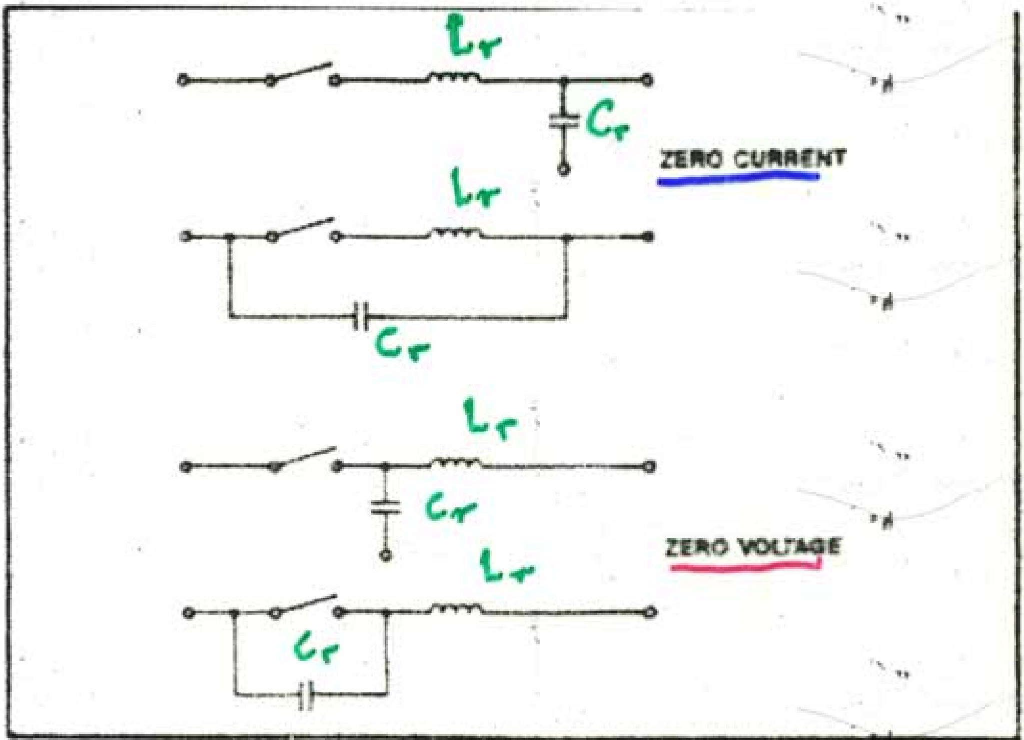
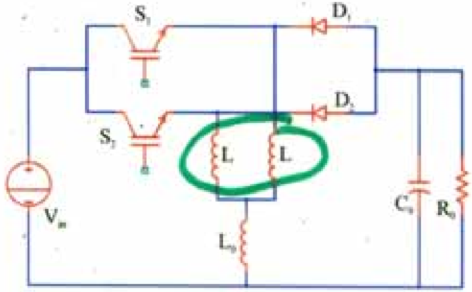
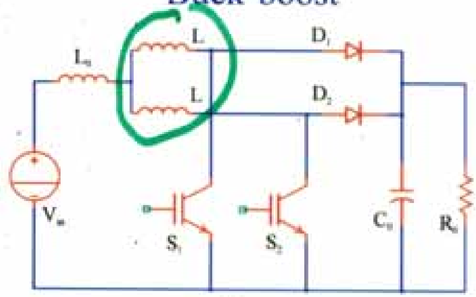


Fig. 7 - Resonant Switches

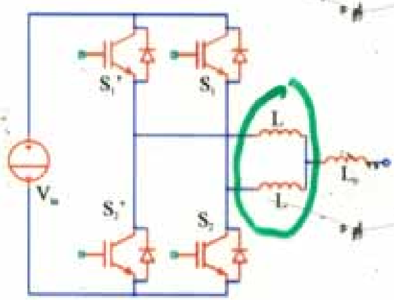
Examples of Other Interleaved ZCT Converters



Buck-boost

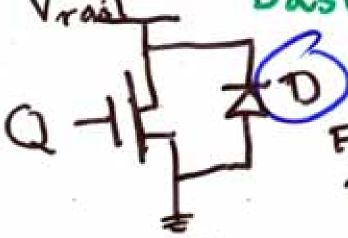


Boost



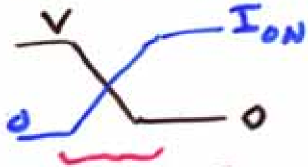
One leg of an
interleaved ZCT bridge
or three-phase inverter

Basic Concept
ZVS



FET off

To go from FET off to FET on



$[V I_{ON} \Delta t]$

$\mathcal{E}_{\text{off to on}}$

$$P_{\substack{\text{off}\\\text{on}}} = f_{sw}\left[\mathcal{E}_{\substack{\text{off}\\\text{on}}}\right]$$

FET diode

if D is placed on ahead of Q by a leading i

$V_Q = 0$ during

$$\mathcal{E}_{\substack{\text{off}\\\text{on}}} \longrightarrow 0$$

Soft switching:

Zero-voltage and zero-current switching

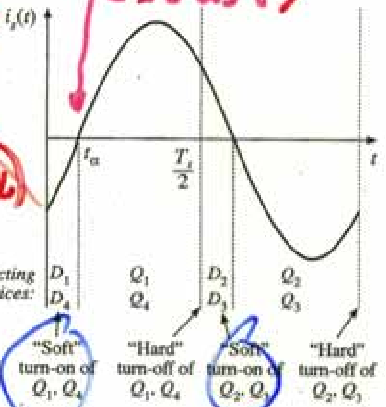
eLi   ice

H bridge: Approx
V is ?
i is ?

- Soft switching can mitigate some of the mechanisms of switching loss and possibly reduce the generation of EMI
- Semiconductor devices are switched on or off at the zero crossing of their voltage or current waveforms

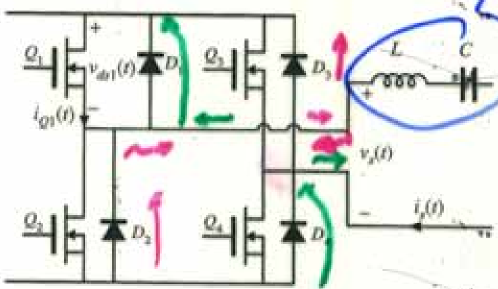
e=0 as i ↑

Angle i is α to Z(tank) IN



Fundamentals of Power Electronics

if tank Q is high i is sine



Conduction sequence: D_1–Q_1–D_2–Q_2

Q_1 is turned on during D_1 conduction interval, without loss

leading i forces $V_a = 0$ (via diode) prior to Q on } E_{off}, E_{on} → 0

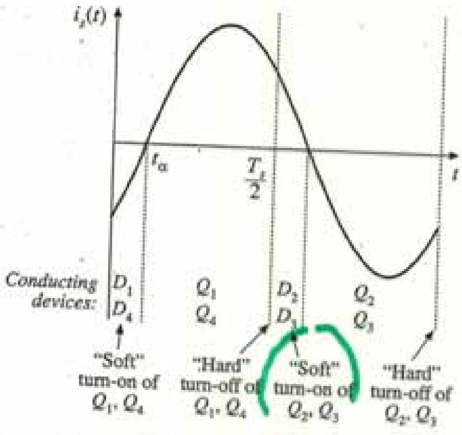
i_s is (+) as shown ⇒ diodes 2 and 3 are on
Forces V_{Q_2} and V_{Q_3} = 0 even if gate drives keep Q_2 and Q_3 off
⇒ as i_{Q_2} and i_{Q_3} increase $i_Q * V_Q * \Delta t \cong E_{SW} = 0$

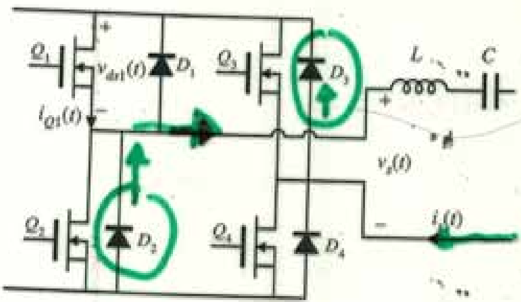
Soft switching: & make zero

Zero-voltage and zero-current switching

Below ZVT turn-on of Q_2 & Q_3

- Soft switching can mitigate some of the mechanisms of switching loss and possibly reduce the generation of EMI
- Semiconductor devices are switched on or off at the zero crossing of their voltage or current waveforms





Conduction sequence: D_1–Q_1–D_2–Q_2

Q_1 is turned on during D_1 conduction interval, without loss

i_s is $-$ ⇒ Diodes 1 and 4 on
Forces V_{Q_1} & $V_{Q_4} = 0$
even if Q_1 and Q_4 are off
⇒ as i_{Q_1} & i_{Q_4} increase $i_a * V_a \Delta t = E_{sw} = 0$

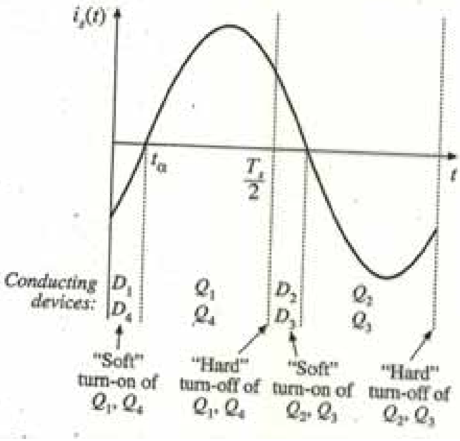
Soft switching:

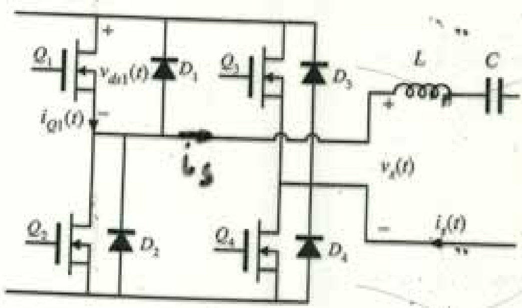
Zero-voltage and zero-current switching

i_s is $+$ ⇒ Diodes 2 and 3 on etc

Soft switching can mitigate some of the mechanisms of switching loss and possibly reduce the generation of EMI

Semiconductor devices are switched on or off at the zero crossing of their voltage or current waveforms





Conduction sequence: D_1–Q_1–D_2–Q_2

Q_1 is turned on during D_1 conduction interval, without loss

Prob 19.1 HW set #C

1. *Analysis of a half-bridge dc–dc parallel resonant converter, operated above resonance.* In Fig. 1, the elements C_B, L_F, and C_F are large in value, and have negligible switching ripple. You may assume that all elements are ideal. You may use the sinusoidal approximation as appropriate.

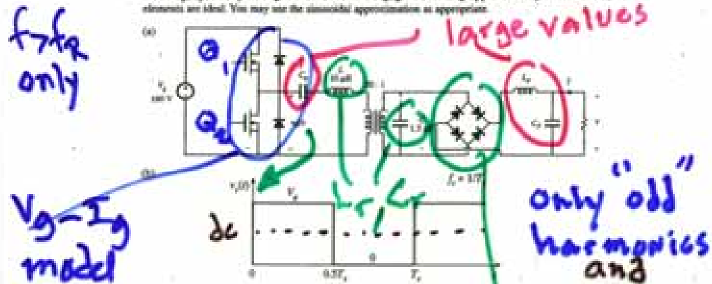


Fig. 1 Half-bridge parallel resonant converter of Problem 1: (a) schematic, (b) switch voltage waveform.

(a) Construct an equivalent circuit for this converter, similar to Fig. 19.22, which models the fundamental components of the tank waveforms and the dc components of the converter input current and output voltage. Clearly label the values and/or give expressions for all elements in your model, as appropriate.

easy to model V_R – I_R

At rated (maximum) load, this converter produces $I = 20$ A at $V = 3.3$ V.

(b) What is the converter switching frequency f_s at rated load?

(c) What is the magnitude of the peak transistor current at rated load?

At minimum load, the converter produces $I = 2$ A at $V = 3.3$ V.

(d) What is the converter switching frequency f_s at minimum load?

(e) What is the magnitude of the peak transistor current at minimum load? Compare with your answer from part (c)—what happens to the conduction loss and efficiency at minimum load?

V_{out} fixed @ 3.3V

$2 \leq I_{out} \leq 20$ A

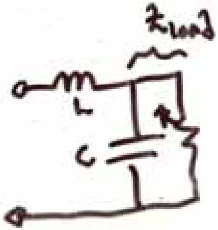
∴ as V_g or I vary requires f to change to keep $V_o = 3.3$V

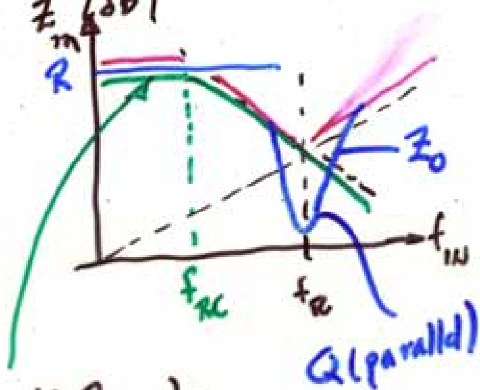
$$V_{out} = \underbrace{\left[\frac{V_{S1}}{V_g}\right]}_{\substack{\text{input}\\ \text{switch}\\ \sim \frac{2}{\pi}}} \underbrace{\left[\frac{V_{R1}}{V_{S1}}\right]}_{\substack{\text{Resonant}\\ \text{Circuit}\\ H(\omega)}} \underbrace{\left[\frac{V_{out}}{V_{R1}}\right]}_{\substack{\text{Rectifer}\\ \sim \frac{2}{\pi}}}$$

$\frac{4}{\pi^2}$ for $\frac{1}{2}$ bridge

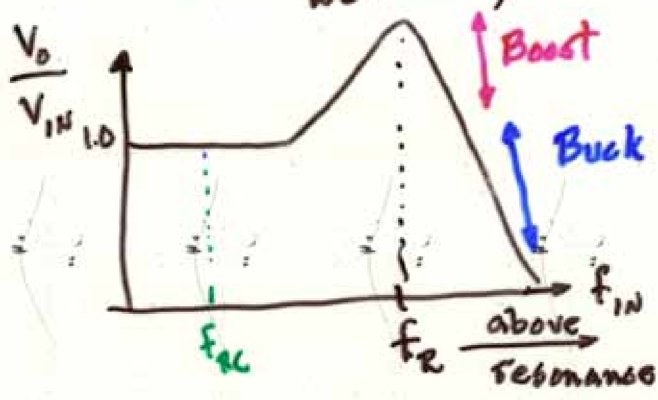
$\frac{8}{\pi^2}$ for full bridge

$$H(\omega) \sim \underbrace{\left(\frac{V_{IN}}{Z_{IN}}\right)}_{i} \underbrace{\left[\frac{1}{\omega C} \,||\, R\right]}_{\text{load}}$$





$$\frac{V_{out}}{V_{IN}} = \frac{\underbrace{\frac{1}{wC} \parallel R}_{Z_{load}}}{\underbrace{wL + \frac{1}{wC} \parallel R}_{Z_{IN}}}$$



Pbm 14.1

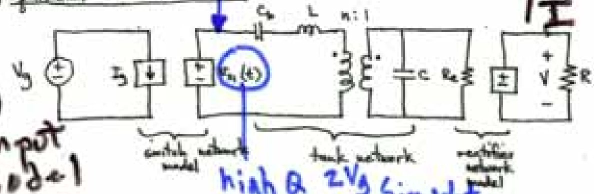
②

Half-bridge dc-dc parallel resonant converter

$2 \le I \le 20$

a) Equivalent circuit model

① Input Model



high Q $\frac{2V_g}{\pi}\sin\omega_s t$

Switch network model:

Model the dc component of $i_g(t)$, and the fundamental component of $v_s(t)$.

$\omega_s = 2\pi / T_{sw}$

$v_s(t)$ contains a dc component (that is removed by C_s), and odd harmonics. Fourier series of $v_s(t)$:

$$v_s(t) = \frac{1}{2}V_g + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2}{\pi}\frac{V_g}{n}\sin(n\omega_s t) \qquad \text{with } \omega_s = \frac{2\pi}{T_s}$$

full bridge $\frac{4}{\pi}\frac{V_g}{n}$

The fundamental component is

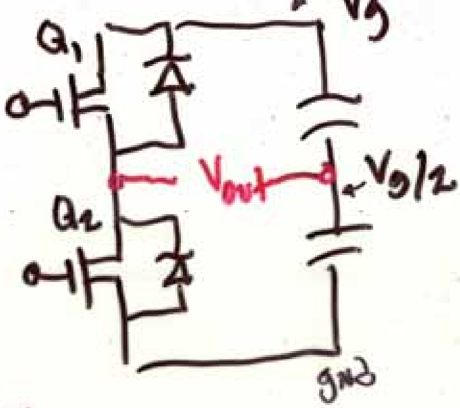
$$v_{s1}(t) = \frac{2V_g}{\pi}\sin(\omega_s t)$$

In response to this applied voltage, the fundamental component of the tank input current is

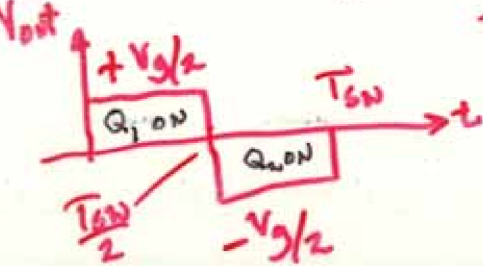
$$i_{s1}(t) = I_{s1}\sin(\omega_s t - \varphi_s)$$

The input current on the dc side of the switch network is given by

$$i_g(t) = \begin{cases} i_s(t) & \text{when } Q_1/D_1 \text{ conduct } (0 < t < \frac{1}{2}T_s) \\ 0 & \text{when } Q_2/D_2 \text{ conduct } (\frac{1}{2}T_s < t < T_s) \end{cases}$$



With C $\frac{1}{2}$ bridge of Pbm 19.) V_{out} is $\pm$ wrt 0



by symmetry only odd harmonics

Look @

1. Input circuit model
2. Rectifier model
3. Tank model

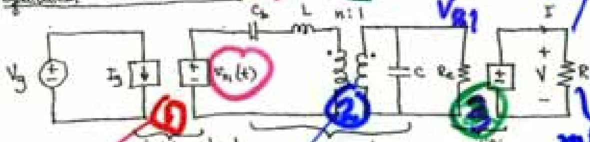
$f_{sw} > f_R$

②

Half-bridge dc-dc parallel resonant converter

a) Equivalent circuit model $\xrightarrow{i_{s1}}$ ← NEW

max load
$I = 20\,A$
$V = 3.3\,V$



min load
$I = 2\,A$
$V = 3.3$

model 1st

$\frac{V_{R1}}{V_{s1}}$

① Switch network model:

Model the dc component of $i_g(t)$, and the fundamental component of $v_s(t)$.

$v_s(t)$ contains a dc component (that is removed by C_s), and odd harmonics. Fourier series of $v_s(t)$:

why?

$$v_s(t) = \frac{1}{2}V_g + \sum_{n=1,3,5,\ldots} \frac{2V_g}{n\pi} \sin(n\omega_s t) \qquad \text{with } \omega_s = \frac{2\pi}{T_s}$$

The fundamental component is

$$v_{s1}(t) = \frac{2V_g}{\pi} \sin(\omega_s t)$$

$|V_{s1}| \sim \frac{2}{3} V_g$ vs full bridge $\frac{4}{\pi}$

In response to this applied voltage, the fundamental component of the tank input current is

Key

$$i_{s1}(t) = I_{s1} \sin(\omega_s t - \varphi_s)$$

$f > f_r$ i lags

The input current on the dc side of the switch network is given by

$$i_g(t) = \begin{cases} i_s(t) & \text{when } Q_1/D_1 \text{ conduct } (0 < t < \frac{1}{2}T_s) \\ 0 & \text{when } Q_2/D_2 \text{ conduct } (\frac{1}{2}T_s < t < T_s) \end{cases}$$

For $f_{op} > f_{LC}$

Average over ½ T_s

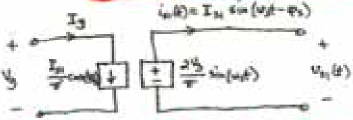
The dc component of $i_g(t)$ is therefore

$$I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)\,dt = \frac{1}{T_s}\int_0^{\frac{1}{2}T_s} I_{s1}\sin(\omega_s t - \varphi_s)\,dt$$

$$= \frac{I_{s1}}{\pi}\cos(\varphi_s)$$

So the half-bridge switch network model is

dc ⟷ ac Input model

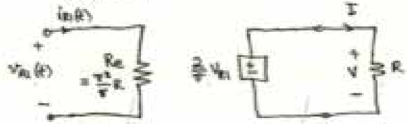


$$\frac{V_{S1}}{V_g} = \frac{2}{\pi}$$

Do next easiest model F.B.R.

3 Rectifier network model

Model the fundamental component of $i_R(t)$, and the dc component of $|v_R(t)|$. The waveforms and resulting model are identical to those derived in the textbook for the parallel resonant converter:



$$\frac{V_{out}}{V_{R1}} = \frac{2}{\pi}$$

with $v_{R1}(t) = V_{R1}\sin(\omega_s t - \varphi_R)$

$i_{R1}(t) = \frac{4I}{\pi}\sin(\omega_s t - \varphi_R)$

$$R_e = \frac{\pi^2}{8}\left[\frac{V_{out}}{I_{out}}\right]$$

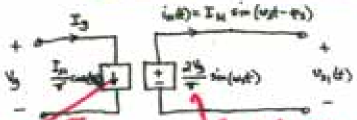
Next resonant network $H(\omega)$

The dc component of $i_g(t)$ is therefore

$$I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)\,dt = \frac{1}{T_s}\int_0^{\frac{1}{2}T_s} I_{s1}\sin(\omega_s t - \varphi_s)\,dt$$

$$= \frac{I_{s1}}{\pi}\cos(\varphi_s)$$

So the half-bridge switch network model is



full $\frac{2I_{s1}}{\pi}$ full bridge $\frac{4V_g}{\pi}$

3 Rectifier network model

Model the fundamental component of $i_R(t)$, and the dc component of $|v_R(t)|$. The waveforms and resulting model are identical to those derived in the textbook for the parallel resonant converter:

with $v_{R1}(t) = V_{R1}\sin(\omega_s t - \varphi_R)$

$i_{R1}(t) = \frac{4I}{\pi}\sin(\omega_s t - \varphi_R)$

Tank

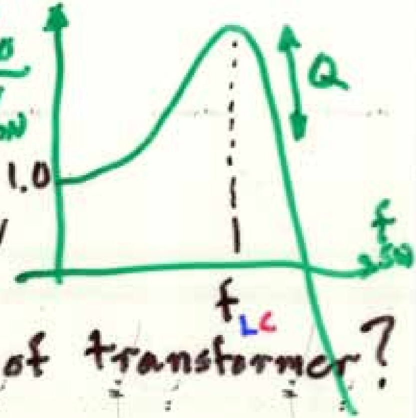
3.3V desired across R_e

big to block DC

load taken across V_c

For $R_e > \sqrt{L/C}$

expect $\frac{V_o}{V_{in}}$



Key

$f_{op} > f_{LC}$

$\Rightarrow i_{tank}$ lags V

Any effects of transformer?

(a) on f

(b) on Q

makes $\omega_{LC} \propto n$

$\Longrightarrow$

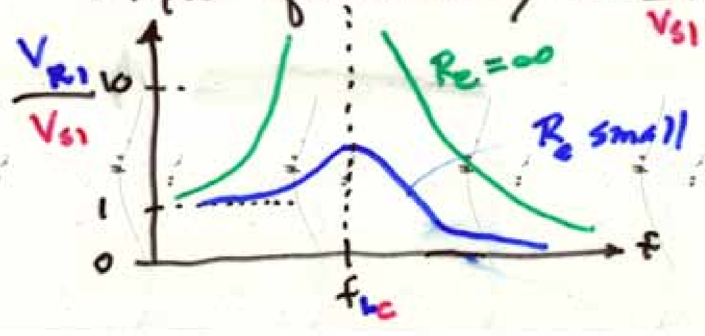
familiar?

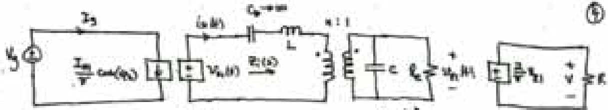
$$\omega_{LC} = \frac{n}{\sqrt{LC}}$$

$$Q_e = \frac{R_e}{Z_0} = \frac{R_e}{\sqrt{\frac{L/n^2}{C}}} = \frac{n\,R_e}{\sqrt{L/C}}$$

$$Q = f(n)$$

Expect qualatatively two $\frac{V_{R1}}{V_{S1}}$





$v_{s1}(t) = \frac{2V_g}{\pi}\sin(\omega_s t)$

$R_e = \frac{\pi^2}{8}R$

full $R_e = \frac{8}{\pi^2}R$

Solution of model for dc conversion ratio $M = \frac{V}{V_g}$:

$$V = \underbrace{\left(\frac{V_{s1}}{V_g}\right)}_{\frac{2}{\pi}}\underbrace{\left(\frac{V_{R1}}{V_{s1}}\right)}_{\|H\|}\underbrace{\left(\frac{V}{V_{R1}}\right)}_{\frac{2}{\pi}} = \frac{4}{\pi^2}\|H(j\omega_s)\|$$

(3) resonant network: Move L and V_{s1} to secondary!

where the transfer function $H(s)$ is

$$H(s) = \frac{1}{n}\,\frac{R_e \| \frac{1}{sC}}{\frac{sL}{n^2} + R_e \| \frac{1}{sC}} = \frac{1}{n}\,\frac{1}{1 + \frac{sL}{n^2 R_e} + \frac{s^2 LC}{n^2}}$$

$V_{s1} \rightarrow \frac{V_{s1}}{n}$

$$= \frac{1}{n}\,\frac{1}{1 + \frac{s}{Q_e\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{with } \omega_0 = \frac{n}{\sqrt{LC}}$$

$L \rightarrow \frac{L}{n^2}$

$$Q_e = \frac{R_e}{n}\sqrt{\frac{C}{L}}$$

$$\|H(j\omega_s)\| = \frac{1}{n}\,\frac{1}{\sqrt{\left(1-\left(\frac{\omega_s}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega_s}{Q_e\omega_0}\right)^2}} = \frac{1}{n}\,\frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

so $$M = \frac{4}{n\pi^2}\,\frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

This equation is used to plot V vs. f_s on the next page, at maximum load and at minimum load.

2A 20A

Analytically $\frac{V_{R1}}{V_{S1}} =$

$$\frac{R_e \parallel \frac{1}{sC}}{\frac{sL}{n^2} + R_e \parallel \frac{1}{sC}} = \frac{Z_{load}}{Z_{in}}$$

$$\frac{R_e/sC}{R_e + \frac{1}{sC}} = \frac{R_e}{R_e C s + 1} = R_e \parallel \frac{1}{sC}$$

$$\left.\frac{R_e}{(R_e C s + 1)\frac{sL}{n^2} + R_e}\right\} \text{from } \frac{Z_{load}}{Z_{in}} \; \frac{1}{R_e C s + 1}$$

$$\left[\frac{s^2 LC}{n^2} + s\frac{L}{R_e n^2} + 1\right]^{-1} \underbrace{\frac{1}{n}}_{\text{from } \frac{V_{S1}}{n}} = \frac{V_{R1}}{V_{S1}}$$

$$\frac{1}{n} \; \frac{1}{\frac{s^2}{\omega_R^2} + \frac{s}{Q\omega_r} + 1} = \frac{V_{R1}}{V_{S1}}$$

Quantative

Chosen values

Element values

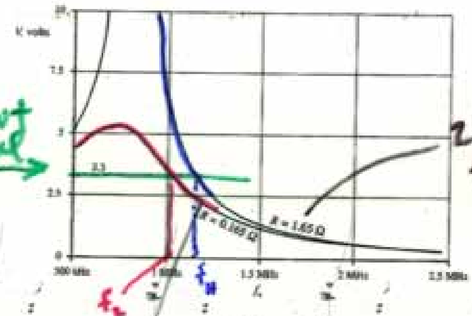
L	10 μH
C	1.5 μF
n	20
V_g	160 V
V	3.3 V

Operating points

at I =	20 A	2 A
R =	0.165 Ω	1.65 Ω
R_e =	0.204 Ω	2.036 Ω
f_0 =	821,873 Hz	821,873 Hz
Q_e =	1.58	15.77
f_s	1,031,696 Hz	1,156,025 Hz

for $V_0 = 3.3$

$R_e \cong \frac{\pi^2}{8} R$



Vout goal

2A load
R ↑
Q ↑ ≈ 16

20A load
R ↓
Q ↓ ≈ 1.6

b) Finding f_s at a given load current

We know that

$$M = \frac{4}{\pi^2} \frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

We want to solve for F:

$$\left(\frac{4}{\pi^2 M}\right)^2 = (1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2 = 1 - 2F^2 + \frac{F^2}{Q_e^2} + F^4$$

So

$$F^4 + F^2\left(\frac{1}{Q_e^2} - 2\right) + 1 - \left(\frac{4}{\pi^2 M}\right)^2 = 0$$

Quadratic formula:

$$F^2 = \frac{-\left(\frac{1}{Q_e^2} - 2\right) \pm \sqrt{\left(\frac{1}{Q_e^2} - 2\right)^2 - 4 + 4\left(\frac{4}{\pi^2 M}\right)^2}}{2}$$

Simplify:

$$F = \sqrt{1 - \frac{1 - \sqrt{1 - 4Q_e^2 + 64Q_e^4/\pi^4 M^2}}{2Q_e^2}}$$

$Re = 0.204$

(1) At rated load, $Q_e = 1.58$ and $f_s = F \cdot f_0 = 1.03$ MHz $\Big\}\ f_L$

(2) At minimum load, $Q_e = 15.8$ and $f_s = 1.156$ MHz $\Big\}\ f_H$

(answers to parts b and d)

$Re = 2.04$

Pbm 19.1 HW set #1 LZ

1. *Analysis of a half-bridge dc–dc parallel resonant converter, operated above resonance.* In Fig. 1, the elements C_B, L_F, and C_F are large in value, and have negligible switching ripple. You may assume that all elements are ideal. You may use the sinusoidal approximation as appropriate.

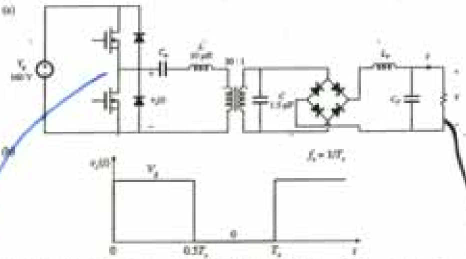


Fig. 1 Half-bridge parallel resonant converter of Problem 1.: (a) schematic, (b) switch voltage waveform.

(a) Construct an equivalent circuit for this converter, similar to Fig. 19.22, which models the fundamental components of the tank waveforms and the dc components of the converter input current and output voltage. Clearly label the values and/or give expressions for all elements in your model, as appropriate.

At rated (maximum) load, this converter produces $I = 20$ A at $V = 3.3$ V.

(b) What is the converter switching frequency f_s at rated load?

(c) What is the magnitude of the peak transistor current at rated load?

At minimum load, the converter produces $I = 2$ A at $V = 3.3$ V.

(d) What is the converter switching frequency f_s at minimum load?

(e) What is the magnitude of the peak transistor current at minimum load? Compare with your answer from part (c)—what happens to the conduction loss and efficiency at minimum load?

Question

i_Q largest for $R_L \to \infty$?

when $f_{op} = f_{LC}$

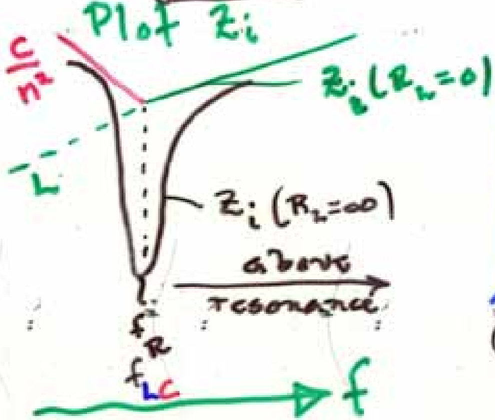
Largest $i_{IN} = i_Q$ when $R_L = \infty$ OR $R_L = 0$

for $f_{op} = f_{LC}$

$R_L = 0$

$R_L = \infty$

Series resonance

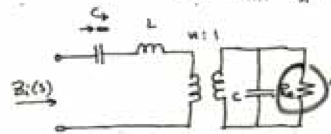


$i_{IN} = i_Q$ largest for $R_L = \infty$ @ $f_{op} = f_{LC}$

Peak transistor current

$$I_{S1} = \frac{V_{s1}}{\|Z_i\|}$$

where $Z_i(s)$ is the tank network input impedance and with $V_{s1} = 2V_g/\pi$

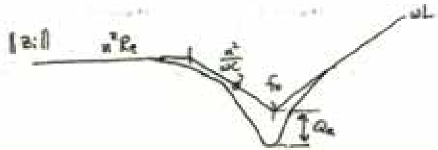


Not R

$$R_e \equiv \frac{\pi^2}{8} R$$

$$Z_i(s) = sL + n^2\left(\frac{1}{sC} \| R_e\right)$$

$$= n^2 R_e \frac{\left(1 + s\frac{L}{n^2 R_e} + s^2\frac{LC}{n^2}\right)}{\left(1 + sR_eC\right)}$$



At maximum load:

$R_e = 0.204\ \Omega$

$f_s = 1.03$ MHz

$Q_e = 1.58$

$\|Z_i\|_{f=f_s} = 36.1\ \Omega$

Peak transistor current $I_{S1} = \dfrac{\frac{2}{\pi}(160)}{36.1} = 2.82$ A

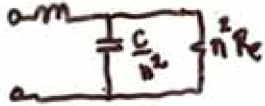
Peak transistor current occurs R_L small or large? (7)

$$I_{s1} = \frac{V_{s1}}{\|Z_i\|}$$ where $Z_i(s)$ is the tank network input impedance and with $V_{s1} = 2V_g/\pi$

$n = 20$

$L = 10\,\mu H$

$C = 1.5\,\mu F$



$$Z_i(s) = sL + n^2\left(\frac{1}{sC} \| R_e\right)$$

$$= n^2 R_e \frac{\left(1 + s\frac{L}{n^2 R_e} + s^2 \frac{LC}{n^2}\right)}{\left(1 + sR_eC\right)}$$

$$f_0 = \frac{1}{\sqrt{L\frac{C}{n^2}}}$$

$$f_0 = \frac{n}{\sqrt{LC}}$$

At maximum load:

$R_e = 0.204\ \Omega$ ← $\underbrace{3.3V/20A}_{\frac{\pi}{8} R_L(\min)}$

$f_s = 1.03$ MHz

$Q_e = 1.58$

$\|Z_i\|_{f=f_s} = 36.1\ \Omega$

Peak transistor current $I_{s1} = \frac{\frac{2}{\pi}(160)}{36.1} = 2.82$ A

At minimum load:

$$R_e = 2.036\ \Omega$$

$$f_s = 1.156\ \text{MHz}$$

$$Q_e = 15.8$$

$$\|Z_i\|_{f=f_s} = 36.0\ \Omega$$

$$I_{s1} = \frac{\frac{2}{\pi}(160)}{36.0} = 2.83\,\text{A}$$

The peak transistor current is essentially independent of load current. Hence, the conduction loss is also independent of load current. This would lead to poor efficiency at light load.

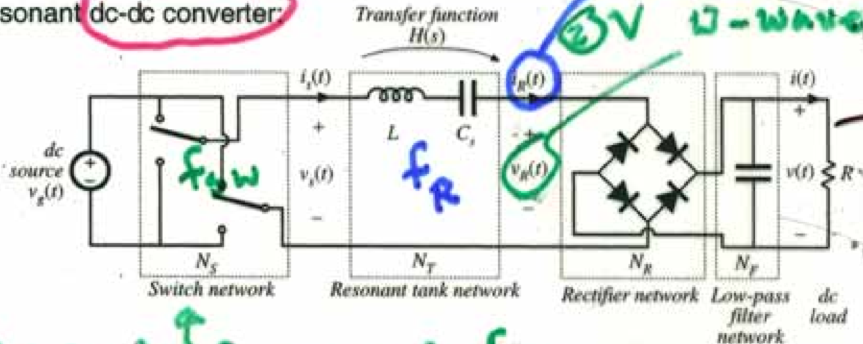
DC → □-wave → Sinewave → DC
rectifier

19.1 Sinusoidal analysis of resonant converters

① switched i Sinewave @ f_r

② V □-wave @ f_{sw}

A resonant dc-dc converter:



— DC out

By control you set f_{sw}

If tank responds primarily to fundamental component of switch network output voltage waveform, then harmonics can be neglected.

Let us model all ac waveforms by their fundamental components.

$\frac{f_{sw}}{f_R}$ determines $|i_R|$

Suprise: V_R and V_S both □-waves

AE Family

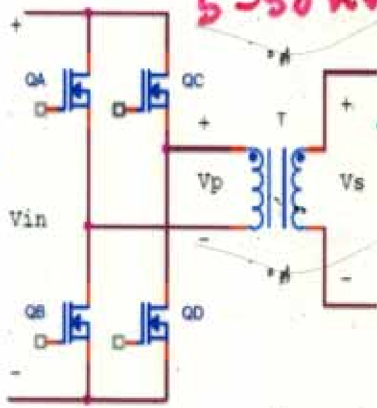
Standard DC/LF/MF Topologies (3-10kW)

- Offline pulse width modulated full-bridge
  - MDX (DC)
  - MDXII (DC)
- Offline phase-modulated full-bridge for greater dynamic range
  - PE (LF)
  - PDX (MF)
- Promising results in the HF range (3-30MHz)

Sucoss to 160 MHz

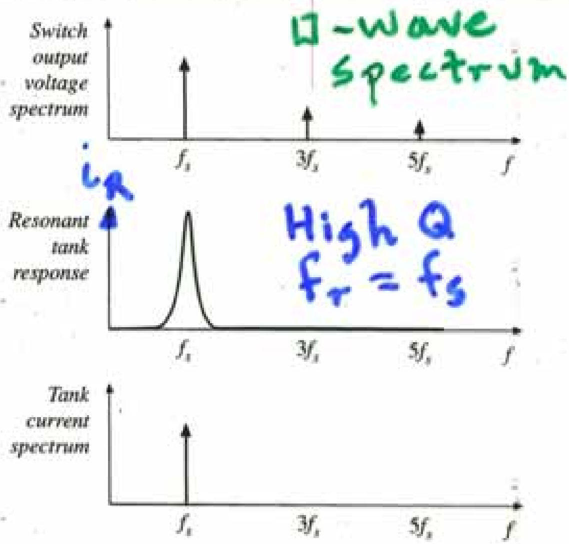
5-50 kW

DC



RF

The sinusoidal approximation



Tank current and output voltage are essentially sinusoids at the switching frequency f_s.

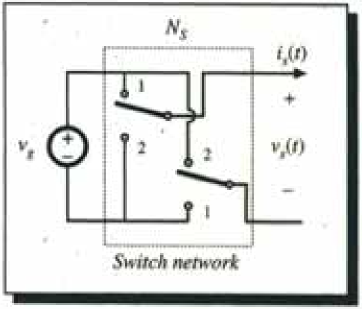
Neglect harmonics of switch output voltage waveform, and model only the fundamental component.

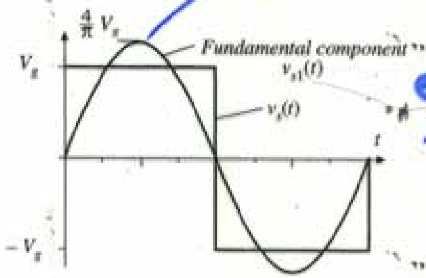
Remaining ac waveforms can be found via phasor analysis.

DC → □-wave → fundamental component

19.1.1 Controlled switch network model

peak 1.3 V_g





equal time $\pm V_g$

If the switch network produces a square wave, then its output voltage has the following Fourier series:

$$v_s(t) = \frac{4V_g}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin(n\omega_s t)$$

The fundamental component is

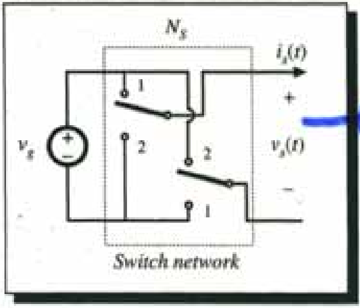
$$v_{s1}(t) = \frac{4V_g}{\pi} \sin(\omega_s t) = V_{s1} \sin(\omega_s t)$$

So model switch network output port with voltage source of value $v_{s1}(t)$

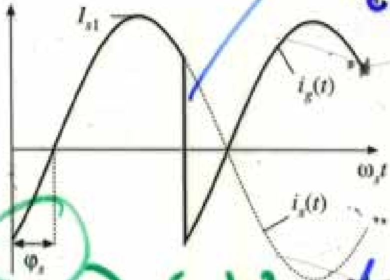
Model of switch network input port

i_s is abruptly switched by control gate drive

For narrow of
① $f_{RZC} \Rightarrow f_{sw}$
② High Q



$\rightarrow Z_{in}$



Set by $\angle Z_{in}$ @ f_{sw}

Assume that switch network output current is

$$i_s(t) = I_{s1} \sin(\omega_s t - \varphi_s)$$

It is desired to model the dc component (average value) of the switch network input current.

$$\langle i_g(t) \rangle_{T_s} = \frac{2}{T_s} \int_0^{T_s/2} i_g(\tau) d\tau$$

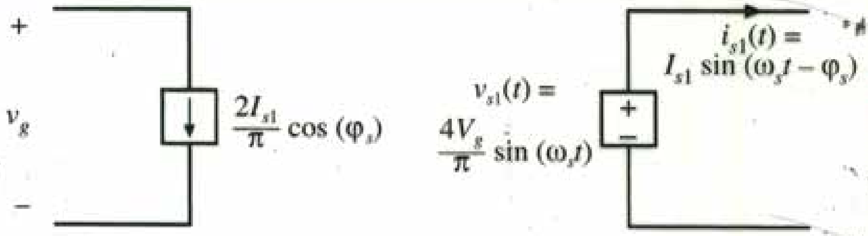
$$\approx \frac{2}{T_s} \int_0^{T_s/2} I_{s1} \sin(\omega_s \tau - \varphi_s) d\tau$$

$$= \frac{2}{\pi} I_{s1} \cos(\varphi_s)$$

$\phi_s = f(f_{sw}/f_r)$

Switch network: equivalent circuit

P_{DC}^{eff}



P_{AC} @ ω_{sw}

- Switch network converts dc to ac
- Dc components of input port waveforms are modeled
- Fundamental ac components of output port waveforms are modeled
- Model is power conservative: predicted average input and output powers are equal

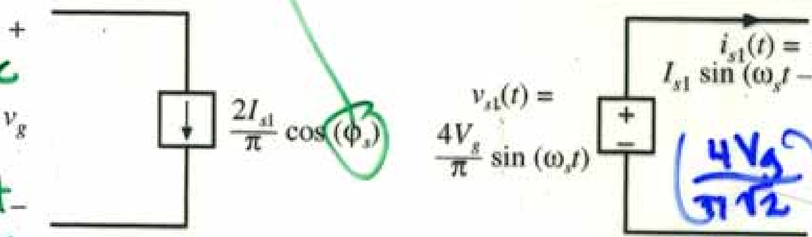
If lossless switches and lossless L C

$P_{DC}^{eff} = P_{AC}$

Φ_s set by switch phasing and $\angle Z_{IN}$

Switch network: equivalent circuit

$P_{IN} = P_{DC} = V_g I^{DC}_{eff}$
constant DC power



$$P^{AC}_{OUT} = \underbrace{\left(\frac{4V_g}{\pi\sqrt{2}}\right)\left[\frac{I_s}{\sqrt{2}}\right]}_{RMS}\cos\Phi_s$$

- Switch network converts dc to ac
- Dc components of input port waveforms are modeled
- Fundamental ac components of output port waveforms are modeled
- Model is power conservative: predicted average input and output powers are equal

$P^{DC}_{IN} = P^{DC}_{out}$ ON <u>average only</u>

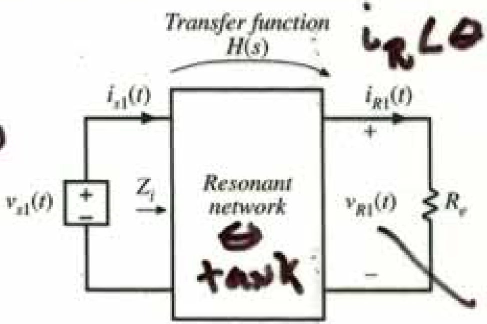
High Q

19.1.3 Resonant tank network

Re is effective load

$i_R \angle 0$

$V_s \angle 0$



$0 \le R_e \le \infty$

short open

$V_R \angle 0$

Model of ac waveforms is now reduced to a linear circuit. Tank network is excited by effective sinusoidal voltage (switch network output port), and is load by effective resistive load (rectifier input port).

Can solve for transfer function via conventional linear circuit analysis.

SW stress each extreem!

Solution of tank network waveforms

Transfer function:

$$\frac{v_{R1}(s)}{v_{s1}(s)} = H(s)$$

Ratio of peak values of input and output voltages:

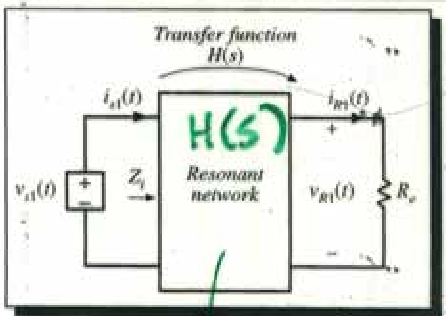
$$\frac{V_{R1}}{V_{s1}} = \left\| H(s) \right\|_{s = j\omega_s}$$

Solution for tank output current:

$$i_R(s) = \frac{v_{R1}(s)}{R_e} = \frac{H(s)}{R_e} v_{s1}(s)$$

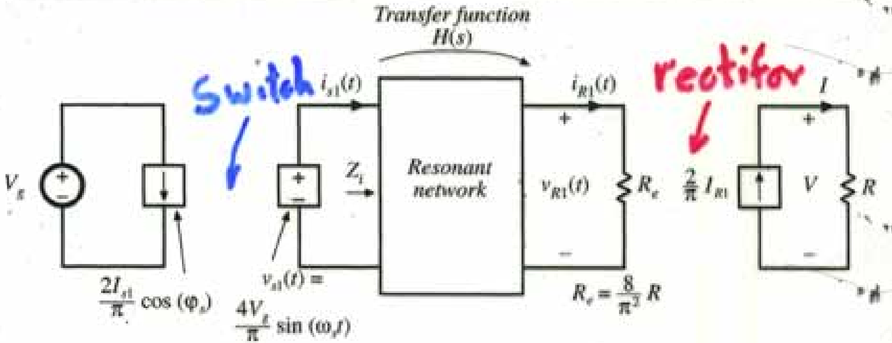
which has peak magnitude

$$I_{R1} = \frac{\left\| H(s) \right\|_{s = j\omega_s}}{R_e} V_{s1}$$



① series RLC
② parallel RLC
③ L-C-C
C-L-L

19.1.4 Solution of converter voltage conversion ratio $M = V / V_g$



$$M = \frac{V}{V_g} = \underbrace{(R)}_{\left(\frac{V}{I}\right)} \underbrace{\left(\frac{2}{\pi}\right)}_{\left(\frac{I}{I_{R1}}\right)} \underbrace{\left(\frac{1}{R_e}\right)}_{\left(\frac{I_{R1}}{V_{R1}}\right)} \underbrace{\left(\| H(s) \|_{s = j\omega_s}\right)}_{\left(\frac{V_{R1}}{V_{s1}}\right)} \underbrace{\left(\frac{4}{\pi}\right)}_{\left(\frac{V_{s1}}{V_g}\right)}$$

Eliminate R_e:

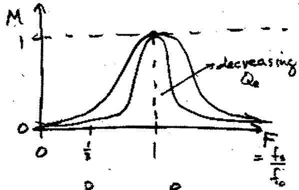
$$\frac{V}{V_g} = \| H(s) \|_{s = j\omega_s}$$

Conversion ratio M

$$\frac{V}{V_g} = \left\| H(s) \right\|_{s = j\omega_s}$$

So we have shown that the conversion ratio of a resonant converter, having switch and rectifier networks as in previous slides, is equal to the magnitude of the tank network transfer function. This transfer function is evaluated with the tank loaded by the effective rectifier input resistance R_e.

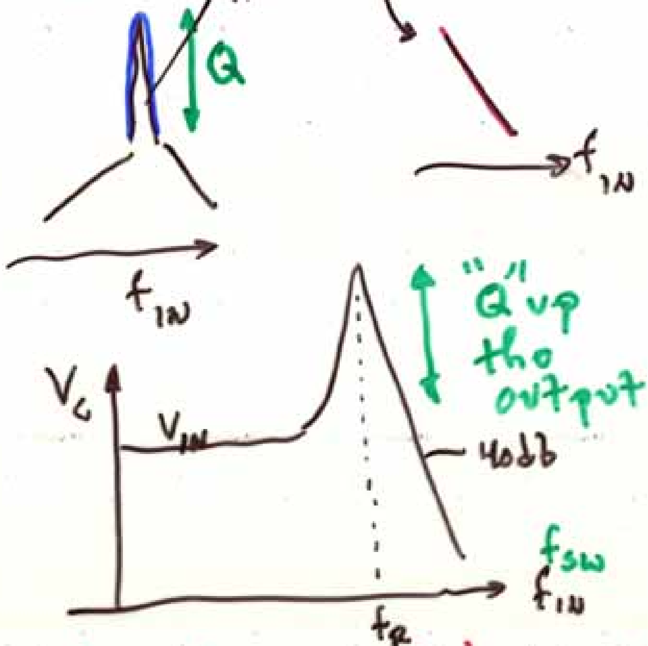
V_R / V_G 5/5



$$Q_e = \frac{R_o}{R_e} = \frac{R_o}{\left(\frac{8}{\pi^2} R\right)}$$

What about $\frac{V_c}{V_g}(f)$?

$V_c = i_{IN} \, Z_c$



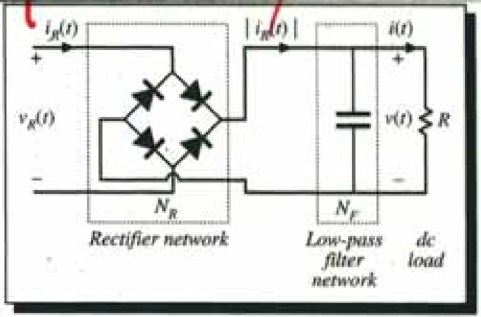
Anticipates parallel resonance

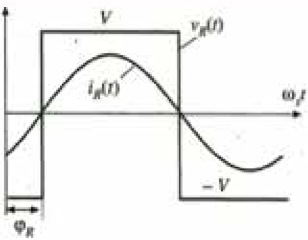
$f_{SW} > f_R$: boost or buck

sinusoidal from tank

rectified i_R

Rectifier dc output port model





Output capacitor charge balance: dc load current is equal to average rectified tank output current

$$\left\langle \left| i_R(t) \right| \right\rangle_{T_s} = I$$

Hence

$$I = \frac{2}{T_S} \int_0^{T_s/2} I_{R1} \left| \sin(\omega_s t - \varphi_R) \right| dt$$
$$= \frac{2}{\pi} I_{R1}$$

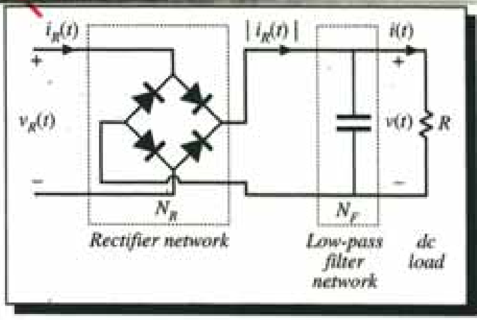
$$|I_R| = \frac{\pi}{2} I_{DC}$$

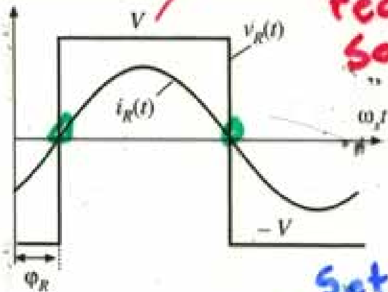
Rectifor ON $V_R = |V_o^{R}|$

filtered @ f_R

19.1.2 Modeling the rectifier and capacitive filter networks

suprise! V_R is □-wave for as the rectifier sees $\pm V_{out}$





Assume large output filter capacitor, having small ripple.

$v_R(t)$ is a square wave, having zero crossings in phase with tank output current $i_R(t)$.

If $i_R(t)$ is a sinusoid:

$$i_R(t) = I_{R1} \sin(\omega_s t - \varphi_R)$$

Set by tank @ f_{sw} / f_R

Then $v_R(t)$ has the following Fourier series:

$$v_R(t) = \frac{4V}{\pi} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n} \sin(n\omega_s t - \varphi_R)$$

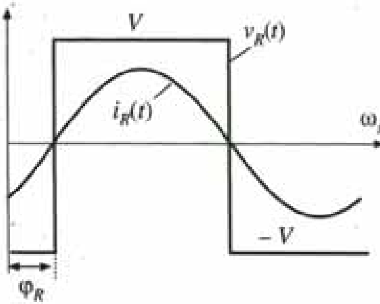
High Q

Sinusoidal approximation: rectifier

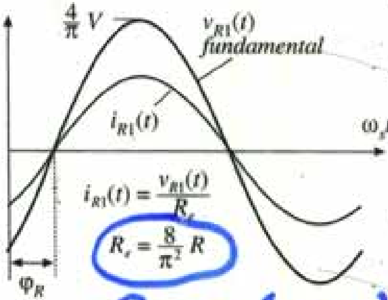
Again, since tank responds only to fundamental components of applied waveforms, harmonics in $v_R(t)$ can be neglected. $v_R(t)$ becomes

$$v_{R1}(t) = \frac{4V}{\pi} \sin(\omega_s t - \varphi_R) = V_{R1} \sin(\omega_s t - \varphi_R)$$

Actual waveforms



with harmonics ignored



Reff of rectifier

Equivalent circuit of rectifier

$\frac{\pi}{2} I_{DC} = |i_R|$

fund: $\frac{4 V_{out}}{\pi}$

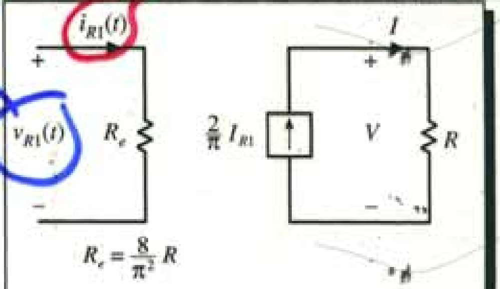
Rectifier input port:

- Fundamental components of current and voltage are sinusoids that are in phase
- Hence rectifier presents a resistive load to tank network
- Effective resistance R_e is

$$R_e = \frac{v_{R1}(t)}{i_R(t)} = \frac{8}{\pi^2}\frac{V}{I}$$

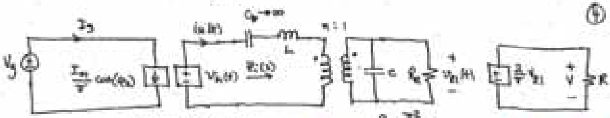
With a resistive load R, this becomes

$$R_e = \frac{8}{\pi^2} R = 0.8106R$$



Rectifier equivalent circuit

All togeather



$v_{s1}(t) = \frac{2V_g}{\pi}\sin(\omega_s t)$ $\qquad$ $R_e = \frac{\pi^2}{8}R$

Last resonant ckt.

Solution of model for dc conversion ratio $M = \frac{V}{V_g}$:

$$V = \underbrace{\left(\frac{V_{s1}}{V_g}\right)}_{\frac{2}{\pi}} \underbrace{\left(\frac{V_{R1}}{V_{s1}}\right)}_{\|H\|} \underbrace{\left(\frac{V}{V_{R1}}\right)}_{\frac{2}{\pi}} = \frac{4}{\pi^2}\,\|H(j\omega_s)\|$$

TANK

Key factor

vs text full-bridge $\frac{8}{\pi^2}$ Eq 19.27

(2) where the transfer function $H(s)$ is

trf

$$H(s) = \frac{1}{n}\,\frac{R_e \parallel \frac{1}{sC}}{\frac{sL}{n^2} + R_e \parallel \frac{1}{sC}} = \frac{1}{n}\,\frac{1}{1 + \frac{sL}{n^2 R_e} + \frac{s^2 LC}{n^2}}$$

Z divider

$$= \frac{1}{n}\,\frac{1}{1 + \frac{s}{Q_e\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \qquad \text{with} \quad \omega_0 = \frac{n}{\sqrt{LC}}$$

$$Q_e = nR_e\sqrt{\frac{C}{L}}$$

new

$Q = f(n)$

$W_0 = f(n)$

$$\|H(j\omega_s)\| = \frac{1}{n}\,\frac{1}{\sqrt{\left(1-\left(\frac{\omega_s}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega_s}{Q_e\omega_0}\right)^2}} = \frac{1}{n}\,\frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

So $$M = \frac{4}{n\pi^2}\,\frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

This equation is used to plot V vs. f_s on the next page, at maximum load and at minimum load.

Vary R_e by factor 10

∴ Q_e by factor 10

(5)

R_{load} range 10X

Element values

L	10 μH
C	1.5 μF
n	20
V_g	160 V
V	3.3 V

Max — min

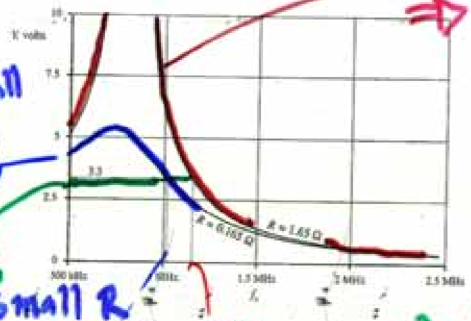
Operating points

at I =	20 A	2 A
R =	0.165 Ω	1.65 Ω
R_e =	0.204 Ω	2.036 Ω
f_0 =	821,873 Hz	821,873 Hz
Q_e =	1.58	15.77
f_s	1,031,696 Hz	1,156,025 Hz

Big Q

min load ⇒ large R

≥ small Max load



V_{out} fixed

small R

f

Max load 1.03 MHz $= f_{op}$

min load 1.15 MHz f_{op}

V_{out} fixed ⇒ $1.03 \le f \le 1.15$

$$R_{load}(min) = \frac{3.3}{2A}$$

$$R_e = \frac{\pi^2}{8}\left(\frac{3.3}{2.0}\right)$$

8

At minimum load:

$R_e = 2.036\ \Omega$

$f_s = 1.156\ \text{MHz}$

$Q_e = 15.8$

$\|Z_i\|_{f=f_s} = 36.0\ \Omega$

$$I_{s1} = \frac{\frac{2}{\pi}(160)}{36.0} = 2.83A$$

The peak transistor current is essentially independant of load current. Hence, the conduction loss is also independant of load current. This would lead to poor efficiency at light load.

99

Suprise?

$\eta \downarrow$ for large R_L

b) Finding f_s at a given load current

We know that

$$M = \frac{4}{\pi^2} \frac{1}{\sqrt{(1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

We want to solve for F:

$$\left(\frac{4}{\pi^2 M}\right)^2 = (1-F^2)^2 + \left(\frac{F}{Q_e}\right)^2 = 1 - 2F^2 + \frac{F^2}{Q_e^2} + F^4$$

so

$$F^4 + F^2\left(\frac{1}{Q_e^2} - 2\right) + 1 - \left(\frac{4}{\pi^2 M}\right)^2 = 0$$

Quadratic formula:

$$F^2 = \frac{-\left(\frac{1}{Q_e^2} - 2\right) \pm \sqrt{\left(\frac{1}{Q_e^2} - 2\right)^2 - 4 + 4\left(\frac{4}{\pi^2 M}\right)^2}}{2}$$

Simplify:

$$F = \sqrt{1 - \frac{1 - \sqrt{1 - 4Q_e^2 + 64Q_e^4/\pi^4 M^2}}{2Q_e^2}}$$

At rated load, $Q_e = 1.55$ — Max load $\Rightarrow$ small R

and $f_s = F \cdot f_0 = 1.03$ MHz

At minimum load, $Q_e = 15.5$ and $f_s = 1.156$ MHz

(answers to parts b and d)

large R