# Lecture 52

## I. Accurately Measuring or Calculating Loop Gain, $T_{mv}$ (s) or $T_{mi}$ (s), by Voltage or Current Injection: Theory versus Experiment

A. Why Fixate on Loop Gain?

B. How to Measure Loop Gain accurately

1. Voltage ,V, injection Method: $\frac{Z_1}{Z_2} < 1$
   a. Art of choosing the injection point
   b. Conditions for $T_m(s) = T(s) = T_C(s)$
   c. Analysis of Two Injection Conditions
      (1) $Z_1$ (looking back) < $Z_2$ (looking forward)
      (2) $|T| >> Z_1/Z_2$
   d. Op amp examples
      (1) V injection @ $Z_{out}$
      (2) Voltage divider's for analyzing $T_{mv}(s)$

2. Current , I, Injection Method: $\frac{Z_2}{Z_1} < 1$
   a. Artful choice for i injection
      $Z_2$(looking forward) << $Z_1$ (looking

back) $|T| > \frac{Z_2}{Z_1}$

3. General Injection Point

$\frac{Z_1}{Z_2} \ll 1$ $\frac{Z_2}{Z_1} \ll 1$ $\frac{Z_1}{Z_2} \approx 1$

v injection i injection ?

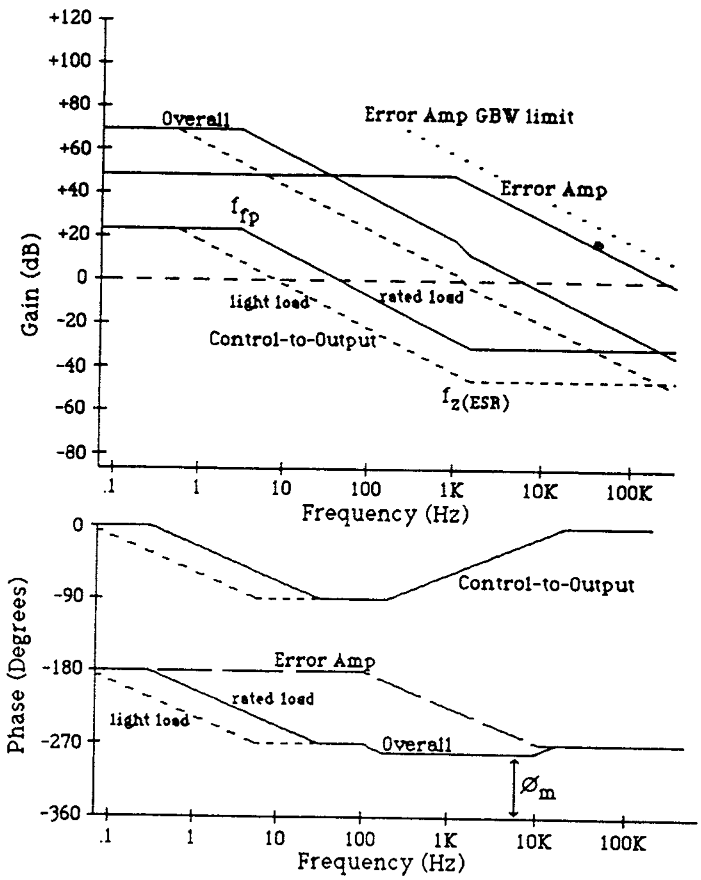
4. Measuring T(s) in Unstable Systems

C. Example: Erickson Pbm 9:10

# I. Accurately Measuring or Calculating Loop Gain, $T_{mv}$ (s) or $T_{mi}$ (s), by Voltage or Current Injection: Theory versus Experiment

## A. Why Measure Loop Gain?

Our intent is to accurately measure loop gain, in order to analyze it and determine if we could improve it. In order to apply $G_C$(s) compensation networks we first have to first know **the true state of the uncompensated T(f)** and then tailor it to achieve better closed loop performance we desire for the converter. For single pole T(f) one would say it is always stable in closed loop and therefore in no need of improvement. However, we show below that single pole T(f) is also a big beneficiary of COMPENSATION.



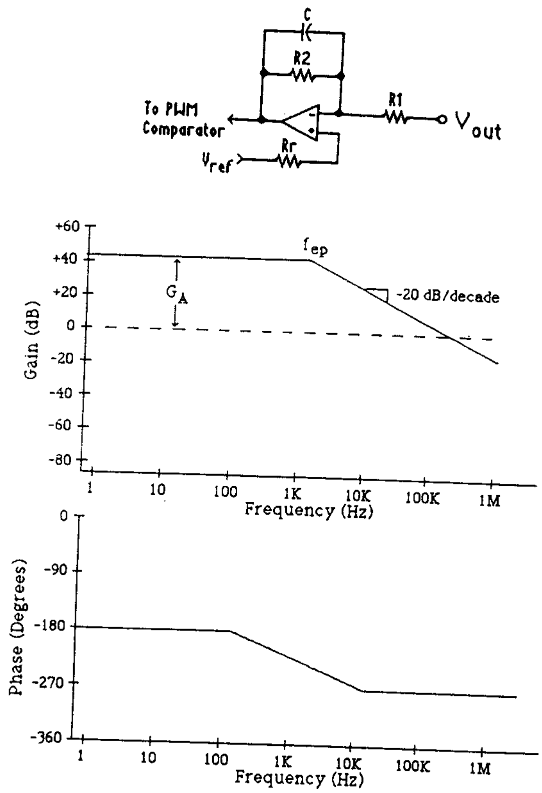

The above Bode plot contains both the uncompensated and compensated versions as well as the gain bandwidth limits of the op-amp employed in compensation networks. The gain curves for the uncompensated T(f) are in the lower portion of the figure, starting from 20 db and with a single pole, at $f_{fp}$. Note the pole location varies with load because $f_{fp}$~ 1/ R(load) C(filter cap). This single pole response is characteristic of current controlled converters as described in Chapter 11 of Erickson, as well as of the DCM mode of voltage controlled converters as described in Erickson chapter 10. **The zero in the T(f) is caused by??** This T(f) would be stable in closed loop operation so why bother with additional compensation to T(f)? Clearly for better closed loop transient response, we would like to INCREASE the $f_C$ of T(f) up to the limit $f_C < f_{SW}/5$. The op-amp $G_C$ plot to accomplish this is also shown in the figure starting at 50 db for DC and possessing a

SINGLE pole at a location $f_{ep}$. The potential problem is that we must be careful that the compensation not exceeds the gain bandwidth product of the op-amp, which is plotted to the far right hand side of the gain plots versus frequency. The overall or compensated T(f) plots are shown on the very top of the figure starting at 70 db ,near DC, and exhibiting an average slope of 20 db/decade throughout the entire frequency region. This is accomplished by carefully placing the $G_C$ single pole compensation, $f_{ep}$, right near the ESR zero location at the frequency, $f_Z$ (ESR). The op-amp provides 180 degrees of phase shift from DC onwards and the single pole will kick-in at $f_{EP}$ to add another 90 degrees starting at the frequency f= $f_{PE}$ /10 and completing the full 90 degrees at f=10 $f_{PE}$ as shown on the top of page 5.

The key design choice is to place

$f_{EP} \sim f_Z(ESR)$

Given this condition of an average slope of 20 db/ decade for the compensated loop gain, the combination of the uncompensated DC gain, $G_{VD}$(uncompensated)=$G_{DC}$ in db units, and the op-amp compensator gain at DC, $G_C$= $G_{XO}$ in db units, will determine the $f_C$ of the overall or compensated loop gain since $G_{XO}$ +$G_{DC}$ = 20 log ($f_{XO}$/ $f_{FP}$). Note in absolute units, we have $A_A$ and $A_{XO}$ for the absolute gain of the op-amp and the original uncompensated converter response respectively. Where $f_{XO}$ is the uncompensated loop gain crossover frequency. The op-amp implementation to achieve a single pole compensation network ,with in-band gain limiting, is shown below.




The amount of **extra gain need below the single compensation pole to hit the desired $f_C$** is given by:

$G_A = G_{XO} + 20 \log ( f_{XO} / f_{EP})$ in db

The resistor value $R_2$ ,is related to $R_1$ by the relation:

$A_A$( in absolute units) = $10^{G(A)/20}$ and $R_2 = A_A R_1$

Finally the value of the $R_1$ and the filter capacitor must satisfy the a simple relationship to the single pole location:

$$\mathbf{f_{EP} = 1/ 2p\, R_1\, C\, A_{XO}}$$

Owing to the high DC gain and high $f_C$ desired we may find that the op-amp gain-bandwidth product MUST BE such as to easily exceed the compensation network gain bandwidth product. This limit of the op-amp is shown as the dashed curve in the gain plots to the far right hand side. The wrong choice of op-amp frequency response could kill the efficacy of this method of $G_C$.

Lets recount two points about T(s) functions and feedback.

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where $T$ is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where $T$ is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.

4. Stability can be assessed using the phase margin test. The phase of $T$ is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

These two points should always be kept in mind as we apply compensation to original T(s) that we encounter. However ACCURATE determination of the original T(s) is crucial. How to we insure that our calculations on T(s) or measurements of T(s) are accurate?? See section B on page 7. Indeed if we have an inaccurate measure of uncompensated T(s), the cure via a new $G_C$ may be worse that the original perceived malady.

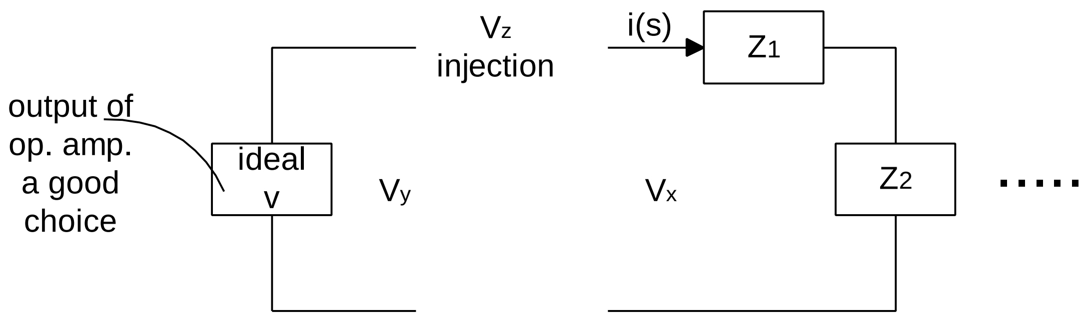
# B. How to Measure or Calculate T(f) Accurately

We will cover both voltage (section 1) and current injection methods (section 2) to either measure or calculate loop gain T(s). Comparisons between theory and experiment are very revealing.

## 1. Voltage Injection Method of Measuring T(s)

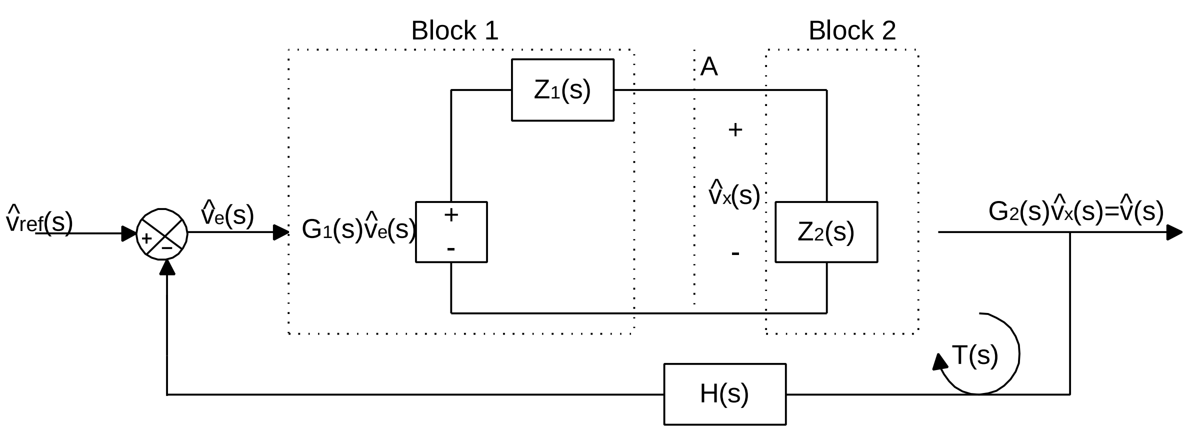
### a. Artful choice of v injection location.

This means a point near an ideal v source where the impedance looking back from the injection point is nearly zero. This is a difficult choice unless we employ an op-amp in the compensator network. $Z_{IN}$ for an ideal op-amp is high and $Z_{OUT}$ is low so it's use offers the location of a near ideal voltage injection point as shown.

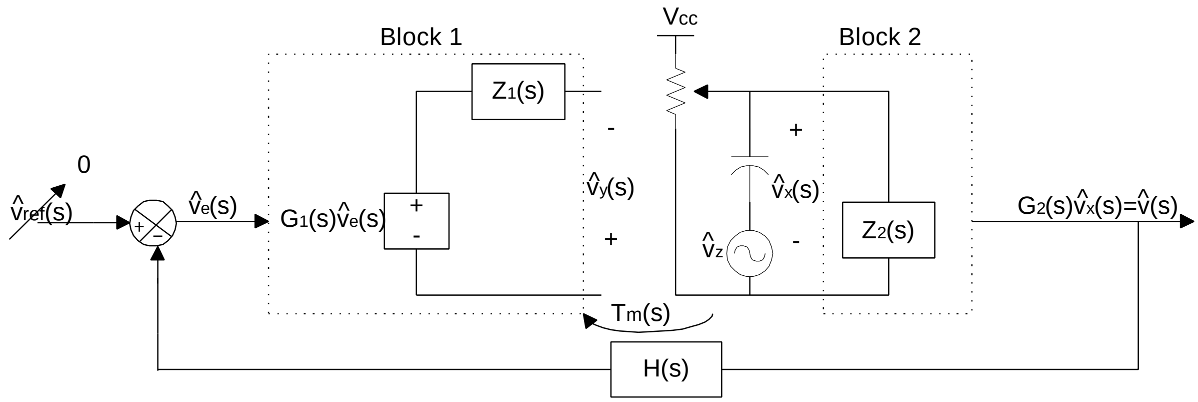



$V_X$ is the loop voltage input and $V_Y$ is the loop voltage output. We inject $V_Z(f)$ and measure $\frac{V_Y(f)}{V_X(f)}$ as a function of frequency. Below if we measure T(s) with a break point between $Z_1$ and $Z_2$ ,we will find: $T(s) = G_1(s) * ( Z_1/(Z_1+Z_2)) G_2(s) H(s)$

**What occurs if $Z_1$ <<$Z_2$?? What about $Z_1$ >>$Z_2$ ??**



In practice when we break the feedback loop we have to be careful to preserve the DC levels that pre-existed before we intruded to make T(f) measurements. We also have to consider that the ideal v source has some series impedance. Finally to get loop gain $V_y/V_x$ via valid superposition rules, we must have ONLY $V_z$ as an input to the system. All other independent inputs must be disabled. That is their ac variation must be zero about their DC values: all other ac V(sources) shorted and all other ac I(sources) opened. We see one such situation below.




$$T_m(s) = \left.\frac{V_y}{V_x}\right|_{\substack{\Delta Vref \approx 0 \\ \Delta Vg \approx 0}}$$

**$T_M$ implies measured values of loop gain and $T_C$ a calculated value of the loop gain**. Parasitic elements will make $T_M$ and $T_C$ differ substantially as will other effects to be discussed below. For a full test of T(s) drive source, $v_Z$, at a variety of frequencies. Because of the effect of loading of block 2 on block 1, the measured loop gain as compared to the actual loop gain is:

$$T_m(s) = T(s)\left[1+\frac{Z_1(s)}{Z_2(s)}\right]$$

**b. Rough Required Conditions for $T_m$ »the actual T(s)**

Only if certain conditions are met will either $T_M$ or $T_C$ be a valid measure of actual conditions.

1. AC Conditions

$$T_{mv} = T_s \text{ if } Z_2 \rangle\rangle Z_1 \quad \text{for all applied frequencies}$$

↓

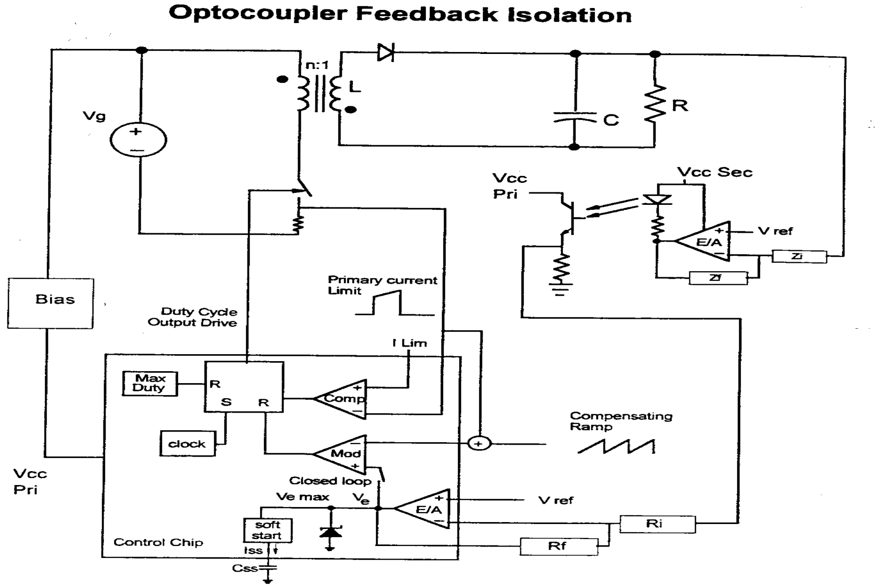
This is why the art of choosing the injection point for voltage is so important

Second big problem for V injection

**2. Maintaining Proper DC Conditions during Measurement**

If T is large especially at low f or dc, as we designed it, then a very small mV dc variation could SATURATE some components as we try to measure T(s). Op amps used in the $G_c(s)$ block are especially susceptible to this undesired low frequency saturation. So we usually **keep the dc loop closed** by injection $V_z$ via a transformer winding. The dc sees the winding as a short and the low frequency/dc loop remains closed and unperturbed. Alternatively we insert ac signals via a big capacitor to allow ac passage and DC blockage. However the DC blockage harms the stabilizing effect of feedback unless we carefully reinsert the DC by a potentiometer. Opto-couplers offer another choice:

**Optocoupler Feedback Isolation**



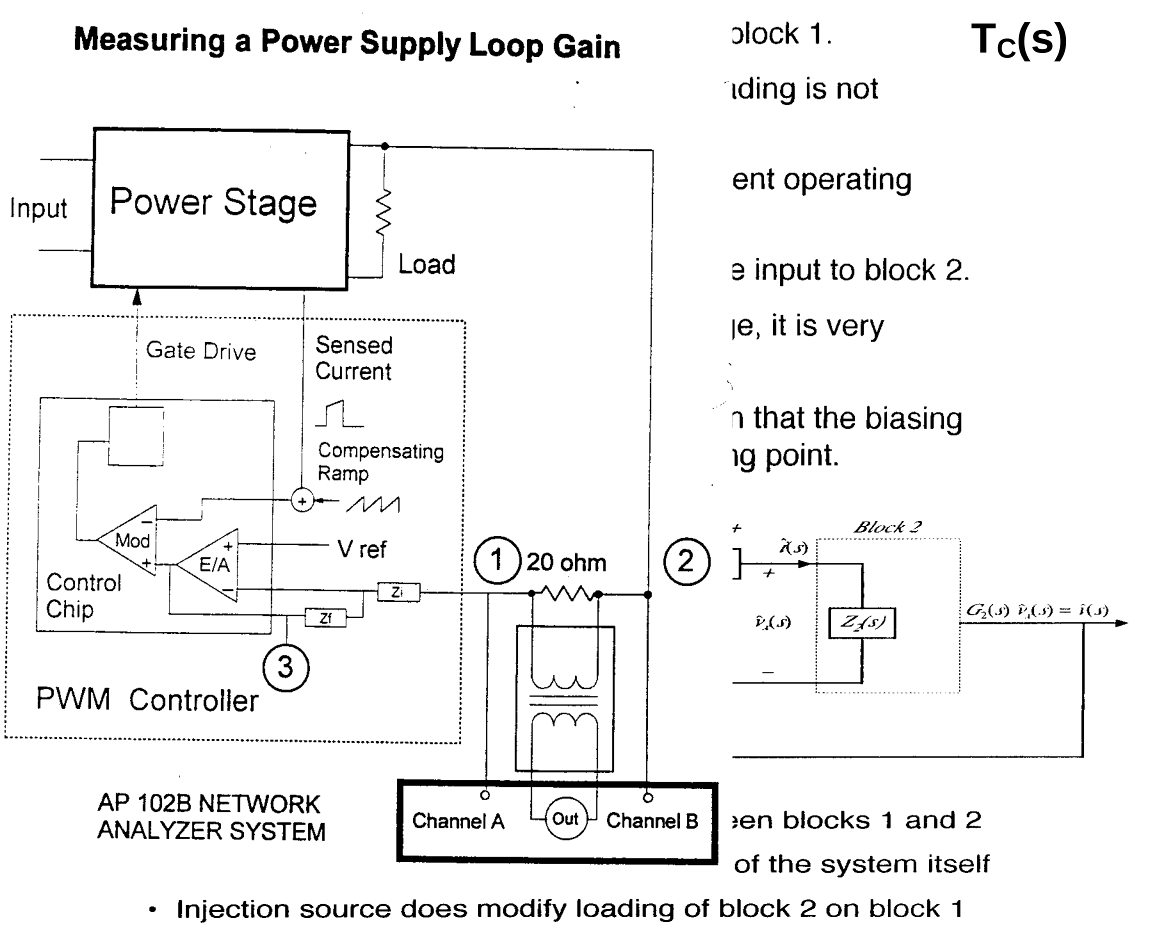

Design Tips:

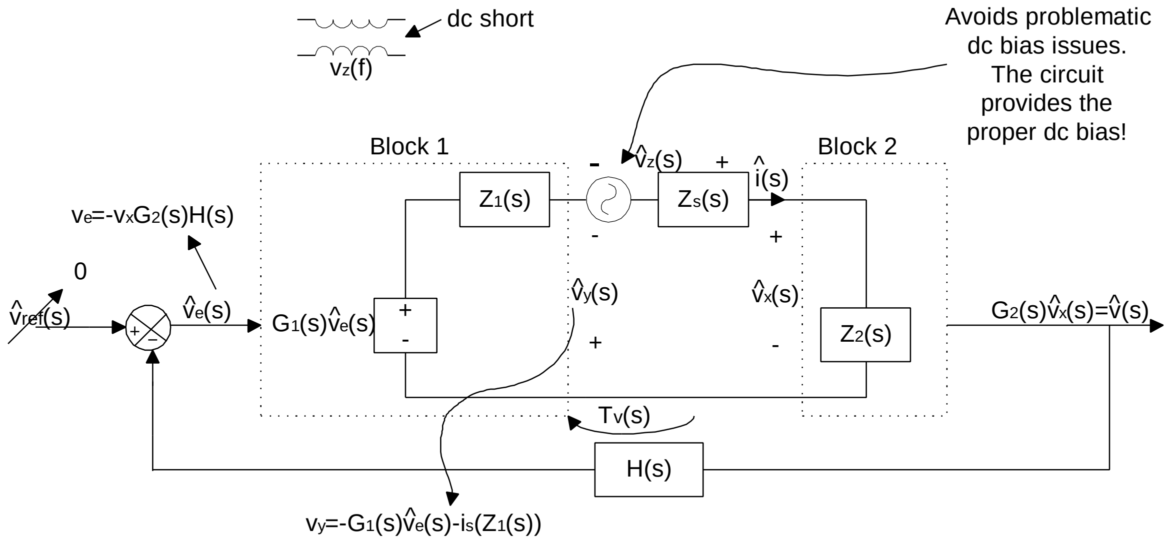
Run optocoupler at upper end of range for collector current for best

For HW #4 what's the advantage of opto isolation for T(s) data??

### 3. The Proper Loop Gain Path: The Shining Path

Third big problem for injection point is the possibility of multiple paths from the injection point to the output. Choose an injection point with the one path to the output - the shining path or sendero luminoso. Finally, we often can make a useful approximation that for T(s) large in the closed loop, $V_c \rightarrow 0$. This is often useful in control loops where you have a mix of control blocks, circuits, and summing points. Setting or assuming $V_c \rightarrow 0$ simplifies the solution of T(s) as it does in op amp circuits with apparent complex feedback paths subsequently being reduced to a simpler circuit. Use if possible, simple V divider analysis between and within blocks as we will show below. In summary:






$V_y$ and $V_x$ are put into the inputs of a network analyzer tuned to the f of $V_z$(f). We can either measure T(s) all at once or do it in tandem sections. If we are artful on the choice of the injection point with (1) $Z_1 << Z_2$ and (2) $T(s) > Z_1/Z_2$ then the actual values of $V_z$ and $z_s$(s) are not relevant either to the measurement nor the calculation of T(s). Give this we would naturally choose $Z_s$(s) big to even further reduce any loading effects.

$$V_e(s) = - V_x(s)\, G_2(s)\, H(s)$$

see block 1

$$- V_y(s) = G_1(s)\, V_e(s) - i(s)\, Z_1(s)$$

$$- V_y(s) = -V_x(s) G_2(s)\, H(s)\, G_1(s) - \underbrace{i(s)}_{\displaystyle \frac{V_x}{Z_2(s)}} Z_1(s)$$

$$\left.\frac{V_y(s)}{V_x(s)}\right|_{\substack{\Delta Vref=0 \\ \Delta Vg=0}} = G_1(s)\, G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)}$$

$$T_{mv}(s) = G_1 G_2 H + \frac{Z_1}{Z_2}$$ as measured in a real system

$$T(s) = G_1 \frac{Z_1}{Z_1 Z_2} G_2 H$$ actual values of T versus f

$$T_{mv}(s) = T(s)\left[1+\frac{Z_1}{Z_2}\right] + \frac{Z_1}{Z_2}$$ lowest smallest measurable

loop gain

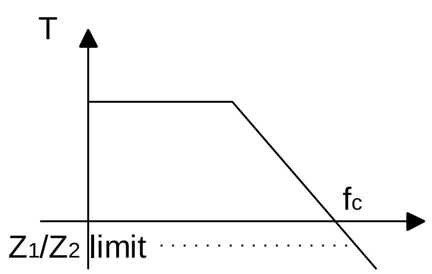
That is measured or calculated T(s) is believable only under certain very specific conditions.

$T_{mv} \approx T(s)$

**iff (1) $Z_1 < Z_2$ for all frequencies of interest**

**(2) T >> $Z_1/Z_2$ at all frequencies of interest**

Clearly $Z_1/Z_2$ versus frequency limits the lower end values of T(s) that can be extracted <u>from</u> $T_m(s)$ with V injection methods and still maintain any accuracy of measurement or calculation.

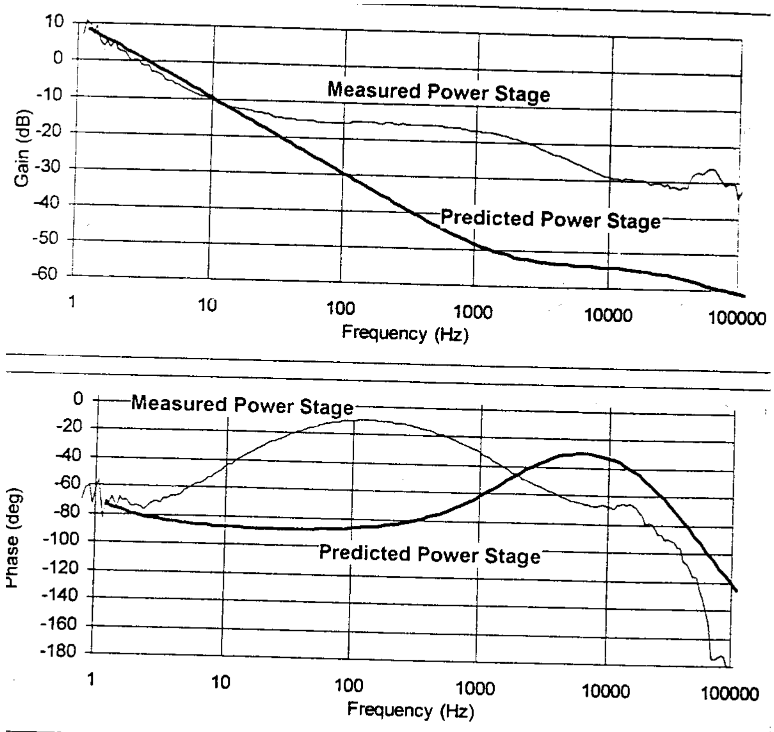



Usually we find for f > $f_c$ we enter a problem area for the V injection method as T is small and the required condition T > $Z_1/Z_2$ may not be met.

One "trick" is to seek a f range where $Z_2$ is very large, then get initial $T_m(s)$ measurements there.

**d. Measurement of T(s) versus Theory**

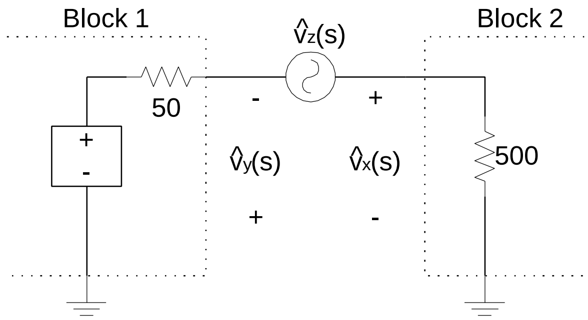
We always compare measurement versus theory and compare to achieve a more holistic understanding of the loop gain.




Which is the plot to believe and under what conditions?? Are experimental Bode plots always correct? Above what is the slope of the expected roll-off and why is the measured different? Can you make some guesses to these questions?

## e. Isolated Op Amp Examples of T(s) Calculations

1. V injection at output of op amp insures low Z looking back from the injection point. Op amp is usually found in the $G_c(s)$ block. It's easy to insert R-C networks around the op-amp to achieve the desired compensation network for an ailing or non-optimum converter open loop Bode plot.




$\frac{Z_1}{Z_2} = \frac{1}{10}$ a good rule of thumb for v injection validity

$T_m(s) = T(s)$

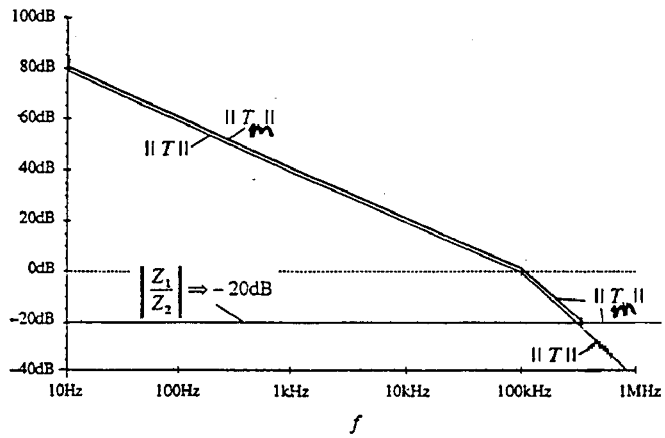
$$T_m(s) = T(s)\left[1 + \frac{Z_1}{Z_2}\right] + 0.1$$

| $\downarrow$ | $\downarrow$ |
|---|---|
| 1.1 or .83 db | -20 db |
| For large $T_m(s)$ $T(s) = T_m \pm 1$db | For $T_m(s)$ below 20db $T_m(s) \neq T(s)$ |

Assume a specific T(s) situation with two well separated poles

$$T(s) = \frac{80\,\text{db}}{(1 + s/10\text{KHz})(1 + \frac{s}{100\text{KHz}})}$$

Now we can make the Bode plots of T(s) and muse about what is good and bad about the uncompensated T(s) and what type of $G_C(s)$ we would employ to improve the compensated T(s) into from what we are starting with. But never use bad T(s) data to start the process.

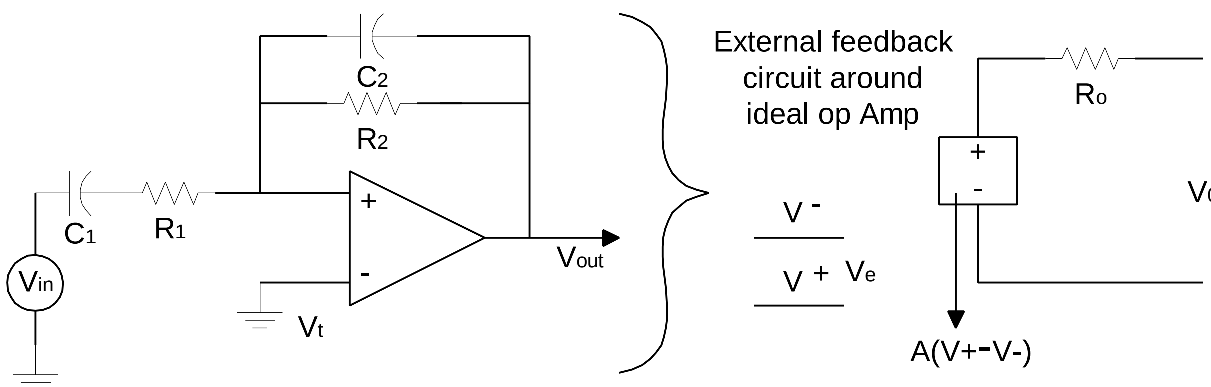



$T_M$ by voltage injection methods ACCURATELY reflects the real T(s) only down to values above –20 db. Below this the measured T(s) is not valid.

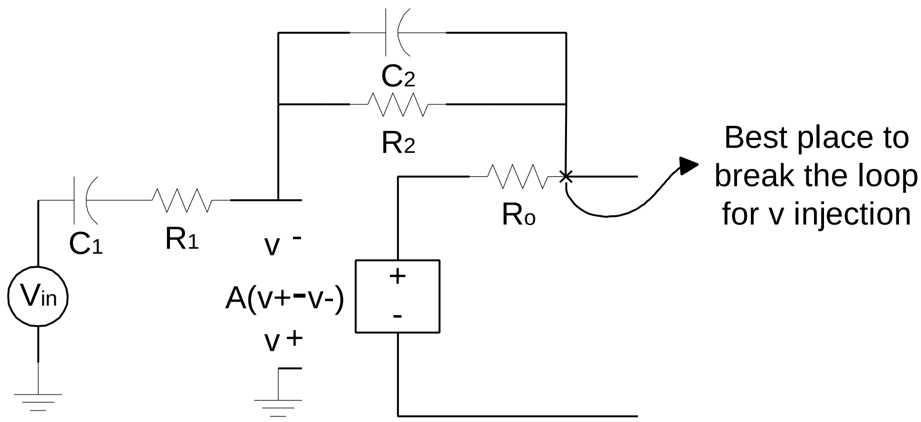
The V injection method is of LIMITED accuracy and must be recognized as such. Lets not tailor $G_C$ to fix a non-existant problem.

## 2. Using $V_c$ ® 0 in Op Amp with Feedback

The T(s) calculation for the $G_C$ block below is made easy by knowing $v_{error}$ between the positive and negative terminals is zero for very large op-amp gain. This is called a "virtual ground".

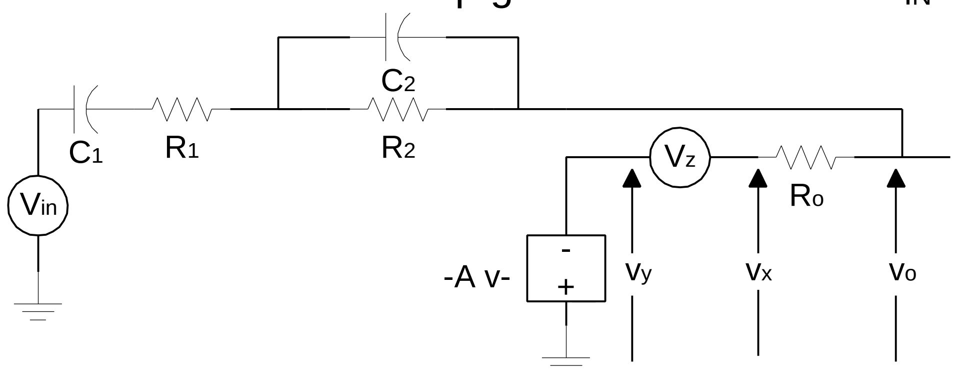



Op amp input has $V_{error} \rightarrow 0$ due to high loop gain. This simplifies the ac circuit analysis as shown below, when $v_{error} = 0$. That is we can use the "virtual ground" concept to reduce the level of complexity of the circuit analysis.




$$T_{mv}(s) = \left.\frac{V_y}{V_x}\right|_{V_1=0}$$

We must remove other independent sources for accurate loop gain measurement! $V_{IN} = 0$.




Start analysis assuming <u>we know</u> $V_x$ to find

$$\frac{V_y}{V_x} = \left(\frac{V_o}{V_x}\right)\left(\frac{V^-}{V_o}\right)\left(\frac{V_y}{V^-}\right) = T_m(s)$$

Visualize the signal flow around the loop instead of doing loop equations. Lets do a series of voltage dividers as follows:

$$\frac{V_o}{V_x} = \frac{\left[\left(\frac{1}{SC_1} + R_1\right) + \left(R_2 \| \frac{1}{SC_2}\right)\right]}{R_o[\text{numerator}]}$$ }simple voltage divider

$$\frac{V^-}{V_O} = \frac{\left(\frac{1}{SC_1} + R_1\right)}{\left[\left(\frac{1}{SC_1} + R_1\right) + \left(R_2 || \frac{1}{SC_2}\right)\right]}$$ }These poles will cancel in

T(s) calculations as we show below.

$$\frac{V_y}{V^-} = A\begin{pmatrix}\text{gain of the} \\ \text{Op Amp}\end{pmatrix}$$ If $A \to \infty$

$V^- \to 0$

$$T_{mv} = \frac{\left(\frac{1}{SC_1} + R_1\right) A}{R_o + \left[\left(\frac{1}{SC_1} + R_1\right) + R_2 || \frac{1}{SC_2}\right]}$$ $=( V_O/V_X)( V^-/V_O)(V_Y/V^-)=V_X/$

This method of calculating $T_m$(s) has

· No complex loop node equations to make mistakes in

· minimum of algebra in which to make further errors

· uses simple V dividers that even I can accomplish

Hence, we have a higher confidence in the final outcome. From $T_m$(s) we still have to do the following to make Bode plots easy:

1. Put in std form $$\frac{T_{mo}\left(1 + \frac{s}{w_z}\right)}{1 + \frac{s}{w_o Q} + \left(\frac{s}{w_o}\right)^2}$$
2. Draw Bode plots with asymptotes around $f_0$
3. Calculate $\phi_m$ at the T(s) unity gain cross-over frequency to predict closed-loop stability.

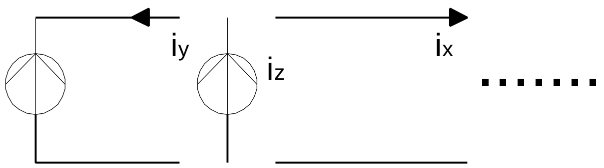
**For HW #4 How would above $G_C$ be useful to tailor T(s)?**

## C. How to (measure/calculate) $T_{mi_i}$(s) or $T_{Ci}$(s)

The open loop gain can be derived in current terms as well.

## 1. Current Injection into the open loop

a. Artful choice of i injection location. This means a point <u>just after</u> an ideal current source. The impedance looking back from the injection is nearly infinite, or impedance looking forward is zero. Hence the ratio or division of current is near unity from $I_Z$ to $I_X$. $I_X$ is the ac input current to the loop and $I_Y$ is the ac output current from the loop



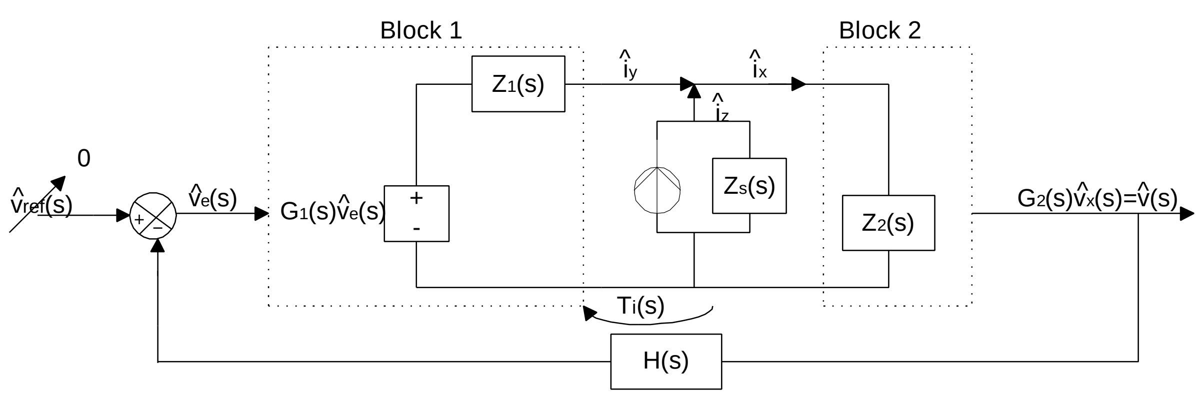

ratio of currents $I_X / I_Z = 1$ is best

$$T_m(s) = \left. \frac{i_y(s)}{i_x(s)} \right|$$

All other independent sources are removed in this calculation. One finds like in voltage injection:

$$T_{mi_i}(s) = T(s)\left[1 + \frac{Z_2(s)}{Z_1(s)}\right] + \frac{Z_2(s)}{Z_1(s)}$$

Loop System and Injection Point



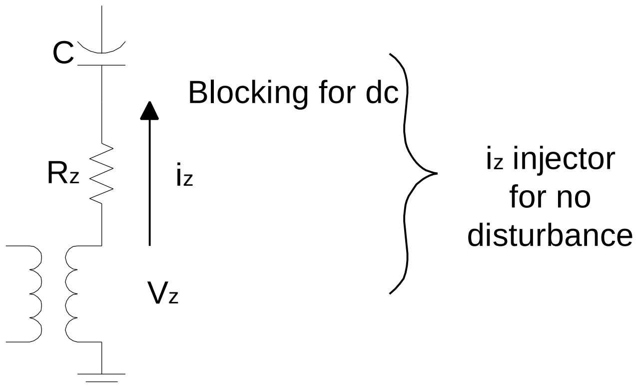

$T_{mi}(s)$ = the actual or real T(s) provided we meet the conditions

1. $Z_2(s) < Z_1(s)$

and

2. T(s) > $\frac{Z_2}{Z_1}$

If the two above conditions are met the exact choice of $Z_S(s)$ doesn't matter. To maintain dc balance we employ transformers or blocking capacitors at the injection point.




In summary, for current injection conditions to be proper:

It can be shown that

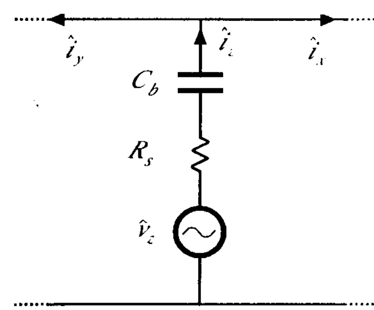
$$T_i(s) = T(s)\left(1 + \frac{Z_2(s)}{Z_1(s)}\right) + \frac{Z_2(s)}{Z_1(s)}$$

Conditions for obtaining accurate measurement:

$(i)$ $\| Z_2(s) \| << \| Z_1(s) \|$, and

$(ii)$ $\| T(s) \| >> \left\| \frac{Z_2(s)}{Z_1(s)} \right\|$

Injection source impedance $Z_s$ is irrelevant. We could inject using a Thevenin-equivalent voltage source:




The question left unanswered is what about a given injection point that we choose to employ. What method should we employ to get the open loop gain?

3. Compare an injection point conditions and decide.

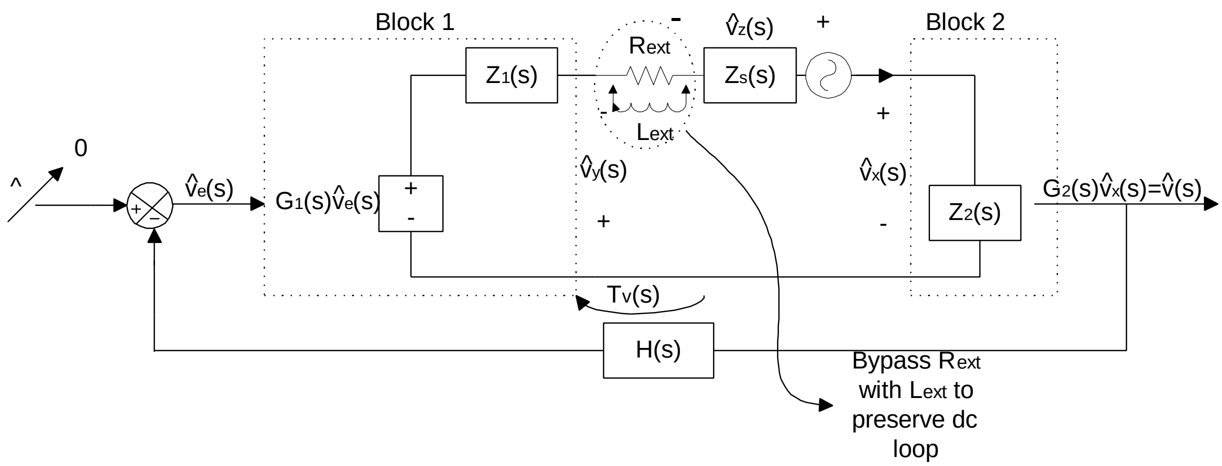
$\frac{Z_1}{Z_2}$ low $\qquad$ $\frac{Z_2}{Z_1}$ low $\qquad$ $\frac{Z_2}{Z_1} \approx 1$

v injection i injection What to do

In later lectures we will cover the case of $Z_1 = Z_2$ separately via the "double null injection method".

## 4. Measuring T(s) in Existing but Unstable Systems

Usually we measure T(s) to avoid instability. To measure $T_m(s)$ in an existing unstable system we first have to stabilize it in order to measure it. Perhaps the easiest way is to kill the loop gain and regain stability is by the insertion of an external impedance, $Z_{ext}$ as shown below. **This instability could break out only under some extreme transient conditions and it may be only transient in nature as well. But it prevents measurements being made.**



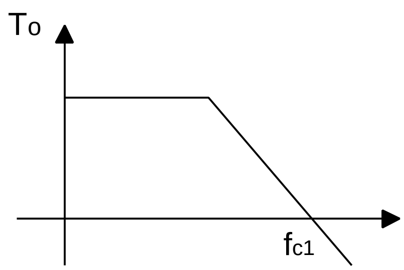

Loop Gain w/o $Z_{ext}$

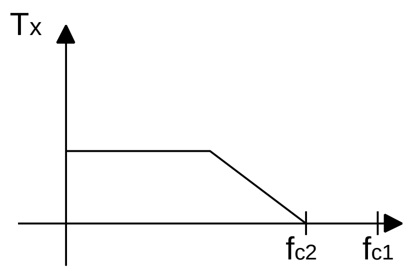
$$T_O \sim \frac{Z_2}{Z_1 + Z_2}$$

Loop Gain with $Z_{ext}$

$$T_X \sim \frac{Z_2}{Z_1 + Z_2 + Z_{ext}}$$

We make $Z_{EXT}$ big to decimate $T_0$( without $Z_{EXT}$) and thus reduce $f_C$ making the previously unstable T(s) stable for the measurement.



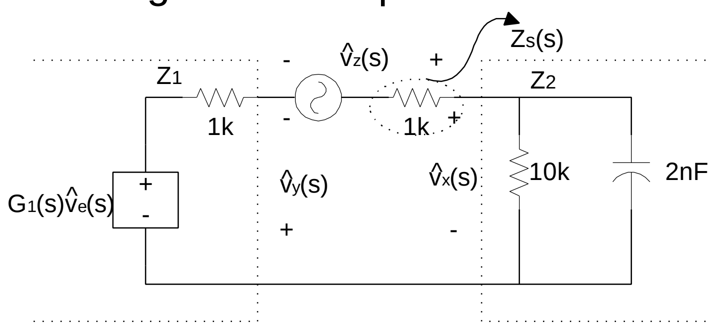



**$f_{c2} << f_{c1}$ Þ Better $f_m$ and should result in a closed loop response free of instability..**

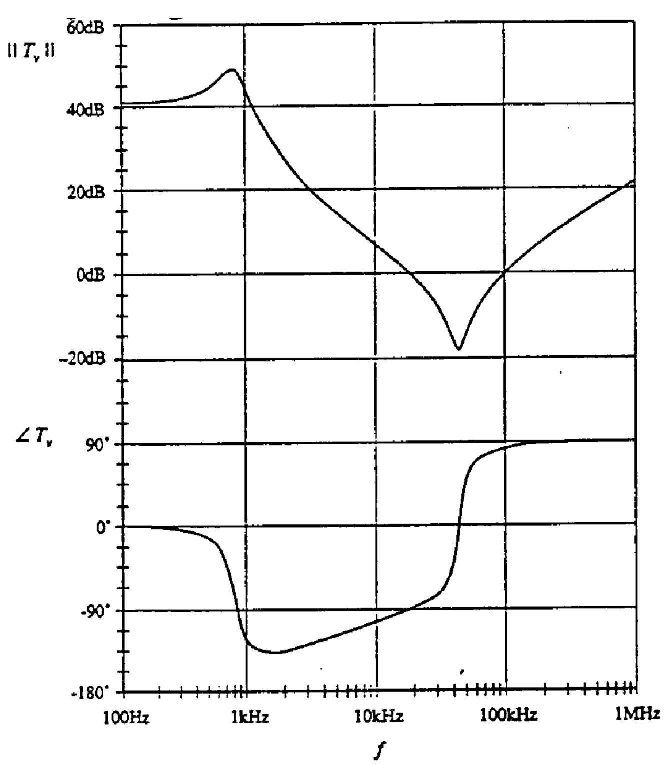
# C. Example: Erickson Pbm 9.10

1. We have a voltage injection point where $\frac{Z_1}{Z_2} \sim \frac{1}{10}$ for all frequencies of interest. What to do with the $T_{MV}(f)$ data we collect? Is it all-believable? Is there a limited frequency range over which we can trust the data?

The voltage insertion point is shown below.




From the insertion point showm we take the $T_{mv}(s)$ data shown below




From measured $T_M(s)$ and associated phase plots, find valid T(s) and $f_{min} \leq f \leq f_{max}$ region of validity for the measured data.

This reduces to a ? of over what f range do we meet the criterion:

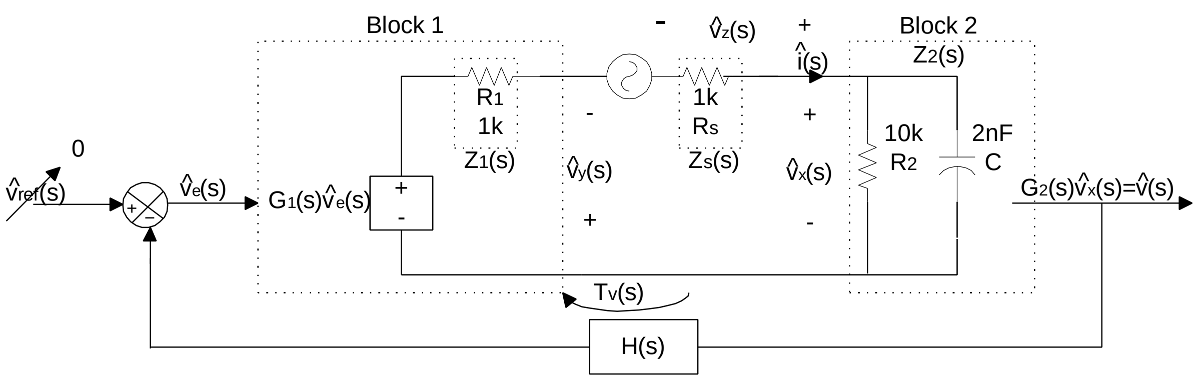
$Z_1(f) << Z_2(f)$. Only there does $T_M(s)$ = actual T(s)

$$T_{mv}(s) = T(s)\left[1 + \frac{Z_1(s)}{Z_2(s)}\right] + \frac{Z_1(s)}{Z_2(s)}$$

For the specific conditions we are considering:

$$\frac{Z_1(s)}{Z_2(s)} = \frac{1K}{10K \| \frac{1}{s\,2nF}} = 0.1 + \frac{s}{5x10^5}$$

$Z_1/Z_2 \approx 1$ for f = 20 kHz and $Z_1/Z_2$ will posses a single zero!
The standard loop block will be as depicted below:




<u>At low f:</u>

$$\frac{Z_1}{Z_2} \sim \frac{R_1 = 1K}{10K \| \frac{1}{s(2nF)}}$$

$Z_2$ is decreasing with f. The frequency where $X_C$ =10K will be an important one:

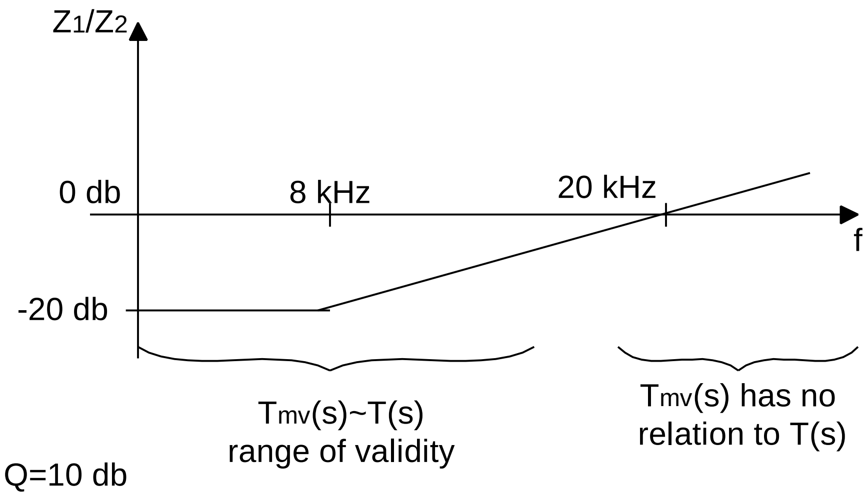
$$\frac{1}{s(2nF)} = 10K \Rightarrow f = \frac{1}{2p\,10^4\,2x10^{-9}} = 8kHz$$

At 8kHz $Z_1 \backslash Z_2$ reduces to 1/5 as $Z_2$ = 10K in parallel with 10 K.

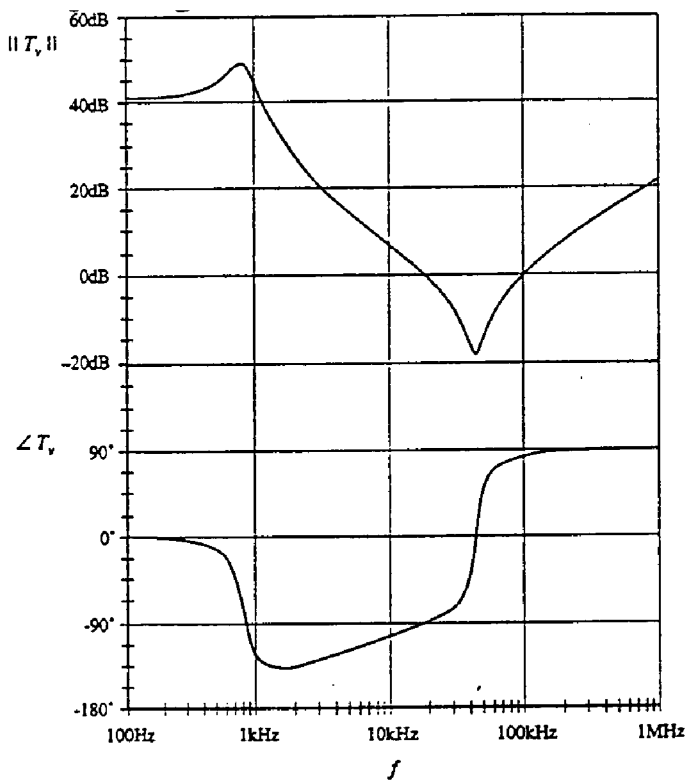
$|Z_2|$ = 10K||10K = 5K. Let's be generous and still consider this a valid condition for $T_{MV}(s)$ to be really T(s).

A still valid condition for $T_{mv}(s)$ is far below 8 kHz $\frac{Z_1}{Z_2} \approx 0.1$ or -20 db. Now @ 8kHz $\frac{Z_1}{Z_2} \sim 0.2$ or -14 db. Finally, $\frac{Z_1}{Z_2} \equiv 1$ or 0 db at f = 20 kHz.

We can plot this ratio by algebra on the graph to yield.




Now the conditions for valid measurements are clearly noted. On the top of page 24 we look at $T_M$ and pontificate as to why the zero seen in the $T_M$ data is NOT BELIEVABLE.




There is a non-believable zero's in $T_M(s)$ because the basic assumption of voltage injection method is invalid for f > 20 kHz since

$$\frac{Z_2}{Z_2} \approx 1$$

Usually T ↓ as f → ∞ And it does not flatten out

From the $T_{mv}(s)$ data <u>below</u> 20 kHz we can guesstimate

- · double pole occurs @ 800 Hz with a "Q" of 10 db
- · single zero occurs @ 3.2 kHz
- · $T_{mvo}$ = 40 db from the DC values

$$T(s) \approx \frac{40\,db(1+s/w_Z)}{(1+\frac{s}{w_o Q}+(\frac{s}{w_o})^2)}$$

In the standard form of the transfer function: $f_o$ = 800 Hz
Q ≈ 10 db or 2.5 and $f_Z$ = 3.2 kHz