Lecture 48
Review of Feedback

HW # 4 Erickson Problems Ch. 9 #'s 7 &9 and questions in lectures

I. Review of Negative Feedback

A. General
1. Overview
2. Summary of Advantages
3. Disadvantages

B. Buck Converter Example
1. Open Loop Transfer Function
a. $G_{vd}(s) = V_o(s)/\hat{d}(s)$
b. $G_{vg}(s) = V_o(s)/V_g(s)$
c. $Z_o(s)$

2. Closed Loop Around V_o & d
a. Loop Gain T(s)

$$\left[\frac{V_{out}}{V_{ref}}\right]_{CL} = \frac{1}{H}\frac{T}{1+T}, \quad [Z_o]_{CL} = \frac{Z_o}{1+T}$$

$$\left[\frac{V_{out}}{V_g}\right]_{CL} = \frac{G_{vg}(s)}{1+T(s)}$$

3. T(s), $\frac{T(s)}{1+T(s)}$ and $\frac{1}{1+T(s)}$
a. T(s) from Laplace Transform
b. T(f) vs f in db units
c. $\frac{T}{1+T}$ from T via Algebra on graph

d. $\frac{1}{1+T}$ plots from T via Algebra on graph

e. $\left[\frac{V_{out}}{V_g}\right]_{CL} \equiv \frac{G_{vd}}{1+T(s)}$ by Algebra or graph

4. Basic Feedback via Pbm 9.2

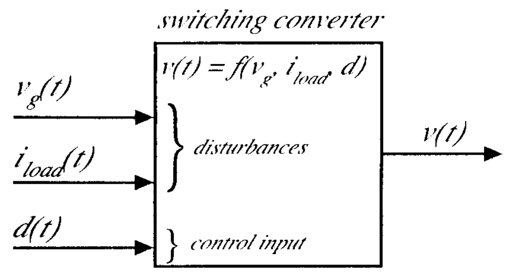
Lecture 48
Review of Feedback

I. Review of Negative Feedback
A. General Issues
1. Overview

We have a switch mode converter, which we wish to stabilize from load and input changes by employing feedback to maintain the output level to a pre-established level.

Objective: maintain constant output voltage $v(t) = V$, in spite of disturbances in $v_g(t)$ and $i_{load}(t)$.

Typical variation in $v_g(t)$: 100Hz or 120Hz ripple, produced by rectifier circuit.

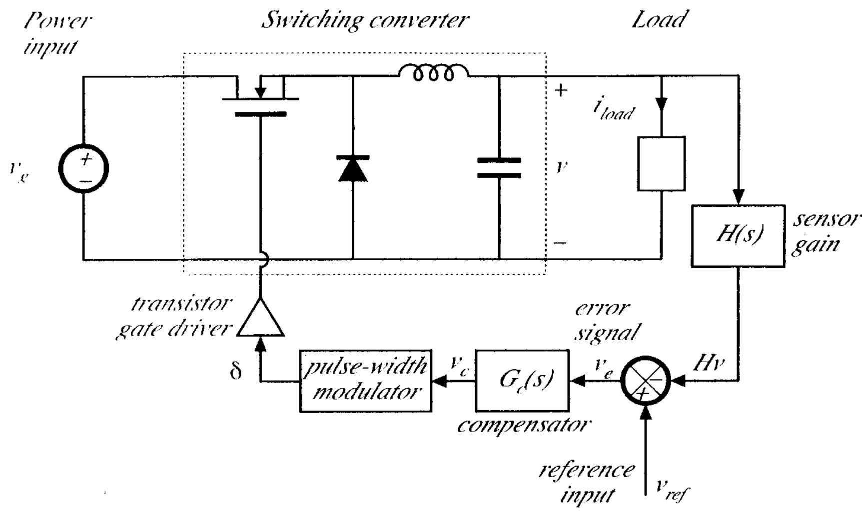


Load current variations: a significant step-change in load current, such as from 50% to 100% of rated value, may be applied.

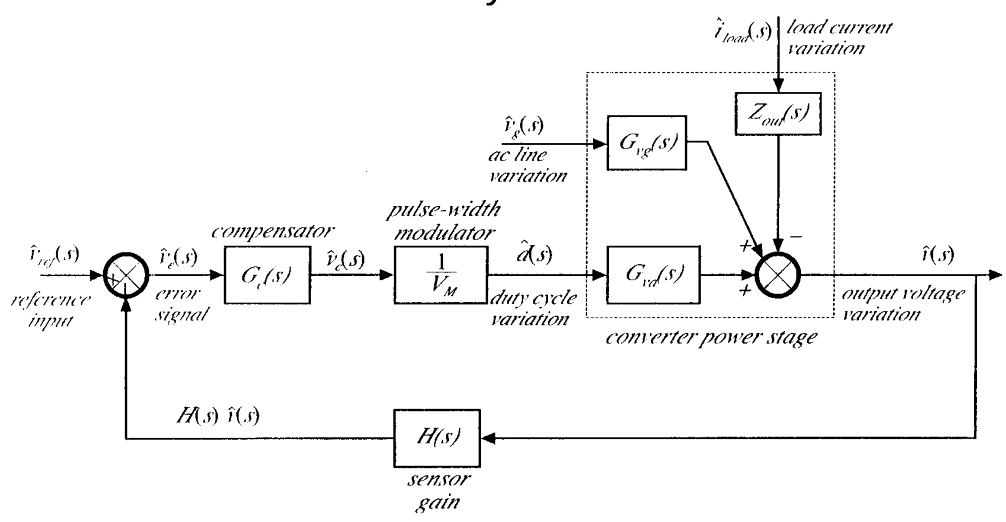
A typical output voltage regulation specification: 5V ± 0.1V.

Circuit elements are constructed to some specified tolerance. In high volume manufacturing of converters, all output voltages must meet specifications.

To implement this feedback we need to sense the output and compare it to a reference value to provide a difference signal, which drives the pulse width modulator to provide the proper value of duty cycle to drive the difference to zero. This is shown on the top of page 4. Note also the use of a compensator-box, which is employed to tailor the open loop response so that when feedback is used to reduce the effect of sudden load and input changes, no oscillation occurs. This use of feedback is artful because the power supply response is slow and if the feedback acts too fast the system may oscillate. The art is to tailor the response of the error amplifier, via the compensator-box components.



Later we will represent each box above by a transfer function, which will both amplitude and phase plots versus frequency called Bode plots. The Bode plot employs logarithms so that when we have a tandem series of transfer functions the combined response is just the sum of the individual Bode responses. In block diagram form we would have a feedback system as shown below.



The block diagram has three components inside the dashed line box on the right. Note that both line voltage variations and load current variations lie outside the feedback loop. Duty cycle variations lie within the voltage feedback loop. We can derive a solution to the block diagram output voltage that includes contributions from V_G, I_{load} and Z_{out} as shown below.

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} \pm \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

which is of the form

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} \pm \hat{i}_{load} \frac{Z_{out}}{1+T}$$

$$with \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M = \text{"}loop\ gain\text{"}$$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

Each of block components determines T(s). While we can easily identify the LOOP GAIN, T(s), for the converter system, we now ask how does it respond to disturbances? And how does feedback on the converter vary as the magnitude of T(s) varies with frequency? Are all frequencies the same for feedback?

Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{d}=0 \\ \hat{i}_{load}=0}}$$

With addition of negative feedback, the line-to-output transfer function becomes:

$$\left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{v}_{ref}=0 \\ \hat{i}_{load}=0}} = \frac{G_{vg}(s)}{1 + T(s)}$$

Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the line-to-output transfer function becomes small.

We have to insure that T(s) is large at frequencies of interest. This is most readily accomplished by employing a high($<10^3$) gain operational amplifier in the feedback loop. Since line frequencies are around 50-0 Hertz and harmonics the DC gain is the determining factor to reduce line frequency variations. What about changes in the output due to load current variations?

Original (open-loop) output impedance:

$$Z_{out}(s) = \pm \left. \frac{\hat{v}(s)}{\hat{i}_{load}(s)} \right|_{\substack{\hat{d}=0 \\ \hat{v}_g=0}}$$

With addition of negative feedback, the output impedance becomes:

$$\left. \frac{\hat{v}(s)}{\pm\, \hat{i}_{load}(s)} \right|_{\substack{\hat{v}_{ref}=0 \\ \hat{v}_g=0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

Feedback reduces the output impedance by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the output impedance is greatly reduced in magnitude.

Here we want T(s) to be large at high frequencies, not just at DC. This insures that the high frequency output impedance with feedback is much less that the open loop output impedance. What about the closed loop reference voltage to output voltage transfer function? In short Z_{OUT} varies with loop gain.

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

$$\left. \frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \right|_{\substack{\hat{v}_g=0 \\ \hat{i}_{load}=0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

If the loop gain is large in magnitude, i.e., $\| T \| >> 1$, then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

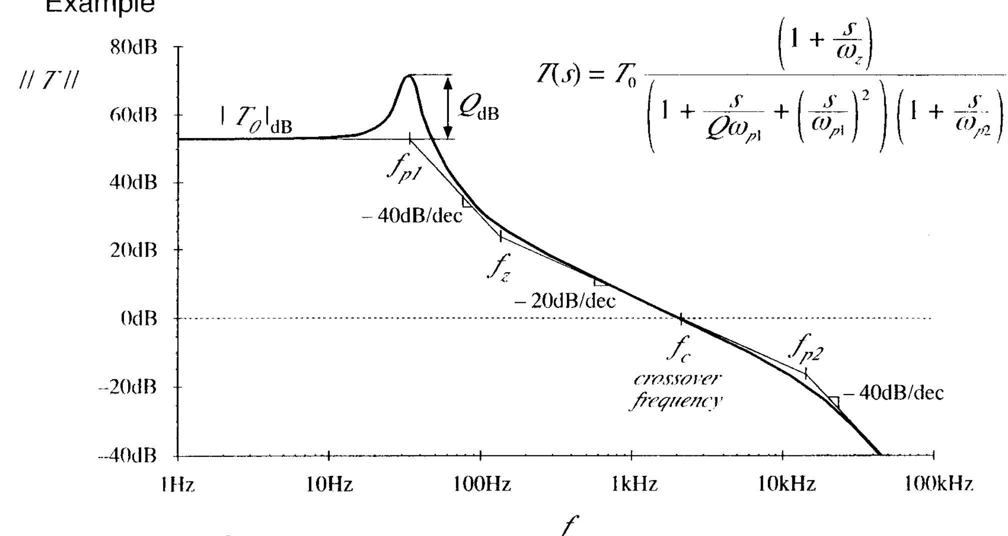
which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

$$\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}$$

That is the output of the converter is some fixed ratio of the reference voltage, if T(s) is LARGE. We can insure this from DC to high frequency by tailoring loop gain T(s). A typical T(s) and its amplitude Bode plot versus frequency is shown in the middle of page 7. All the benefits of feedback only occur when T(s) is LARGE. Since T(s)= H(s) G_C(s) G_{VD}(s) /V_M . We normally rely on a large value of H(s) from the sensor gain block, which often includes an operational amplifier and a compensation network to tailor the loop gain. The H(s) block is in the low power portion of the loop and easy to work with. H(s) often possesses several isolated poles and several isolated zeros. The G_{VD}(s) block is the control duty cycle to output voltage transfer function and usually possesses two poles at the same frequency,ω_{P1}. All togeather the loop gain would contain three poles and one zero as shown below If H(s) had but one pole and one zero.

Example



At the crossover frequency f_c, $\| T \| = 1$

When the loop gain goes to unity all the benefits of feedback are lost. This has implications as discussed below.

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } \| T \| >> 1 \\ T & \text{for } \| T \| << 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{1}{H(s)}$$

This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T/(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s) G_{vd}(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

It also has effects on how the feedback loop effects the line voltage and output current disturbances.

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & for \| T \| >> 1 \\ 1 & for \| T \| << 1 \end{cases}$$

Below the crossover frequency: $f < f_c$ and $\| T \| > 1$

Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\| T \|$

Above the crossover frequency: $f > f_c$ and $\| T \| < 1$

Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances

2. Summary of Feedback Advantages

$$\frac{V_o}{V_{in}} = \frac{A_{0L}}{1 + b\, A_{0L}} \approx \frac{1}{b}$$

1. Advantages
   - V_o variations due to A_{0L} variations are eliminated
   - V_o variations due to L, R & C “tolerances” are reduced for elements inside A_{0L}
   - V_o changes due to power supply variations are reduced

by $\dfrac{1}{1+A_{0L}b}$

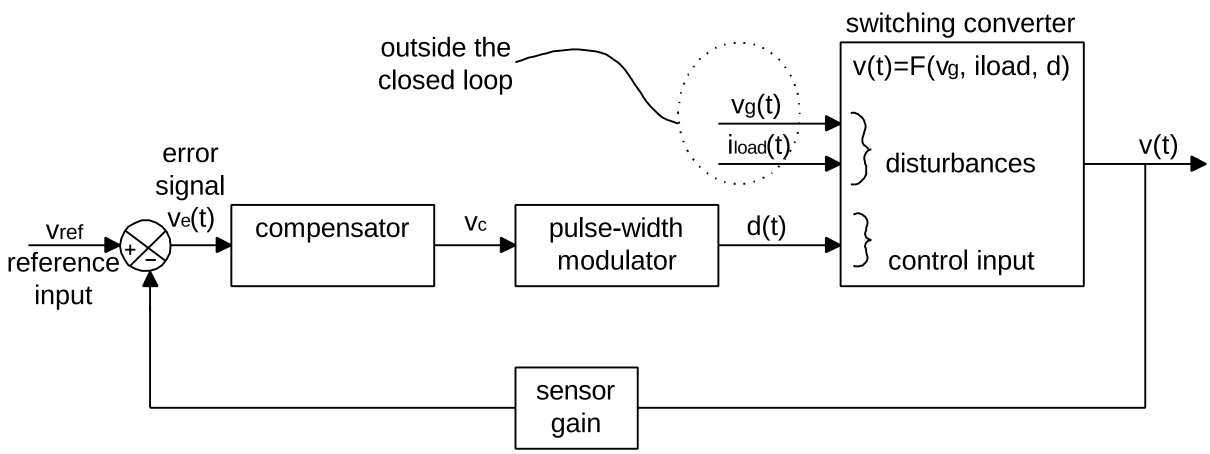
- Z_o reduced by $1 + A_{0L}\beta$ therefore I_o variations, less ΔV_o · Z_{in} increased by $1 + A_{0L}\beta$
- V_o changes due to switch changes over time and temp.

3. Feedback Oscillation and Compensation

- After we close the feedback loop, sometimes, oscillation/instability occurs for frequencies where $A\beta = 1 < 180^o$ or NEARBY this condition. To be safe from undesired oscillation the rule of thumb we will develop later will be:

If $A\beta| = 1$ we require $<A\beta$ be 76^o from 180^o.

To achieve the proper phase shift at unity gain for T(s), we employ a f compensation element inside the feedback loop. This is only a nominal additional expense - it can be done in low power sections.

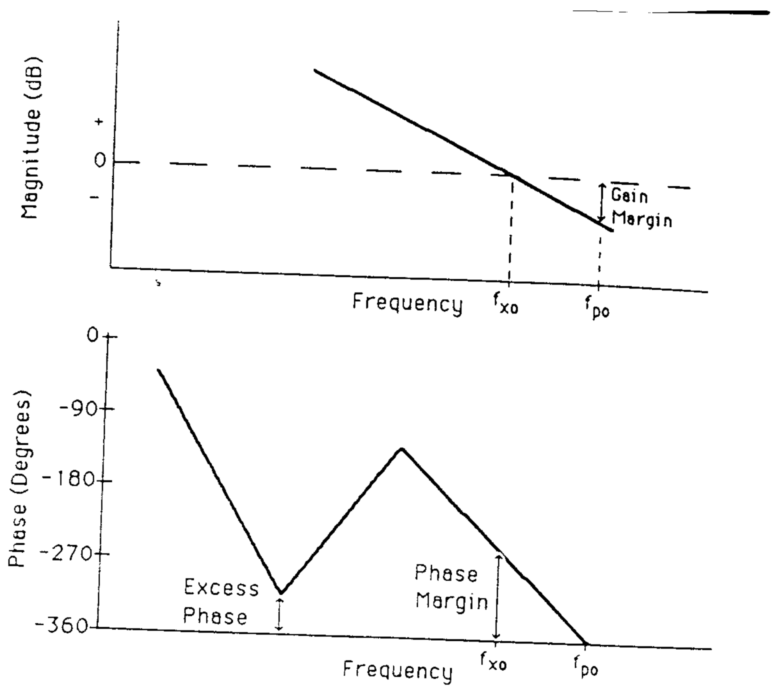


The COMPENSATOR box will contain four possible transfer functions listed in the table below to tailor the LOOP GAIN to achieve both **load regulation and transient response**.

Compensation Type	Load Regulation	Transient Response
Single-pole	Good	Poor
Single-pole with in-band gain limiting	Fair	Good
Pole–zero	Good	Good
2-pole–2-zero	Good	Good

Clearly to achieve both goals, pole-zero combinations are best.

The subject of closed loop stability will be given in detail in Lecture 49, but below we briefly outline the major issues in anticipation. We define phase margin, gain margin and excess phase in the figure below, which contains both amplitude and



phase plots of the OPEN LOOP gain, T(s).

Changes in gain, ΔG, and changes in phase Δϕ along a gain slope are shown for a 20 db per decade slope in the figure below. We will often have to be quantitative about gain and phase changes as we move along gain slopes from one frequency to another. In particular we will add COMPENSATION to the open loop gain via the compensation block to get the proper amplitude and phase plots in order to achieve **the two goals of feedback both improved load regulation and improved transient response.** This is artful, as we must trade one benefit off against anothe**r.**

Feedback Loop Compensation

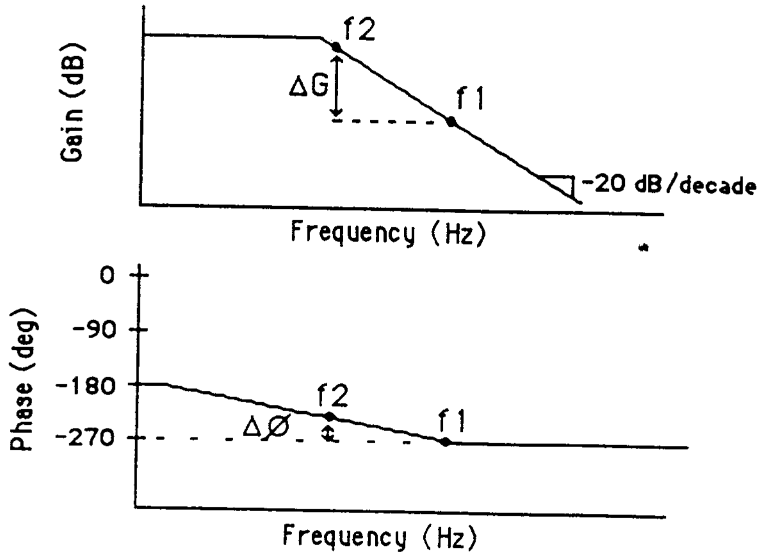


Figure B–8 Illustrating how the mathematical tools are used.

Remember that the compensation box is in series in the loop gain so it is able to modify and tailor both the amplitude and phase responses of T(s) individually. Phase shifts generally kick in at 1/10 the break frequencies for amplitude changes and persist till 10 times the break frequency for amplitude changes. To quantify both phase differences and amplitude differences at changing frequencies in compensation networks we show two rules of thumb below.

-20 db gain slopes have:

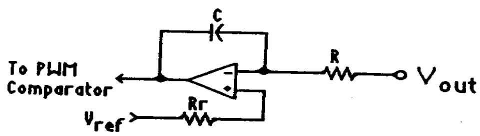
1. $\Delta G(f_2 - f_1) = 20 \log (f_2/f_1)$
2. $\Delta\phi(f_2 - f_1) = \tan^{-1}(f_{2/}\, f_1)$

-40 db gain slopes have:

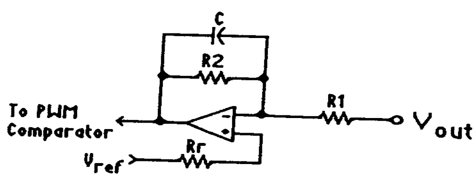
1. $\Delta G(f_2 - f_1) = 40 \log (f_2/f_1)$
2. $\Delta\phi(f_2 - f_1) = 2 \tan^{-1}(f_{2/}\, f_1)$

What a proper amplitude and phase plot for the open loop gain to achieve both **improved load regulation and improved transient response** will be given later in Lecture 49.

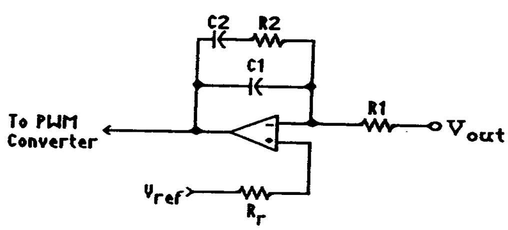
SHOW for HW # 4 that the four op amp circuits below provide the following: (a) a single pole filter, (b) a single pole with gain limiting for achieving both flat high and low frequency response, (c) a single pole/zero combination and (d) a two pole/two zero combination. **Draw both the gain and phase plots for all four circuits indicating the location of all poles and zeros**. Use you junior electronics to solve each op amp circuit.



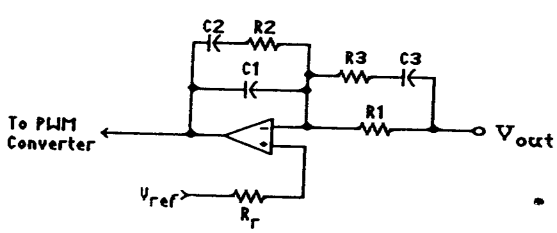
(a)



(b)



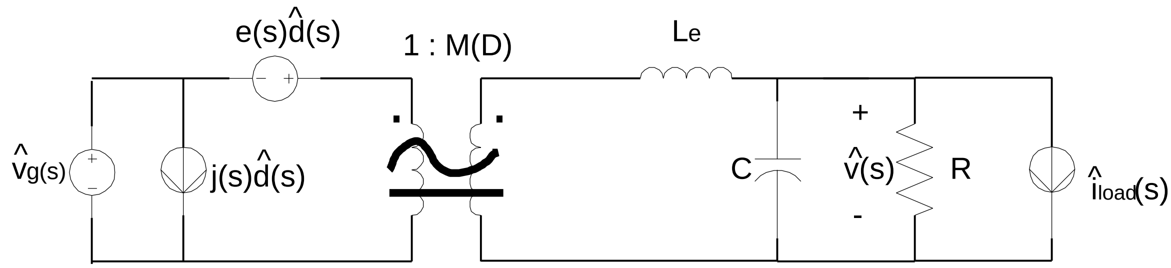
(c)



(d)

B. Buck Converter Example

1. **Open Loop**: Three independent inputs: $\hat{d}$, $\hat{V}_g$, $\hat{i}_{liter}$



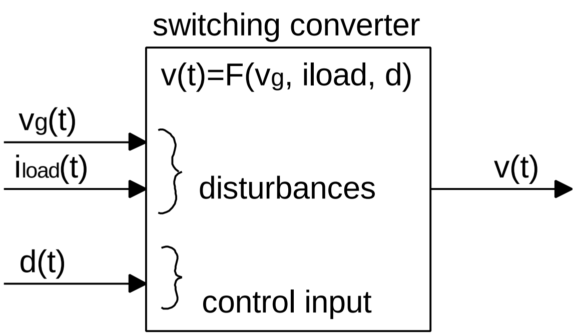
Open loop relations

$$\hat{V}_{out} \equiv G_{vd}(s)\,d(s) + G_{vg}(s)\,V_g(s) - Z_o\,i_l(s)$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{v_g=0\,i_l=0} \quad \left.\frac{\hat{v}(s)}{v_g(s)}\right|_{\hat{d}=0\,i_l=0} \quad \left.\frac{\hat{v}(s)}{i_l}\right|_{v_g=0\,\hat{d}=0}$$

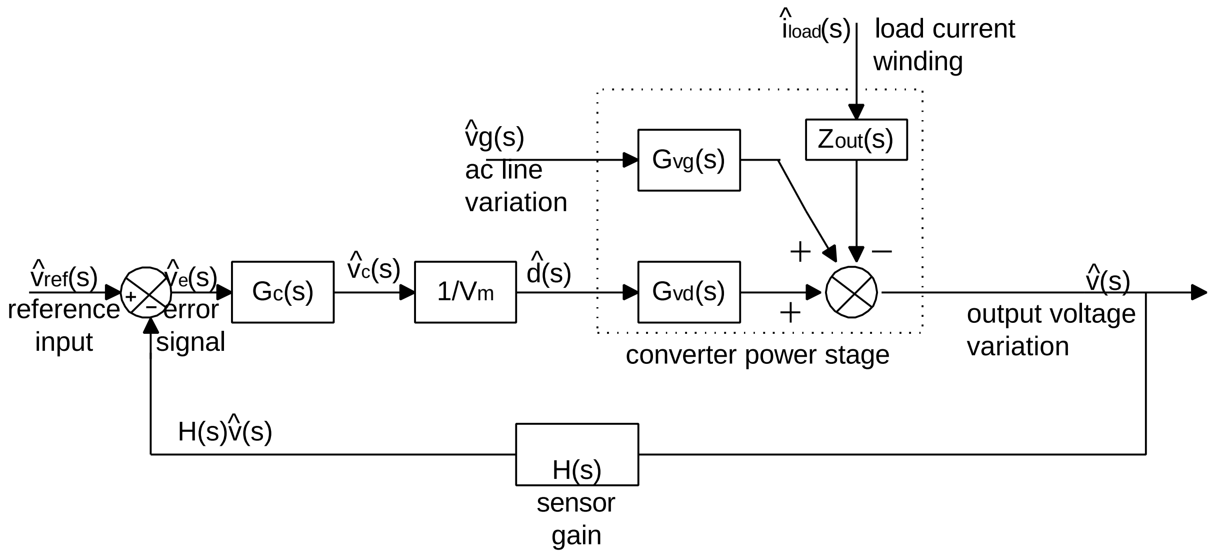
Open loop conditions



V_o is also dependent on frequency of input variation

2. **Closed Feedback Loop** around V_o & V_{ref}

Compare V_o to V_{ref} to drive the duty cycle to drive $V_o \equiv V_{ref}$
Duty cycle control is “the old way.” Modern converters use the “current programmed mode” described in chapter11 of Erickson in addition to the voltage feedback loop. Still we start here, with a simple voltage feedback loop. The system is as shown below:
Open loop w.r.t. i_{load} and V_g



Closed loop w.r.t. (V_o, V_{ref}) via $\hat{d}$ control and the pulse width modulator loop, ultimately ΔV_g, ΔV_{ref} and Δi_{load} will cause ΔV_{out} variation. In the closed feedback loop we find:

$$T \equiv \text{loop gain} = \frac{H\,G_c\,G_{vd}}{V_D} = T(f)$$

↓ varies with frequency

Now Inside Loop Outside Loop

$$\hat{V}_{out} = \frac{\hat{V}_{ref}}{H}\,\frac{T}{1+T} + \hat{V}_g\,\frac{G_{vg}(s)}{1+T} - i_{out}\,\frac{Z_o}{1+T}$$

If |1+T| is 1000 for frequencies around the mains, then

$\frac{\Delta V_o}{\Delta V_g}$ is reduced by 1000 by the application of feedback

⇒say 2V 2nd harmonic @120Hz on V_g only causes 2 mV change in V_o with the loop closed.

$\frac{\Delta V_o}{\Delta i_{out}}$ is also reduced by 1000 ⇒ Z_o with feedback is effectively 1000 times smaller, when we achieve T= 1000 over a limited frequency range.

$$\frac{\hat{V}_{out}}{\hat{V}_{ref}} = \frac{1}{H} \Rightarrow$$ For stable output use(frequency/temperature) compensated resistors etc. 0.1% resistor is only 50 cents

3. Frequency Response of the Feedback Loop

There are three parts: Loop Gain T(s),

$$T(s)/1+T(s) \ \& \ \frac{1}{1+T(s)}$$

a. Consider only T(s) with three Poles and One Zero

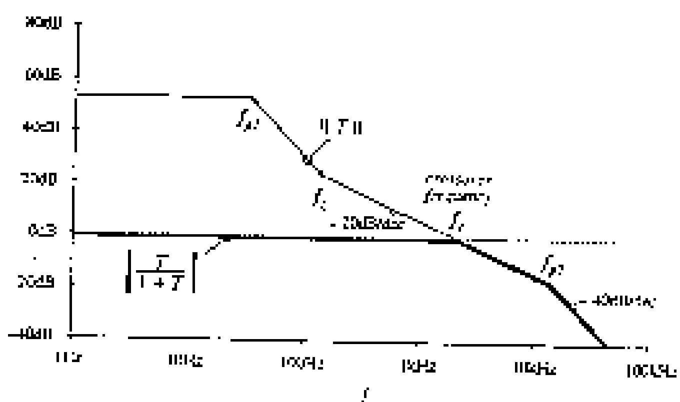
$$T(s) = \frac{T_o \ 1+S/W_z}{\left[1 + \frac{s}{QW_p1} + (\frac{s}{W_{p1}^2})\right]\left[1+s/W_{p_2}\right]}$$

Use low Q approximation to solve for well separated poles or quadratic equation for close roots. If in cubic form just employ a root solver - like on a HP 48 calculator, to get all three poles.

b. Draw Amplitude of T(s) in db vs frequency: T(f)

Use log-log plots, then express all amplitudes in db. We can from pole and zero locations draw both T and T/1+T by hand to a first approximation.

c. T/1+T comes easily from T via Algebra on graph



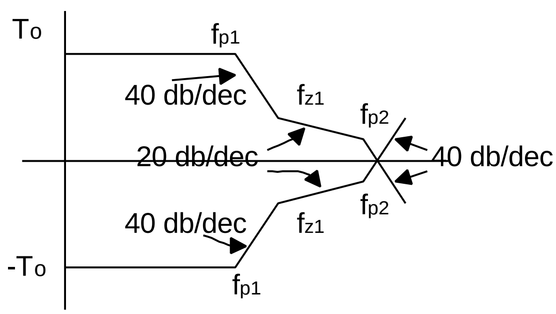
T large: $\frac{T}{1+T} \rightarrow 1$ or "0" db

T small: $\frac{T}{1+T} \rightarrow T$

⇒ Easier to do on graph than using algebra!

Asymptote with a double pole will have a "Q peak" as we saw on page 7. This "Q peak" will have no effect on T/1+T plots since when T is very large T/1+T → 1 anyway. However this "Q peaking of T(s)" will effect other plots such as $\frac{G_{vg}}{1+T}$, $\frac{Z_o}{1+T}$ as we will see below.

d. Consider how to plot 1/1+T from T



Easy to plot 1/T from T

T large: $\frac{1}{1+T} \rightarrow \frac{1}{T}$ ⇒ Algebra on a graph for 1/1+T

T small: $\frac{1}{1+T} \rightarrow 1$ ⇒

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$G_{vd}(s) \quad G_{vg}(s) \quad Z_{out}(s)$

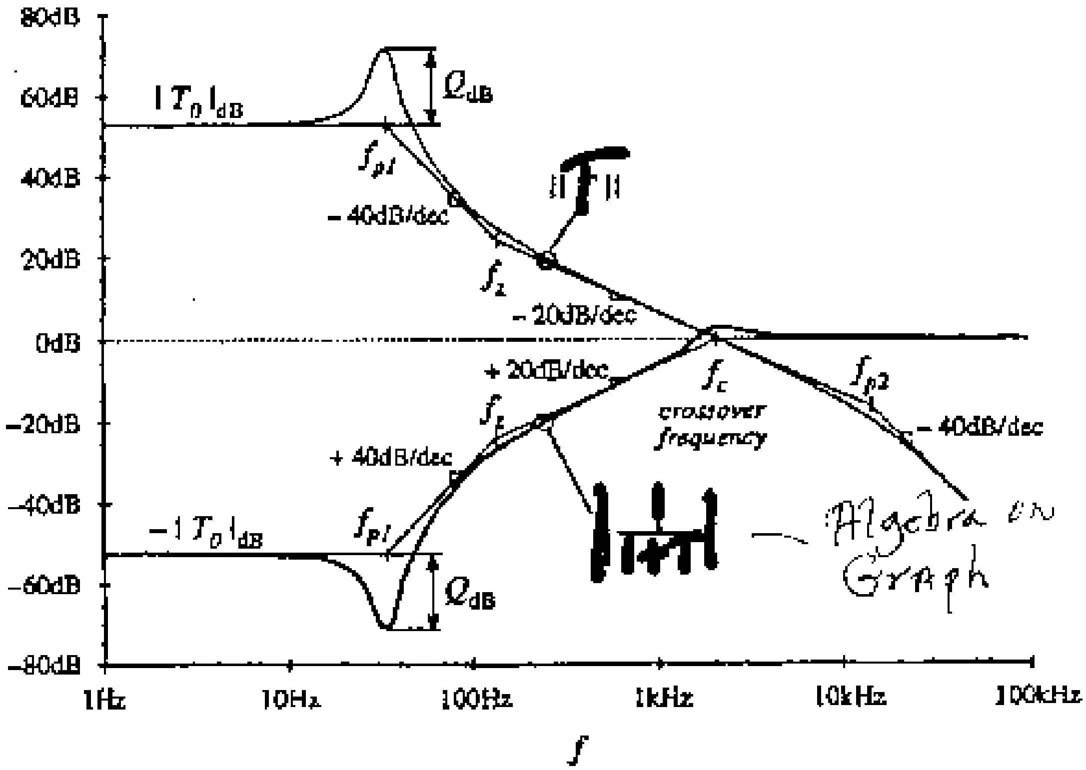
Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \quad \frac{G_{vg}(s)}{1+T(s)} \quad \frac{Z_{out}(s)}{1+T(s)}$

The loop gain:

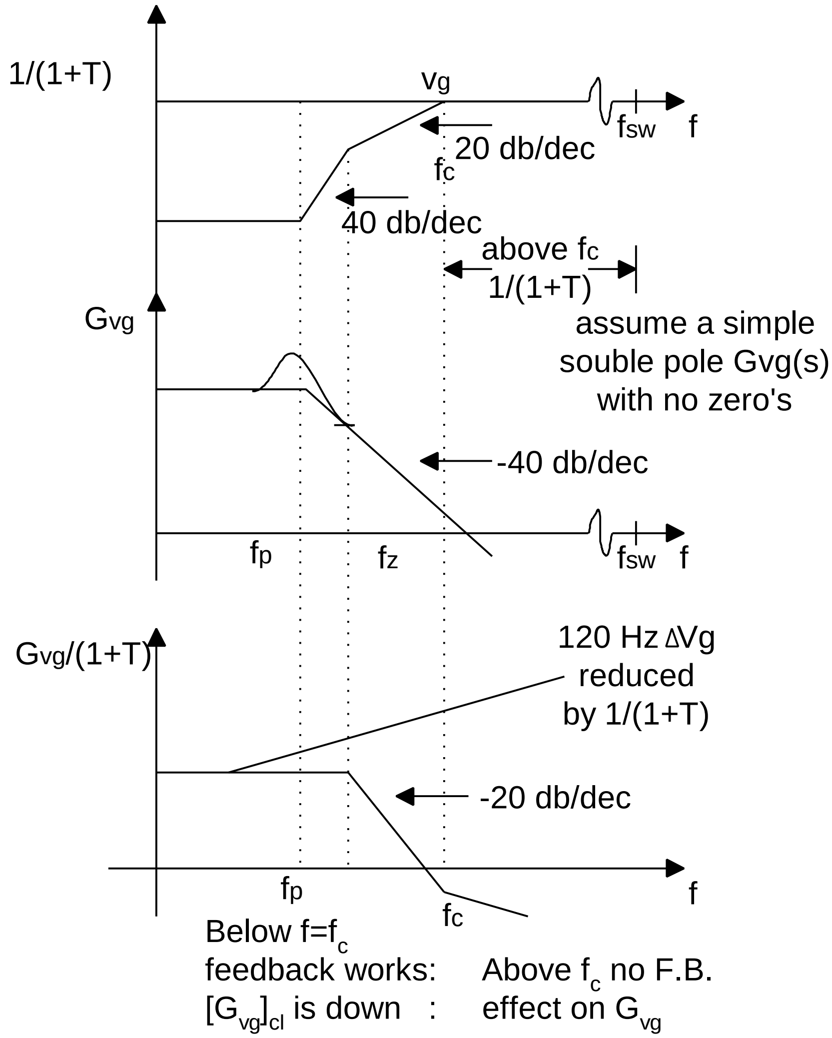
$T(s)$

Next we include the "Q peaking of T(s)" in T and 1/ (1+T)



Given $\frac{1}{1+\mathrm{T}}$ plots versus frequency lets see its effect on a typical open loop G_{VG}(s) factor when we close a feedback loop around G_{VG}(s). The closed loop converter characteristics are found in a three-step process. First plot the 1/(1+T) characteristic versus frequency, from known T versus frequency plots by algebra on the graph. Next plot the open loop G_{VG}(f) characteristics, which have a simple double pole at the same frequency break point. Finally combine the product of the two terms togeather by algebra on the graph.

From DC to the location of the double pole the product is just G_{VG} divided by 1/(1+T). Since above the double pole location ,f_P , G_{VG} falls at 40 db per decade while 1/(1+T) rises at 40 db per decade the product is constant. When the zero at f_Z kick's in in 1/(1+T) the product falls off at 20 db per decade all the way to the crossover frequency of G_{VG} when the product is in the negative db region. See plots below.



4. Let's next consider a simple example. Basic feedback conditions are shown in a complex system such as that of Pbm. 9.2 from Erickson. See page 19

The flyback converter system of Fig. 9.41 contains a feedback loop for regulation of the main output voltage v_1. An auxiliary output produces voltage v_2. The dc input voltage v_g lies in the range 280V ≤ v_g ≤ 380V. The compensator network has transfer function

$$G_c(s) = G_{cm}\left(1 + \frac{\omega_1}{s}\right)$$

where $G_{cm} = 0.05$, and $f_1 = \omega_1/2\pi = 400$Hz.

(a) What is the steady-state value of the error voltage $v_e(t)$? Explain your reasoning.

(b) Determine the steady-state value of the main output voltage v_1.

(c) Estimate the steady-state value of the auxiliary output voltage v_2.

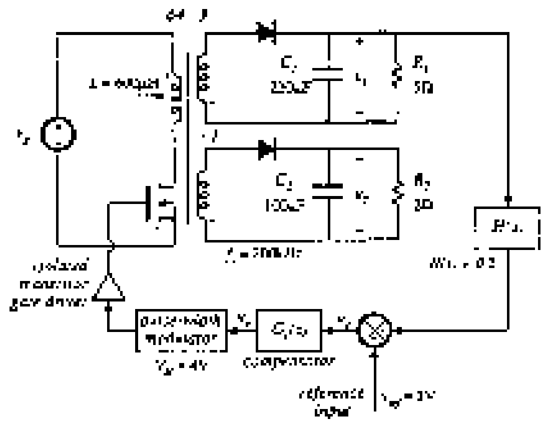


Fig 9.41

a. V(error) should ideally be zero! $\Rightarrow G_c(f=0) \rightarrow \infty$

b. $$\frac{V_{out}}{V_{ref}} = \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \rightarrow \frac{1}{H(s)} \text{ for } T \rightarrow \infty$$

$$V_{out} = \frac{V_{ref}}{H(s)} = \frac{3}{1/5} = 15\,V$$

c. V_2 out = ? n:n_1 n:n_2

V_1 = 15V with 64:3 Trf all else the same for 64:1

V_2 = 1/3 V_1 = 5V