

## Lecture 47

# Switch Mode Converter Transfer Functions: $T_{vd}(s)$ and $T_{vg}(s)$

### A. Guesstimating Roots of Complex Polynomials( this section is optional)

1. Quick Insight  $n=2$
2.  $n^{\text{th}}$  order case
3. Cuk example
4. Forth Order Filter  
Problem 8.19

### B. Buck Boost $T_{vd}(s)$ and $T_{vg}(s)$ Analysis

1. Circuit Model
2. Loop Equations
3.  $T_{vg}(s)$  in standard form : Two poles
4.  $T_{vd}(s)$  in standard form : RHP Zero and two poles
5. Specific Buck-Boost Design
  - a. DC conditions : CCM
  - b. AC Bode plots : CCM

### C. Right Half Plane Zeros

1. Transient Response
2. Origin of RH plane Zero in differing delays
3.  $T(s)$  of a single Zero insight
4. Phase Margin Limitations with RH Plane Zero

## Lecture 47

# Switch Mode Converter Transfer Functions: $T_{vd}(s)$ and $T_{vg}(s)$

### A. Guesstimating Roots of Complex Polynomials : $P(s)$

#### 1. Quick Insight

$$\begin{aligned} a_2 s^2 + a_1 s + 1 &= (1 + a_1 s) \left(1 + \frac{a_2}{a_1} s\right) \\ &= 1 + s \left[ a_1 + \frac{a_2}{a_1} \right] + a_2 s^2 \end{aligned}$$

OK approximation only if  $\frac{a_2}{a_1} \ll a_1$

Why bother?

Left Side Roots  
hard to guess

Right Side Roots are easy to guess

$$s_1 = -\frac{1}{a_1} \quad \text{low f root coeff. if } a_1 \text{ is big}$$

$$s_2 = -\frac{a_1}{a_2} \quad \text{high f root if } a_1 \text{ is big}$$

This is the Widely separated root approximation:

$$\begin{array}{c} \text{X} \text{-----} \text{X} \\ s_1 = 1/a_1 \leftarrow \text{Means} \rightarrow s_2 = a_1/a_2 \\ \text{This is valid if } a_2/a_1 \ll a_1 \end{array}$$

Can we generalize this to higher order polynomials?

Can we avoid root finder programs? It's really academic as root finders are widely available on calculators.

#### 2. $n^{\text{th}}$ order case

a. ideal case for approximation

$$R(s) = 1 + a_1 s + a_2 s^2 + a_n s^n \quad \text{All } a\text{'s are real}$$

Form is arbitrary but USEFUL

Beat/Cajole/Force into standard form

$R(s) = (1 + t_1s)(1+t_2s) \dots(1+t_ns) \rightarrow$  easy to see by expanding out all the factors.  $t$ 's or time constants are either real or complex

$$a_1 = t_1 + t_2 + \dots t_n$$

$$a_2 = t_1(t_2 + t_3 + \dots t_n) + t_2(t_3 + t_4 \dots t_n)$$

$$a_3 = t_1 t_2 (t_3 + t_4 + \dots t_n) + t_2 t_3 (t_4 + t_5 + \dots t_n)$$

.

.

$a_n = t_1 t_2 t_3 \dots t_n \rightarrow$  easy to see by expanding out factors. **We**

**ask under what conditions can solutions be easily**

**approximated?** We can use above for approximating roots if the roots are real and well separated. Then only the first term of each equation is dominant.

$$|t_1| > |t_2| > |t_3| > \dots > |t_n|$$

This is a **big if** and the goal is to be clever in choosing the  $t$ 's in descending order when one does the factoring by hand of polynomials. Few cases are ideal.

If successful,

$$a_1 = t_1 \text{ -----} \rightarrow t_1 = a_1$$

$$a_2 = t_1 t_2 \text{ -----} \rightarrow t_2 = a_2/a_1 \quad \} \text{ Works for simple second order insight case}$$

$$a_3 = t_1 t_2 t_3 \text{ -----} \rightarrow t_3 = a_3/a_2$$

.

.

.

$$a_n = t_1 t_2 \dots t_n \text{ ----} \rightarrow t_n = a_n/a_{n-1}$$

In summary this assumes no inequalities are violated.

**if**  $\rightarrow |a_1| > \left| \frac{a_2}{a_1} \right| > \left| \frac{a_3}{a_2} \right| \ll \dots \ll \frac{a_n}{a_{n-1}}$  and if

the  $P(s) = 1 + a_1s + a_2s^2 + \dots a_ns^N$

$P(s)$  must be in this standard form then the polynomial can be approximately factored by :

$$P(s) = (1 + a_1 s) \left(1 + \frac{a_2}{a_1} s\right) \left(1 + \frac{a_3}{a_1} s\right)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{lowest f} & \text{second f} & \text{third highest f} \\ \text{pole} & \text{pole} & \text{pole} \end{array}$$

### b. What if one inequality is not met

ideal:  $P(s) = (1 + a_1 s) \left(1 + \frac{a_2}{a_1} s\right) \dots \left(1 + \frac{a_n}{a_{n-1}} s\right)$

$$|a_1| \gg \left| \frac{a_2}{a_1} \right| \gg \left| \frac{a_3}{a_2} \right| \dots \gg \left| \frac{a_n}{a_{n-1}} \right|$$

Non-ideal case where one inequality is not satisfied we leave the corresponding roots in quadratic form:

$$\frac{a_k}{a_{k-1}} \gg \frac{a_{k+1}}{a_k}$$

$$P(s) = (1 + a_1 s) \left(1 + \frac{a_2}{a_1} s\right) \dots \left[1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right] \dots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

The quadratic equation below

$$\left[1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right]$$

$$\begin{array}{cc} \downarrow & \downarrow \end{array}$$

we solve as a usual quadratic } Valid only for this pair of roots only }  $a_k > a_{k-1} * a_{k+1}$

When several inequalities are violated then we must use cubic or higher order roots and often the roots are close in value.

**c. Cubic Polynomial:** In general for a cubic polynomial this may occur two ways  $1 + a_1s + a_2s^2 + a_3s^3$

### 1. First and Second Close Roots Case

First guess  $(1 + a_1s)(1 + \frac{a_2}{a_1}s)(1 + \frac{a_3}{a_2}s)$

---X-----X-----X-----

$s_1$        $s_2$

close pair

$s_3$

isolated root

$(1 + a_1s)(1 + \frac{a_2}{a_1}s)$

solution

not valid

$(1 + \frac{a_3}{a_2}s)$

still valid

approximation

Use:  $(1 + a_1s + a_2s^2)$

$$s_3 = -\frac{a_2}{a_3}$$

to solve for unknown

$s_1$  and  $s_2$  just

solve quadratic

quadratic validity

$(1 + a_1s)(1 + \frac{a_2}{a_1}s)$

$$a_k > a_{k-1} a_{k+1}$$

$1 + (a_1 + \frac{a_2}{a_1})s + a_2s^2$

$$k = 1$$

$$a_1 > \frac{a_2}{a_1} - a_1^2 > a_0 a_2$$

$$a_1^2 > a_2$$

Summary:

$1 + a_1s + a_2s^2$

$1 + \frac{a_3}{a_2}s$

⇓

$s_1$  and  $s_2$

⇓

$s_3$

### 2. Second and Third Roots Close Case

---X-----X-----X-----

$s_1$

$s_2$

$s_3$

Original equation  $1 + a_1s + a_2s^2 + a_3s^3$

$$(1+a_1s) \quad \left(1 + \frac{a_2}{a_1}s\right) \quad \left(1 + \frac{a_3}{a_2}s\right)$$

Isolated root  
valid approximation

Invalid First guess  
Close roots  
Not valid

$$\left(1 + \frac{a_2}{a_1}s\right)\left(1 + \frac{a_3}{a_2}s\right)$$

Use:  $1 + \left(\frac{a_2}{a_1}\right)s + \left(\frac{a_3}{a_1}\right)s^2$

$$1 + \left(\frac{a_2}{a_1} + \frac{a_3}{a_2}\right)s + \frac{a_3}{a_1}s^2$$

Use this quadratic to solve for  $s_2$

and  $s_3$  only if

provided

$$\frac{a_3}{a_2} \ll \frac{a_2}{a_1}$$

$$a_k^2 > a_{k-1} a_{k+1}$$

$$a_2^2 > a_1 a_3$$

$$k = 2$$

$$a_2^2 > a_1 a_3$$

Summary:

$$(1 + a_1s)$$

$$1 + \frac{a_2}{a_1}s + \frac{a_3}{a_1}s^2$$

↓

$$s_1 = -1/a_1$$

yields  $s_2$  and  $s_3$

### 3. Cubic Polynomial Case for CuK

CuK converter with D, D' left out for simplicity

$$G(s) = \frac{G_o}{1 + s\left(\frac{L_1 + L_2}{R}\right) + s^2 L_1 C + s^3 \frac{L_1 L_2 C}{R}}$$

$$\begin{matrix} \downarrow & & \downarrow & & \downarrow \\ a_1 & & a_2 & & a_3 \end{matrix}$$

How to factor D(s) depends on circuit values. We want (but not always valid)

$$D(s) = \left(1 + a_1s\right)\left(1 + \frac{a_2}{a_1}s\right)\left(1 + \frac{a_3}{a_2}s\right)$$

$$T(s) = \left[ 1 + s \left( \frac{L_1 + L_2}{R} \right) \right] \left[ 1 + s \frac{RCL_1}{L_1 L_2} \right] \left[ 1 + s \frac{L_2}{R} \right]$$

a.  $a_1 > \frac{a_2}{a_1} > \frac{a_3}{a_2}$  ideal case of well separated roots

If  $L_1$  is chosen  $\gg L_2$  the approximate factorization is OK because:

$\frac{L_1}{R} > RC > \frac{L_2}{R}$  for the ideal case of well separated roots

$\left[ 1 + s \frac{L_1}{R} \right] [1 + s RC] \left[ 1 + s \frac{L_2}{R} \right] \rightarrow$  easy to get roots from circuit

elements. Consider  $D(s) = (1 + a_1 s) (1 + \frac{a_2}{a_1} s) (1 + \frac{a_3}{a_2} s)$

Whereas if no approximations are made:

$$\left[ 1 + s \left( \frac{L_1 + L_2}{R} \right) \right] \left[ 1 + s \frac{RCL_1}{L_1 + L_2} \right] \left[ 1 + s \frac{L_2}{R} \right]$$

What if you find the second inequality is violated  $a_1 \gg \frac{a_2}{a_1} \not\gg \frac{a_3}{a_2}$

due to component choices!

$$\frac{L_1 + L_2}{R} \gg RC \frac{L_1}{L_1 + L_2} \text{ but not } \gg \frac{L_2}{R}$$

This will require a different approach.

b. Second and Third Terms Roots Close

Then for easy root solving we break  $T(s)$  into two terms: one with a single real root and the second with two possibly complex roots.

Still valid

Use this Quadratic

$$\left[ 1 + s \left( \frac{L_1 + L_2}{R} \right) \right] \left[ 1 + sRC \frac{L_1}{L_1 L_2} + s^2 C^* L_1 || L_2 \right]$$

Again this is valid only if both  $L_1 \gg L_2$  and  $L_1/R \gg RC$

$$\left[1 + s \frac{L_1}{R}\right] \left[1 + sRC + s^2 CL_1\right]$$

↓

If also we find low Q for the quadratic then we get two well separated roots from the quadratic term as well

### c. First and Second Terms Have Close Roots

This is a case when the first inequality is violated.

$$\frac{L_1 + L_2}{R} \gg \frac{RC L_1}{L_1 + L_2} \gg \frac{L_2}{R}$$

The inequalities are the same as  $L_1 > L_2$  and  $RC > L_2/R$ . It is no longer required that  $L_1/R \gg RC$

Return to Quadratic  $(1 + s \frac{L_2}{R})$  form for first and second roots  $s_1$

and  $s_2$ . We still have a valid approx. for  $s_3 = -a_2/a_3 = L_2/R$  and the quadratic is simply

$$\left(1 + s \left(\frac{L_1 + L_2}{R}\right) + s^2 L_1 C\right) = \left(1 + s \frac{L_1}{R} + s^2 L_1 C\right)$$

↓

Solve this quadratic for  $s_1$  and  $s_2$

## 4. Fourth Order Polynomial from a two section L-C Filter

Erickson Pbm. 8.19.

### a. Two Section L-C Filter

A two-section L-C filter has the following transfer function:

$$G(s) = \frac{1}{1 + s \left(\frac{L_1 + L_2}{R}\right) + s^2 (L_1 (C_1 + C_2)) + s^3 \left(\frac{L_1 L_2 C_1}{R}\right) + s^4 (L_1 L_2 C_1 C_2)}$$

The element values are:

R 50 mW

C<sub>1</sub> 680 mF

C<sub>2</sub> 4.7 mF



$$\begin{array}{ccc} L_1 & 500 \text{ mH} & L_2 & 50 \text{ mH} \\ & & & 1 \end{array}$$

$$G(s) = \frac{1}{1 + s \left( \frac{L_1 + L_2}{R} \right) + s^2 [4(C_1 + C_2) + L_2 C_2] + s^3 \left( \frac{L_1 L_2 C_1}{R} \right) + s^4 (L_1 L_2 C_1 C_2)}$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ a_1 & a_2 & a_3 & a_4 \end{array}$$

$$R = 50 \text{ m}\Omega$$

$$C_1 = 680 \text{ mF}$$

$$C_2 = 4.7 \text{ mF}$$

$$L_1 = 500 \text{ mH}$$

$$L_2 = 50 \text{ mH}$$

(a) Factor  $G(s)$  into approx. real and quadratic poles.

Compute  $a_1$   $a_2$   $a_3$   $a_4$  and  $t_1$   $t_2$   $t_3$   $t_4$

$$a_1 = 0.011$$

$$t_1 = a_1 = 0.011$$

$$a_2 = 342.6 \times 10^{-9}$$

$$t_2 = \frac{a_2}{a_1} = 31.15 \times 10^{-6} \text{ Non ideal sequence}$$

$$a_3 = 340 \times 10^{-12}$$

$$t_3 = \frac{a_3}{a_2} = 991.4 \times 10^{-6} \text{ we must use a}$$

quadratic to solve  $s_2, s_3$

$$a_4 = 80 \times 10^{-18}$$

$$t_4 = \frac{a_4}{a_3} = 235.3 \times 10^{-9}$$

Check:  $a_1 \gg \frac{a_2}{a_1}$  but  $\frac{a_2}{a_1}$  is not  $> \frac{a_3}{a_2}$

$a_1 \gg \frac{a_2}{a_1} \gg \frac{a_1 a_4}{a_2^2} \gg \frac{a_4}{a_3}$  Valid to break out one quadratic

$$(\Rightarrow) 0.011 \gg 31.15 \times 10^{-6} \gg 2.81 \times 10^{-6} \gg 235.3 \times 10^{-9}$$

$$\Rightarrow G(s) = \frac{1}{(1 + a_1 s) \left( 1 + \frac{a_2}{a_1} s + \frac{a_3}{a_1} s^2 \right) \left( 1 + \frac{a_4}{a_3} s \right)}$$

$$\approx \frac{1}{(1 + \frac{s}{w_o})(1 + \frac{s}{Qw_1} + \frac{s^2}{w_1^2}) + (1 + \frac{s}{w_2})}$$

Compare coefficients, we get

$$w_o = \frac{1}{0.011} = 90.9 \text{ rad/s} \Rightarrow f_o = \underline{14.47 \text{ Hz}}$$

$$w_1 = \sqrt{\frac{a_1}{a_3}} = 5687.96 \text{ rad/s} \Rightarrow f_1 = \underline{905.3 \text{ Hz}}$$

$$Q_{w_1} = \frac{a_1}{a_2} \Rightarrow Q = \underline{5.65 \text{ db}}$$

$$w_2 = \frac{a_3}{a_4} = 4255319 \text{ rad/s} \Rightarrow f_2 = \underline{677.3 \text{ KHz}}$$

Justification:

Actual exp.  $G(s) = \frac{1}{a_0 + a_1s + a_2s^2 + a_3s^3 + a_4s^4}$

$$\begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ A_0 & A_1 & A_2 & A_3 & A_4 & \end{array}$$

Approx. exp.

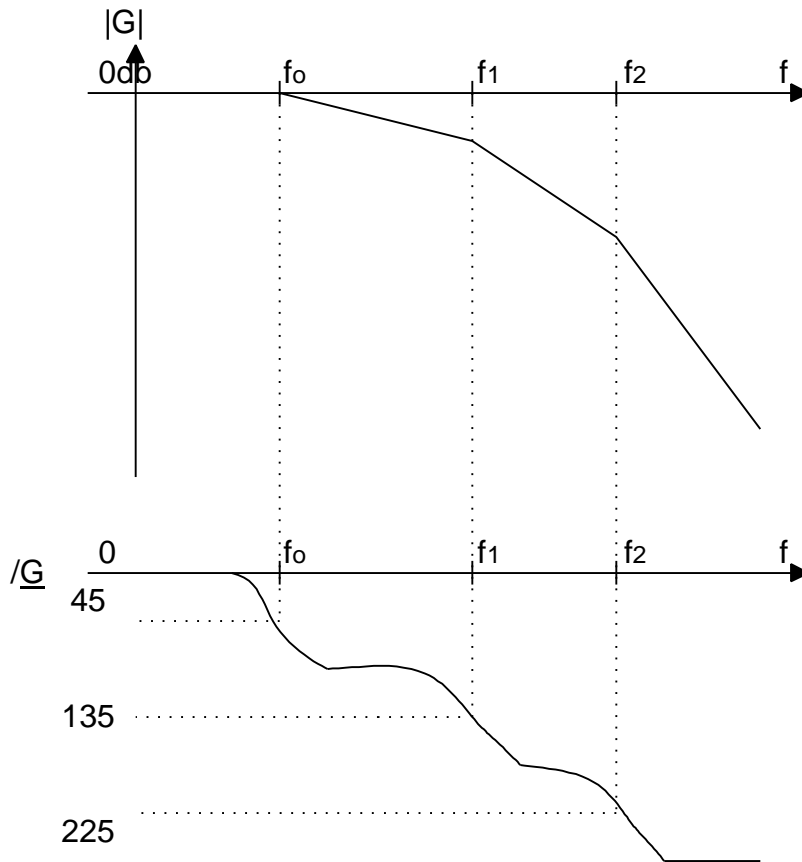
$$G(s) = \frac{1}{1 + (\frac{a_2}{a_1} + a_1 + \frac{a_4}{a_3})s + (a_3 + a_2 + \frac{a_2 a_4}{a_1 a_3})s^2 + (a_3 + \frac{a_4}{a_3} + \frac{a_4 a_2}{a_4 s^2})s^3 + \frac{a_4}{a_4} s^4}$$

$$\begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ A_0 & A_1 & A_2 & A_3 & A_4 & \end{array}$$

Numerical values:

Pole Finder

	Actual	Factoring Approx.
A <sub>0</sub>	1	1
A <sub>1</sub>	1.1 x 10 <sup>-2</sup>	1.103 x 10 <sup>-2</sup>
A <sub>2</sub>	3.43 x 10 <sup>-7</sup>	3.76 x 10 <sup>-7</sup>
A <sub>3</sub>	3.4 x 10 <sup>-10</sup>	3.4008 x 10 <sup>-10</sup>
A <sub>4</sub>	7.99 x 10 <sup>-17</sup>	7.99 x 10 <sup>-17</sup>



We wish to reduce  $Q$  from 5.65 db to 2 db without changing break frequencies. Is this possible?

(c) Now  $Q = 2$

$$\text{From 1, 2} \rightarrow \frac{a_2}{a_1} = \frac{1}{Qw_1}, \quad \frac{a_3}{a_1} = \frac{1}{w_1^2} \rightarrow w_1 = \sqrt{\frac{a_1}{a_3}}$$

$$\Rightarrow Q \sqrt{\frac{a_1}{a_3}} = \frac{a_1}{a_2} = \frac{\sqrt{a_1 a_3}}{a_2}$$

$$\text{but } C_1 \gg C_2 \Rightarrow$$

$$L_1 \gg L_2 \Rightarrow \text{approx. } \begin{aligned} a_1 &\approx \frac{L_1}{R} \\ a_2 &\approx L_1 C_1 \\ a_3 &\approx \frac{L_1 L_2 C_1}{R} \\ a_4 &\approx L_1 L_2 C_1 C_2 \end{aligned}$$

$$\text{Then } Q = \frac{\sqrt{\frac{L_1}{R} \frac{L_1 L_2 C_1}{R}}}{L_1 C_1} = \frac{1}{R} \sqrt{\frac{L_2}{C_1}}$$

$$w_0 = \frac{1}{a_1} = \frac{R}{L_1}$$

$$w_1 = \sqrt{\frac{a_1}{a_3}} = \frac{1}{\sqrt{L_2 C_1}}$$

$$w_2 = \frac{a_3}{a_4} = \frac{1}{RC_2}$$

So  $Q$  depends only on  $R$ ,  $L_2$ ,  $C_1$ . Let's change  $L_2$  and  $C_1$ .  
Can't change  $R$  as it changes  $w_0$

$$R = 50 \text{ m}\Omega, \quad Q = 2$$

$$\Rightarrow L_2 = 0.01 C_1 \text{ and } C_1 = \frac{10}{w_1} = \frac{10}{5688} = 1.76 \text{ mF}$$

$$L_2 = 0.01 C_1 = \underline{17.581 \text{ mH}}$$

Check:

$$w_1 = \sqrt{\frac{a_1}{a_3}} = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C_1}} = 5687.96 \text{ rad/s}$$

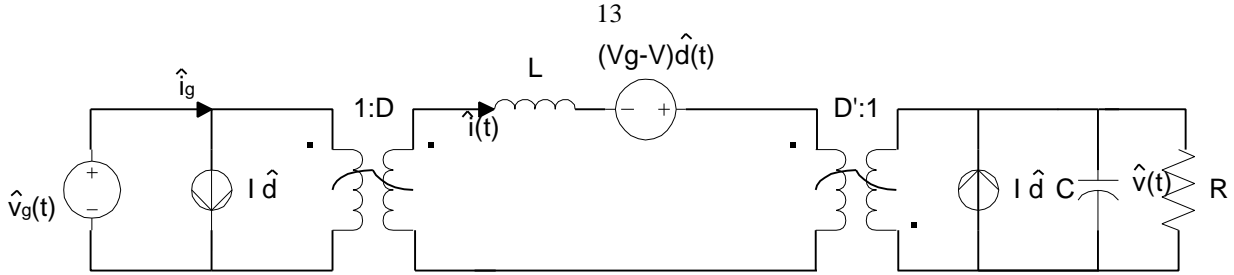
$$\Rightarrow f = 905.3 \text{ Hz} \Rightarrow \text{same}$$

## B. Buck-Boost $T_{vd}(s)$ and $T_{vg}(s)$ Analysis

This is the crucial material for this lecture as part A can be replaced by using a root solver on a basic calculator.

### 1. Equivalent ac Small Signal model

The Buck-Boost small signal circuit model was developed earlier.



The buck-boost small signal equations follow.

Small-signal ac equations of the buck-boost converter, derived in section 7.2:

$$L \frac{d\hat{i}(t)}{dt} = D\hat{v}_g(t) + D'\hat{v}(t) + (V_g - V)\hat{d}(t)$$

$$C \frac{d\hat{v}(t)}{dt} = -D'\hat{i}(t) - \frac{\hat{v}(t)}{R} + I\hat{d}(t)$$

$$\hat{i}_g(t) = D\hat{i}(t) + I\hat{d}(t)$$

We aim to develop these equations to yield **two separate transfer functions** as described below  $G_{Vg}$  and  $G_{Vd}$ :

The converter contains two inputs,  $\hat{d}(s)$  and  $\hat{v}_g(s)$  and one output,  $\hat{v}(s)$

Hence, the ac output voltage variations can be expressed as the superposition of terms arising from the two inputs:

$$\hat{v}(s) = G_{vd}(s)\hat{d}(s) + G_{vg}(s)\hat{v}_g(s)$$

The control-to-output and line-to-output transfer functions can be defined as

$$G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s)=0} \quad \text{and} \quad G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\hat{d}(s)=0}$$

This is our goal to extract these two functions for the buck-boost from the equivalent circuit. Setting one input to zero will result in great simplifications to the loop equations of the full circuit.

2. Loop Equations:

$$\hat{i}_g(t) = D\hat{i}(t) + I\hat{d}(t) \quad (\text{for the first circuit loop on left above})$$

$$L \frac{d\hat{i}}{dt} = D v_g(t) + D'v(t) + (v_g - v)\hat{d} \quad (\text{middle circuit loop})$$

$$C \frac{dv}{dt} = -D\hat{i} - \frac{\hat{v}(t)}{R} + I\hat{d}(t) \quad (\text{output loop})$$

Next do the Laplace transform for the small signal output voltage.

$$\hat{v}(s) \equiv G_{vd}(s)\hat{d}(s) + G_{vg}(s)\hat{v}_g(s)$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \frac{\hat{v}(s)}{\hat{d}(s)} \Big|_{\hat{v}_g(s)=0} & & \frac{\hat{v}(s)}{\hat{v}_g(s)} \Big|_{\hat{d}(s)=0} \end{array}$$

Simple but tedious algebra leads to:

$$sC\hat{v}(s) = -\frac{D'}{sL} \left( D\hat{v}_g(s) + D'\hat{v}(s) + (V_g - V)\hat{d}(s) \right) - \frac{\hat{v}(s)}{R} + I\hat{d}(s)$$

$$\hat{v}(s) = \frac{-DD'}{D'^2 + s\frac{L}{R} + s^2LC} \hat{v}_g(s) - \frac{V_g - V - sLI}{D'^2 + s\frac{L}{R} + s^2LC} \hat{d}(s)$$

write in normalized form:

$$\hat{v}(s) = \left( -\frac{D}{D'} \right) \frac{1}{1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}} \hat{v}_g(s) - \left( \frac{V_g - V}{D'^2} \right) \frac{\left( 1 - s\frac{LI}{V_g - V} \right)}{1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}} \hat{d}(s)$$

Or in the form, that we will use later, after putting DC analysis in.

$$\hat{v}(s) = -\frac{V_g - V - sLI}{D'^2 + s\frac{L}{R} + s^2LC} \hat{d}(s) - \frac{DD'\hat{v}_g(s)}{D'^2 + s\frac{L}{R} + s^2LC}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \text{yields } T_{vd}(s) & & \text{yields } T_{vg}(s) \\ T_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} & & T_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} \end{array}$$

### 3. Line to Output Transfer Function

Hence, the line-to-output transfer function is

$$G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} \Big|_{\hat{d}(s)=0} = \left(-\frac{D}{D'}\right) \frac{1}{1 + s \frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}}$$

which is of the following standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

In standard form, such that coefficients of  $s^0$  are unity,  $\Rightarrow$

The line (mains) to output transfer function is:

$$T_{vg}(s) = \left(\frac{-D}{D'}\right) \frac{1}{1 + s \frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}} \text{ Pole location in the dynamic transfer}$$

function depend on the **choice of DC operating point, D**.

$$L_{\text{eff}}(D) \equiv L/(D')^2 = f(D, L, C, R).$$

In standard quadratic pole pair form:

$$T_{vg}(s) = \frac{T_{go} \text{ (constant low f gain)}}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \text{ with Q in db units}$$

Identify like terms

$$T_{go} \equiv -\frac{D}{D'} \quad \text{Buck-boost Gain @ dc}$$

$$\omega_0 \equiv \frac{1}{\sqrt{L_{\text{eff}} C}} = \frac{D'}{\sqrt{LC}} \quad \text{The dynamic transfer function rollover}$$

frequency depends on the choice of DC operating point  $f(D')$

$$\frac{1}{Q\omega_0} = \frac{L_{\text{eff}}}{R}$$

$$Q = D'R \sqrt{\frac{C}{L}} \quad \text{Note also that the second order Q also depends on}$$

the DC operating point chosen for the converter  $Q = f(D')$

**4. Control to Output Transfer Function:**  $T_{vd}(s) \equiv \frac{\hat{v}(s)}{\hat{d}(s)} \Big|_{\hat{v}_g(s) = 0}$

$$T_{vd}(s) = f(D', V_g, V, I, L, C, R)$$

↓

All DC voltages/currents as well as D and component values of L and C effect the dynamic response.

In standard form such that  $s^0$  coefficients are unity:

$$T_{vd}(s) = \left[ -\frac{V_g - V}{D'^2} \right] \frac{1 - s \frac{LI}{V_g - V}}{1 + s \frac{L_{eff}}{R} + s^2 L_{eff} C}$$

or in the standard form below,

$$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} \Big|_{\hat{v}_g(s) = 0} = \left( -\frac{V_g - V}{D'^2} \right) \frac{\left( 1 - s \frac{LI}{V_g - V} \right)}{\left( 1 + s \frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2} \right)}$$

Standard form:

$$G_{vd}(s) = G_{d0} \frac{\left( 1 - \frac{s}{\omega_z} \right)}{\left( 1 + \frac{s}{Q\omega_0} + \left( \frac{s}{\omega_0} \right)^2 \right)}$$

Poles of  $T_{vd}(s)$  are the same as poles of  $T_{vg}(s)$ , but the zero is unique. **It is a right half plane zero.** We effect  $w_z$  location by choice of all DC values for  $V_g, V, I$  etc.  $w_z(\text{RHP ZERO}) = D'R/DL$  we will show below. Compare to standard. quadratic form:

$$\frac{T_{d0}(\text{constant low f gain})[1 - s/w_z]}{1 + \frac{s}{Q\omega_0} + \left( \frac{s}{\omega_0} \right)^2} = T_{VD}(s)$$

It's easy to design the DC gain for Buck-Boost given  $V(\text{out})$  and



the duty cycle.

$$T_{do}(dc) = \frac{V_g - V}{D'^2} = -\frac{V_g}{D'^3} = \frac{V}{DD'^2}$$

Recall for Buck-Boost DC conditions:  $\frac{V}{V_g} = -\frac{D}{D'}$  and  $I = -\frac{V}{R} \frac{1}{D'}$

Be sure to note that I(DC) is in inner loop of the buck-boost circuit model on page 12. Then we find from the DC conditions the zero location:

$$\omega_z = \frac{V_g - V}{LI} = \frac{D'}{D} \frac{R}{L} \quad \text{right-half plane}$$

$$G_{i/o} = -\frac{V_g - V}{D'^2} = -\frac{V_g}{D'^3} = \frac{V}{D D'^2}$$

$$\omega_z = \frac{V_g - V}{LI} = \frac{D' R}{D L} \quad (\text{RHP})$$

$$\omega_0 = \frac{D'}{\sqrt{LC}}$$

$$Q = D'R \sqrt{\frac{C}{L}}$$

— Simplified using the dc relations:  $V = -\frac{D}{D'} V_g$   
 $I = -\frac{V}{D'R}$

Next we try to get more specific and quantitative.

## 5. Specific Buck-Boost Design

$$\begin{array}{ll} D = 0.6 & R = 10 \\ V_g = 30V & L = 160\text{mH} \\ & C = 160\text{mF} \end{array} \quad \left. \vphantom{\begin{array}{l} D \\ V_g \\ \end{array}} \right\} f_{sw} =$$

### a. We know DC operating conditions

$$V = -\frac{.6}{.4} 30 = -45V$$

$$I = \frac{-45}{10} \frac{1}{.4} = 11.25A \quad \text{in middle loop of model, on page 12.}$$

We also find  $I_{\text{out}} = -4.5A$  in the outer loop of the page 12 circuit.  
Recall that  $I(\text{middle}) = I_{\text{out}} + I_{\text{in}}$ ? for CCM conditions

**b. AC values in std. transfer functions** (using db for plots) for these DC conditions.

First a  $T_{\text{VG}}(s)$  having only poles is considered.

$$G_{\text{go}} = \frac{D}{D'} = 1.5 \text{ or } 3.5 \text{ in db units}$$

$$f_o = \frac{\omega_o}{2p} = \frac{D'}{2p\sqrt{LC}} = 400 \text{ Hz}$$

$$Q = D'R\sqrt{\frac{C}{L}} = 4 \text{ in normal units or } 12 \text{ in db units}$$

$$T_{\text{vg}}(s) = - \frac{D}{D'} = 1.5 = 3.5\text{db}$$

$$1 + \frac{s}{(12)(2pf_o)} + \left(\frac{s}{2pf_o}\right)^2$$

$\downarrow \quad \downarrow$   
 Q    400 Hz    low resonance

$$Q = D'R\sqrt{C/L} = 4 = 12\text{db}$$

Now for a  $T_{\text{VD}}(s)$  having a right hand plane zero we have a new set of values.

$$G_{\text{do}} = \frac{V}{D(D')^2} = 469 \text{ or } 53.4 \text{ db}$$

$$f_z = \frac{D'R}{2pDL} = 6.6 \text{ kHz}$$

$$T_{\text{vd}}(s) = \frac{G_{\text{do}} \left(1 - \frac{s}{\omega_z}\right)}{1 + \frac{s}{(12)2pf_o} + \left(\frac{s}{2pf_o}\right)^2}$$

$\downarrow \quad \downarrow$

Q in db    400 Hz

In summary:

Suppose we are given the following numerical values:

$$\begin{aligned} D &= 0.6 \\ R &= 10\Omega \\ V_g &= 30V \\ L &= 160\mu H \\ C &= 160\mu F \end{aligned}$$

Then the salient features have the following numerical values:

$$\begin{aligned} |G_{g0}| &= \frac{D}{D'} = 1.5 \Rightarrow 3.5\text{dB} \\ |G_{i0}| &= \frac{|V|}{D D'^2} = 469V \Rightarrow 53.4\text{dBV} \\ f_0 &= \frac{\omega_0}{2\pi} = \frac{D'}{2\pi\sqrt{LC}} = 400\text{Hz} \\ Q &= D'R\sqrt{\frac{C}{L}} = 4 \Rightarrow 12\text{dB} \\ f_z &= \frac{\omega_z}{2\pi} = \frac{D'R}{2\pi DL} = 6.6\text{kHz} \end{aligned}$$

If L goes to small values(DCM) what happens to the zero location?

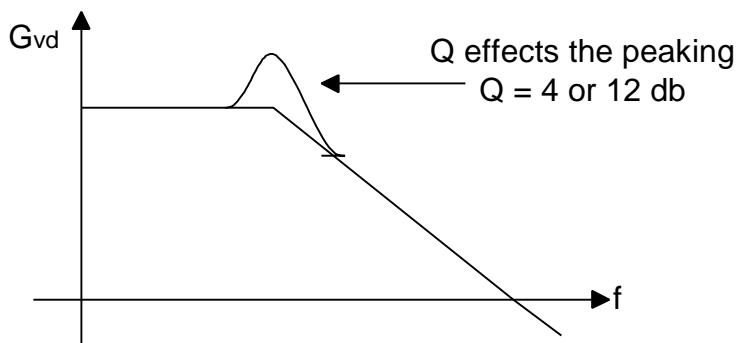
In  $G_{vd}(f)$  Phase Plots we will find

Two poles cover  $180^\circ$  over a frequency range, where now Q is in real units and not in db

$$f_{\text{lower}} = 10^{-\frac{1}{2Q}} f_0 = 10^{-1/8}(400) = 300\text{Hz}$$

$$f_{\text{upper}} = 10^{+\frac{1}{2Q}} f_0 = 10^{1/8}400 = 533\text{Hz}$$

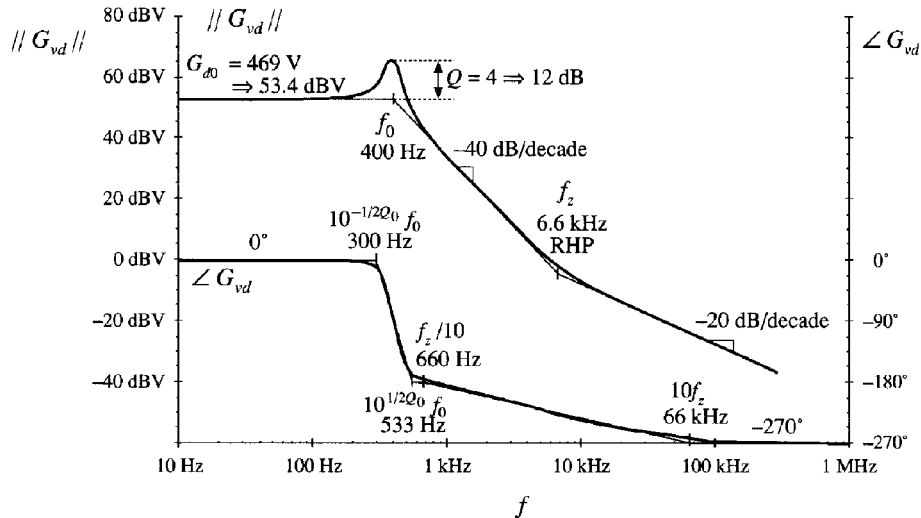
In  $G_{vd}(f)$  Gain Plots we will find:    We have a double pole at 400 Hz showing a resonant “Q peak”.



The RHP zero will add amplitude above  $f_z = 6.6$  kHz but will cause additional phase lag threatening stability if we use feedback.

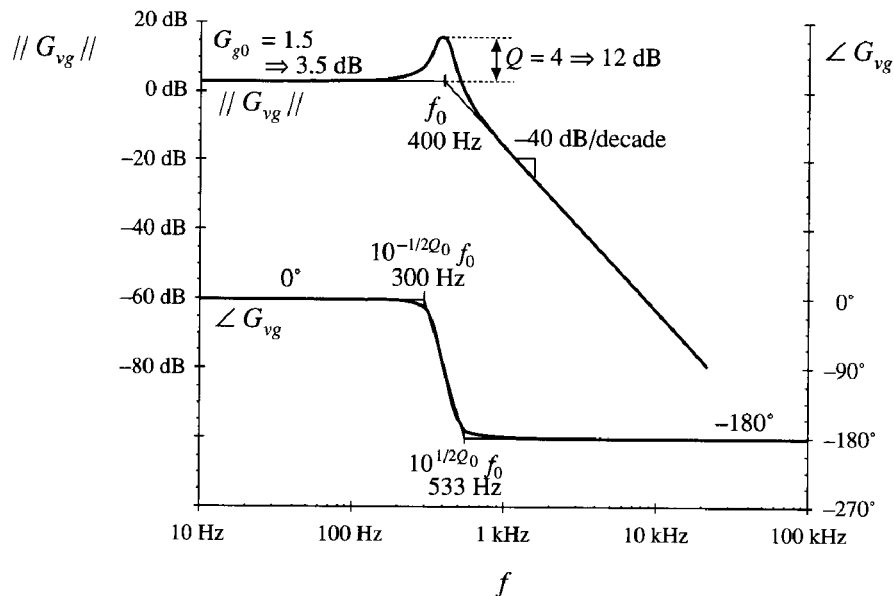
## Below are plotted $|G_{vd}|$ and $\angle G_{vd}$

Note carefully the unity gain line at "0" db.



RH plane zero starts to contribute phase angle at 660 Hz and LIMITS phase margin at unity gain. Additional phase contribution starts at  $f_z/10$  @ 660 Hz.

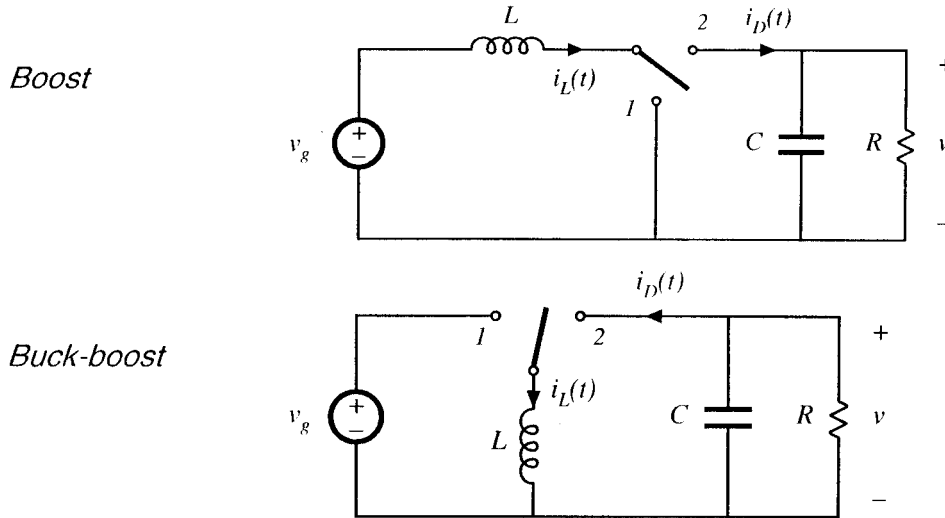
The  $G_{vg}$  amplitude and phase is much simpler as shown below.



Here the  $f$  margin at unity gain is also small but not as bad as  $T_{vd}$ .

### C. Right Half Plane Zero Issues and Indicators of This

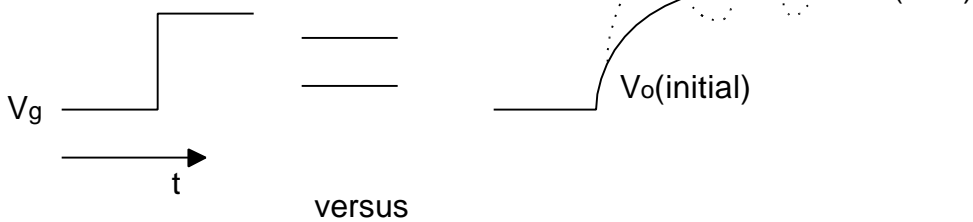
$$\langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s}$$



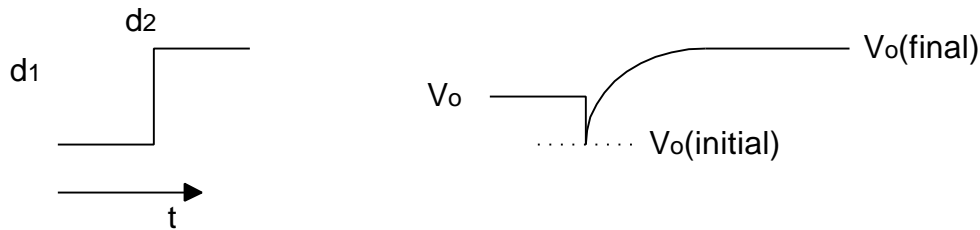
The above two switch mode circuits have right-half plane zeros.

#### 1. Transient Response Indicators of a RHP zero

a. Typical of 2nd order poles



b. Typical of (RH Plane/zero)



This RH plane zero occurs for several PWM dc-dc converters

- boost
- buck-boost
- flyback

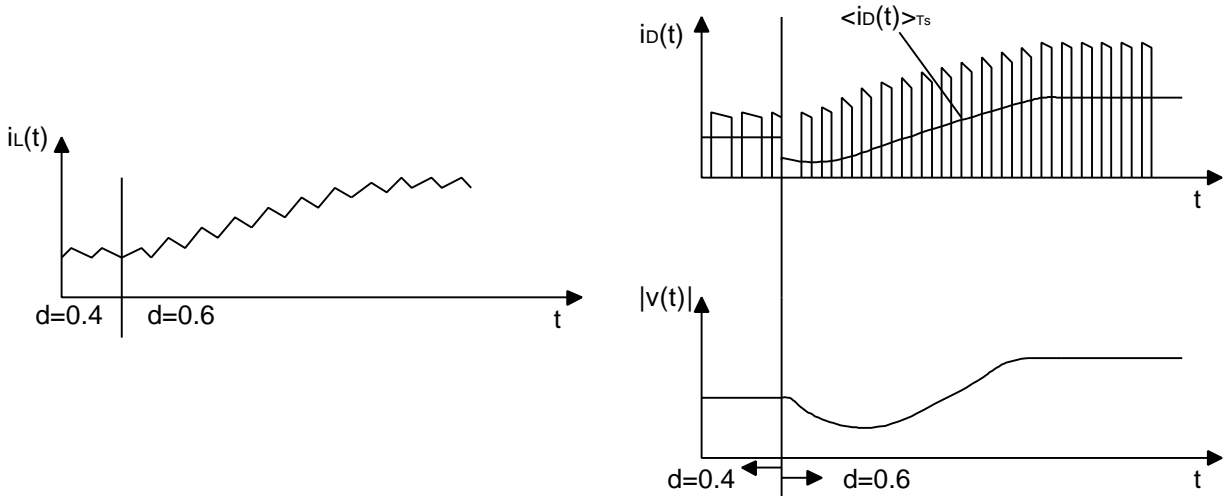
It does not occur for buck converter. WHY?

## 2. Origin of RH Plane Zero

Differing time delays for  $D'$  which changes  $V_o$  immediately &  $i_m$  which changes  $V_o$  only slowly due to switching over several switch cycles, causing initial  $V_o$  to fall for all converters with output driven by  $i_m$ . We have therefore the case that: ( $i_m$ ) stays the same over short  $\Delta t$  time periods but  $D$  and  $D'$  change fast.  $D$  changes instantaneously by the control circuit knob, this instantaneously lower  $D'$  and thus  $V_o$  goes down initially and only later increases when the effect of bigger  $D$  makes it to the output.

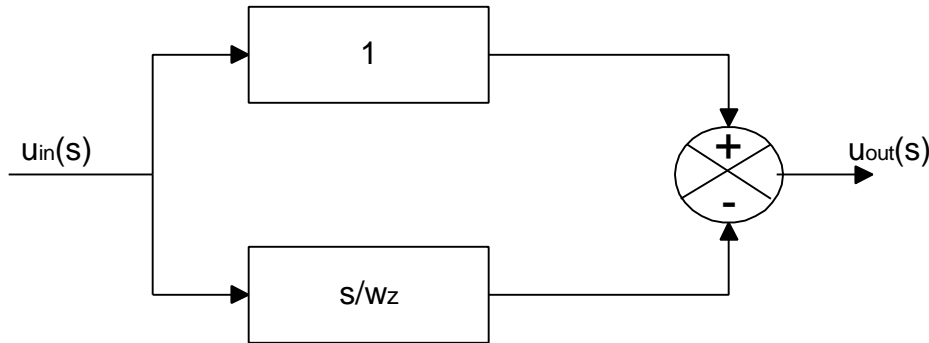
- \*  $i_{Lm}$  cannot change instantaneously
- \* time interval  $d' = 1-d$  can change instantaneously which is the duration  $i_m$  is delivered to the output. This causes a phase lag and momentary drop in  $V_o$  followed by subsequent increase in  $V_o$  sudden change in  $d$  that builds up  $i_m$  slowly.

$i_L$  varies slowly  $i_D$  duration changes instantaneously but  $i_{Lm}$  does not  $\Rightarrow V_o \downarrow$  then  $\uparrow$

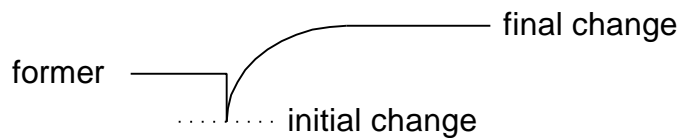


### 3. $T(s)$ of a single right –half plane zero insight

$\frac{V_o}{V_d} \sim (1 - s/w_z)$  A RHP zero acts as two parallel paths to  $V_o$



Overall transient response:



### 4. Phase Margin Limitations of Any Closed Loop with RHP Zero Present

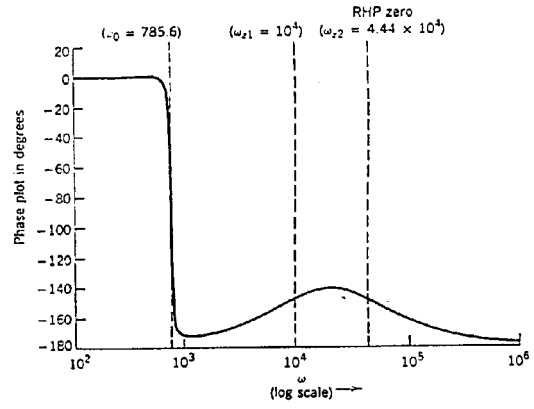
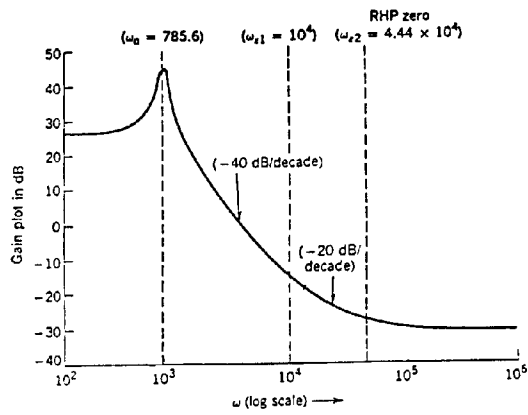
$$A_{CL} = \frac{A_{oL}}{1 + A\beta} \quad \text{Want } (A\beta) = 1$$

Then  $\angle A\beta$  far from  $180^\circ$

How far?  $76^\circ$  away (for conservative designer),  $60^\circ$  for average designer and  $10^\circ$  away for poor designer!

Flyback converter bode plots:

Oddity is gain (low f) is non-linear w.r.t.  $V_d$   
 f range with -40db/decade drop depends on low f gain. Not enough  $\phi$  margin come close to instability at unity gain  $\phi \rightarrow 180^\circ$



Note the unity gain location at "0" db.

Flyback operating DCM does not have right half plane zero!  
 Without LH plane zero above the phase margin would be nil.  
 Moreover, RH plane zero returns phase to  $180^\circ$ .