LECTURE 44
Averaged Switch Models II and Canonical Circuit Models

A. **Additional Averaged Loss-less Switch Models**

1. Single Transformer Circuits
    a. buck
    b. boost
2. Double Cascade Transformer Models:
    a. Buck – Boost
    b. Flyback
3. Bridge Inverter
4. Phase Controlled Rectifier

B. **Additional Averaged. Switch Models With Losses**

1. Buck
    a. Switching Power Loss
    b. MOS R_{ON} & V_D Pbm. 7-17a
2. Boost with R_{ON} & V_D Pbm. 7-17b
3. Buck-Boost with R_{ON} and V_D Pbm. 7-17

C. Canonical Models

1. Overview
2. Example of buck-boost Canonical Form
3. General Case

LECTURE 44
Averaged Switch Models II and Canonical Circuit Models

A. Additional Averaged. Switch Model Lossless Models

1. Single Transformer

Below we give final results, but not details of the intermediate steps, for averaged-switched models of some basic converters both with and without losses.

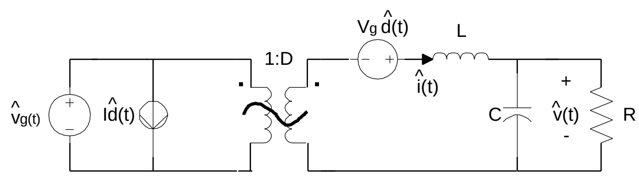
a. Lossless models

First we examine converters without any losses from Inductor-resistance, R_L, DC switch losses of several types , and active switching loss due to transient V-I effects. All losses can be easily added to the loss-less models later.

1. Single transformer models

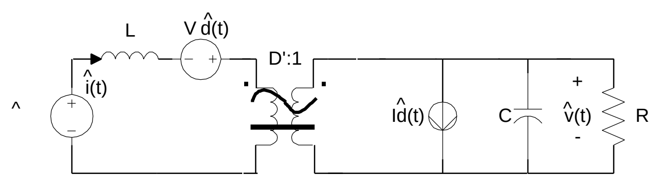
Pbm. 7-17a adds effects of R_{on} and V_D see pg. 11 herein for more details

Buck:



Pbm. 7-17b adds effects of R_{on} and V_D see pg. 12 herein for more details

Boost:

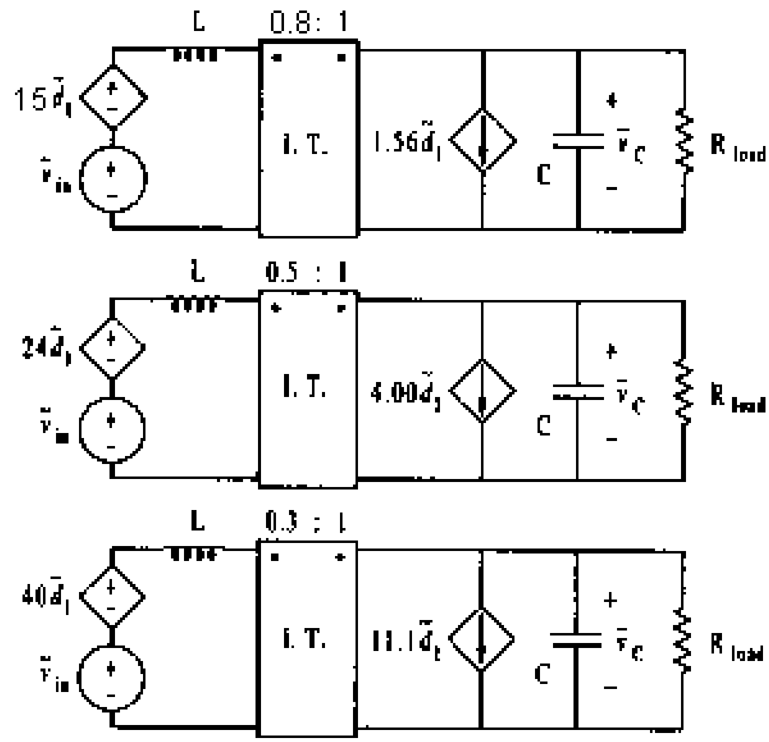


Consider a boost converter with 12V input and a 12Ω load resistor. The nominal operating points are in the table below.

Boost Converter Nominal Operating Points for D_1 = 20%, 50%, and 70%

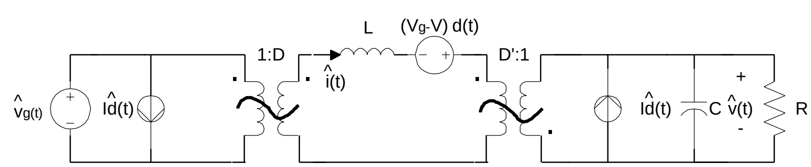
D_1	$1 - D_1$	V_{out} (V)	I_{out} (A)	I_{in} (A)
0.20	0.80	15.0	1.25	1.56
0.50	0.50	24.0	2.00	4.00
0.70	0.30	40.0	3.33	11.11

The three corresponding small signal equivalent circuits for small signal analysis are shown again below to make a clarification. Model parameters depend on the chosen operating point while model type depends on converter type.

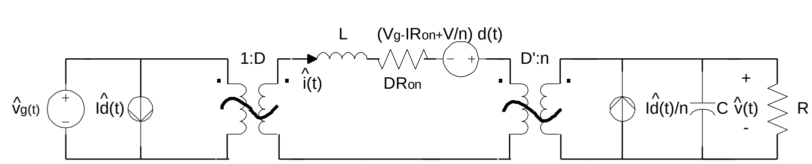


2. **Double cascade transformer models**

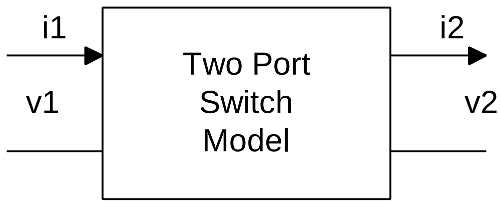
Buck-Boost:
Pbm. 7-17c adds effects of R_{on} and V_D see pg. 14



Flyback:

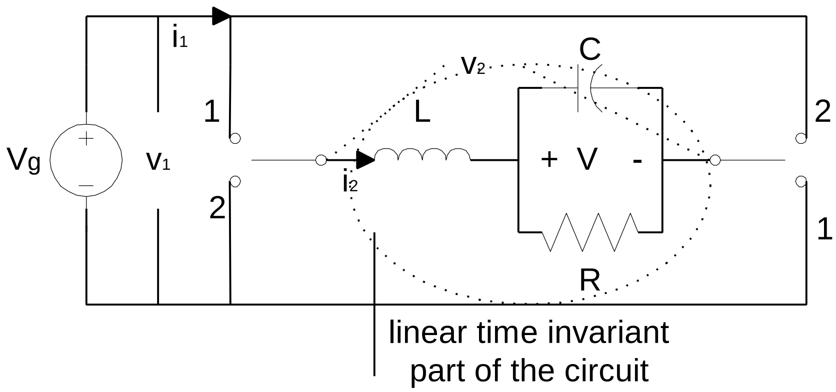


3. Choose a two port model of the TWO switches in the Bridge type inverter. This is not covered in Erickson Ch. 7. But let's do it on our own as it will be useful later for dc to ac conversion.



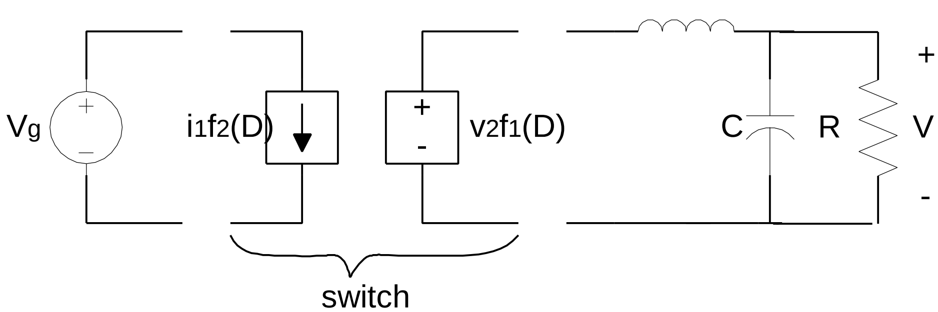
sign of i_2 more natural out of port

Bridge Inverter:

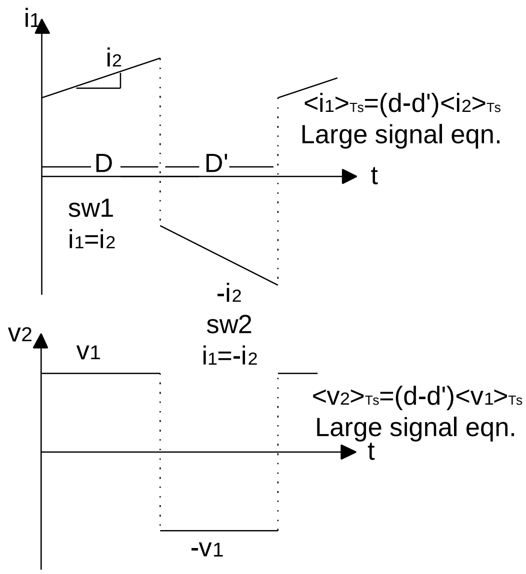


Choose as <u>independent</u> variables: V_1 - V_g, i_2 - i_1

choose as <u>dependent</u> variables: V_2, i_1



$i_1 = f_1(d,V_1,\ i_2)$ Draw i_1 and V_2 waveforms
$V_2 = f_2(d,V_1,\ i_2)$ $\rightarrow$ versus time and average



$<i_1>_{Fs} = d<i_2> + d'<-i_2>$

$<i_1>_{Ts} = (d-d')<i_2>$

large signal equation

$<V_2>_{Ts} = (d-d')<V_2>$

large signal equation

Next perturb and linearize the large signal equations:

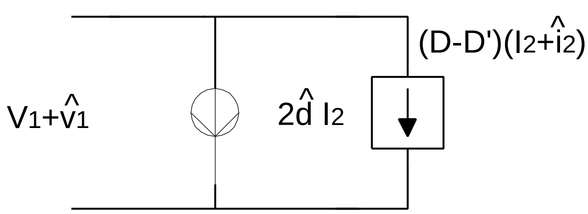
$$<V_g> \;=\; V_g + \hat{V}_g, \quad <i_2> \;=\; I_2 + \hat{i}_3 \quad d \;=\; D + \hat{d}$$

Input

$$<i_1> \;=\; [(D+\hat{d}) - (D-\hat{d})](I_2 + \hat{i}_2)$$

$$= \underbrace{(D-D')(I_2 + \hat{i}_2)}_{\downarrow} \;+\; \underbrace{2\,d\,I_2}_{\downarrow} \;+\; \text{Higher Order Terms}$$

dependent source independent source

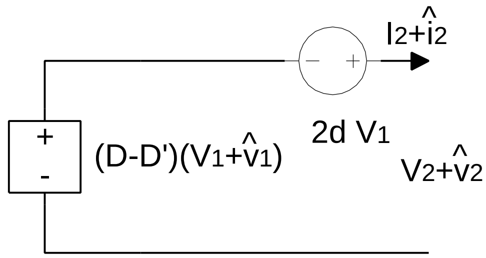


Output

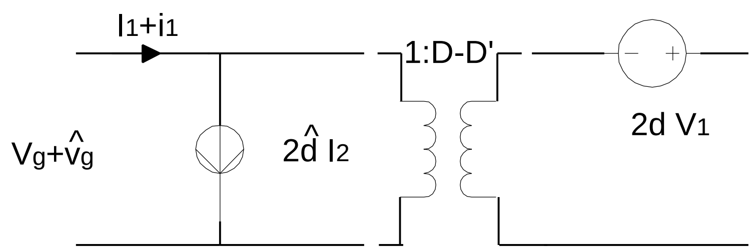
$$<v_2> \;=\; [(D+d) - (D-d)][V_1 + \hat{v}_1]$$

$$= \underbrace{(D-D')(V_1 + \hat{v}_2)}_{\downarrow} \;+\; \underbrace{2\,dV_1}_{\downarrow}$$

dependent source independent source



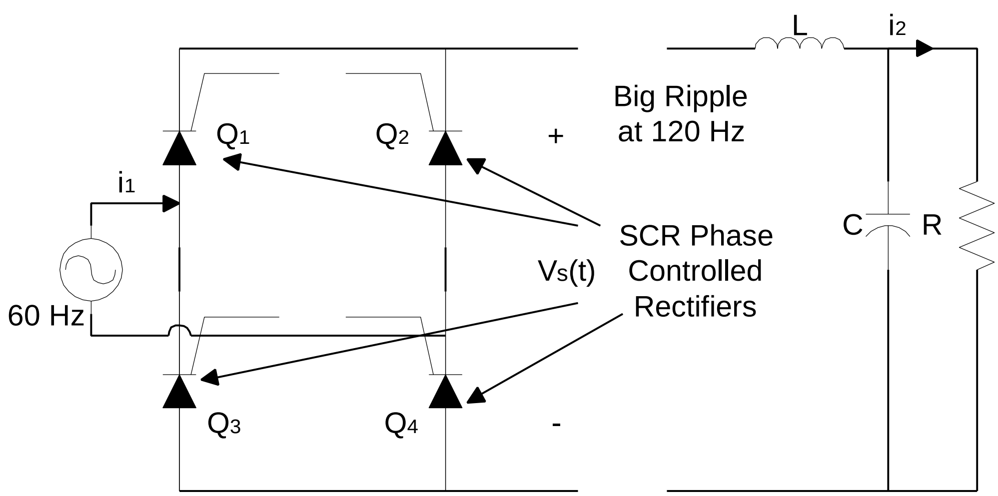
Put both input/output loops together using an AC/dc transformer



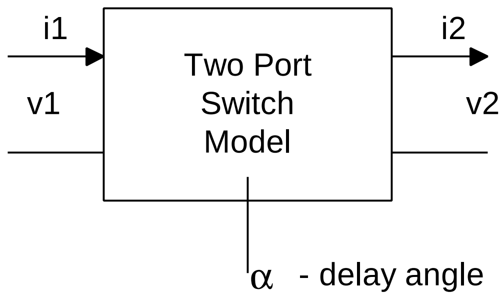
This is another averaged switch model for our analytical toolbox.

4. Phase Controlled Rectifier

This looking ahead to Chapters 15-of Erickson let's now model the phase controlled rectifier switching. This is also not covered in Erickson Ch. 7. Let's do it.



Q_1 or Q_2 is always on if L is large $i_2(t) \approx$ DC with a 120 Hz ripple SCR's go on with delay angle a. Replace all four SCR's, with a two port network.



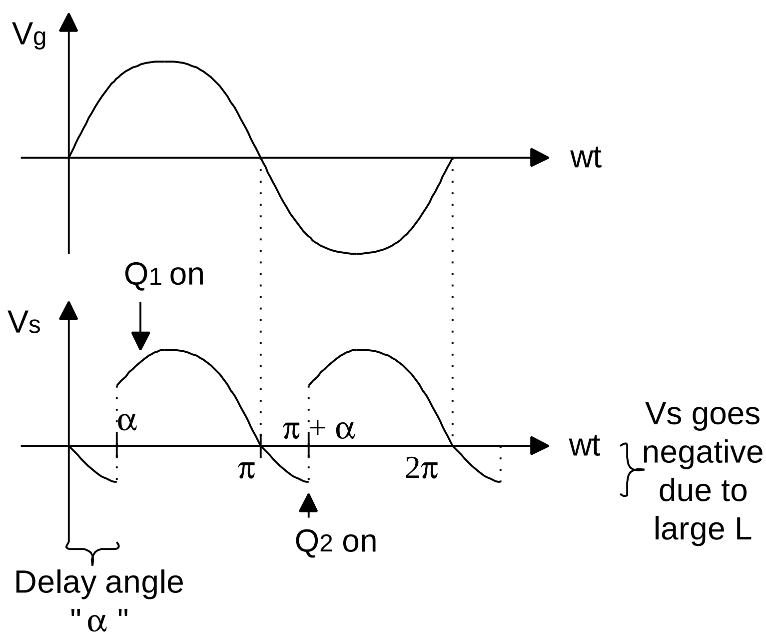
$V_2 = f(\alpha)$

Independent variables : $i_2 \sim i_L$

$v_1 \sim v_g$

Dependent variables: $v_2 \sim v_s$

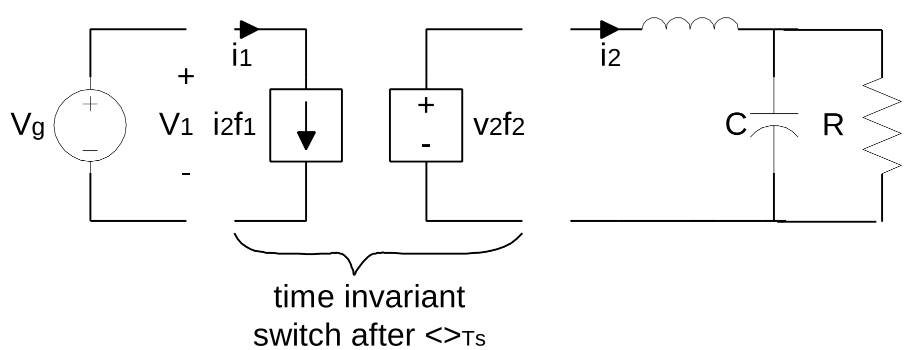
$i_1 \sim i$



Averaged Switch.
Model

$i_1 = f_1(i_2, v_1, \propto)$

$v_2 = f_2(i_w, V_1, \mu)$



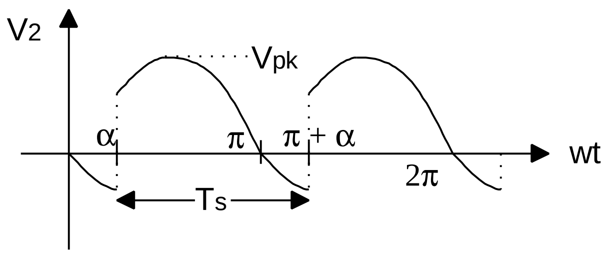
Ultimate goal will be $V_o/d = f(\omega)$

<u>AC side of rectifier</u>:

Q_1 on $i = i_1 = i_L = i_2$ } Square wave
Q_3 on $i_1 = - i_L = - i_2$ }

Later we will determine harmonic content. For now leave alone do NOT take average. We will NOT get input model today, only the simplier output model.

<u>DC Side of Rectifier</u>



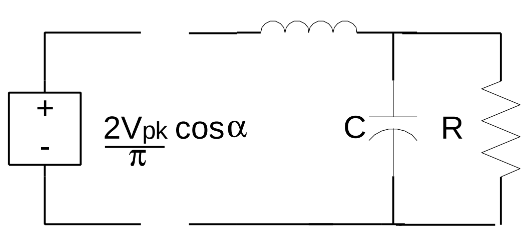
For the output model

$$< V_2 >_{Ts} = \frac{1}{T_s} \int_0^{T_s} V_2(t)\, dt$$

$$= \frac{1}{p} \int_{\propto}^{p+\propto} V_2(q)\, dq$$

$$< V_2 >_{Ts} = \frac{2\, V_{pk}}{p} \, \mathrm{Cos} \propto$$

Average Circuit Model: Large Signal



<u>Perturb and Linearize</u>

$$< V_2 > \; = \; V_2 + \hat{v}_2 , \quad < \propto > \; = \; A + \hat{\propto}$$

$$< V > \; = \; V + \hat{v} \quad < V_{pk} > \; = \; V_{pk} + \hat{v}_{pk}$$

$$\frac{2}{p} \left[V_{pk} + \hat{V}_{pk} \right] \left[Cos(A + \hat{\propto}) \right] \equiv \; < V_2 >_{Ts}$$

$$\downarrow$$

CosA cos∝ - SinA sin∝

Use Taylor series for (Cosα, Sinα)

$$CosA \left[1 - \frac{a^2}{2!} + .. \right] - \left[a - \frac{a^3}{3!} + .. \right] SinA$$

CosA - a SinA

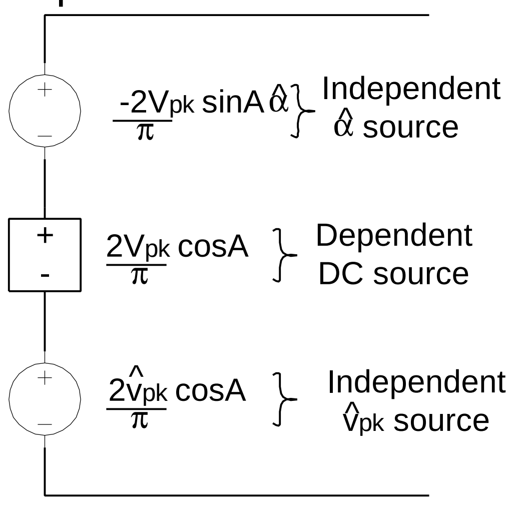
$$< V_2 >_{Ts} \; = \; \frac{2}{p} V_{pk} CosA - \frac{2}{p} V_{pk} \; a \; SinA$$

$$+ \frac{2}{p} \hat{V}_{pk} \; CosA$$

$$\downarrow \qquad\qquad \downarrow$$

one DC term   two 1st order linear terms

Output Circuit for SCR Switches



The point to take away is that circuit averaging works for phase timed as well as PWM. It is a powerful modeling aid for complex switches.

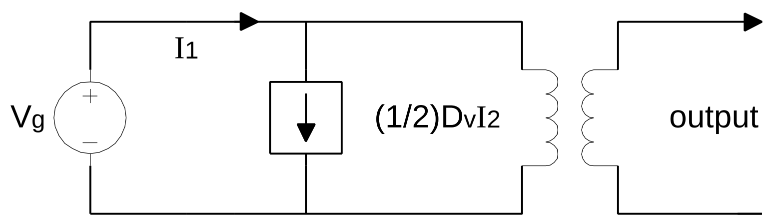
B. Switching Losses of Diode and Transistors

1. Buck Converter

Recall that non-zero switch times gave us additional losses for the converter efficiency calculations

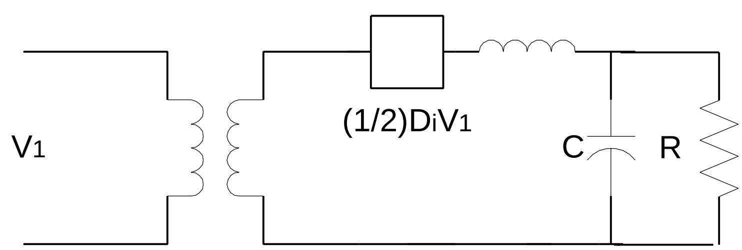
a. Solid state switching losses.

Current sink loss occurs in input circuit via a dependent source



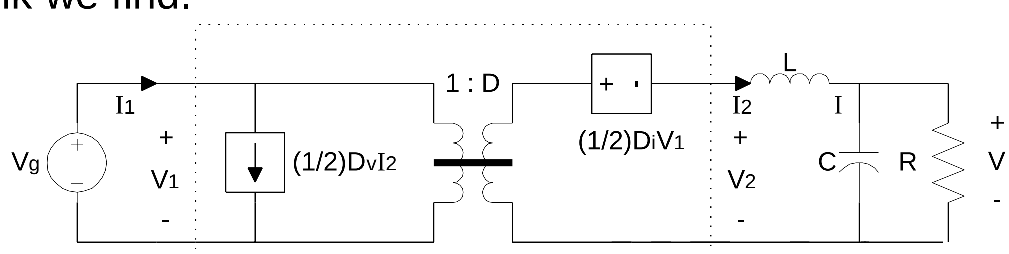
$$D_v \equiv \frac{t_{vf} + t_{vr}}{T_s}$$

Voltage sink loss occurs in the output circuit via a dependent voltage source.



$$D_i \equiv \frac{t_{ir} + t_{if}}{T_s}$$

All together for the buck including both switching loss terms in the circuit model. That is both the input current sink and output V sink we find.



Other dc losses can be easily added to above switch loss model. R_L for example for the inductor goes in the output loop in series. Hysterisis loss and core loss due to eddy currents goes in the input circuit just before the primary as R_C in parallel with $D_V I_S$. Transistor dc/rms loss is included via V_D and R_D. Etc. etc

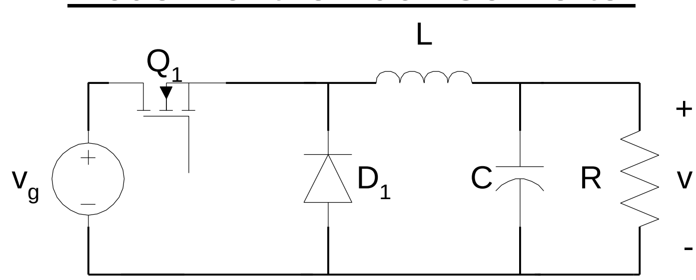
b. **Transistor Ron and diode V_D included in average Switch models for the buck.**

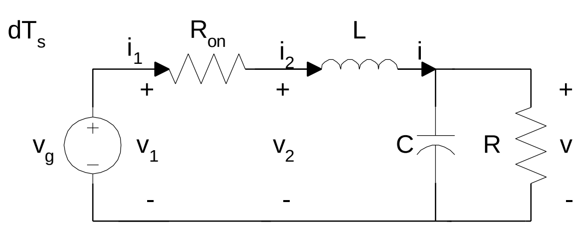
Erickson Problem 7.17

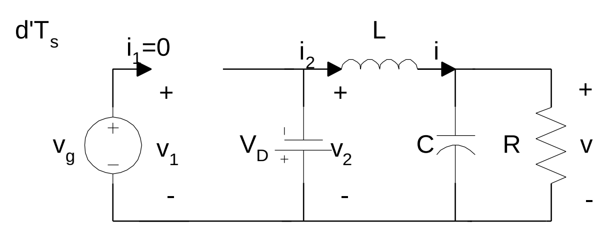
Transistor on resistance, R_{on}, and diode forward voltage drop, V_D, included in Average Switch Models for Both the Buck and Boost Circuits.

1) Buck:

<u>Model For the Buck Converter</u>



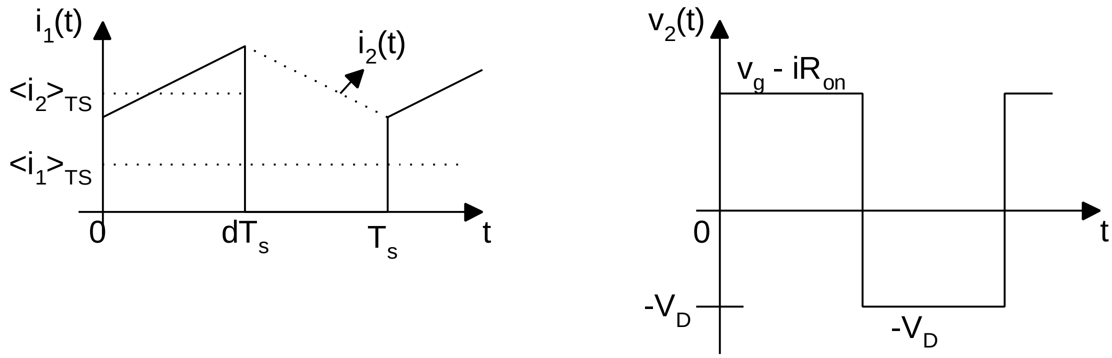




i) Choose: $v_1(t)$ and $i_2(t)$ for Independent Variables
$v_2(t)$ and $i_1(t)$ for Dependent Variables

Below are the Plots versus Time for the dependent variables, $v_2(t)$ and $i_1(t)$, with respect to the independent variables, $v_1(t)$ and $i_2(t)$.

The time dependent waveforms are:



ii) Express $v_2(t)$ and $i_1(t)$ in terms of $i_2(t)$, $v_1(t)$ and $d(t)$:
on $dT_s \Rightarrow v_1(t) = v_g(t)$, so $\underline{v_2(t)=d(t)(v_g(t)-i_2(t)Ron)+d'(t)(-V_D)}$ AND: $\underline{i_1(t) = d(t)i_2(t) + d'(t)*0}$

iii) Average $v_2(t)$ and $i_1(t)$ with respect to one period T_s:

a) $\langle i_1(t)\rangle_{Ts} = d(t)\langle i_2(t)\rangle_{Ts}$,

b) $\langle v_2(t)\rangle_{Ts} = d(t)(\langle v_g(t)\rangle_{Ts} - \langle i_2(t)\rangle_{Ts}Ron) + d'(t)(-V_D)$

iv) Perturb and Linearize:

$\langle v_1(t)\rangle_{Ts} = \langle v_g(t)\rangle_{Ts} = V_1 + \hat{v}_1$, $d' = D' - \hat{d}$,

$\langle v_2(t)\rangle_{Ts} = V_2 + \hat{v}_2$, $\langle v(t)\rangle_{Ts} = V + \hat{v}$,

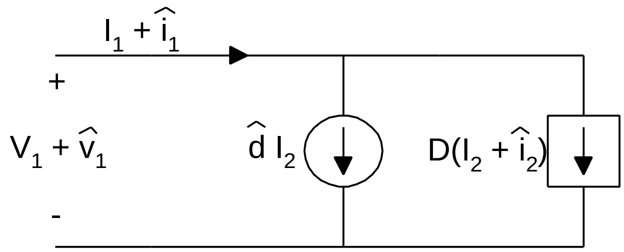
$\langle i_2\rangle_{Ts} = I_2 + \hat{i}_2$, $\langle i_1\rangle_{Ts} = I_1 + \hat{i}_1$, $d = D + \hat{d}$,

Get Equivalent Circuits

$\Rightarrow$Current Generator:

$\langle i_1\rangle_{Ts} = (D + \hat{d})(I_2 + \hat{i}_2) = D(I_2 + \hat{i}_2) + \hat{d}I_2 + [\hat{d}\,\hat{i}_2]$® **0**

Input Circuit:

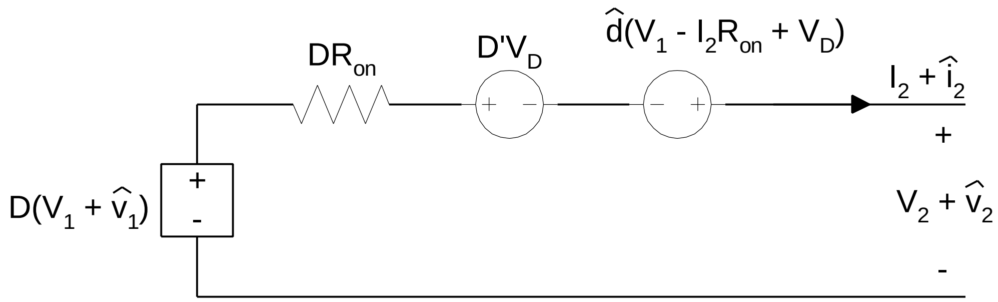


$\Rightarrow$ Voltage Generator:

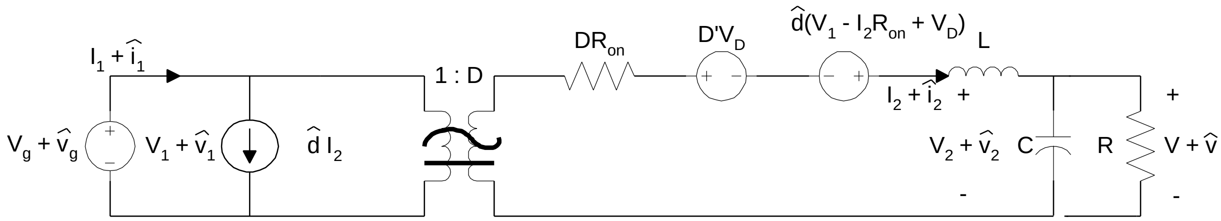
$<v_2(t)>_{Ts} = V_2+\hat{v}_2 = (D+\hat{d})[(V_1+\hat{v}_1)-R_{on}(I_2 + \hat{i}_2)]-(D'-\hat{d})V_D =$

$= D(V_1+\hat{v}_1)-DR_{on}(I_2+\hat{i}_2)-D'V_D+\hat{d}(V_1-I_2R_{on}+V_D)+\mathbf{[}\hat{d}\,\hat{v}_1 \mathbf{-} \hat{d}\,\mathbf{R_{on}}\hat{i}_2\mathbf{]®\ 0}$

Output Circuit:

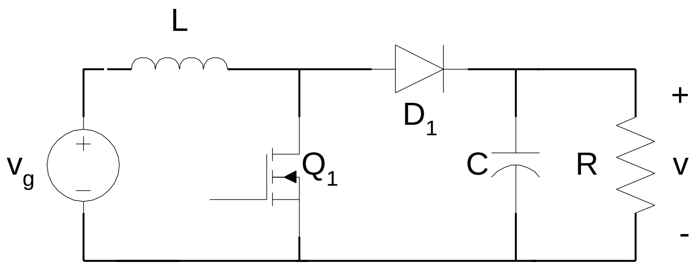


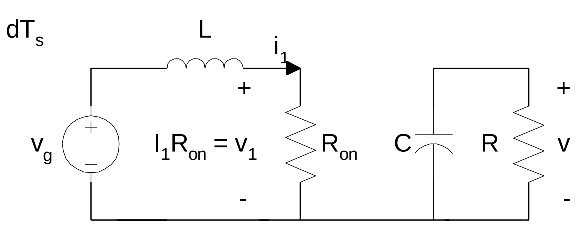
v) Combine them to form the full Buck Model with losses R_{on} and V_D:

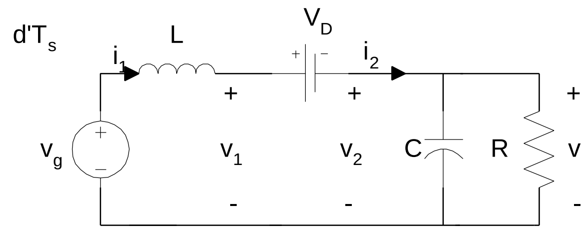


2) Boost:

Model For the Boost Converter





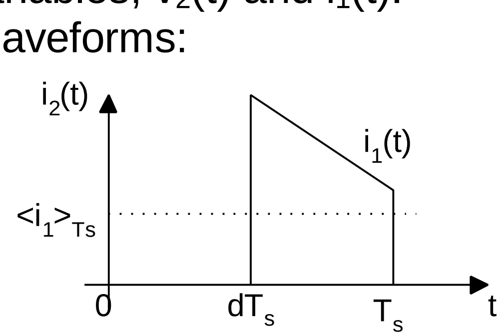


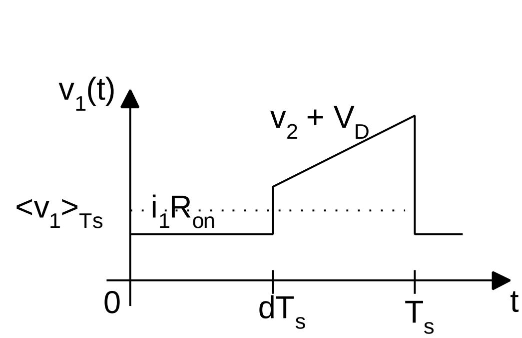
i) Choose: $v_2(t)$ and $i_1(t)$ for Independent Variables

$v_1(t)$ and $i_2(t)$ for Dependent Variables

Below are the Plots versus Time for the dependent variables, $v_1(t)$ and $i_2(t)$, with respect to the independent variables, $v_2(t)$ and $i_1(t)$.

Waveforms:





ii) Express $v_1(t)$ and $i_2(t)$ in terms of $i_1(t)$, $v_2(t)$ and $d(t)$:

$\Rightarrow$ $\underline{v_1(t) = d(t)(i_1(t)Ron)+d'(t)(v_2(t)+V_D)}$

AND: $\underline{i_2(t) = d(t)*0 + d'(t)i_1(t)}$

iii) Average $v_1(t)$ and $i_2(t)$ over one period T_s:

a) $<i_2(t)>_{Ts} = d'(t)<i_1(t)>_{Ts}$,

b) $<v_1(t)>_{Ts} = d(t)(<i_1(t)>_{Ts}Ron) + d'(t)(<v_2(t)>_{Ts}+V_D)$

iv) Perturb and Linearize:

$<v_1(t)>_{Ts} = V_1 + \hat{v}_1$, $d' = D' - \hat{d}$, $d = D + \hat{d}$,

$<v(t)>_{Ts} = <v_2(t)>_{Ts} = V_2 + \hat{v}_2$, $<v_g(t)>_{Ts} = V_g + \hat{v}_g$,

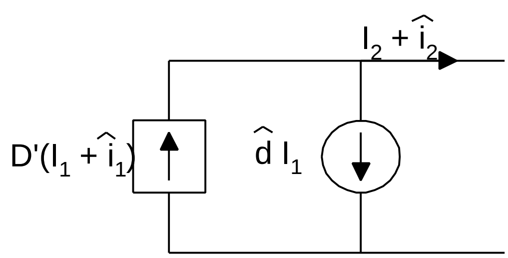
$<i_2>_{Ts} = I_2 + \hat{i}_2$, $<i>_{Ts} = <i_1>_{Ts} = I_1 + \hat{i}_1$,

Get Equivalent Circuits

$\Rightarrow$Current Generator:

$<i_2>_{Ts} = (D - \hat{d})(I_1 + \hat{i}_1) = D(I_1 + \hat{i}_1) - \hat{d}I_1 - \mathbf{[\hat{d}\,\hat{i}_1]}$**® 0**

OR:

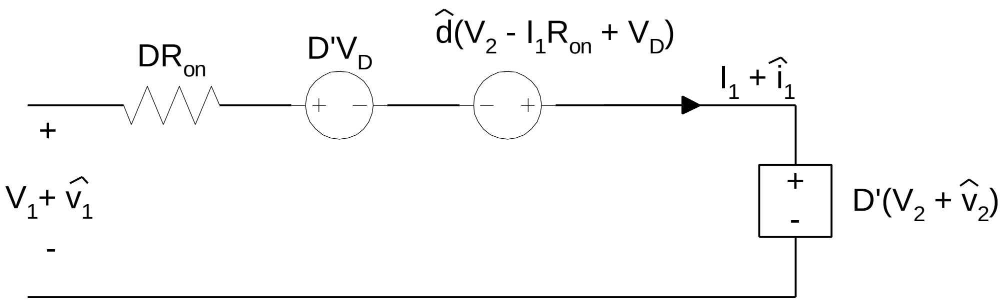


$\Rightarrow$Voltage Generator:

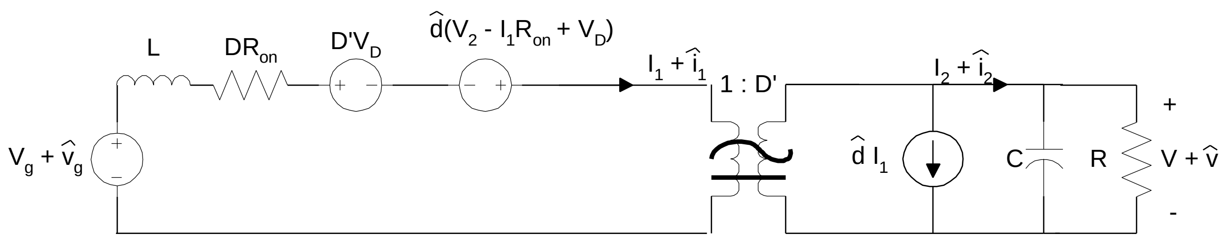
$<v_1(t)>_{Ts} = (D+\hat{d})R_{on}(I_1 + \hat{i}_1) + (D'-\hat{d})[(V_2 + \hat{v}_2) + V_D =$

$=D'(V_2 + \hat{v}_2)+DR_{on}(I_1 + \hat{i}_1)+D'V_D-\hat{d}(V_2-I_1R_{on}+V_D)-\mathbf{[\hat{d}\,\hat{v}_2 - \hat{d}\,R_{on}\hat{i}_1]}$**® 0**

OR:



v) Combine them to form the full Boost Model with losses R_{on} and V_D:



Do the Buck-Boost with R_{on} and V_D for HW#3

C. Canonical Models

1. Overview

Having seen several converter AC models, we now consider a canonical circuit model that could be used for all converters. The canonical model would contain:

- the ac/dc transformer M(D)
- An EFFECTIVE low-pass filter to remove f_{SW}
- Independent sources which represent the duty cycle variations

Clearly, the parameters of the model would change for each converter topology. However, as we use the same topology the changes in parameters would allow for a better comparison between converter types.

All PWM CCM dc-dc converters perform the same basic functions:

- Transformation of voltage and current levels, ideally with 100% efficiency
- Low-pass filtering of waveforms
- Control of waveforms by variation of duty cycle

Hence, we expect their equivalent circuit models to be qualitatively similar.

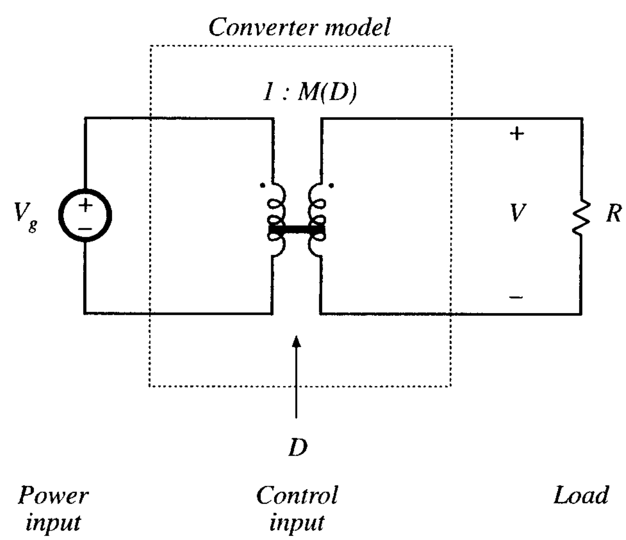
Canonical model:

- A standard form of equivalent circuit model, which represents the above physical properties
- Plug in parameter values for a given specific converter

There are five steps in the development of a canonical model:

1. *Transformation of dc voltage and current levels*

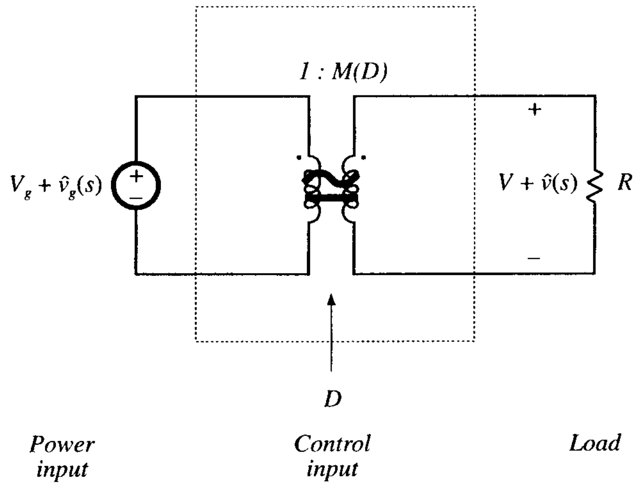
- *modeled as in Chapter 3 with ideal dc transformer*
- *effective turns ratio $M(D)$*
- *can refine dc model by addition of effective loss elements, as in Chapter 3*



This models only the steady-state or DC behavior. On the top of page 18 we include AC variations.

2. *Ac variations in $v_g(t)$ induce ac variations in $v(t)$*

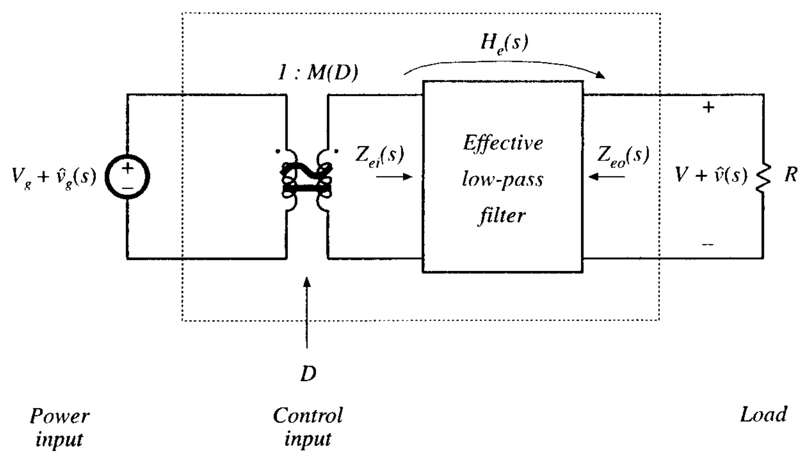
- *these variations are also transformed by the conversion ratio $M(D)$*



The next step is the introduction of a low-pass filter.

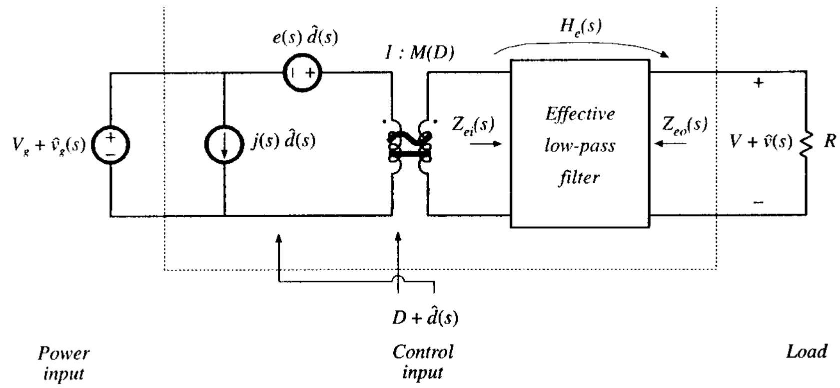
3. *Converter must contain an effective low-pass filter characteristic*

- *necessary to filter switching ripple*
- *also filters ac variations*
- *effective filter elements may not coincide with actual element values, but can also depend on operating point*



L_e will be the effective filter inductor, which will have contributions from both the circuit inductance, the duty cycle, and the actual filter inductor.

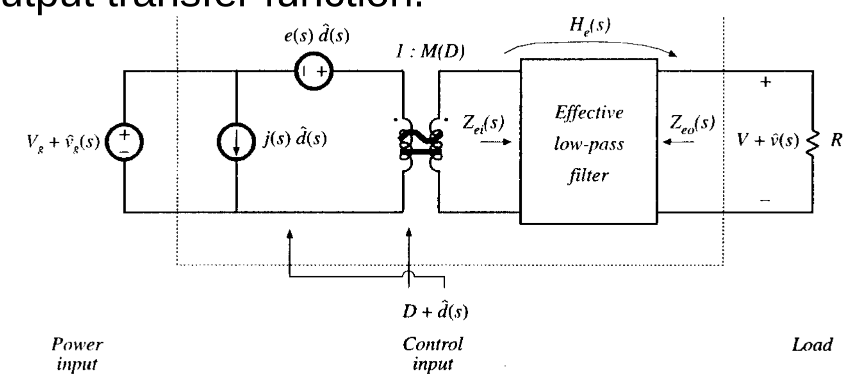
Next is the control input, d ,variations which induce ac variations at the converter output. We showed in all our small signal AC models that the two independent sources of current and voltage respectively could fully represent duty cycle variations as shown on the top of page 19.



4. *Control input variations also induce ac variations in converter waveforms*

- *Independent sources represent effects of variations in duty cycle*
- *Can push all sources to input side as shown. Sources may then become frequency-dependent*

In general the canonical model can be solved for two types of transfer functions: $G_{vg}(s)$ and $G_{vd}(s)$. $G_{vg}(s)$ will be the line or mains to output transfer function. $G_{vd}(s)$ will be the control to output transfer function.



Line-to-output transfer function: $$G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} = M(D)\,H_e(s)$$

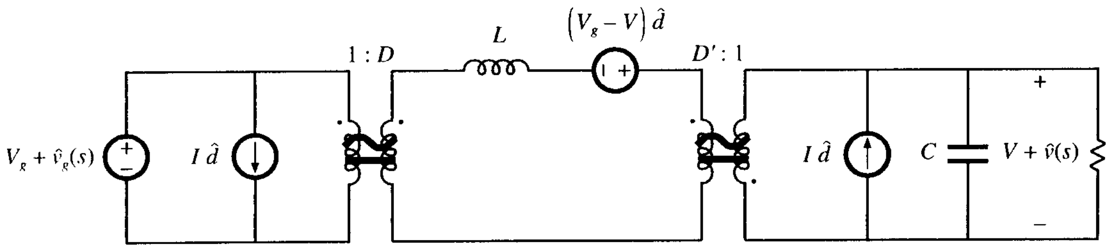
Control-to-output transfer function: $$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} = e(s)\,M(D)\,H_e(s)$$

Knowing the above canonical form has all sources in the input circuit and the EFFECTIVE filter circuit in the output, means we can take our prior hard work with AC models and re-construct them to fit the canonical form above.

2. **Example of buck-boost Canonical Form**

We start with the prior AC model and tell the moves we must make to get into canonical form:

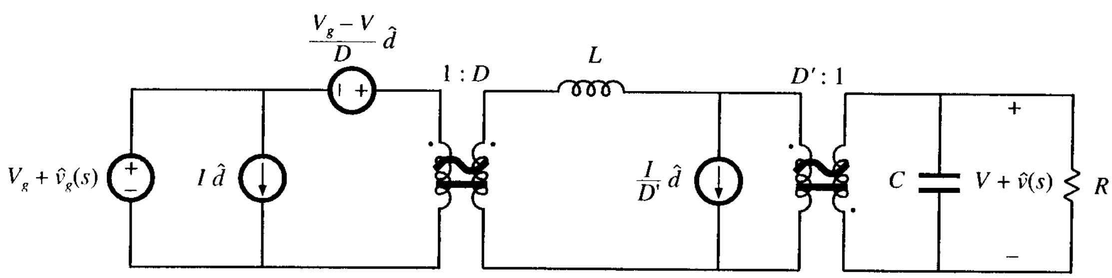
Small-signal ac model of the buck-boost converter



- Push independent sources to input side of transformers
- Push inductor to output side of transformers
- Combine transformers

There will five steps in the process:

Push voltage source through 1:D transformer
Move current source through D':1 transformer

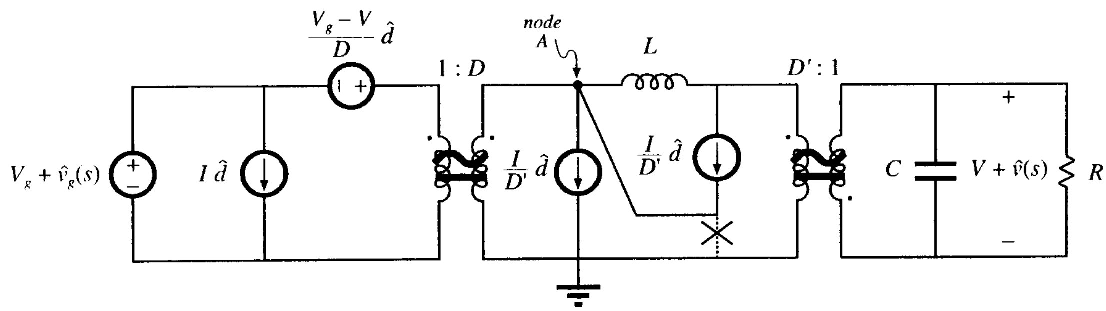


We next move the current source just placed in the middle loop from the secondary circuit to the primary circuit. This will take several steps to accomplish. First we move the current source to the left of the inductor in the middle loop by breaking the ground connection and moving it to a new current source on the left of the inductor as shown on the top of page 21. This leaves the current in node A unchanged and loop equations unaltered.

How to move the current source past the inductor:

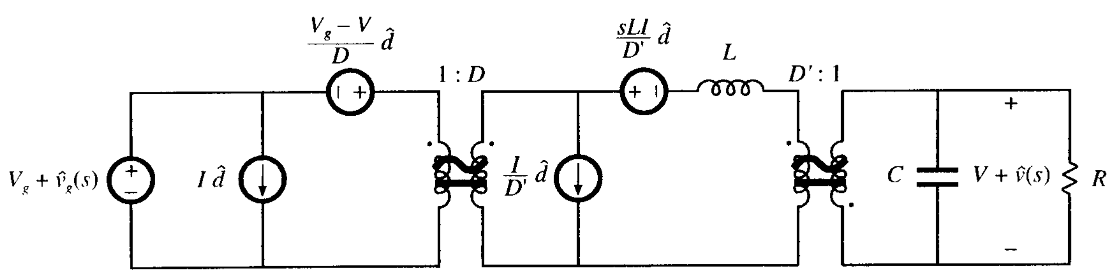
Break ground connection of current source, and connect to node A instead.

Connect an identical current source from node A to ground, so that the node equations are unchanged.



Step three replaces the current source and impedance on the right side of the middle loop by a Thevenin equivalent circuit.

The parallel-connected current source and inductor can now be replaced by a Thevenin-equivalent network:

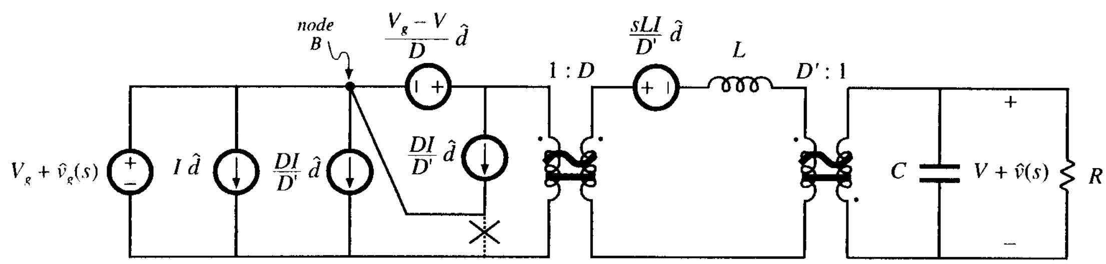


Step 4 is shown on the top of page 22 and it involves moving all independent sources to the input circuit, just as the canonical model. After that is accomplished we will also need to move the inductor in the middle loop to the output loop to have the same form as the canonical model.

Now push current source through 1:D transformer.

Push current source past voltage source, again by:

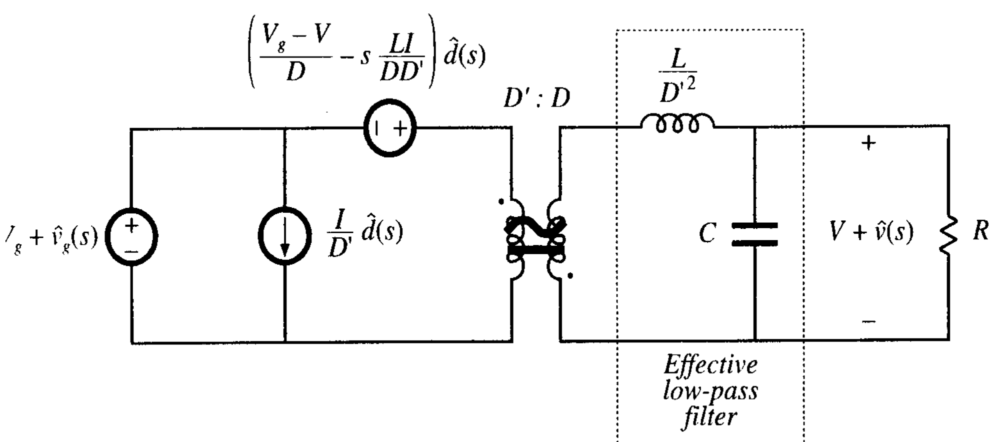
- *Breaking ground connection of current source, and connecting to node B instead.*
- *Connecting an identical current source from node B to ground, so that the node equations are unchanged.*
- *Note that the resulting parallel-connected voltage and current sources are equivalent to a single voltage source.*



Step 5 will yield the final canonical circuit:

Push voltage source through 1:D transformer, and combine with existing input-side transformer.

Combine series-connected transformers.



It's worth noting that the uniqueness of the buck-boost circuit in the input portion is all in the coefficients of the voltage source, e(s), and the current source, d(s).

Voltage source coefficient is:

$$e(s) = \frac{V_g + V}{D} - \frac{s\,LI}{D\,D'}$$

Simplification, using dc relations, leads to

$$e(s) = -\frac{V}{D^2}\left(1 - \frac{s\,DL}{D'^2\,R}\right)$$

Pushing the sources past the inductor causes the generator to become frequency-dependent.

In the output portion, the uniqueness is all in the EFFECTIVE inductor value for the low-pass filter, L_e.

3. General Case

A more general comparison of all three basic converter topologies is found below. Note in particular the right–half plane zero's in the e(s) coefficients.

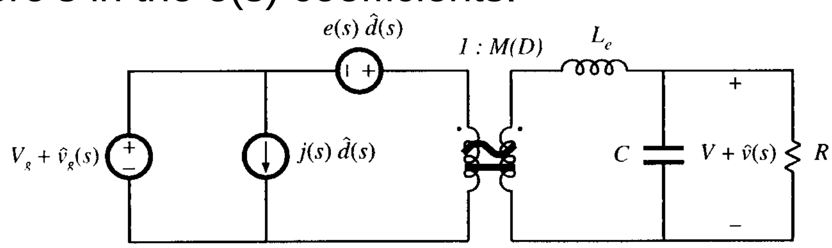


Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	$M(D)$	L_e	$e(s)$	$j(s)$
Buck	D	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$	$\frac{L}{D'^2}$	$V\left(1 - \frac{s\,L}{D'^2\,R}\right)$	$\frac{V}{D'^2\,R}$
Buck-boost	$-\frac{D}{D'}$	$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{s\,DL}{D'^2\,R}\right)$	$-\frac{V}{D'^2\,R}$

HW #3 is due next class-time