LECTURE 39

CCM to DCM Boundary Conditions

HW #2 DUE next time

A. CCM to DCM Transition Boundary via **D versus $I_{av}/(I_{OB})_{max}$ Plots** with V_{IN} / V_{OUT} as a parameter

1. Overview
2. Buck DCM to CCM Boundary Plot
3. Boost DCM to CCM Boundary Plot
4. Buck-Boost DCM to CCM Boundary Plot
5. Illustrative Steady-State Example

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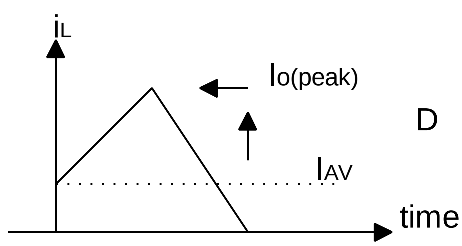
A. CCM to DCM Transition Boundary via **D versus $I_{av}/(I_o)_{max}$** Plots

1. Overview

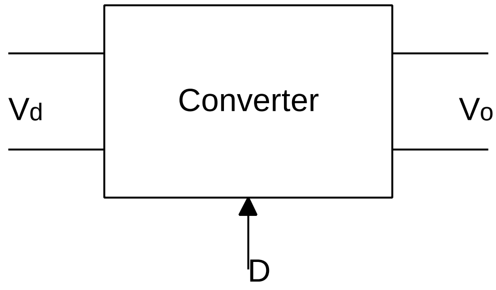
In CCM we have only two portions of the duty cycle: D_1 and D_2 which give rise to two circuit topologies during the switch cycle. D_1 is actively set by the control circuitry, defaulting the value of $D_2 = 1 - D_1$. In DCM one or more additional circuit conditions are met that result in a DIFFERENT DC TRANSFER FUNCTION:

a) Unipolar diode conduction in one of the switches
b) Low I_o(DC) and high ripple i_L in the current waveform

With both are present then DCM of operation can occur with three circuit topologies present over the switch cycle.



Whenever I_o(peak) > I_{AV} and uni-directional switches present ⇒ DCM operation occurs with its mixed desirable and undesirable features both on a DC and AC basis.



$I_o(\text{peak}) = f_1(D)$; $I_{AV} = f_2(D)$;
$f_1(D) \neq f_2(D)$

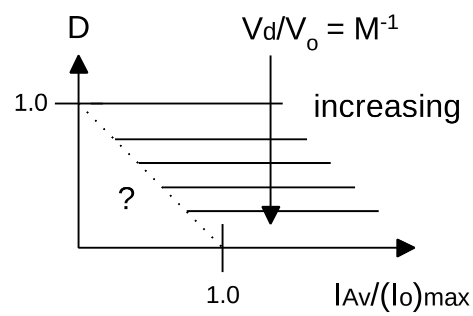
Both I_o(peak) and I_{AV} depend on the duty cycle D. But each is a unique function of D for each circuit topology. So to set an inequality between them sets up a range of duty cycles.

Hence, when the duty cycle, D, is such that I_o(peak) = I_{AV} we have a transition or border region mapped out between DCM and CCM of operation. This will be unique for each circuit topology. In short when the ratio: $\frac{I_{AV}}{(I_o)_{peak}} \approx 1$ the DCM to CCM boundary occurs.

Often in applications we require V_o = constant for a converter while V_g (raw DC input) and D (duty cycle) are adjusted accordingly to achieve this, usually by output driven feedback loops. From our prior work on equilibrium conditions, lossless CCM of operation has an ideal output V source characteristic with I_o not effecting V_o. That is V_0 is constant for all I_0. $V_o = M(D)V_{in}$ only, with no I_o dependence.

We will contrast this with the DCM of operation which has a non-ideal V_o source with I_o effecting V_0. Since for DCM, $V_o = M(D, R_L)V_{in}$. Moreover, as we saw in lecture 38 V_o(DCM) usually exceeds V_o(CCM) for a fixed duty cycle.

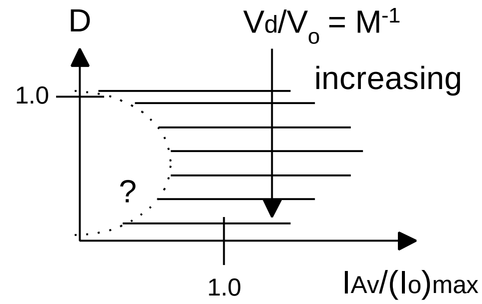
We aim in lecture 39 to reconcile these two equilibrium relations by plotting out the CCM to DCM boundary conditions versus duty cycle. The CCM-DCM boundary transition is best seen by plotting for each circuit topology the following: duty cycle, D, on the ordinate or y-axis versus the ratio $I_{AV}(D)/I_o$(peak) on the abscissa or x-axis. We will get unique plots for the three major converters as shown below in anticipation of the results we will derive herein later.



The above dashed line is CCM-DCM boundary transition

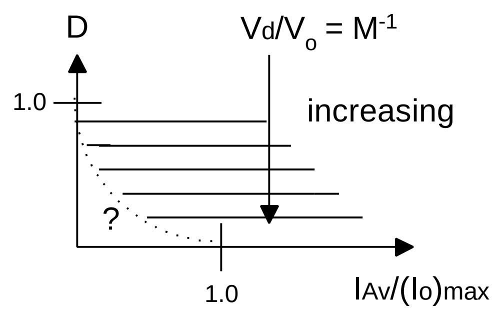
for the <u>Buck</u> topology. For DCM operation various D values are possible to the left of the dashed curve and we will determine them. Note the ideal (load independent) voltage source characteristics of the CCM will not follow into the DCM region of operation.

Next is the boost topology.



The above dashed line is CCM-DCM boundary transition for the <u>boost</u> topology in steay-state. To the left of the dashed line we will derive the non-ideal source characteristics, as compared to the ideal CCM voltage source curves.

Next is the buck-boost.



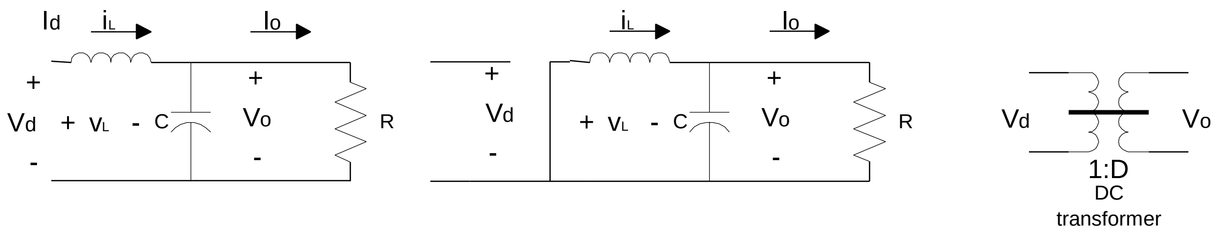
The above dashed line is CCM-DCM boundary transition for the b<u>uck-boost</u> topology. Depending on the operating Mor DC gain, a unique D is set for the CCM-DCM transition. To the left of the transition boundary we must derive the non-ideal DCM curves. In Lecture 38 we solved for V_{out} in DCM of operation by using intuitive linear analysis as well as by solving the quadratic equations resulting from balance conditions in the three circuit topologies. Herein we concentrate on the goal of **clearly defining the border between the two regions under all operating conditions**.

We will first calculate for each circuit topology the unique relationships for $I_{AV} = f_1(D)$ and $I_o(peak) = f_2(D)$. These are two distinct relations for each topology. Then we will plot D versus $I_{AV} / I_o(max)$ and on this plot delinate with a dashed curve the ratio $I_{AV}(D)/I_{peak}(D) = 1$. This will sketch out the locus of points for the CCM-DCM boundary transition or the dashed boundary curve.

1. Buck or step down topology CCM to DCM Boundary

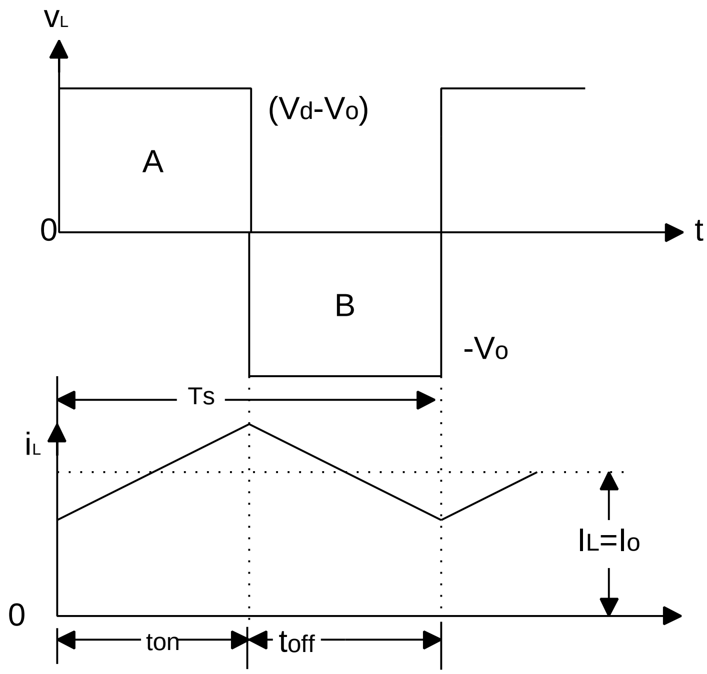
We let $V_D = V_{in}$ be the input voltage. Often this is the rectified mains. We will first review CCM conditions and then find $I_{critical}$ at the CCM to DCM boundary.

a. **Review of CCM DC Transfer Function and Inductor Waveforms**

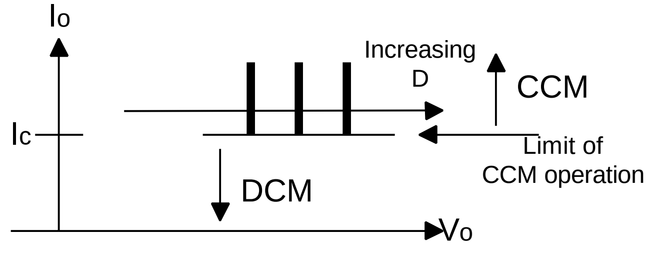


Use $\langle V_L \rangle_{T_s} = 0$ $\qquad \dfrac{V_o}{V_d} = \dfrac{t_{on}}{T_s} = D \qquad \dfrac{I_o}{I_d} = \dfrac{1}{D}$

During the interval t_{on} $V_L = V_d - V_0$ while for the time t_{off} $V_L = -V_0$. The buck converter operates CCM for the case I_o (average) > $I_{CRITICAL}$. The individual circuit conditions will set both critical and average currents. In short for high DC levels of I_L and low levels of ΔI_L(ripple), we have only uni-polar inductor current. On the other hand when I_o(equivalent DC level) goes below a critical level, $I_{CRITICAL}$, we have the possibility of bi-polar inductor current. With uni-polar switches present this may cause the onset of the DCM of operation when one of the switches turns off inadvertently. Below $I_0 = I_{CRITICAL}$ the ideal CCM of operation is no longer valid. Our goal is to quantify the CCM to DCM boundary.

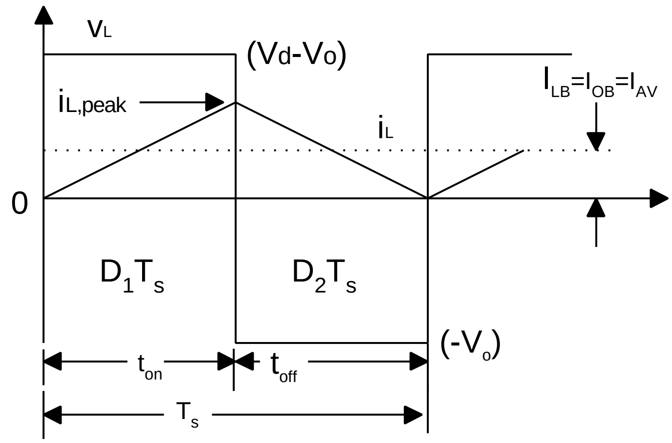


As easily seen in the waveform versus time plots of I_L, for I_o < I(critical) DCM occurs and for I_o > I(critical) we get CCM operation. So we know already if $I < I_c \equiv$ I(critical) we are just beginning DCM and at the ultimate limit of CCM. This allows us to quantify the boundary transition as a function of duty cycle, D. This boundary is shown schematically in the I_o - V_o plot below of an ideal buck. Solid lines are CCM operation up to the limit of DCM where I_o - V_o plots are no longer vertical lines. This was also shown in our overview plot of the buck circuit, where we plotted D versus the ratio I_{AV} /I_0(max). The boundary plot varies with D,V_{in} and V_0 in a complex way we will derive below.



b. **At the Boundary I = $I_{CRITICAL}$**

Below we show the i_L current waveform just as it hits zero and tries to go negative.



The effective DC inductor current is defined by the three equivalent parameters: $I_{LB} = I_{OB} = I_{AV}$. Where the subscript B refers to the boundary of CCM to DCM

The critical current for the buck converter is $\frac{D\,T_s}{2L}(V_d - V_o)$.

We can set the inequality:

If $\Rightarrow I_{LB} = I_{av} \leq \frac{D\,T_s}{2L}(V_d - V_o)$ DCM occurs as discussed above. We have several ways to satisfy the inequality. For we can choose: T_{SW},V_d,V_0,L and D to meet the inequality. There are two major paths for I_{AV} < I(critical) and each will give a unique DCM to CCM boundary plot as we will see below for the two separate practical cases: V_d = constant and V_0 = constant. In <u>either case</u> if I_{AV} decreases i_L will try to go negative <u>but</u> the uni-polar diode switch will not allow it

$\underline{V_d\text{(input) Constant}}$

A buck converter driving a DC motor often has $V_o = DV_d$ and V_d is fixed but D varies to control V_0. In this case we find at the CCM to DCM edge

$$I_{av} = \frac{T_S V_d}{2L} D(1-D) = I_{LB}$$

$\underline{V_o\text{(output) Constant}}$

A buck converter supplying DC power to a computer often has V_o fixed via external feedback that varies D and to compensate for any V_d variation and

Since $V_d = V_o/D$ we can find I_{LB}

$\underline{V_d\text{ Constant}}$

By plotting I_{AV} versus D

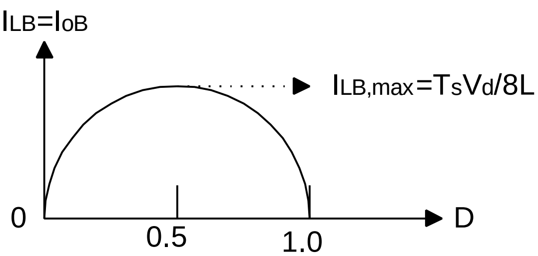
$\underline{V_o\text{ Constant}}$

I_{LB}

$$I_{AV} = \frac{T_S V_o}{2L}(1-D) \text{ at}$$

DCM to CCM edge

We find I_{LB} or I_{AV} is max. at D = 1/2



For fixed V_o; $(I_{vo})_{max}$ for a buck converter occurs at D = 0. But D=0 means $V_d = V_{in}$ = infinite ,via $I_{LB}(max) = I_o/D$

$I_{LB}(max) = I_{AV}(max) = T_{sw}V_d/8L$ we can rewrite our expressions

For $(I_{LB})_{max}$ for V_d constant **For $(I_{LB})_{max}$ for V_o Constant**

$$\frac{T_S V_d}{2L} \qquad\qquad \frac{T_S V_o}{2L}$$

$$\downarrow \qquad\qquad \downarrow$$

$I_{AV} \equiv 4(I_{LB})_{max} D(1-D)$ $\qquad$ $I_{AV} = (I_{LB})_{max} * (1-D)$

If we divide the period $1-D = D'$ into D_2 and D_3 one can show that $I_L = 0$ in DCM at time $D_1 + D_2$ as shown on page 10

$$D_2 = \Delta_1 = \frac{I_{AV} = I_o}{4(I_{AV})_{max} D}$$

From this we solve for D

in terms of the same ratio $I_{AV}/(I_{AV})_{max}$ or $I_{AV}/(I_{LB})_{max}$

$$\frac{V_o}{V_D} = \frac{D^2}{D^2 + \frac{1}{4}\frac{I_{AV}}{(I_{V_D})_{max}}}$$

$$D = \frac{V_o}{V_d}\left[\frac{I_{AV}/I_{max}}{1 - V_o/V_d}\right]^{1/2}$$

We can plot V_o/V_d(y-axis) versus D

D vs $\frac{I_{AV}}{I_{max}}$ for $\frac{V_d}{V_o}$ as a parameter as on page 10

$\frac{I_{AV}}{I_{max}}$ (x-axis) for V_d = constant

<u>V_d Constant</u>

$$(I_{vLB})_{max} = \frac{T_s V_d}{8L}$$

$$I_{AV}(D) = 4(I_{LB})_{max} D_1(1 - D_1)$$

<u>V_o Constant</u>

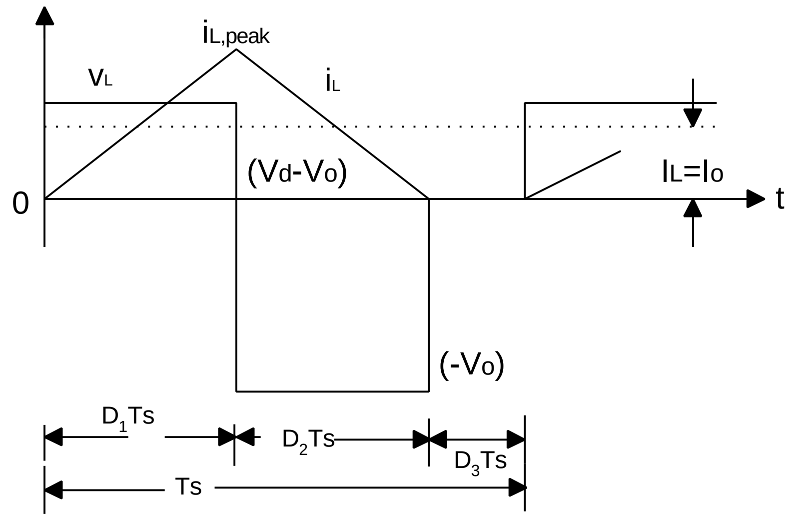
$$(I_{LB_o})_{max} = \frac{T_s V_o}{2L}$$

$$I_{AV}(D) = (I_{vLB_o})_{max} 1 - D_1$$

In <u>either case</u> if I_{AV} decreases i_L will try to go negative <u>but</u> the diode will not allow it. When the diode stops conducting we go from two known periods of switching, D and 1-D, to three periods (D_1, D_2 and D_3), only one of which D_1T_s is known as shown below from the active switch drive. D_2 and D_3 are set by circuit conditions not by switch drive conditions.

c. Into the DCM Region: Beyond the Boundary

We will have three independent time periods within the switch cycle as shown on page 10. Note that D_2 is not equal to 1- D_1 and the third period D_3 is another unknown.



We can try to estimate D_2 from the two equations that are true for all V_d(input) and V_0(output) cases.

EQ1.$\langle V_L \rangle_{Ts} = 0$, $(V_d-V_o)D_1T_s + (-V_o)\, D_2T_s = 0$

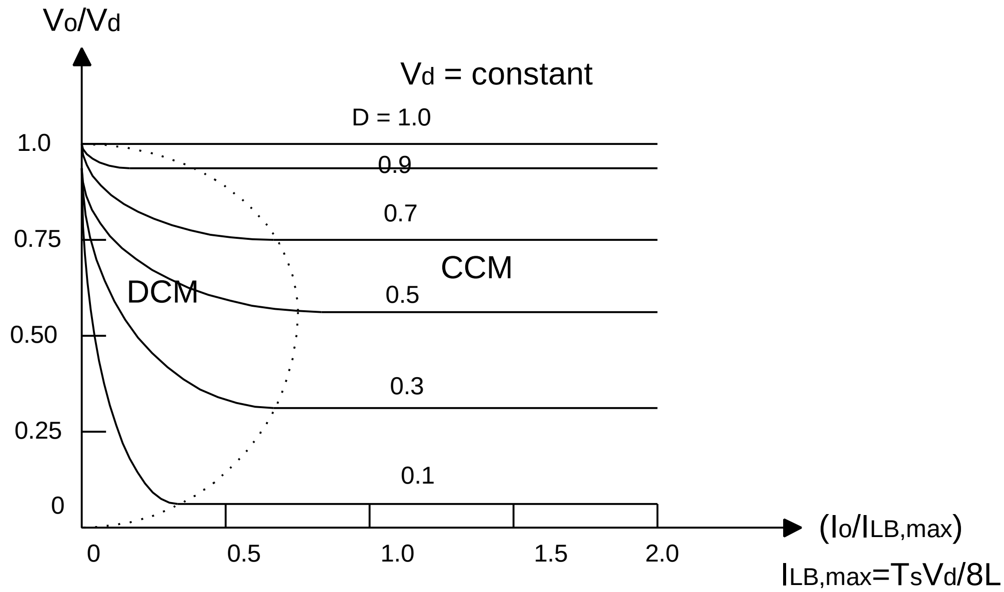
$\frac{V_o}{V_d} = \frac{D_1}{D_1 + D_2}$, <u>Note</u> **EQ 2** $(i_L)_{peak} = V_o/L\; D_2T_s$

$$I_{AV} = \frac{1}{2}(i_L)_{peak}(D_1 + D_2) = \frac{V_o T_s}{2L}(D_1 + D_2)\, D_2$$

Equations 1 and 2 are both employed to make plots of D versus $\left(\frac{I_0}{I_{LB}(max)}\right)$, where I_{LB}(max) a scaling factor is either $T_sV_d/(8L)$ for V_d = constant or $T_sV_o/(8L)$ for V_0 = constant.

1. DCM to CCM Boundary for a Buck with V_d (Constant): where the x-axis scale factor I_{LB}(max) = $T_sV_d/(8L)$

- We have an ideal Buck V_o source for CCM only which has V_o/V_d fixed for all possible load currents up to the DCM boundary as shown
- Dashed curve is the calculated CCM - DCM boundary and it occurs only at low load current.



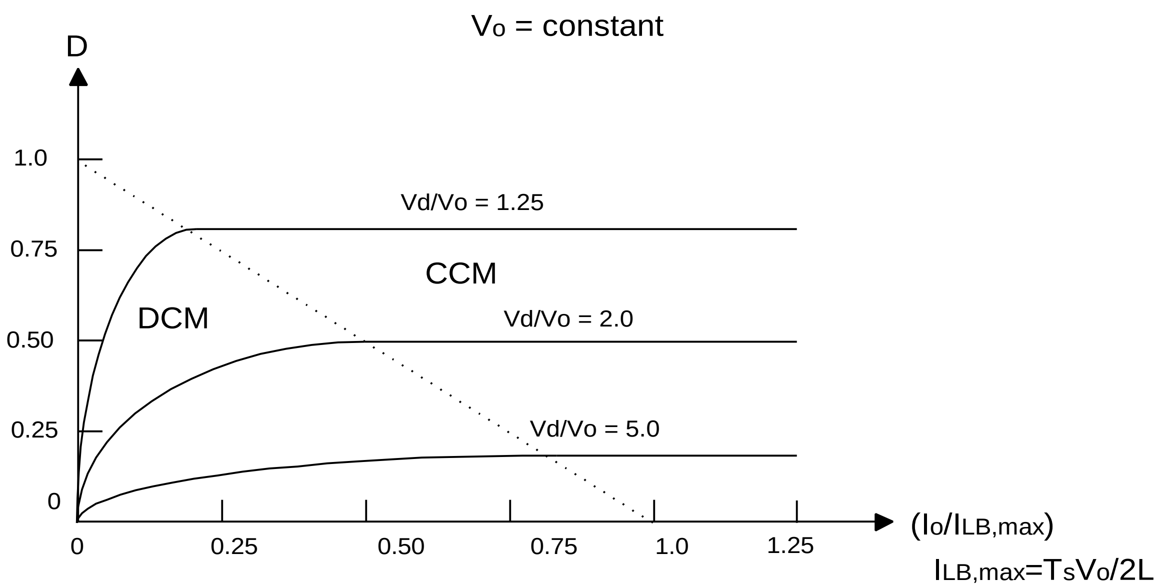
The DCM to CCM boundary depends on both D and I_{AV}/I_{max}. Note how V_0 /V_d changes in DCM versus load current and is constant in CCM. We will revisit this in Chapter 10 of Erickson, especially Problem 10.3 which graduate students are to do for homework #2 now, undergraduates will do it later.

Next we do D vs. I_o/I_{LB} plots for V_o fixed conditions.

2. DCM to CCM Boundary for(<u>V_o Constant) the x-axis scale factor</u> I_{LB}(max) = T_sV_o/(8L)

- We have an ideal buck V_o source for CCM only. We have fixed V_d/V_o for all load currents up to the DCM boundary as shown on page 12.
- Dashed curve is the derived boundary from CCM to DCM. At the boundary we find : $I_{AV} = I_{max}(1-D)$

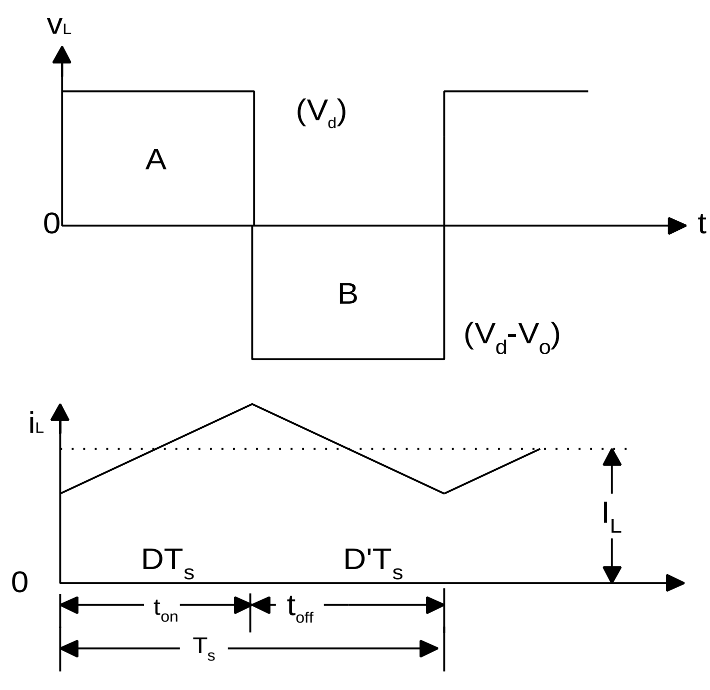
Note the special case on the x-axis $D = 0 \Rightarrow \dfrac{I_{AV}}{I_{max}} = 1.0$

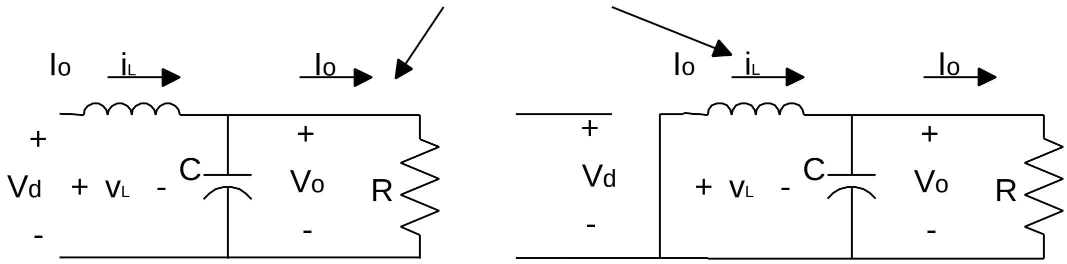


Similar plots of the DCM to CCM boundary can be made for the other two basic converter topologies as we will do below.

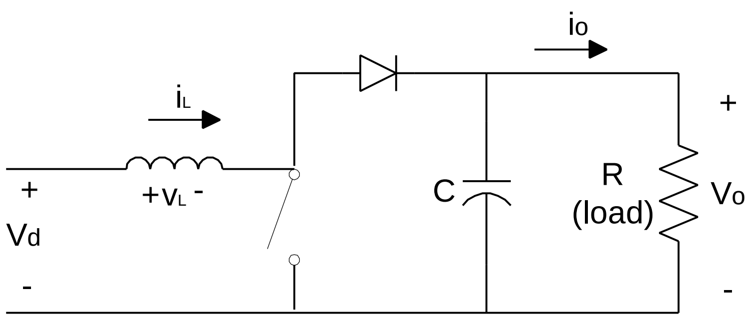
2. Boost or Step-up Topology

a. Review of CCM V_0 /V_d Transfer Function and Inductor Waveforms





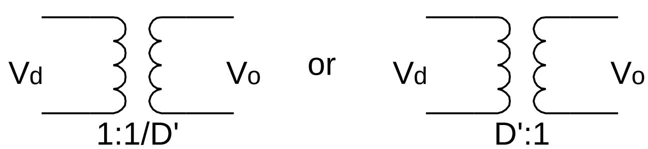
The above switch on and switch off circuit topologies **are WRONG** for the boost circuit. For HW#2 please draw the correct circuit topologies, that the equations below will satisfy by using the boost circuit below in two switch states.



$$\langle V_L \rangle_{Ts} = 0;\ V_d D + (V_d - V_o)(1-D) = 0;\ \frac{V_o}{V_d} = \frac{1}{1-D} = \frac{1}{D'}$$

Moreover for the lossless converter $I_{out} = I_{in}(1-D)$

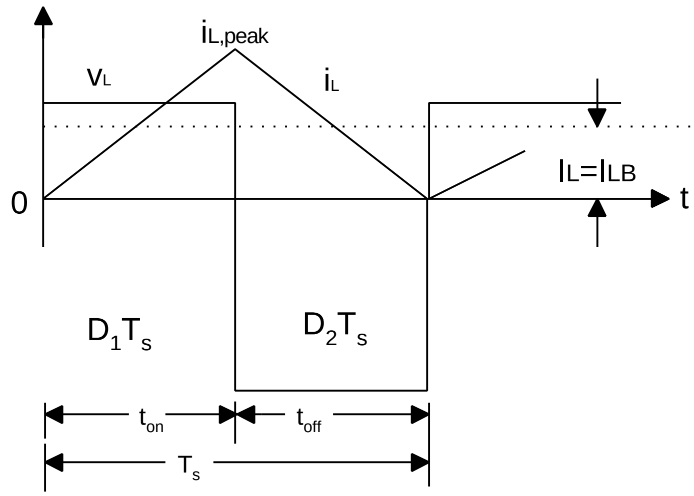
The DC transformer model for the CCM boost topology would be:



b. **At the DCM to CCM Boundary**

At the CCM-DCM boundary the DC current reduces until the ac current tries to go negative and the uni-directional switch cannot follow. I_{LB}(average)= $I_{AV} \equiv \frac{1}{2} i_L(\text{peak}) = \frac{V_d}{2L} D_1 T_s$

This equation is valid just at the boundary.



The average output current, I_{LB}(average),also occurs at CCM-DCM boundary. We choose to examine the case V_o(constant) and V_d(input) or V_{in} varies which corresponds to a crude rectified ac mains for V_{in} and a feedback circuit where V_0 is kept constant. We will find I_{LB}(boundary), I_{OB}(output at the boundary) versus the D_1 duty cycle set by the timing

I_{LB}(average at the boundary) $I_d = I_{AV} = \frac{T_s V_o}{2L} D_1(1-D_1)$ The maximum occurs at D_1 = ½ and is called I_{LB}(max). Now we learned before that the inductor current equals I_{in} for a boost converter. We also know at the boundary that $I_{out} = I_{in}(1-D_1)$ using the simple CCM relation in steady-state.

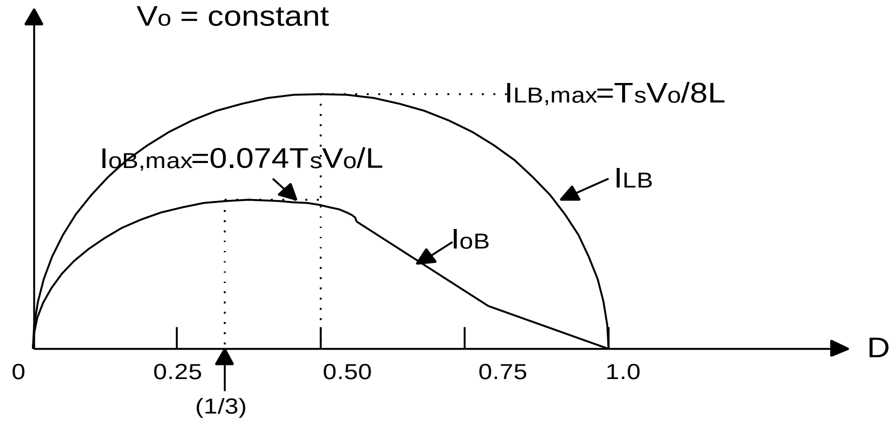
$$I_{OB} = I_{out} = I_0 = i_L(1-D_1) = \frac{T_s V_o}{2L} D_1(1-D_1)^2$$

EQU. #1 at the border

$$(I_{AV})_{max} = \frac{T_s V_o}{8L} \text{ occurs at } D_1 = \frac{1}{2}$$

Equ #2 at the border $(I_{oB})_{max} = \frac{2}{27} \frac{T_s V_o}{L}$ at $D_1 = \frac{1}{3}$

Where 2/27=0.74. Plotting these two equations versus D on page 15 we find $(I_{AV})_{max}$ occurs at D=0.5,while $(I_o)_{max}$ occurs at D=0.33; as shown.



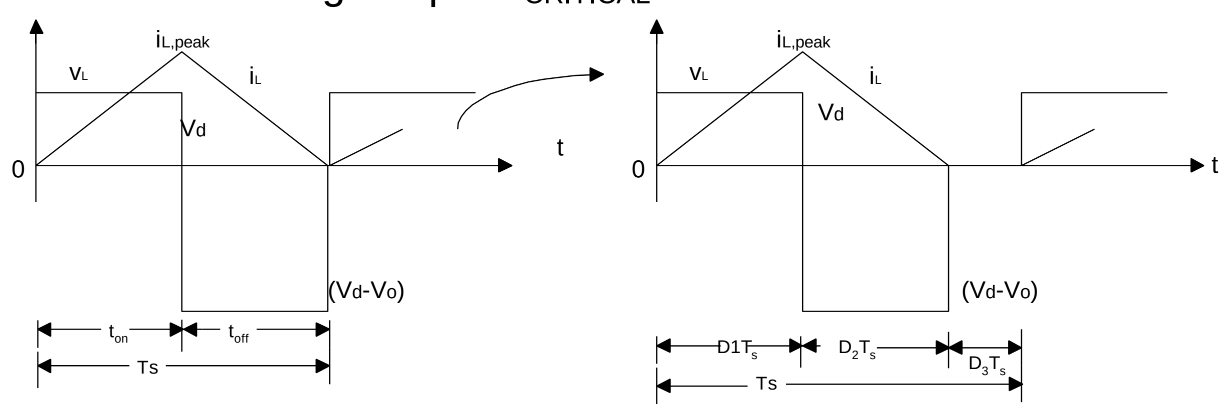
Rewriting both currents in terms of the maximum values versus D we find:

$I_{AV}(D) = 4D_1(1-D_1)\,(I_{AV})_{max}$ $\qquad I_o(D_1) = 27/4\ D_1\,(1-D_1)^2\,(I_o)_{max}$

When i_L tries to go negative we go from the left curve below to the right:

c. **Into the DCM Region: Beyond the Boundary**

Assume that as the output current decreases that both V_d and D are unchanged up to $I_{CRITICAL}$.



The volt-sec balance on the inductor$\langle V_L \rangle_{Ts} = 0$ in DCM case to the right on the above figure gives:

$$D_2 \neq 1 - D_1,\ D_2 = 1 - D_1 - D_3$$

$$V_d\, D_1 T_s + (V_d - V_o)\, D_2 T_s = 0$$

This leads to equation #3

Eq #3 $\frac{V_o}{V_d} = \frac{D_2 + D_1}{D_2}$ $\Rightarrow$ In the lossless case $P_o = P_d$

$\frac{I_o}{I_d} = \frac{D_2}{D_1 + D_2}$ $= I_{out}/I_{in}$

Next we turn to the average or DC conditions for I_{in} or I_d at the boundary via the simple triangle rule Average value= ½ $I_{peak}\ \Delta t$

$$(I_d)_{AV} = (I_L)_{AV} = \frac{V_d}{2L} D_1 T_s [D_1 + D_2]$$

$\downarrow$ $\downarrow$

1/2 peak total time duration(D_1 +D_2) for I>0

The average output current is related to the average input current:

$$(I_o)_{AV} \equiv (I_d)_{AV} * \frac{D_2}{D_2 + D_1}$$

Since I_0 (average at the boundary) in terms of V_d ,D_1 and D_2 is: **Eq#4** $(I_o)_{AV} = \frac{T_s V_d}{2L} D_1 D_2$

Using Equations 1,3 and 4 above we find

$$D = f\left(\frac{V_o}{V_d}\right),\ I_o / I_{max}$$

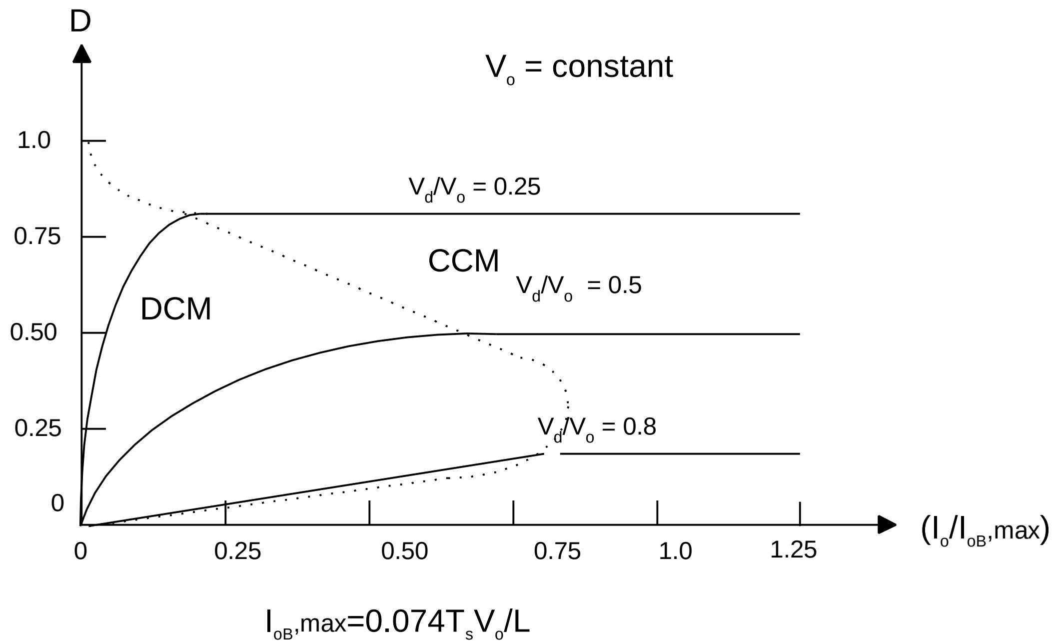
$$D(\text{Boost}) = \left[\frac{4}{27}\ \frac{V_o}{V_d}\ \left(\frac{V_o}{V_d} - 1\right)\ \frac{I_o}{I_{max}}\right]^{1/2}$$

If we plot D (y-axis) versus the now familiar ratio I_o/I_{oB}(max) on the x-axis where I_{OB}(max) =2/27 $T_{sw}V_0/L$:

- We achieve an ideal boost V_o source for CCM only, where V_0 is flat versus I_o/I_{OB}.
- The dashed curve is the derived DCM-CCM Boundary in steady-state. Note that the D required to achieve DCM of operation changes with V_0 and I_0/I_{OB}(max) conditions Below we consider additional salient points

$D \rightarrow 0$, $I_o/(I_o)_{max} \rightarrow 0$. For all V_0/V_d curves

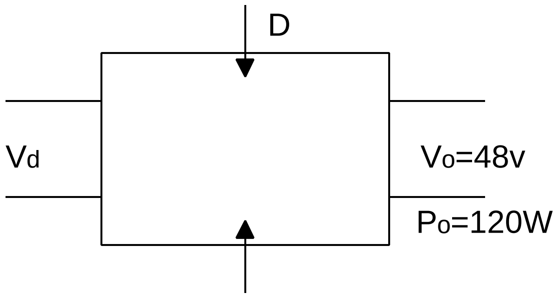
- For $I_0/I_{OB}(max) > 0.9$ only CCM occurs with ideal V source curves as shown below



<u>Warning</u>: We have assumed that we have an operating feedback loop so that in DCM, V_o is kept constant during each T_s by varying D, $V_o = V_d/1\text{-}D$. If there is no feedback however then at light load $V_o \rightarrow$ dangerously high.

<u>d. **Example:**</u>**For the 120 W Boost Converter below we choose C very large so we always have DC output**.

V_d(input) is crude DC and varies from 12 to 36 V but D changes, via an undisclosed feedback loop, to keep the output fixed at 48 V.



In steady state the output current in steady state is:

$$(I_o) = \frac{120\,W}{48} = 2.5\,A$$

f_s = 50 KHz, T_s = 20 msec

We are asked to find L(max) which **insures DCM operation** only. We want to avoid entirely CCM of operation. That is the choice of L must not exceed L(max). $L < L_{MAX}$ to insure DCM of operation. Using CCM equations, which are valid only at the border, we find the range of D required.

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$ for V_o (fixed) and $12 \le V_d \le 36$.

Then we find $\frac{1}{4} \le D \le \frac{3}{4}$ in order to keep V_o fixed at 48 V.

$$I_o = \frac{T_s V_o}{2L} D(1-D)^2$$

We solve for L_{max} and note the smallest L_{max} would occur at D=0.75

$$L \le \frac{T_s V_o}{I_o(\min)} 0.75(0.25)^2.$$

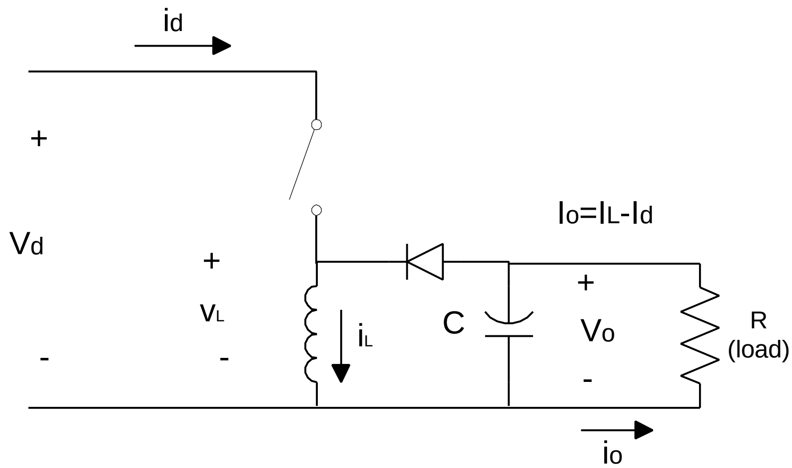
Hence for a given T_s = 20 msec, V_o = 48 V, and I_o = 2.5 A, we find:

$L \le$ 9 mH guarantees we always operate INSIDE the edge of the CCM to DCM boundary **on the DCM side**.

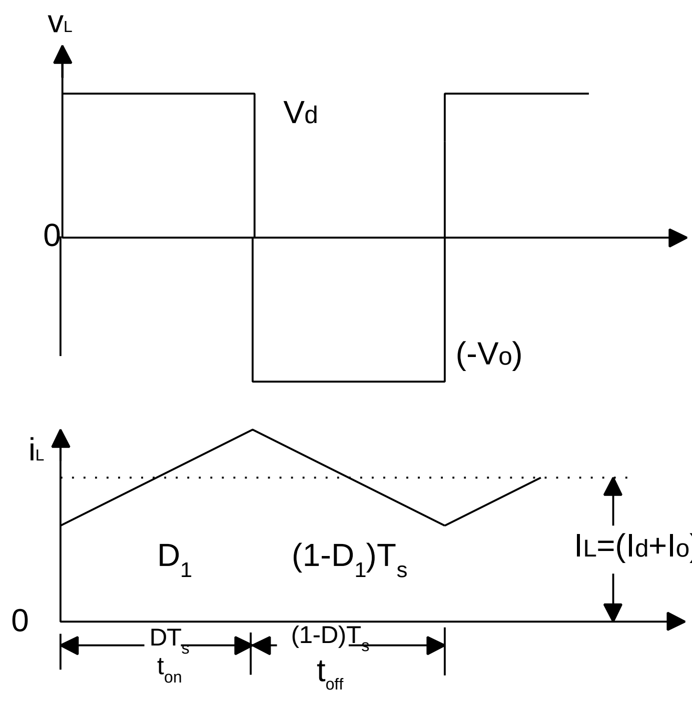
2. Buck-Boost Topology

a. Review of CCM V_0/V_d Transfer Functions and Inductor Waveforms

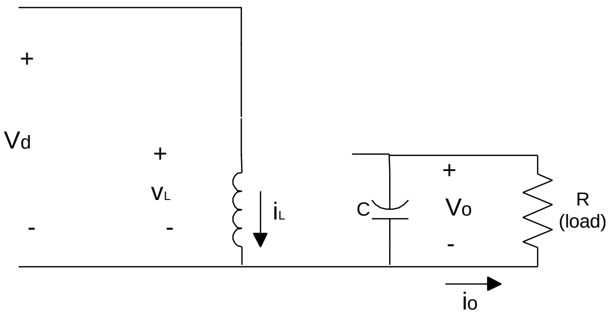
The basic buck-boost circuit with switches is shown below:

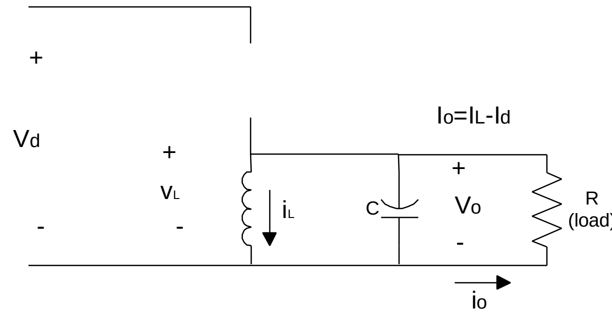


In the CCM of operation the V_L and I_L waveforms for buck-boost are shown below.



The corresponding topologies for the two CCM switch conditions are:

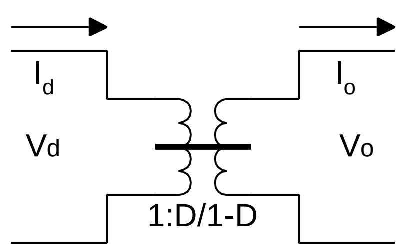




In steady-state the volt-sec on the inductor yields:
$<V_L>_{Ts} = 0$; $V_d\, DT_s + (-V_o)(1\text{-}D)\, T_s$;

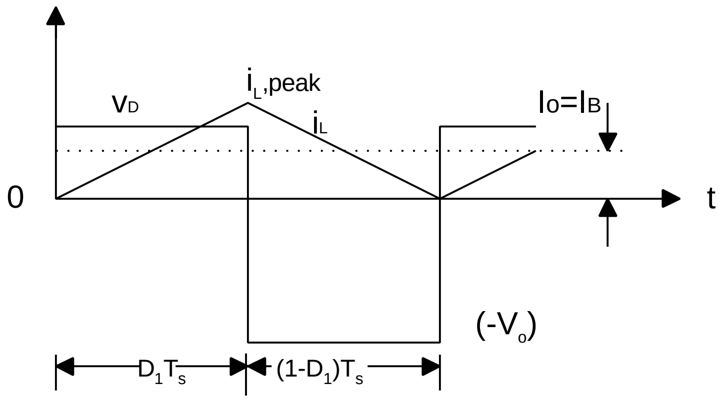
Hence we find $\frac{V_o}{V_d} = \frac{D}{1-D}$ and for a loss free converter we also know $\frac{I_o}{I_D} = \frac{1-D}{D}$

The DC Transformer Model for the Buck-boost in Steady – State is:



b. **At the DCM Border**

At the CCM-DCM Boundary the inductor current, i_L, just reaches zero and tries to go negative but the circuit diodes will not allow it. We now write expressions for the inductor current average, I_{AV}(boundary), versus V_d and D as well as versus V_0 and D.



I(average at boundary)=$I_{AV} \equiv \frac{1}{2} i_L(\text{peak}) = I_d =$

I_{LB}(average)= $I_{av}(V_d, D) = \frac{T_s D_1}{2} \frac{V_d}{L}$

Max at D = 0

↓

Using at the DCM to CCM boundary

$V_d = V_o \frac{(1-D_1)}{D_1} \Rightarrow = \frac{T_s V_o}{2L}(1-D_1) = I_{AV}$ (boundary)=$I_{AV}(V_0,D)$.

In the buck-boost if I_C(capacitor)=0, the output current is: $I_0 = I_d(1-D)/D$ and at the border we find:

$I_o = I_L - I_d = \frac{T_S V_o}{2L}(1-D_1)^2$, which is Max at $D_1 = 0$

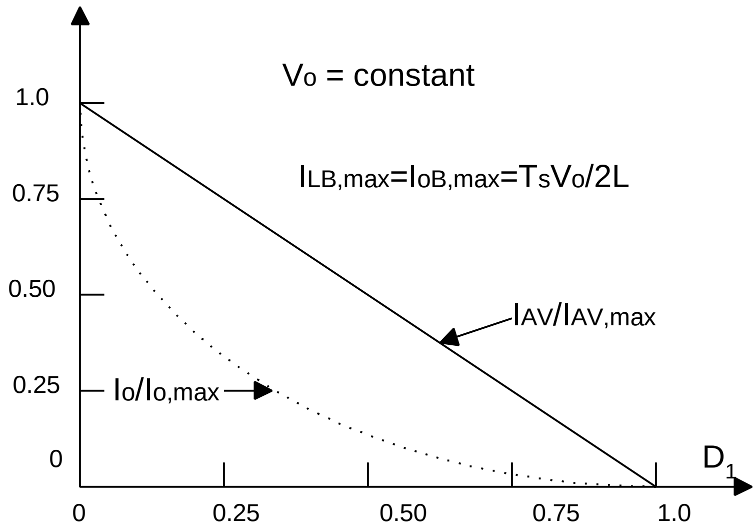
$I_{LB}(\max) = I_{OB}(\max) = T_{SW} V_0 / 2L$

$(I_{AV})_{\max@ D=0} = \frac{T_s V_o}{2L}$ *and* $I_{AV} = (I_{LB})_{\max}(1-D_1) = I_{LB}(D)$

$(I_{OB})_{\max@ D=0} = \frac{T_s V_o}{2L}$ *and* $I_{OB} = (I_o)_{\max}(1-D_1)^2 = I_{OB}(D)$

Assuming V_0 is constant with respect to D(via feedback), if we plot

$\frac{I_{AV}}{(I_{AV})_{\max}} = 1-D$ we find the linear solid line connecting the two axii as shown on page 22.



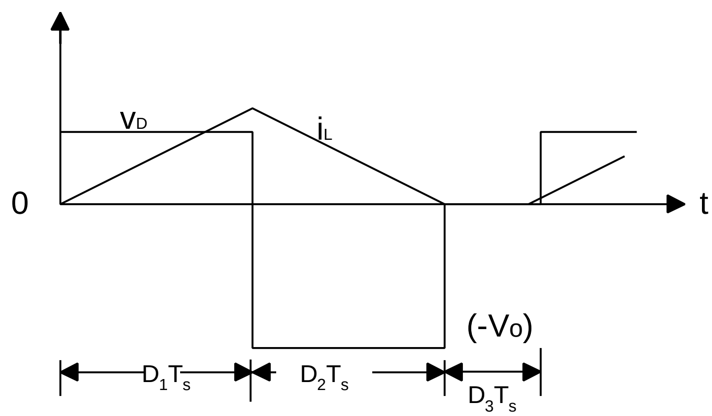
If we plot

$\frac{I_{OB}}{(I_{OB})_{\max}} = (1 - D)^2$ we find the dashed line shown above

c. In the DCM Region: Beyond the Border

If i_L tries to go negative in the buck-boost circuit the uni-polar diode prevents it. So DCM occurs with three time periods D_1,D_2, and D_3 shown below. Doing volt-sec balance on the inductor in the three periods we find:

$<V_L> = 0$; $V_dD_1T_s + (-V_o) D_2T_s = 0$; Where $D_2 \neq 1 - D_1$; Rather $D_2 = 1 - D_1 - D_3$



$\frac{V_o}{V_d} = \frac{D_1}{D_2}$ $\Rightarrow$ In a loss less converter operating in DCM

P_o(out) = P_d(input)

$\frac{I_o}{I_d} = \frac{D_2}{D_1}$ $\Rightarrow$We can calculate the value at the border

$$(I_L)_{AV} = \frac{1}{2}(i_L)_{peak}(D_1 + D_2)$$

↓ Height ↓ time

$$= \frac{V_d}{2L} D T_s (D_1 + D_2)$$

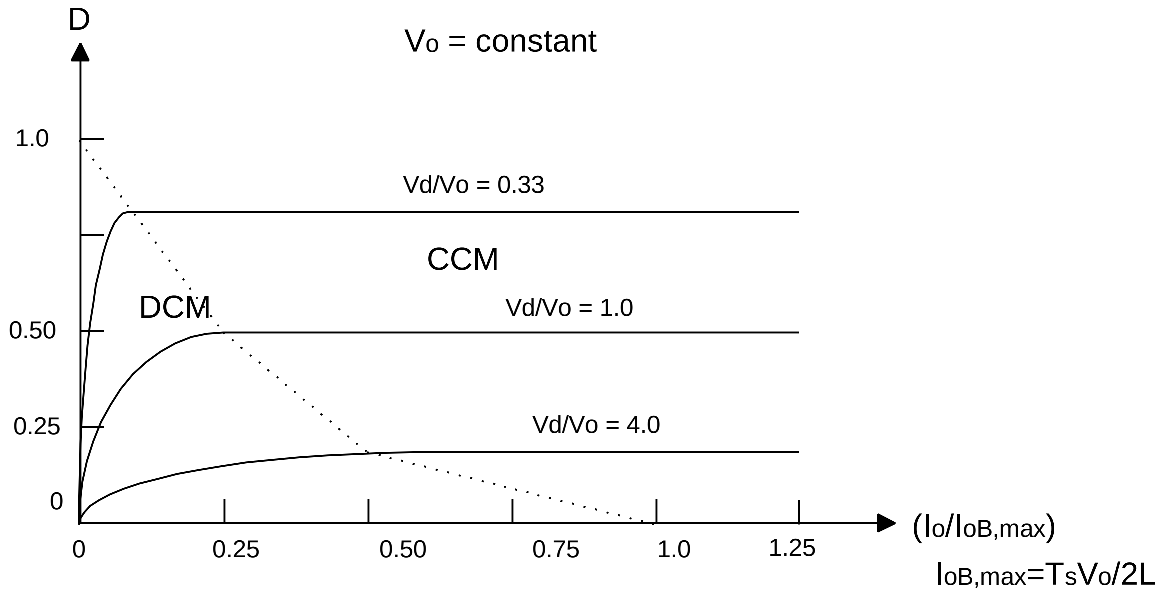
One can plot the duty cycle at the DCM to CCM border, D, on the y-axis for V_0 = constant as a function of the output current,I_0, on the x-axis. We scale the x-axis as I_0 / I_{OB}(max). Where $I_{OB} = T_{SW} V_0 / 2L$. In short,

$$D = f\left(\frac{I_o}{(I_o)_{max}}, \frac{V_o}{V_d}\right)$$

$$D = \frac{V_o}{V_d}\left[\frac{I_o}{(I_{OB})_{max}}\right]^{1/2} = D(DCM\ to CCMBorder)$$

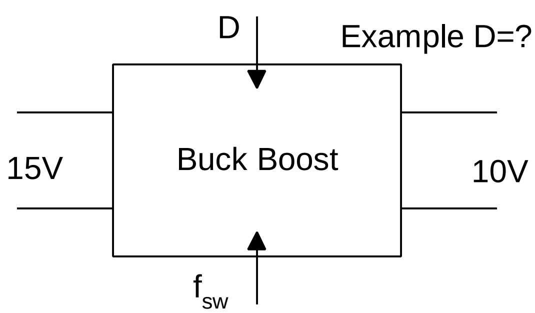
<u>Summary of Buck-Boost CCM to DCM Border</u>

1. We have an ideal V_o source for CCM operation which is flat with I_o/I_{OB}(max) throughout the CCM region.
2. The dashed curve is the CCM-DCM boundary which varies position with both the ratio $V_d /.V_0$ and the ratio I_0 / I_{OB}(max)
3. Note that V_d / V_0 versus D is no longer flat in the DCM region of operation. $D \rightarrow 0$ pins all curves to one operating point. I_0 */I_{OB}(max)=0 for all V_d / V_0 curves as shown on page 24.*



5. Simple Numerical Example

Consider the 10 Watt buck-boost converter below with f_{SW}= 20 kHz and with a steady-state output voltage at 10 V. We have a load such that the current drawn is 1.0 A at the output. C is assumed very large so V_0 = constant and L = 1/20 mH. **CAN you tell if this is a DCM or CCM equilibrium condition??**



$P_o = 10$ W
$I_o = 1$ A

$f_{sw} = 20$ KHz, $T_s = 0.05$ msec

Find: D to achieve desired operation & determine if it is CCM or DCM.

First assume CCM operation occurs:

$$\frac{V_o}{V_g} = \frac{10}{15} = \frac{D}{1-D}$$ which implies that for given steady-

state circuit conditions above D = 0.4, but only if we really operating CCM, but we may not be. To determine at the outset the mode of operation we first find $I_{CRITICAL}$ for the buck-boost circuit as follows:

$$I(\text{critical}) \;=\; \frac{T_S V_O}{2L} \;=\; \frac{0.05\,\text{m}\sec{}^*10}{2(0.05\,\text{mH})}$$

I(critical) = 5 A for this buck-boost

I_o(at DCM-CCM boundary) is given by the well known and simple CCM equation.

$I_o = (I_o)_{max}(1\text{-}D)^2 = 5(0.6)^2 = 1.8$ A which is < 5 A so we cannot be operating CCM as we first assumed.

Surprise! ⇒ DCM not CCM operation is occurring in steady-state.

But if we have DCM operation the D value is then given by a different steady-state relationship:

$$D(DCM) \;\equiv \frac{V_O}{V_d}\sqrt{\frac{I_O}{(I_O)_{max}}}$$

$$= \frac{10}{15}\sqrt{\frac{1.0}{5}}$$

D(DCM) = 0.3.

Finally, For HW#2 Due next time:

1. Answer any questions asked throughout lectures 37-39.
2. Erickson Chapter 5 and 10
    a. Graduate Students: Problems 5.4, 5.14 and 10.3.
    b. Undergraduates: Problems 5.4 and 5.14.