Lecture 38

Establishing DCM V_{out}/ V_{in} Quadratic Relations

A. DCM DC Transfer Functions via Quadratic Relations

1. Overview of the DCM Operation Conditions
2. V_o for the Buck
3. V_o for the Boost
4. V_o for the Buck-Boost
5. Summary
6. $L_{critical}$ for the Three Major Circuits
7. Critical Capacitance: $C_{critical}$

B. Homework Solutions and Hints

1. Problem 5.1
2. Problem 5.9
3. Hints for Problem 5.4
4. Hints for Problem 5.14

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A. DCM DC Transfer Functions via Quadratic Relations

1. Overview

The figure below summarizes DCM operation for all three major converters: buck, boost and buck-boost using transistor and diode switches. Note I_L(inductor) goes to zero each switch period and hence is insured to start form zero at the start of each switch period. We will show i_L(on far left), i_{in}(middle plot), and i_d(far right) for each circuit versus time. Note that V_o in DCM comes only from solving a quadratic equation not from M(D), assuming V_{in} is constant over the switch period. $P_{in} = P_{out}$ is also assumed. We will use linear variations of all currents as a good first approximation.

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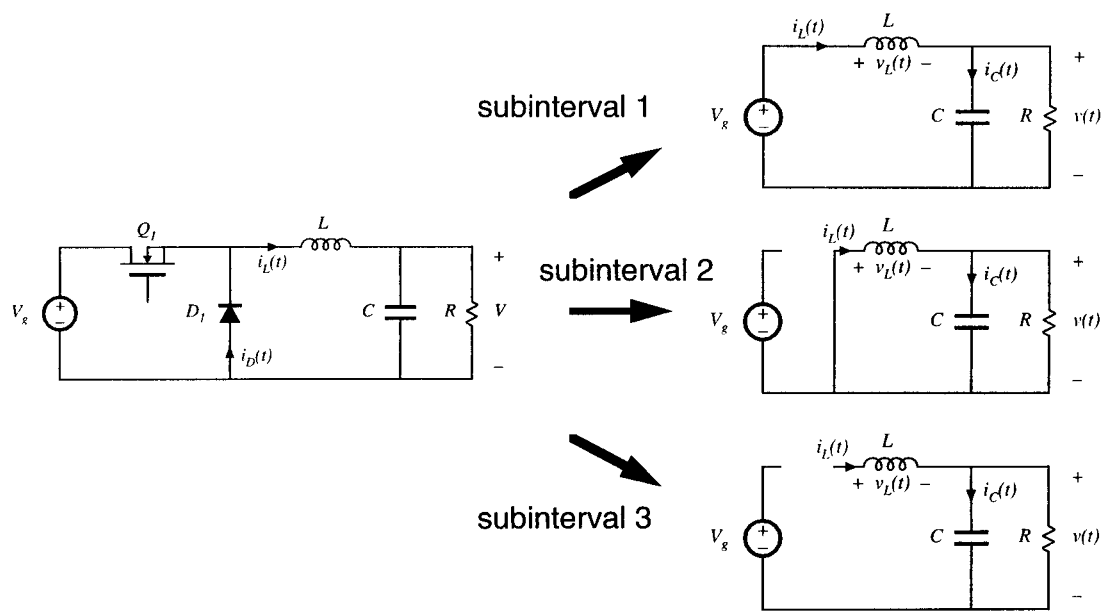
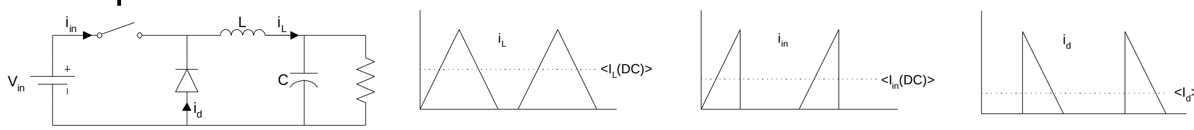
1. V_o Buck



The three circuit topologies for the buck are as above:

The circuit and associated waveforms are shown below in DCM operation.



$v_L = D_1(V_{in} - V_{out}) + D_2(-V_{out})$, $(D_1 + D_2)V_{out} = D_1V_{in}$

$I_L(DC) = V_{out}/R = ½(D_1 + D_2)\, i_L(max)$

b. Derivation of DCM DC Transfer Function

Since D_1T_s is set by the control circuit and V_{in} is assumed constant, the peak inductor current is from the far left waveform:

$$i_L(\text{peak}) = D_1T_s\left[\frac{(V_{in} - V_{out})}{L}\right]$$

From i_{in} triangular waveforms in the middle plot we calculate:

$$I_{in}(\text{av}) = \frac{D_1}{2}\, i_L(\text{peak}) = D_1^2 T_s\left[\frac{(V_{in} - V_{out})}{2L}\right]$$

$$P_{in}(\text{av}) = V_{in}(\text{constant})I_{in}(\text{av}) = D_1^2 T_s\left[\frac{\left(V_{in}^2 - V_{in}V_{out}\right)}{2L}\right]$$

For lossless converters we can say the following:

$P_{in} = P_{out} = V^2_{out}/R$

Yielding a quadratic equation in V_{out}.

Solving:

$$V_o(\text{DCM Buck}) = \frac{-D_1^2 V_{in} R T_s}{4L} + D_1 V_{in}\sqrt{\frac{RT}{2L} + \frac{R^2 T_s^2 D_1^2}{16L^2}}$$

c. Alternative Derivation

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2} \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V) \qquad \text{(from capacitor charge balance)}$$

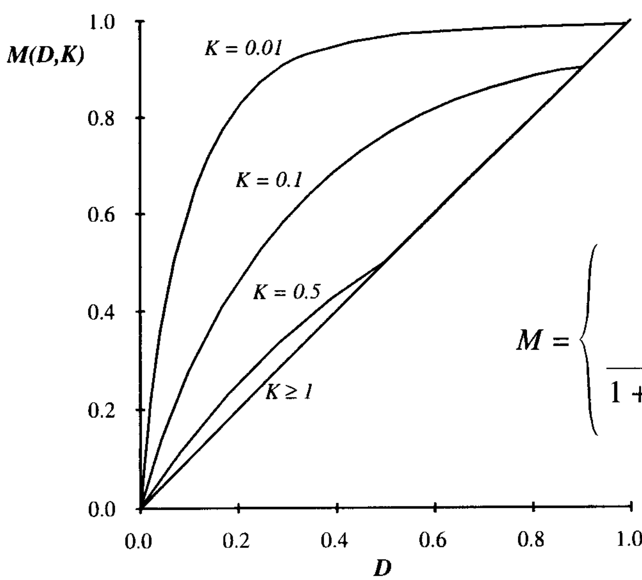
Eliminate D_2, solve for V:

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}$

We get this from doing out steady-state balance. The resulting plot for the buck DCM transfer function vs. D is:



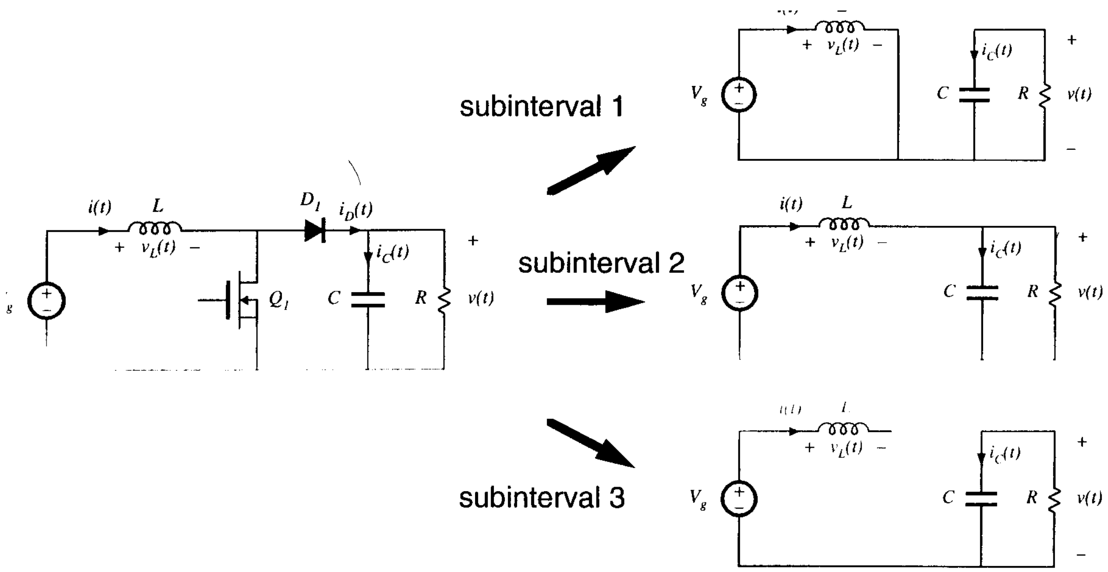
$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

Again note the trend for the DCM DC transfer function to be bigger for all D values than the CCM transfer function.

3 .V_o Boost in DCM Operation

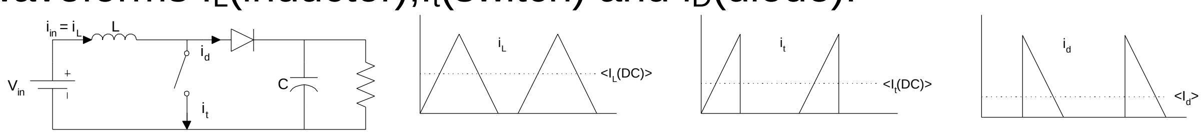
a. General

The three circuit topologies for the DCM boost are shown below:



b. Derive DCM DC Transfer Function

We also show below the three boost circuit current waveforms I_L(inductor),I_t(switch) and i_D(diode):



$v_L = D_1(V_{in}) + D_2(V_{in} - V_{out})$, $(D_1 + D_2)V_{in} = D_2V_{out}$
$I_L(DC) = I_{in}(DC)= ½(D_1 + D_2)\ i_L(max)$

Since D_1 is set by the control circuit the and V_{in} is fixed the peak inductor current from the left side plot is:

$$i_L(\text{peak}) = \frac{D_1T_sV_{in}}{L}$$

From the middle I_{in} plot vs. time in the middle we can find:

$$I_{in}(av) = I_L(av) = \frac{D_1 + D_2}{2} i_L(\text{peak})$$

$$I_{in}(av) = \frac{(D_1 + D_2)V_{in}D_1T_s}{2L}$$

$$P_{in}(av) = V_{in}(\text{fixed})I_{in}(av) = \frac{(D_1 + D_2)V_{in}^2D_1T_s}{2L}$$

$$P_{in}(av) = P_{out}(av) = V_o^2/R_L$$

Both D_2 and V_{out} are unknown.

Yielding a quadratic equation in V_{out}. Fortunately the sawtooth diode current waveform drives the load so that

$$I_{out}(av) = V_{out}/R = \frac{D_2}{2} i_{diode}(peak)$$

Which sets $D_2 = \frac{2V_o L}{RV_{in} D_1 T}$ and we obtain a new quadratic equation for V_o.

Solving:

$$V_o(\text{Boost DCM}) = \frac{V_{in}}{2} + \frac{V_{in}}{2}\sqrt{1 + \frac{2RT_s D_1}{L}}$$

c. Alternative DCM Transfer Function Derivation

Two equations and two unknowns (V and D_2):

$$V = \frac{D_1 + D_2}{D_2} V_g \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R} \qquad \text{(from capacitor charge balance)}$$

Eliminate D_2, solve for V. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

This can be solved via the quadratic formula as shown on page 7.

Solving for V_o /V_g(input) we find:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive V, while other leads to negative V. Select positive root:

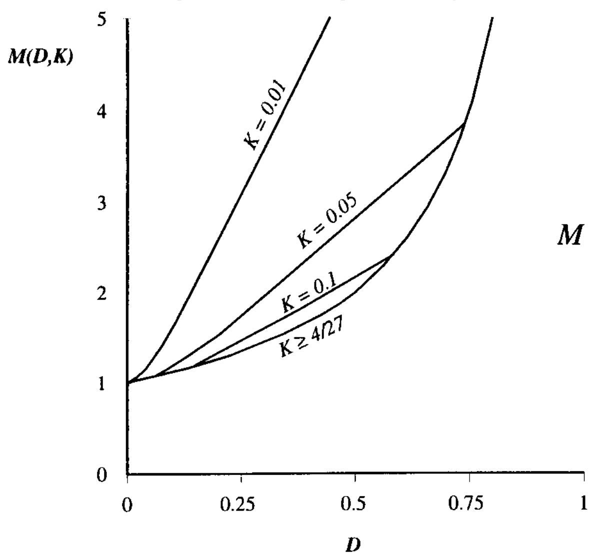
$$\frac{V}{V_g} = M(D_1, K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

This comparative plot again shows the DCM DC transfer



$$M = \begin{cases} \dfrac{1}{1-D} & for\ K > K_{crit} \\ \dfrac{1 + \sqrt{1 + 4D^2 / K}}{2} & for\ K < K_{crit} \end{cases}$$

Approximate M in DCM:

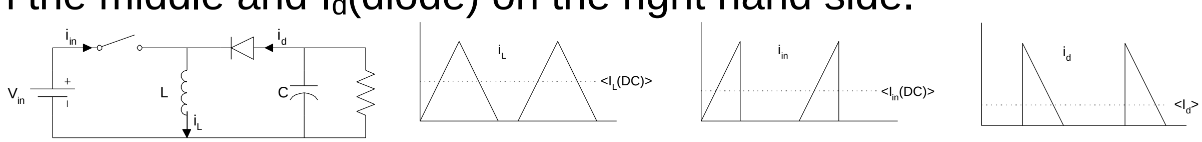
$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

function lying above the CCM values for all D.

4. V_o Buck-Boost DCM Operation

a. General

We skip to the current waveforms I_L(inductor) on the right, I_{in} in the middle and I_d(diode) on the right hand side.



$v_L = D_1(V_{in}) + D_2(V_{out})$, $D_1V_{in} = -D_1V_{out}$

$I_L(DC) = I_{in}(DC) + I_{out}(DC)$

b. Derive the DCM DC Transfer Function

Since D_1 is known and V_{in} is constant the peak inductor current when the Tr is on is:

$$i_L(\text{peak}) = \frac{D_1 T_s V_{in}}{L}$$

From the sawtooth $i_{in}(t)$ waveform:

$$I_{in}(\text{av}) = \frac{D_1}{2} i_L(\text{peak}) = \frac{V_{in} D_1^2 T_s}{2L}$$

$$P_{in}(\text{av}) = V_{in}(\text{const}) I_{in}(\text{av}) = \frac{V_{in}^2 D_1^2 T_s}{2L} = P_{out} = V_o^2/R$$

Solving for V_{out} directly:

$$V_o(\text{Buck-Boost DCM}) = -D_1 V_{in}\sqrt{\frac{RT_s}{2L}}$$

c. Alternative derivation

For HW# 2 YOU Derive the buck-boost transfer function using the proper balance equations

DCM operation occurs for small L, L < L_c(critical), i_L goes to zero before the end of the cycle and when the Tr goes on i_L always starts from zero. That is the ratio RT_s/L has to be below a critical level to avoid DCM and remain in CCM operation.

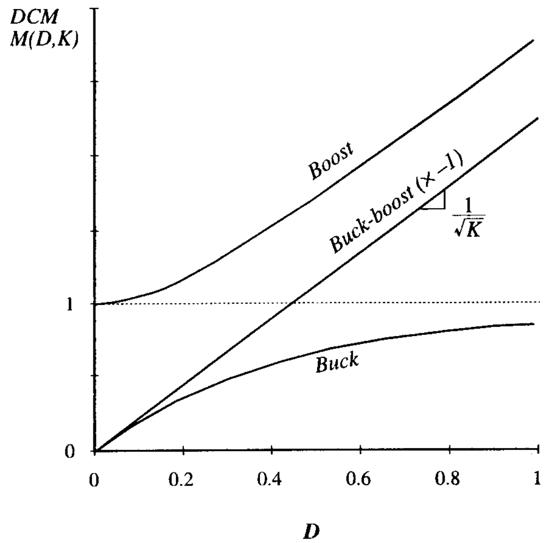
5. A summary would include:

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	DCM $M(D,K)$	DCM $D_2(D,K)$	CCM $M(D)$
Buck	$(1-D)$	$\frac{2}{1+\sqrt{1+4K/D^2}}$	$\frac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\frac{1+\sqrt{1+4D^2/K}}{2}$	$\frac{K}{D}M(D,K)$	$\frac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\frac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\frac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.

This could also be plotted as shown below:



- DCM buck and boost characteristics are asymptotic to $M = 1$ and to the DCM buck-boost characteristic
- DCM buck-boost characteristic is linear
- CCM and DCM characteristics intersect at mode boundary. Actual M follows characteristic having larger magnitude
- DCM boost characteristic is nearly linear

Next we get expressions for L_c(critical) for each converter topology.

6. L_c(critical) for the Three Major Circuits

We expose the DCM / CCM boundary in terms of an L_c(critical). When $L = L_c$ the i_L waveform is always triangular within T_s, so that $D_1 + D_2 = 1$. At this value of L we can also use the DCM expressions for V_o. Setting the two V_o expressions equal sets L_c.

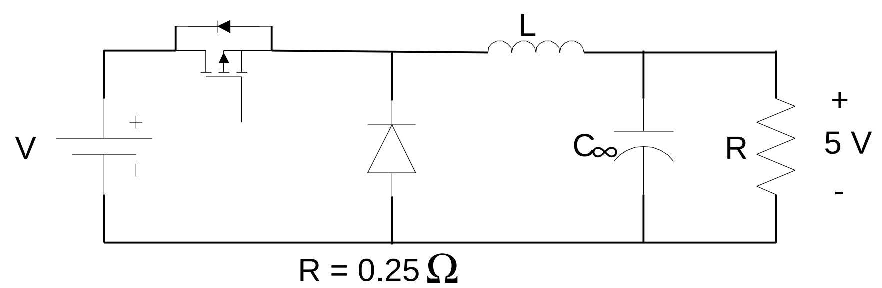
a. Buck critical L

For $V_o = D_1V_{in}$ (at DCM-CCM boundary)

$$\frac{V_{in}^2 D_1}{R} = \frac{D_1^2 T_s}{2L_c}\left[V_{in}^2 - D_1 V_{in}^2\right]$$

$$L_c = \frac{RT_s}{2}\left(1 - D_1\right)$$

The buck circuit below provides 100W at 5V with a 48V input.



What is L_c? At the DCM-CCM boundary using $V_{out} = D_1V_{in}$ yields $D_1 = 5/48$ and $D_2 = 43/48$. $P_{av} = 100$ implies $I_{out}(av) = 20A$ and $\Delta i_L(peak) = 2I_{av} = 40A$.

$e = L_c di/dt$ where $dt = D_1 10\mu s$

$43 = L_c*40/(5*50/48)$ or $L_c = 1.1\ \mu H$

For $L < L_c$ DCM operation for $L > L_c$ CCM operation.

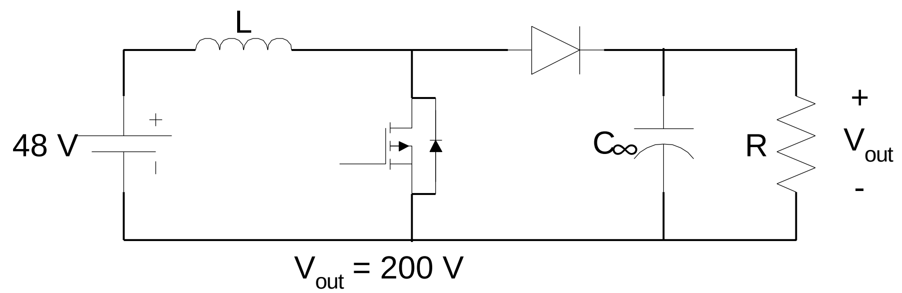
b. **Boost critical L**

For $V_o = V_{in}/(1-D_1)$ (at DCM-CCM boundary)

$$\frac{V_{in}^2}{(1-D_1)^2} = \frac{V_{in}^2 D_1^2 T_s R}{2L_c} + \frac{V_{in}^2}{1-D_1}$$

$$L_c = \frac{D_1 R T_s}{2}(1-D_1)^2$$

In the boost circuit below $V_{in} = 48V$, $V_o = 200V$, $L = 15\ \mu H$ and $f_{sw} = 50$ kHz ($T_s = 20\ \mu s$)



What is load P(min) so that $L \geq L_c$? At the DCM / CCM boundary using $V_{out} = V_{in}/(1-D_1)$ yields $D_1 = 0.76$ and $D_2 = 0.24$. When the Tr is on V_L is 48V for a time $dt = 0.76*20\ \mu s$.

$V_L = Ldi/dt$

The di range during this time is $2I_L(av) = 2I_{in}(av)$

$2I_L(av) = di > 48*0.76*20 / 15\ \mu s = 48.64$

The load power must be at least 48*48.6 / 2 = 1167 W.

c. **Buck-Boost critical L_c**

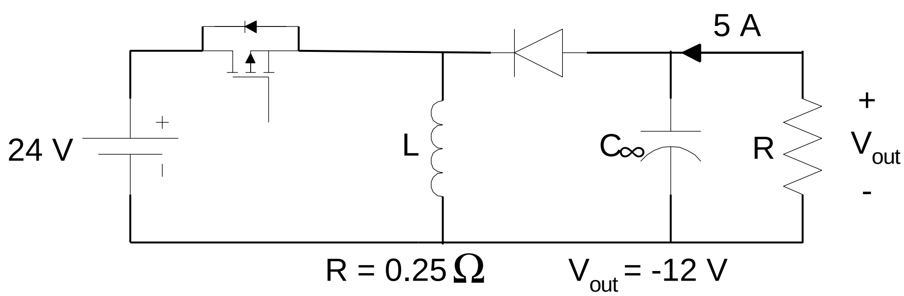
For $V_o = D_1V_{in}/(1-D_1)$ (at DCM-CCM boundary)

Likewise from the DCM relations

$$V^2_{out} = \frac{V_{in}{}^2 D_1{}^2 RT_s}{2L_c} = \frac{V_{in}{}^2 D_1{}^2}{(1-D_1)^2}$$

$$L_c = \frac{RT_s}{2}(1-D_1)^2$$

The buck-boost circuit below has V_{in} = 24V, V_o = -12V and provides 60 W on average.



Find L_c vs. f_{sw}. $I_{out}(av)$ = 60/12 = 5A. For V_{in} = 24V $I_{in}(av)$ = 25A. Now $I_L = I_o + I_D$ = 7.5A provided we are in CCM and

$L > L_c$. Then i_L is a triangle wave from 0 to 2. I_{av} = 15A over the time D_1T_s. Again $V_o = V_{in}D_1 / (1-D_1)$ yields D_1 = 1/3.

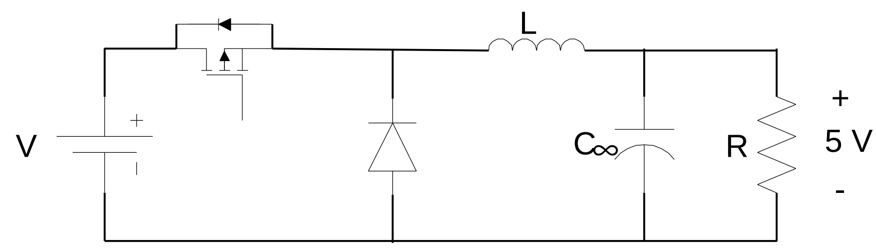
$V_L = Ldi/dt = L*dI_{av} / D_1T_s$

$24 = L_c*15 / (1/3)(f_{sw})$

$Lc = 8Ts / 15 = 8 / (15)f_{sw}\ (\mu H)$

$L_c(1kHz) = 533\ \mu H$ but $L_c(1MHz) = 0.5\ \mu H$. Clearly higher f_{sw} is desired to make L_c as small as possible.

For **HW#2** use the buck converter shown below:

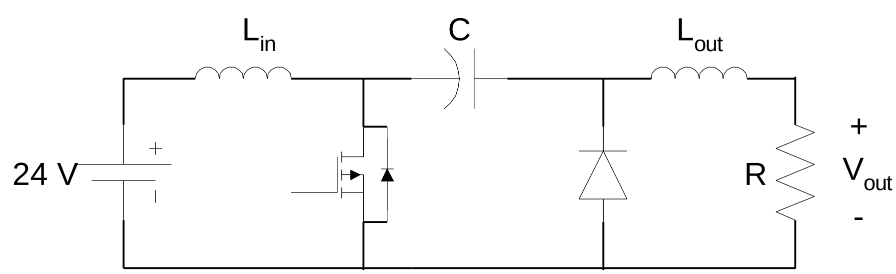


R = 0.25 Ω

A buck converter is designed for nominal 48 V input and 5 V output. It switches at 100 KHz. In practice the input can be anywhere between 30 V and 60 V. the load power ranges between 10 W and 200 W. What is L_{crit} for this converter? Conversely, what is the maximum inductance that will ensure discontinuous mode under all allowed conditions?

5. Critical Capacitance C_c(critical)

$C_{CRITICAL}$ is the capacitance required to keep $V_c > 0$ for all circuit conditions. The Cuk circuit shown below has V_{in} = 24V, V_o = -12, f_{sw} = 200 kHz and provides 120 W.



f_{switch} = 200 kHz V_{out} = -12 V

R = 1.2Ω L_{in}, L_{out} ⟶ ∞

What is C_c(critical)?

I_o(av) = 120/12 = 10A, R_L = 1.2Ω, D_1 = 1/3 and I_{in}(av) = 120/24 = 5A. At the CCM / DCM boundary the boost portion provides a voltage on average across the capacitor V_{in}/(1-D1) = 36V which is the sum of the input and output (12+24).

For C_c the V_c will vary from 0 to $2V_c(av)$ = 72V while the transistor is on for

D_1T_s = (1/3)5 μs.

$i_c = C_c*dV/dt = [C_c*2V_c(av)] / [(1/3)*5] = C_c*72*(3) / 5$

C_c = 0.23 μF

At C_c we get V_o = -12 for V_{in} = 24V. Will this V_o increase / decrease as $C \leq C_c$? What if $C = C_c/2$ = 0.116 μF?

Since we have DCM V_c will ramp down to zero while the Tr is on. When V_c goes negative the diode goes on while the Tr is on. This causes $D_1 + D_2 > 1$. While the diode is on for a time

$\Delta t(1-D_1)T_s$ = 3.33 μs. The voltage varies from 0 to $2(V_{in} + V_{out})$.

$I_c = I_{in} = P_{in} / V_{in} = C\Delta V/\Delta t$

1. $(P_{in}/V_{in}) = C_c*2(V_{in} + V_{out})/3.33\mu s$
2. $P_{in} = P_{out} = V_o^2/R$

Combining 1. and 2. we obtain DCM:

$V_o^2 - 2V_o - 48 = 0$

V_{out} = 8V

$P_{out} = V_o^2/1.6$ = 53.33 W

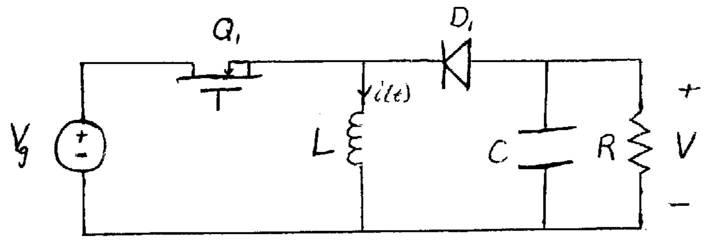
I_{in} = 2.2A

Hence V_o drops for $C < C_c$

C. Homework Solutions and Hints

1. Problem 5.1

Erickson Problem 5.1: Buck-boost is given on pages 14-16



(a) From prob. 2.1:

$V = -\frac{D}{D'} V_g$; $I = \frac{D V_g}{(D')^2 R}$; $\Delta i = \frac{D T_s}{2L} V_g$

Knowing For CCM: $I > \Delta i$

DCM: $I < \Delta i$

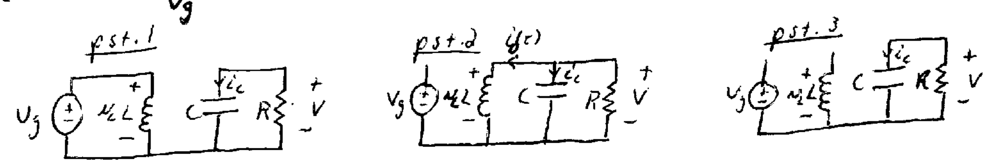
We want DCM thus $I < \Delta i$ or $\frac{D V_g}{(D')^2 R} < \frac{D T_s V_g}{2L}$ ①

Relating $K < K_{crit}(D)$ we solve for K and $K_{crit}(D)$

From ① simplified ⇒ $(D')^2 > \frac{2L}{R T_s}$ or $\frac{2L}{R T_s} < (D')^2$

$K = \frac{2L}{R T_s}$ and $K_{crit} = (D')^2$

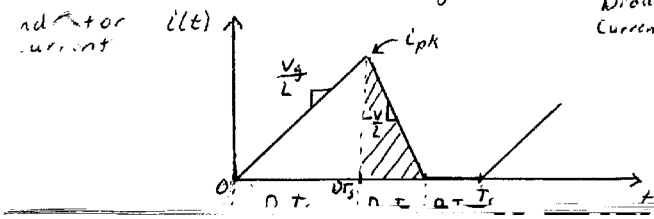
(b) Find $\frac{V}{V_g}$ for DCM:

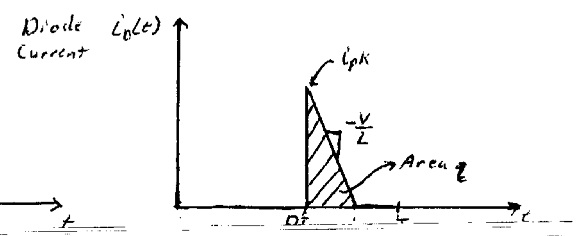


$\langle v_L \rangle = D_1(V_g) + D_2(V) + D_3(0) = 0$ rearranging ⇒ $V = \frac{-D_1 V_g}{D_2}$ ①

$\langle i_c \rangle = i_D + \frac{V}{R} = 0$ or $\langle i_D \rangle = -\frac{V}{R}$

No dc current through "C".





AND: $\langle i_D \rangle = \frac{1}{T_s}\int_0^{T_s} i_D(t)\,dt = \frac{Area\,q}{T_s} = \frac{\frac{1}{2}\, i_{pk}\, D_2 T_s}{T_s}$ 8

$i_{pk} = \left(\frac{V_g}{L}\right)(D_1 T_s)$ thus: $\langle i_D \rangle = \frac{(1) V_g D_1 D_2 T_s}{2L} = \frac{V_g D_1 D_2 T_s}{2L}$

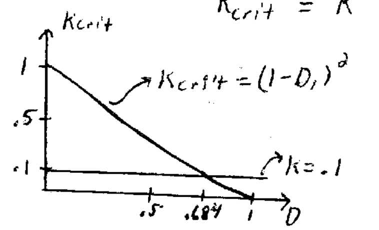
relating $\langle i_D \rangle$ from earlier $\Rightarrow \langle i_D \rangle = \frac{V_g D_1 D_2 T_s}{2L} = \frac{-V}{R}$

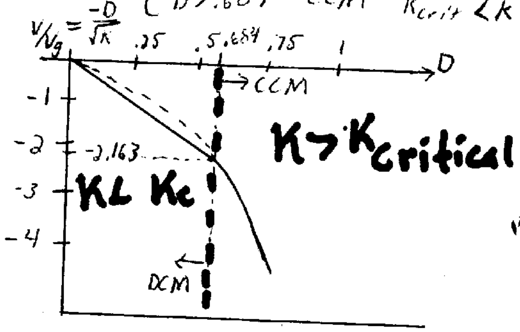
Knowing from (a) $V = \frac{-D_1}{D_2} V_g$: $\frac{V_g D_1 D_2 T_s}{2L} = \frac{V_g D_1}{D_2 R}$ or $D_2 = \sqrt{\frac{2L}{RT_s}} = \sqrt{K}$

$M = \frac{V}{V_g} = \frac{-D_1}{D_2} = \frac{-D_1}{\sqrt{K}}$

(c) $K = 0.1$, plot V/V_g over $0 \le D \le 1$

From (b) $(D_2)^2 = K$ or $\underbrace{(1-D_1)^2}_{K_{crit}} = \underbrace{.1}_{K}$; $D_1 \cong .684$ $\begin{cases} D < .684 & DCM \quad K_{crit} > K \\ D > .684 & CCM \quad K_{crit} < K \end{cases}$

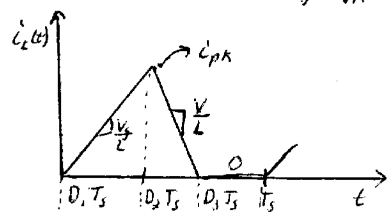


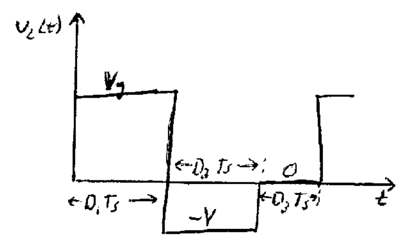


(d) $K = .1$, $D_1 = .3$ Knowing $D_2 = \sqrt{K} = \underline{.316}$ and $D_3 = 1 - D_1 - D_2 = \underline{.3838}$

All three time periods known

$i_{pk} = \frac{V_g}{L} D_1 T_s \Rightarrow \frac{V}{V_g} = \frac{-D}{\sqrt{K}} \cong \underline{.95}$





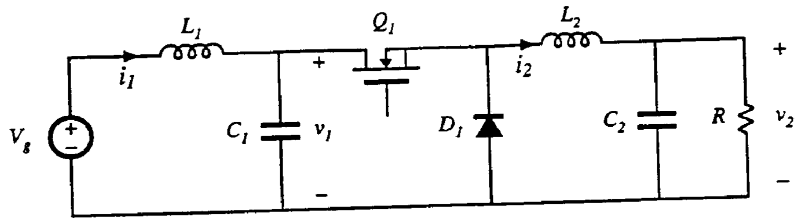
(e) $R \to \infty$; $K = \frac{2L}{RT_s} \to 0$ $K < k_{crit}$ DCM. $M = \frac{V}{V_g} = \frac{-D_1}{\sqrt{K}} \to -\infty$ For $D_1 > 0$

V tends toward $\underline{-\infty}$. Since V becomes very large it probably exceeds the transistor and Diode ratings. To avoid this a minimum load can be used to prevent $R \to \infty$ or a control network to reduce D to 0 when V becomes too l...

2. Erickson Problem 5.9

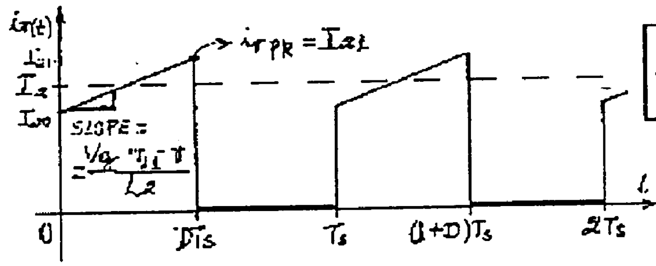
EMI Noise Issues from Converters to Mains: Example of Input Filter PWM dc-dc buck converter

Erickson Problem. 5.9



a) WHEN THE TRANSISTOR Q_1 IS ON (D_1 OFF), THE TRANSISTOR CURRENT $i_T(t)$ IS EQUAL TO $i_2(t)$. THIS CURRENT $i_2(t)$ INCREASES LINEARLY AS WELL AS $i_T(t)$. WHEN Q_1 IS OFF (D_1 ON), THE TRANSISTOR CURRENT IS ZERO. SO $i_T(t)$ IS SHOWN BELOW:

$\Rightarrow v_{L2} = V_g - v_{L1} - v$

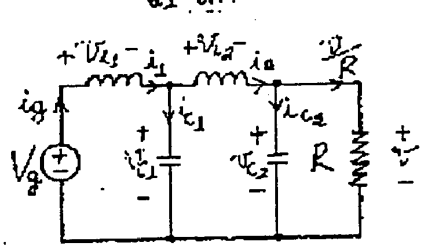


$$\frac{di_2}{dt} = \frac{V_g - v_{L1} - v}{L_2}$$

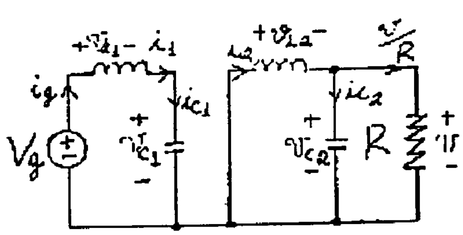
SLOPE

b) LET'S MAKE INDUCTOR VOLT-SECOND BALANCE & CAPACITOR CHARGE BALANCE ON THE CIRCUIT OF FIG. 2.32:

Q_1 ON:



D_1 ON:



$\Rightarrow L_1$ VOLT-SEC BALANCE: (Obs.: SRA ⇒ SMALL RIPPLE APPROXIMATION)

$$\frac{\lambda_1}{T_s} = 0 = D(V_g - V_{C1}) + D'(V_g - V_{C1}) = 0 \Rightarrow \boxed{V_{C1} = V_g} \quad (1)$$

(NO SRA!) ⇒ SRA IS NOT USEFUL!

$\Rightarrow L_2$ VOLT-SEC BALANCE:

$$\frac{\lambda_2}{T_s} = 0 = D(V_{C1} - V) + D'(-V) = 0 \Rightarrow \boxed{V = DV_{C1}} \quad (2) \quad \text{USE SRA!}$$

WITH (1) $\Rightarrow \boxed{V = DV_g}$

⇒ C_1 CHARGE BALANCE:

$\frac{Q_1}{T_s} = 0 = D(I_1 - I_2) + D'(I_1) = 0 \Rightarrow \boxed{I_1 = DI_2}$ ③ USE SRA!

WITH ④ & ② ⇒ $\boxed{I_1 = D\frac{V}{R}}$ & $\boxed{I_1 = D^2\frac{V_g}{R}}$

⇒ C_2 CHARGE BALANCE:

$\frac{Q_2}{T_s} = 0 = D\left(I_2 - \frac{V}{R}\right) + D'\left(I_2 - \frac{V}{R}\right) = 0 \Rightarrow \boxed{I_2 = \frac{V}{R}}$ ④ ⇒ AVOID SRA!

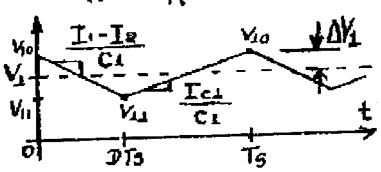
(NO SRA!)

Summary of V_1 versus time

$V_1 = V_g$, $V = DV_g$, $I_1 = D\frac{V}{R} = D^2\frac{V_g}{R}$, $I_2 = \frac{V}{R}$

$2\Delta v_1 = \left(\frac{I_1}{C_1}\right)(D'T_s)$

$\Delta v_1 = \frac{V_g D^2 D' T_s}{2RC_1}$



Now conditions @V_1 for DCM as well as Δi_1, Δi_2 and ΔV_2

c) LET'S EVALUATE THE RIPPLE FOR: L_1, L_2, C_1 & C_2:

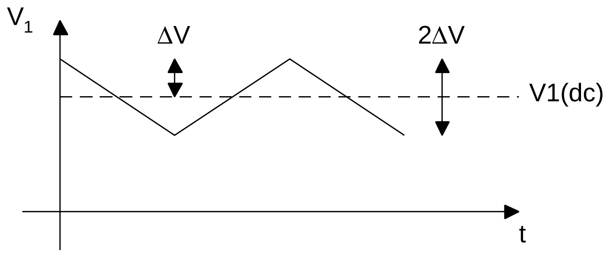
⇒ L_1 ⇒ NO SRA ⇒ $\boxed{\Delta i_1 = \frac{\Delta v_{C1} \cdot T_s}{8L_1}}$ ⑤ ⇒ DUAL FOR EQ. (2-60).

⇒ L_2 ⇒ USE SRA ⇒ $2\Delta i_2 = \left(\frac{V_g - V}{L_2}\right)(DT_s)$ ⇒ $\boxed{\Delta i_2 = \frac{(V_g - V)DT_s}{2L_2}}$ ⑥

⇒ C_1 ⇒ USE SRA ⇒ $2\Delta v_{C1} = \left(\frac{I_1}{C_1}\right)(D'T_s)$ ⇒ $\boxed{\Delta v_{C1} = \frac{\frac{V}{R}D(1-D)T_s}{2C_1}}$ ⑦

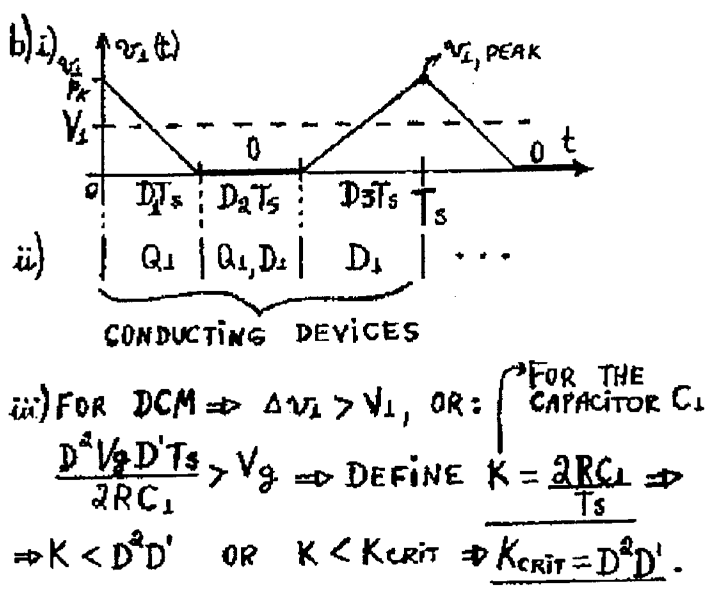
⇒ C_2 ⇒ NO SRA ⇒ $\boxed{\Delta v_{C2} = \Delta v = \frac{\Delta i_2 \cdot T_s}{8C_2}}$ ⑧ ⇒ EQ. (2-60), P. 32.

Now determine when the DCM occurs by plotting $v_1(t)$ with DC value and ripple



$$V_1(\text{dc}) \quad K = \frac{2RC_1}{T_s}$$

Find Q_1 conducts but when V_1 tries or goes negative diode conducts. K(critical) = D^2D'.

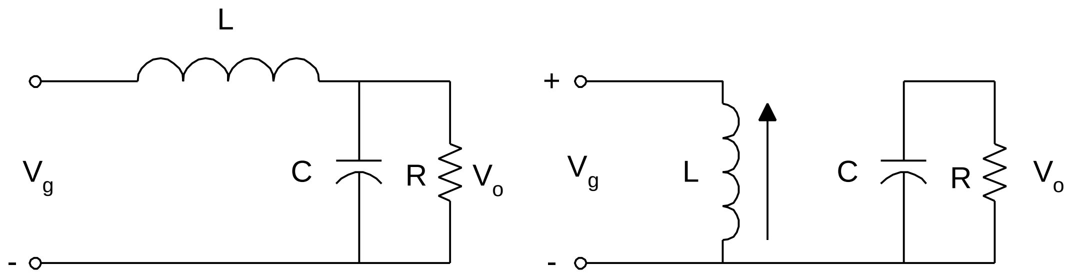


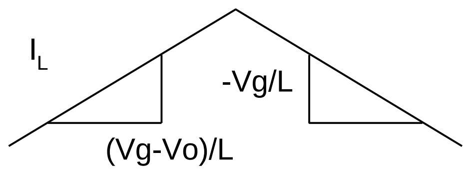
3. Hints for Erickson Problem 5.4

Watkins Johnson Converter in CCM

(See Erickson's Chapter 6 page 137for the #6 topology and timing)

DT_s	$D'T_s$
Q_1, Q_2 on	Q_1, Q_2 off
D_2, D_2 off	D_1, D_2 on

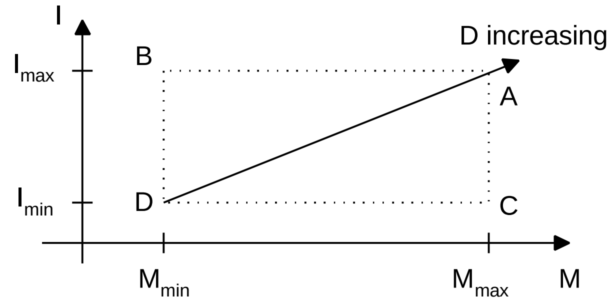




Note: Be careful when dividing by a negative number. If you divide by a negative number you must change the sign of the operator ⇒ -5a < b or a > -b/5 but not a < -b/5, Try a=1 and b=1

4. Hints for Erickson Problem 5.14

DCM Boost
Draw a plot to find "worst case" DCM operation or nearly CCM. $D_3 = 0.1$ is minimum



Fill in the box for all cases of I vs. M.
<u>Four Boundary Points</u>

A. V_g(min), Maximum D values, M_{max}, I_{max}, k_{max}, P_{max}

I_{max} & Max D $\Rightarrow$ worst case
$D_3 = 0.1$ is lowest $\equiv$ Point A

B. $V_g(max)$, M_{min}, I_{max}, k_{max}, P_{max}
Intermediate D values

C. $V_g(min)$, Intermediate D, M_{max}, I_{min}, k_{min}, P_{min}

D. $V_g(max)$, M_{min}, I_{min}, k_{min}, P_{min}, D_{min}

A is choice for boundary of CCM $\leftrightarrow$ DCM
and $D_3 = 0.1$ at A as per statement in problem $D_3(min) = 0$.

From here use steady state DCM Boost equations

$$D_1 = f(k,M) = \sqrt{k(m-1)M}$$

$D_2 = f(D_1, k) = ??$ {Lots of algebra}

Use D_1 above and: $$k = \frac{2L}{RT_S}$$

D_2 should be: $$\sqrt{\frac{2LI}{V_g T_S (M-1)}}$$

<u>Show all work!</u>

Set $D_3 = 0.1 = 1 - D_1 = D_2$

$$D_3 = 1 - M\left[\frac{2LI}{T_S V_g (M-1)}\right]^{1/2}$$

D_1 at point A $\equiv \sqrt{k_{max}(M_{max}-1)M_{max}}$

$\Downarrow$ $\Downarrow$

Use equations on a spreadsheet

$$D_1 = \sqrt{\frac{2LI(M-1)}{T_s V_g}}$$ and specs

<u>Gives L around 5μH</u>

Output capacitor-

1. Use i_D vs. time to get Q
   $2\Delta v_c \approx Q/C \rightarrow$ Estimate C for $\Delta v = 2v$ or $\pm 1V$

2. Use i_{peak} - I vs. time to get Q
   $2\Delta v_c = Q/C \rightarrow$ Estimate C for $\pm$ 1 V ripple.

The following is a list of equations used to derive the spreadsheet values. Where a, b, c, and d refer to points A, B, C, and D respectively.

$$Ts := \frac{1}{fs} \quad Imin := \frac{Pmin}{V} \quad Imax := \frac{Pmax}{V} \quad Rmin := \frac{V}{Imax} \quad Rmax := \frac{V}{Imin}$$

$$Mmax := \frac{V}{Vgmin} \quad Mmin := \frac{V}{Vgmax} \quad L := \left(\frac{1 - D3}{Mmax}\right)^2 \cdot Ts \cdot Vgmin \cdot \frac{Mmax - 1}{2 \cdot Imax}$$

$$Da := \sqrt{2 \cdot L \cdot Imax \cdot \frac{Mmax - 1}{Ts \cdot Vgmin}} \qquad Db := \sqrt{2 \cdot L \cdot Imax \cdot \frac{Mmin - 1}{Ts \cdot Vgmax}}$$

$$Dc := \sqrt{2 \cdot L \cdot Imin \cdot \frac{Mmax - 1}{Ts \cdot Vgmin}} \qquad Dd := \sqrt{2 \cdot L \cdot Imin \cdot \frac{Mmin - 1}{Ts \cdot Vgmax}}$$

$$Ka := \frac{2 \cdot L \cdot Imax}{Ts \cdot Mmax \cdot Vgmin} \quad Kb := \frac{2 \cdot L \cdot Imax}{Ts \cdot Mmin \cdot Vgmax} \quad Kc := \frac{2 \cdot L \cdot Imin}{Ts \cdot Mmax \cdot Vgmin} \quad Kd := \frac{2 \cdot L \cdot Imin}{Ts \cdot Mmin \cdot Vgmax}$$

$$Kcrita := Da \cdot (1 - Da)^2 \quad Kcritb := Db \cdot (1 - Db)^2 \quad Kcritc := Dc \cdot (1 - Dc)^2 \quad Kcritd := Dd \cdot (1 - Dd)^2$$

$$ipka := \frac{Vgmin \cdot Da \cdot Ts}{L} \quad ipkb := \frac{Vgmax \cdot Db \cdot Ts}{L} \quad ipkc := \frac{Vgmin \cdot Dc \cdot Ts}{L} \quad ipkd := \frac{Vgmax \cdot Dd \cdot Ts}{L}$$

$$D2a := \sqrt{\frac{2 \cdot L \cdot Imax}{Vgmin \cdot Ts \cdot (Mmax - 1)}} \qquad D2b := \sqrt{\frac{2 \cdot L \cdot Imax}{Vgmax \cdot Ts \cdot (Mmin - 1)}}$$

$$D2c := \sqrt{\frac{2 \cdot L \cdot Imin}{Vgmin \cdot Ts \cdot (Mmax - 1)}} \qquad D2d := \sqrt{\frac{2 \cdot L \cdot Imin}{Vgmax \cdot Ts \cdot (Mmin - 1)}}$$

$$D3a := 1 - Da - D2a \quad D3b := 1 - Db - D2b \quad D3c := 1 - Dc - D2c \quad D3d := 1 - Dd - D2d$$

$$C := \frac{(ipka - Imax)^2 \cdot D2a \cdot Ts}{4 \cdot ipka \cdot 1}$$