Lecture 36

Transformer and AC Inductor Odds and Ends

A. REVIEW of transformer design

B. "AC" Inductor Design (Erickson, Ch. 14)

C. Transformer Design Nine Step Flow Chart

D. Transformer Heat Flow Analysis

We will need to find an optimum B(core) to balance wire versus core losses

1. Wire Winding Loss P_{cu}(loss) ~ $1/B_{max}^2$
2. Core Loss P_{core}(loss) ~ $B_{max}^{2.6}$
3. **Find: B_{Opt}(core), L_l and T_{core}**

$T(\text{core}) \equiv P_{total}R_\theta + T_{amb}$

4. Leakage Inductance: L_l

Core Geometry Effects

E. Example of a mains ac transformer

1. E-I Core Example
2. General Transformer Core:

A good website for both transformers and AC inductors is www.rencousa.com

LECTURE 36

Transformer and AC Inductor Odds and Ends

A. Review of Transformer Design

1. Total Cu Loss for all "k" transformer wire windings

$$P^{\text{total}}_{\text{of all windings}}(\text{Cu}) = \frac{r(\text{MLT})}{W_A \; k_u} * \left[\sum_{k=1}^{k} \frac{N_k^2 I_k^2}{a_k}\right]$$

assumes all wire windings are the same size and $(\text{MLT})_k = (\text{MLT})_j$ on same core diameter

choose a_k for each to minimize the TOTAL wire loss

Re-express power loss in terms of the **total primary current**

$I_p(\text{total}) = \sum_k \frac{N_k}{N_p} I_k$, Where each I_K is set by the load Z_K

(N_k/N_p) has a denominator N_p(primary winding) which is fixed and equal to: $N_p = \frac{l\,p[volt - \sec]}{2 \; B_{pk} \; A_c}$ for V_p applied on the primary windings on a switched basis, $l = \int V_p \, dt$.

Now we express Ptotal in terms of n_p and I_{total}.

$P_{total}(\text{Cu}) = \frac{r(\text{MLT})}{W_A \; K_u} \; n_p^2 \; I_{total}^2$ The key point to notice is that $n_p^2 \sim \frac{1}{B_{pk}^2}$.

We find $P(\text{Cu}) \sim 1/B^2$ explaining Why as B(core) ↓ P(Cu wire) ↑. In other words, to achieve low B we need to increase the number of copper wire turns, which increases wire losses.

$$P_{\text{all windings}}^{\text{total of}}(cu) = \frac{r\ l_p^2 I_p^2(\text{total})}{4K_u} \quad \frac{(MLT)}{W_A A_c^2} \quad \frac{1}{B_{pk}^2}$$

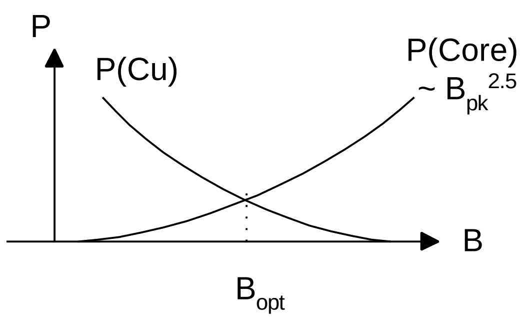
$\downarrow$ **circuit and wire spec's** $\quad\downarrow$ core geometry $\quad\downarrow$ **core flux**

2. Use a spreadsheet to find total power loss using both expressions P(Cu) and P(core) to express P_T. Then take the derivative of each term and set the sum equal to zero.

$$P_T = P_{loss}(cu) + P(\text{core loss})$$

a. With dP(core)/dB = $b\, K_{fe} B^{b-1} A_c \ell_c$

b. $dP(Cu)/dB = -2\left[\frac{r\ l_1^2 I_p^2(\text{total})}{4K_u}\right]\frac{MLT}{W_A A_c^2}\ B^{-3}$



To find the B_{opt} we find B that solves for the for the case

$$\frac{dP}{dB} = 0 = \frac{dP(cu)}{dB} + \frac{dP(core)}{dB}$$

Take the sum and solve for B_{opt}.

$$B_{opt} = \left[\frac{r\ l_p^2 I_p^2(\text{total})}{2K_u}\left[\frac{MLT}{W_A A_c^3 l_e}\right]\left[\frac{1}{b K_{fe}}\right]\right]^{\frac{1}{b+2}}$$

Use a spreadsheet to evaluate B_{opt} from various transformer values in this circuit application. Next plug B_{opt} into P(cu) + P(core) and rearrange into an equality.

$$\frac{W_A (A_c)^{(2(b-1)/b}}{(MLT) l_e^{(2/b)}} \left[(\frac{b}{2})^{-(b/(b+2)} + (\frac{b}{2})^{(2/(b+2)} \right]^{-(b+2)/b} = \frac{r\ l_p^2 I_{tot}^2 K_{fe}^{(2/b)}}{4 K_u (P_{tot})^{((b+2)/b)}}$$

↑

↑

core geometric constant **circuit specs**

To simplify assume b factor » 1, core material (β, K_{gfc}) and we find the following inequality where K_{gfe} is a core parameter catalogued in a core data base.

$$K_{gfe} \geq \frac{r\ l_p^2 I_{tot}^2 K_{fe}^{(2/b)}}{4 K_u (P_{tot})^{((b+2)/b)}}$$

After selecting a core material and geometry from a core database with K_{gfe} > specified value we have A_c, W_A, l_c & MLT all determined from specific core chosen.

Given the core choice and all the core parameters, then B_{opt} can be calculated via a spreadsheet. From B_{opt} we get N_p(opt).

$$N_p(\text{optimum}) = \frac{l_p [volt - \sec]}{2\ B_{opt}\ A_c}$$

All other secondary turns N_k followfrom $\frac{V_p}{N_p} = \frac{V_k(\text{desired})}{N_k(\text{required})}$

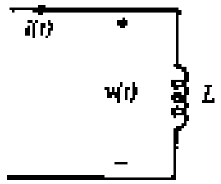
For each winding N_k the area A_k from the core window employed is a fraction of window area of core W_A given via $a_k = \frac{V_k I_k}{P(\text{total})}$

Next we choose the wire AWG# from A_{wk} values obtained via

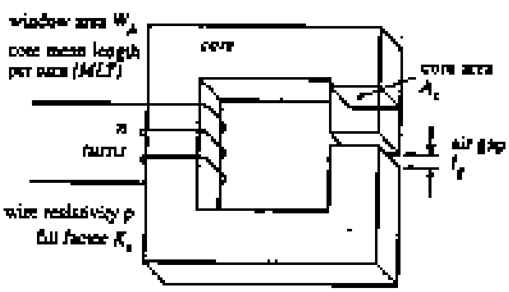
$$A_{wk}(\begin{matrix}\text{wire for k}\\ \text{winding}\end{matrix}) = \frac{K\ W_A a_k}{N_k}$$

B. "AC" Inductor Design (Erickson, Ch. 14)

Compare this seven-step design for AC inductors to the four-step design for “filter” Inductors we did in lecture 33.

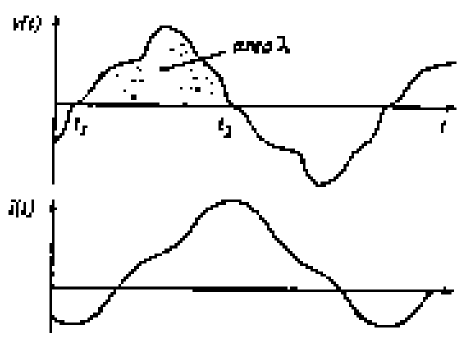


$$L = \frac{N^2}{\Re_g}, \quad \Re_g = \frac{\ell_g}{m_o A_g}$$



$$B(\text{core}) = \frac{l_p(v - \text{sec})}{2 N_p A_g}$$

$$l_p = \int V_L \, dt$$



$$R_L\left(\begin{matrix}\text{of Cu wire} \\ \text{in the primary}\end{matrix}\right) = \frac{r N_p^2 \, \text{MLT}}{K_u W_A}$$

$$P(\text{cu}) = I_p^2 \, R_L \sim \frac{1}{B^2(\text{core})}$$

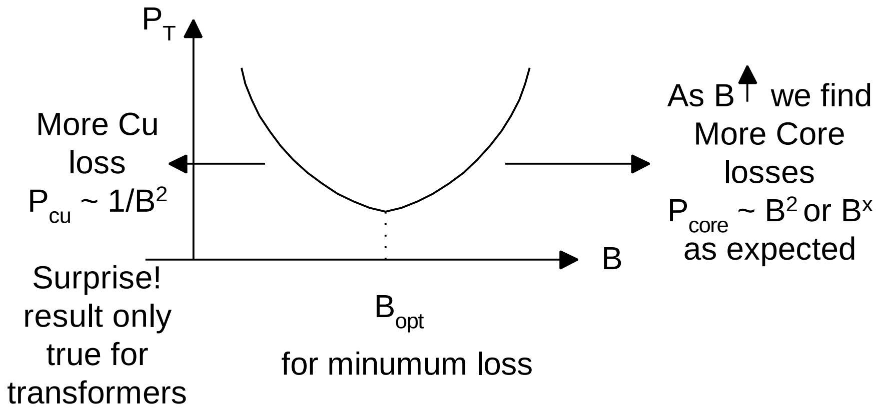
The AC inductor total loss has two components wire I^2R loss and core loss. This AC inductor is very similar to a transformer.

$$P\left(\begin{matrix}\text{total} \\ \text{loss}\end{matrix}\right) = P(\text{cu}) + P(\text{core})$$

$$\uparrow \qquad\qquad \uparrow$$

$$\sim 1/B^2 \qquad \textbf{Iron} \sim \mathbf{B^{2.6}}$$

P(loss) = P(core loss) + P(wire loss)



We aim for choosing $B_{optimum}$ for Minimum Total Loss.

$$B_{optimum} \Rightarrow N_{optimum} = \frac{l}{2 B_{opt} A_c} \quad \text{for } P_T \text{ minimum}$$

Note: $P_{cu} \sim 1/B^2$ is only strictly true for transformer windings or for some AC inductors such as inductors in resonant converters or some flyback inductors. We can trade Fe and Cu to minimize total loss via this B optimum condition in terms of a transformer core parameter K_{gfc} which gives all the required inductor parameters and circuit currents.

$$K_{gfc} \geq \frac{r\; l_p^2 I_p^2 K_{f\,e}^{\;2/b} \,/\, b}{2\, k_u [P(\text{total})]^{\frac{b+2}{b}}}$$

We then employ a **seven-step approach** to AC inductor "L_{AC}" Design as outlined below. (1) **Use a spreadsheet and a core data base to guide <u>selection of the optimum inductor core</u>**

Allowed total power dissipation	P_{tot}	(W)
Winding fill factor of chosen wire	K_u	
core loss exponent	β	
Core loss coefficient	K_{fe}	(W/cm^3T^β)

All the core dimensions are expressed in cm:

core cross-sectional area	A_c	(cm^2)

core winding area	W_A	(cm^2)
mean length per turn around core	MLT	(cm)
magnetic path length in core	l_e	(cm)
peak ac flux density	B_{max}	(Tesla)
wire areas of various windings	A_{w1}, A_{w1}...	(cm^2)

2. **Evaluate peak flux density for the core of part one**

$$B_{max} = \left[10^8 * \frac{r\ l^2 I^2(\text{total})}{2 K_u} \frac{MLT}{W_A A_c^3 \ell_e} \frac{1}{b K_{fe}} \right]^{\frac{1}{b+2}}$$

Using only I_{ac} compare B_{max} to core B_{sat} to avoid saturation

3. **Number of turns of wire for minimum total loss**

$$n = \frac{l}{2\ B_{max}\ A_c} 10^4$$

4. **Air gap length to be cut in inductor core**

$$l_g = \frac{m_o A_c n^2}{L} 10^{-4}$$

With A_c specified in cm^2 and l_g expressed in meters. Alternatively, the air gap can be indirectly expressed via a core parameter called the specific inductance,A_L, (in mH/1000 turns units):

$$A_L = \frac{L}{n^2} 10^9$$

5. **Check for core saturation including both an I_{DC} pedestal with an I_{ac} modulation**

If the inductor contains a dc component I_{dc}, then the peak total flux density B_{pk} is greater than the peak ac flux density B_{max}. Both amp-turns and volt-sec limits add together to set the maximum B. The peak total flux density, in Tesla, is given by:

$$B_{pk} = B_{\max} + \frac{LI_{dc}}{n}$$

6. Evaluate copper wire size

$$A_w \le \frac{K_u W_A}{n}$$

A winding geometry can now be determined, and copper losses due to the proximity effect can be evaluated. If these losses are significant, it may be desirable to further optimize the design by re-iterating the above steps. We account for proximity losses by increasing the effective wire resistively to the value $\rho_{eff} = \rho_{cu} P_{cu}/P_{dc}$ where P_{cu} is the actual copper loss including proximity effects, and P_{dc} is the copper loss obtained when the proximity effect is negligible.

7. Check the total power loss in the inductor due to both wire windings and core losses.

$$P_{cu} = \frac{rn(MLT)}{A_W} I^2$$

$$P_{fe} = K_{fe} B^b{}_{\max} A_c l_e$$

$$P_{tot} = P_{cu} + P_{fe}$$

C. Transformer Design Nine Step Flow Chart

This section is optional and not essential. It offers an alternative to the Erickson approach. It assumes that eddy currents are negligible. If eddy currents are not small we could account for them via an effective AC resistance of the wires.

We can estimate the required S_{max}(V-A) of a transformer core as follows. S= I_p(RMS) V_P(RMS). Use Faraday's law $V_P = N_P A_C \omega B$(core) and the primary current expression, I_P, in terms of current density and primary wire area, $A_{Cu,pri}$, to find:

$S = N_{pri} A_{core}\, \omega\, B(core)\, J(RMS)\, A_{cu,pri} / 2^{1/2}$

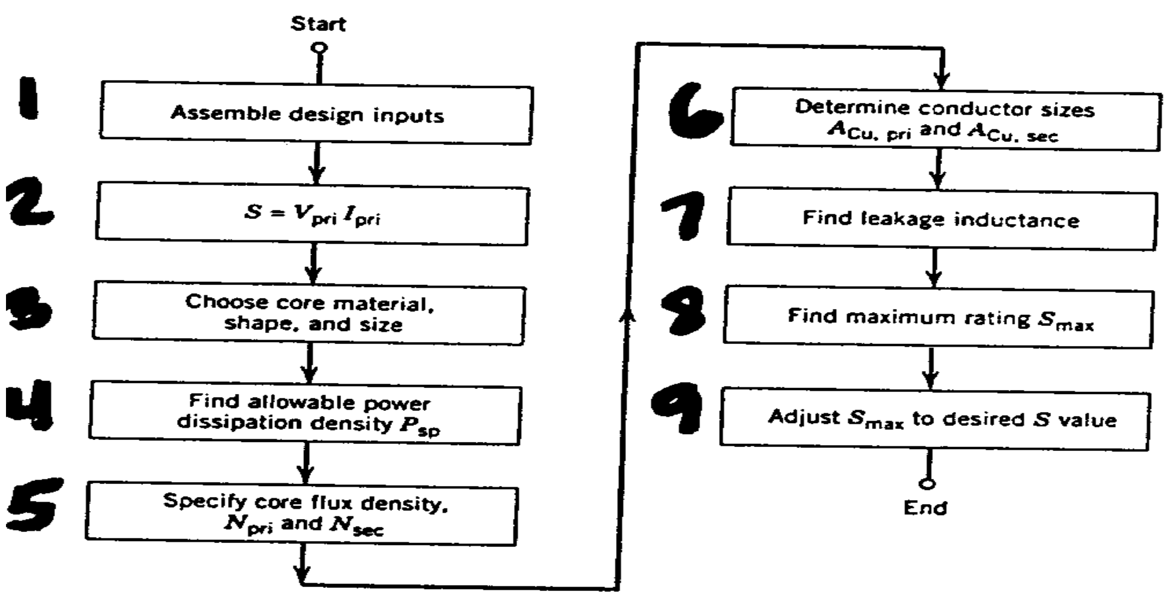
Finally, we can express the primary conductor area in terms of the core wire winding area via:

$A_{Cu,pri} = K_{cu} A_W$(wire winding area of core) /2 N_{pri}.

This then yields an expression:

$$\mathbf{S_{max}(transformer) = 2.22\, K_{cu} f\, A_c\, A_W J_{rms} B_{max}}.$$

In transformer design, we need to insure that $S < S_{max}$.. If S_{max} is too much greater than S_p then to save money we need to step down the size of the chosen transformer core. If $S_p > S_{max}$ then increase the size of chosen transformer core to meet thermal and electrical needs and specifications.



1. Assemble design inputs from PWM converter requirements

V_p, I_p, we need to know the waveforms for Vp and Ip on the transformer primary to extract DC, peak and rms values. $n = \frac{N_p}{N_s}$,

To start we only know the turns ratio n we haven't the chosen the absolute turn numbers for any windings. The chosen operating f of currents in the windings and T_{max} allowed for core as well as

T(ambient) sets the stage for calculating the minimum product R_c(thermal of core) * P_c(heat flow from both core and wire losses) such that the core surface temperature never exceeds the limit set.

2. $S = V_{pri}I_{pri}$

We need to use maximum expected values throughout. The volt-amp rating of the transformer acts as a conservative starting point;
$V*I_p = S_p$
Use this to select a core which has a larger allowable S due to the thermal limits issues raised in 1..

$T_s(\text{core}) = T_A + P(\text{loss in W}) * R_c(°C/W)$

3. Choose core material, core shape, and core size

From a core database we can pick core size based upon the S value from step 2. We also must choose the wire type here as well in order to select the core with adequate wire winding window, K_{Cu} . From this core we have associated set of core parameters such as:S, $R_{\theta,}$ and P(W/cm^3). We also get B_c (max) from the saturation value of the core at the operating frequency. We use B_{max} to set the absolute levels of both N_p and N_s turns ratios. The tradeoff between copper and core occurs here via Faraday's equation.

$$N_p = \frac{V_{prim}}{A_c \, w \, B(max)} +$$

If later we find that the core chosen is inadequate we will come back and EITHER choose a new core size or we will choose a different wire type.

4. Find allowable power dissipation density P_{sp}

Use B_{max} ,the operating frequency and nomograph graph from core manufacturer, which is only good for sinusoidal currents and not for squarewaves etc, to estimate the core energy losses..

$P_{core}(W) = P_{sp} * V_c(\text{core volume})$

5. Specify core flux density and wire type to be employed

The core data base will determine B or we use the thermal limits and the core power loss to set B. From this B value the absolute value of the primary turns are then estimated.

Typical Core Database

Table 30-4 Database of Core Characteristics Needed for Transformer Design

Core No.	Material	$AP = A_w A_c$	R_θ $\Delta T = 60°C$	P_{sp} at $T_s = 100°C$	J_{rms} at $T_s = 100°C$ and P_{sp}	$\hat{B}_{rated}$ at $T_s = 100°C$ and 100 kHz	$2.22 k_{Cu} f J_{rms} \hat{B} A_w A_{core}$ ($f = 100$ kHz)
•	•	•	•	•	•	•	•
8[a]	3F3	2.1 cm^4	9.8°C/W	237 mW/cm^3	$3.3 \sqrt{\frac{R_{dc}}{k_{Cu} R_{ac}}}$ A/mm^2	170 mT	$2.6 \times 10^3 \sqrt{\frac{k_{Cu} R_{dc}}{R_{ac}}}$ V-A
•	•	•	•	•	•	•	•

This table helps tells us how much room is needed for each coil in the limited area wire winding window. The wire type chosen to employ depends on the total wire winding area, A_W or W_A and the allocation between primary, A_P, and secondary, A_S, winding areas. The total winding area is then:

$$A_W = A_P + A_s = N_{pri} A_{cu,pri\ wire} / k_{Cu} + N_{sec} A_{cu,sec\ wire} / k_{Cu}$$

6. Determine conductor sizes $A_{cu,pri}$ and $A_{Cu,sec}$

Below we assume eddy current losses are small. Knowing the open area of the core available for wire windings we split it between N_p and N_s in the simple case of two windings. We demand $J_p^2 = J_{sec}^2$ to achieve the desired wire sizes, if the primary and secondary waveforms are of the same type. The chosen wire geometry factor K_{cu} is included, to find the wire area of the primary and secondary wires based upon the constraint.

$$K_{cu} \left(\frac{I_p}{A_p(cu)} \right)^2 = K_{cu} \left(\frac{I_{sec}}{A_{sec}(cu)} \right)^2 = K_{Cu} J^2$$

Generally for the same wire type in both coils,

$$A_{cu,pri}/ A_{cu,sec} = N_{sec} / N_{pri}$$

Use A_{cu}(prim) & A_{cu}(sec) from wire tables to get the **closest** American (AWG #) or European wire size. Again this may involve iteration as the nearest wire size is an artful choice.

7. Find leakage inductance from core geometry and how we interleave wire windings

$$L_l = \frac{N_p^2}{\Re_{\ell p}} \frac{1}{p^2}$$

Where p is the # of prim/sec interfaces in interleaves winding set-up for the coils. $\Re_{l\,p}$ is the reluctance of the air path for the primary winding. Tailor core/windings to get desired L(leakage) for leakage flux at the primary. At this time one could also try to MINIMIZE proximity effects in the wire windings.

8. Find maximum rating S_{max} of Selected Core

Now we are able to calculate the factor $S_{max} = 2.2\ f\ K_{cu}A_cA_wJ_{rms}B_{max}$ for the selected core and compare it to S_P. $S_p = V_pI_p$ should be $\leq S_{max}$. If S_{max} is **too much greater** than S_p then to save money we need to step down the size of the chosen transformer core and repeat the above design proceedure. If $S_p > S_{max}$ then increase the size of chosen transformer core to meet thermal and electrical spec's.

9. Adjust S_{max} to desired S Value

a. General Comments on this artful balance

If $S_p >> S_{max}$ choose next bigger core or operate present core with additional cooling such as a fan.

If $S_{max} >> S_p$ then choose a smaller cheaper core:

b. Specific Illustrative Case: f_{sw} = 100 kHz, I_p = 4A rms and V_p = 300 Vrms for a sinusoidal excitation. Consider a 4:1 step down transformer. The thermal issues are T_{max}(core)=100 and the ambient temperature is 20 degrees Celsius.

Start the ten step iterative design process with specific numbers

1. Assemble design inputs

Consider we wish to step-down transformer in our PWM converter circuit $\frac{N_p}{N_{sec}} = 4$. For thermal considerations we state T_{max}(core) = 100 and T_{amb} = 40 degrees Celsius.

2. $S = V_{pri} I_{pri}$

S_p = 1200 V-A rms

3. Choose core material shape, and size

Pick core from manufacturers catalog a transformer core with S above 1200 V-A and capable of 100 kHz operation. A ferrite core double E shape of the type given in the table of data is chosen. That is a=1 cm E core has S_{max}>1200 V-A for all values of K_{Cu} larger than 0.2

4. Find allowable power dissipation density P_{sp}

From table of power loss for that specific core material

$$P\left(\frac{W}{cm^3}\right) = 237 \ \frac{mW}{cm^3}.$$

Note also that specific core material has a thermal resistance,R_Q(Double E) = 10^oC/W and the core cross-sectional area for the flux paths is $A_c = 1.5*10^{-4}$.

5. Specify core flux density N_{pri} and N_{sec}

To meet the peak current specifications and still avoid saturation B(max) = 170 mT from the core data base. This sets the absolute numbered primary turns via Faraday's equation:

$$N_p = \frac{V_p(peak)}{A_c \quad w \quad B(max)} = 26.5$$ and N_2=6 if we round off N_P to 24 turns which is divisible by 6. Again this is arbitrary. If we choose N_P=28 then N_2 would be 7.

Where $A_c = 1.5*10^{-4}$, w(in rad/sec) = $2\pi*10^5$ and $V_{peak} = 300\sqrt{2}$.
This yields N_1 = 24 and N_2=6. But it could equally be N_p = 28 and N_s = 7. Try both. But higher N_p means lower B_{max} so try this first.

6. Determine copper wire conductor sizes $A_{Cu, pri}$ and $A_{Cu,sec}$

$$J_{rms} = 3.3\sqrt{\frac{R_{dc}}{K_{cu}R_{ac}}}$$

from the core database.
For the chosen rectangular wire we know: K_{cu} = 0.6

$\frac{R_{ac}}{R_{dc}}$ depends on proximity effects. Lets assume full winding interleaving so for assumed ratios of

$$\frac{R_{ac}}{R_{dc}} = 1.5 \qquad J_{rms} = \frac{3.3}{\sqrt{0.6*1.5}} = \frac{3.5A}{mm^2}$$

Now for both primary and secondary wires to hit this opt J value we can determine the wire size of each coil:

$$\frac{I_p}{A_p} = \frac{I_s}{A_s} = J \qquad A_p = 1.15\ mm^2$$,likewise A_s = 4.6 mm^2

From wire tables we can find standard wire gauges close to these areas. We assume that the windings are as follows. The primary is composed of four sections of 6 turns each and the secondary of three sections of 2 turns each. This means there will be six primary-secondary interfaces Next we ask--How flexible is wire winding sectionalization or interweaving to achieve both low L(leakage) and small proximity effects in the windings.

7. Find leakage inductance

$\mu_o = 4 * 10^{-9}$, $N_p = 24$ and for the chosen geometry $l_w = 9$ and $b_w = 0.7$. hence L(leakage) is fully specified.

$$L_l = \frac{m_o \quad N_p^2 \quad l_w \quad b_w}{3 h_w \quad p^2}$$

We choose p for 6 prim/sec interfaces and compared to p = 1.

$\frac{L_l}{7.2 m\text{H}} \quad \frac{P}{1}$ Base line leakage inductance for p = 1 is 8.µH.

Whereas for p= 6 we get 0.2 .µH or **36 times lower leakage inductance**. We do benefit from proper wire winding sectionalization

8. Find maximum rating S_{max} for the chosen core in the iterative design

From core data base

$S_{max} = 2.6 * 10^3 \sqrt{\frac{K_{cu}}{R_{ac} / R_{dc}}}$; For wire width $K_{cu} = 0.6$,

$R_{ac}/R_{dc} = 1.5$

$S_{max} = 1644 > 1200$ V-A from S_p. This could be a problem. Consider that if we used Litz wire instead for the coils then $K_{cu} = 0.3$ and we could reduce S_{max} as follows:

$S_{max} = 1644 \sqrt{\frac{.3}{.6}} = \frac{1644}{\sqrt{2}} = 1424\,\text{K}$ Still too high a value.

Adjust S_{max} to desired S value

$S_{max} \geq S_p$

$\frac{1644}{1200} = only\, 37\%$ over rated, *we could live with this*

We could lower S_{max} via several routes:

1. Fewer primary turns to reduce copper losses. A new bigger size core could handle higher ϕmax due to lower N_p, Consider for example $N_1 = 20$, $N_2 = 5$ yet maintain $\frac{N_1}{N_2} = 4$

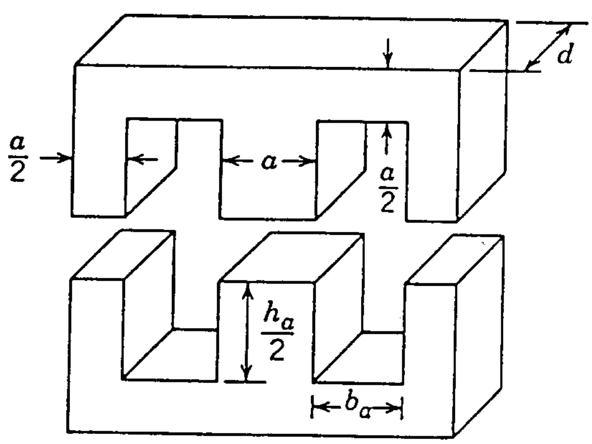
2. Use larger area Cu wire

FOR HW#1 give some other suggestions.

D. Transformer Heat Flow Analysis

This section is also optional for undergraduates.

1. Overview



Middle leg carries ϕ while outer legs carry $\phi/2$. Therefore areas of core legs are unequal.

Table 30-1 Geometric Characteristics of a Near Optimum Core for Inductor/Transformer Design

Characteristic	Relative Size	Absolute Size for a = 1cm
Core Area A_{core}	$1.5a^2$	$1.5cm^2$
Area product AP = A_wA_c	$2.1a^4$	$2.1cm^4$
Core Volume V_{core}	$13.5a^3$	$13.5cm^3$
winding Volume V_w [a]	$12.3a^3$	$12.3cm^3$
total surface area of assembled inductor/transformer	$59.6a^2$	$59.6cm^2$

Notes: [a]Total volume is estimated as the volume in the winding windows $2A_w(d+0.4a)$ plus the two rectangular volumes $A_w(a+0.4a)$, one on wither side of the core, and four-quarter circle cylinders (radius b_w and height h_w). The 0.4a factors are included to allow for finite thickness of the bobbin.

[b]Total surface area is assumed to be composed of the outer area of the core ($50.5a^2$) plus the area of the top and bottom flanges ($5.9a^2$) of the windings plus the area of the rounded quarter-cylinder corners of radius 0.7a and height 2a (total area of four quarter-cylinders is $8.8a^2$) minus the core area covered by the four quarter-cylinder corners [total area = $4(2a)(0.7a) = 5.6a^2$].

If we employ this core in a transformer we specify a 4:1 turns ratio and a sinusoidal excitation @ 100 kHz. Consider a generic magnetic core with dimensions shownon page 16. The core has a black exterior with a thermal emissivity, $\varepsilon = 0.9$ The wire winding area, $A_w = 140\ mm^2$ for the dimension "a" = 1cm

Primary Coil	**Secondary Coil**
$I_{primary} = 4A$ rms	$I_s = 16A$
$V_{primary} = 300V$	$V_s = 300/4$

We further specify total # of turns:

$N_{primary} = 32$	$N_{sec} = 8$

Use for wire Litz wire $K_{cu} = 0.3$

We split core wire winding window between primary and secondary coils. Each winding must have the same J to achieve equal heat loss from both coils ,so wire size is different in the primary and in the secondary. The combination of wire and core loss will act as a heat source. The radiative and convective heat flow from the core will result in an equilibrium core temperature, which must not exceed 100 °C. T(ambient) = 40°C in the flow heat equation is assumed.

T(core) = T(ambient) + P(W)R(°C/W)

2. Find: B(core), L_l and T_{core}

We employ the wire area formula for both coils where the wire type, K_{Cu} and the core wire winding area, A_W are known as well as the number of primary turns:

$$A_{cu}(\text{primary}) = \frac{K_{cu} A_W}{2 N_{prim}} = \frac{0.3(140)}{2\ \ 32} = 0.64\ mm^2$$

For HW #1find the AWG# and the equivalent European wire specification.

$$A_{cu}(\text{secondary}) = \frac{K_{cu} A_W}{2\ \ N_{sec}} = \frac{0.3(140)}{(2)(8)} = 2.6\ mm^2$$

This choice of wire gauge insures equal $J_p = J_s$ and both windings have

$$J = \frac{4\,A}{0.64} = \frac{16\,A}{2.6} = 6.2 \; A/\,mm^2$$

This choice also guarantees winding power loss/cm^3 is the same for primary and secondary coils, P/V= ρJ. Now let's be quantitative in the winding and core loss calculations.

a. <u>Copper Winding Loss</u>

$P = 22\, K_{Cu}\, J^2_{rms} * V_w$(wire winding window volume) = 3.1W

Next we deal with the associated core losses due to J_p and J_s as well as

V_{in} = 300V at 100 kHz.

b. Magnetic <u>Core Loss</u>

V(peak) = $N_p A_c w B_c$(max) Where $A_c = 1.5*10^{-4}\ m^2$, w(omega) = $2\pi*10^{+5}$

For V_p = 300 V we find:B_c(max) = 0.14 Tesla for a given core area.

From core material (3F3) charts.

$P(\frac{mW}{cm^3}) \sim 140 mW$, Now for a = 1 cm we know total core volume is 13.5 cm^3.

P(total) ≈ 1.9W due to core losses.

Total Loss = P(wire) + P(core) = 3.1 + 1.9 = 5.0 W.

c. Core Temp of Transformer: T_c(core)

$$T_{max} - T_{amb} \equiv P_{total} \; [R_q]$$

$T_{amb} = 40^oC$, P_{total} = Core 1.9 + winding 3.1 = 5.0,

$R_\theta = 10^oC/W$ for simple radiative convective cooling

$T_{max} = 90^oC$

What happens to T(core) if we go over I_{rms}(primary) = 4 say to I_{rms} = 5A due to a load current increase?

$$P(\text{winding}) = \left(\frac{5}{4}\right)^2 3.1 = 4.8\,\text{W}$$

Assuming P_{core} and R_θ are the same to a first approximation.
$T_{max} = 108^oC$ Careful it's just over the 100^oC rule.

Solution:
Forced air cooling for the core to reduce the thermal resistance $R_\theta\downarrow$

Note in transformer the primary current I↑ but B_{max} is the same since $B_{max} \sim V_{in}$ not I(primary) which is unchanged regardless of the load current. In an inductor in stark contrast if $I_L\uparrow$ then $B_L\uparrow$ and $P_{core}\uparrow$. Not so for a transformer.

3. **Leakage Inductance:** $L_{\ell_{primary}}$

Here we focus on core geometry effects.

$$L_{\ell p} = \frac{N_p^2}{\Re_{\ell p}} \qquad \Re_{\ell p} = \frac{\ell_p}{m_o \; A_{window}}$$

Note we use the flux path over hw = l_p which is in the core air window. Likewise the flux path area is the air window area l_w*b_w.

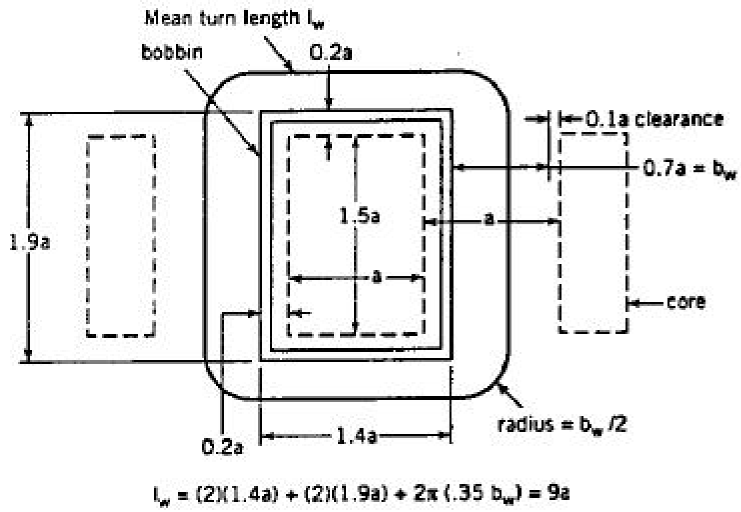


Figure 30-14 Top view of the double-E core bobbin showing the mean length l_w of a single conductor turn (winding) assuming the winding window is completely filled.

$$\Re_{\ell p} = \frac{3h_w}{m_o \ell_w b_w}$$

For

$$L_\ell = \frac{m_o N_p^2 \ell_w b_w}{3hw}$$

For a = 1 cm we find

With l_w = 9*1cm,

b_w = 0.7, $N_p^2 = (32)^2$ and

$\mu_o = 4\pi * 10^{-9}$

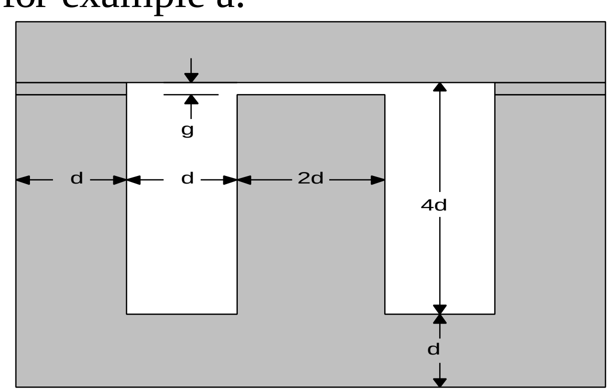
$$L_\ell = 14mH$$

E. Example of a mains ac transformer

We will do two examples below of mains transformers at 50-60 Hz operating frequency.

1. E-I Core Example:

This core type is available in a variety of dimensions, governed by the parameter d. It is desired to use such a core to produce a **500 VA isolation transformer** with input and output at he same voltage. If the wire size is chosen to keep $J \le 200$ A/CM2, what size core will be needed for this application? E-I laminated core for example a.



Depth d

Since this transformer is intended for line frequency use, a laminated metal structure will be chosen for the core material as it is cheaper. The window area in the core figure is $4d^2$. For 500 W at 220 V and output voltage, the wire should be able to carry 2.27 A. This implies a wire size of about #16 AWG, which has a cross-sectional area of 1.309 mm^2.

There are two windings of N turns each, for a total of 2N turns which must fit in the wire winding window. With a fill factor of 0.5, the window area should be double the copper area, or 2 X *2N X* 1.309 mm^2,Thus $4d^2 > 5.236N\ mm^2$. With d given in meters, $N < 7.64 \times 10^5 d^2$.

The magnetic area based on the center leg is $2d^2$, and the core should be chosen large enough to avoid saturation. For metal laminations, $B_{sat} = 1.5$ T. The sinusoidal input of 220 V_{RMS} has a peak value of 311 V, hence from Faraday's law:

$$\frac{311V}{100pN2d^2} < 1.5T,\ N > \frac{0.330}{d^2}$$

It can be shown that if d is less than about 2.5 cm, **it is not possible to meet both constraints on N simultaneously**. Substituting d = 0.0254 m, saturation can be avoided with 512 turns. This number of turns will fit **if the fill factor rises to 0.52**. If a lower fill factor is vital, a larger core must be used. This is the smallest value of d, and therefore the smallest core of this type of material, that meets the requirements. The two wire windings will be wound around the center post on top of each other, rather than the two legs for two reasons. First, the transformer will be smaller if the windings are located in the center. Second, if both windings are on the center post, leakage flux effects are much lower than if the windings are on separate legs. The air of the core would be set to g = 0 for a transformer.

2. General Transformer Core:

A transformer core with saturation flux density of 1.5 T is used for a 60 Hz application. The core area is 1000 mm^2, and the window area is 3800 mm^2. The magnetic path length is 400 mm, and the core volume is 500 cm^3. The mean length per turn is 200 mm. The core material has power loss given by

$$P = 7x10^{-6}B^2f^2 \text{ W/cm}^3$$

Choose wire sizes to provide a 120 V to 20 V voltage transformation at the **highest possible power**.

To avoid core saturation, requires that the peak volts per turn not exceed $wB_{sat}A_{core}$. The 120 V AC winding requires no more than 0.565 V/turn. With 170 V peak from 120 RMS, at least 300 turns will be necessary. For the 20 V winding, 50 turns will be necessary. To maximize power, the largest possible wire should be used.

Estimate next both the copper and core losses. With a fill factor of 0.5 and two windings, an area of (3800 mm^2)/4 = 950 mm^2 is available for the copper in each winding. On the 120 V side, the wire area can be up to (950 mm^2)/300 = 3.17 mm^2. The nearest even size wire is #14 AWG, with an area of 2.08 mm^2. On the 20 V side, the wire area can be (950 mm^2)/50 of 13.3 mm^2. Given the high number of turns, the current density must not be too high. If $J \leq 200$ A/cm^2, then the 120 V winding can support up to 4.16 A. The transformer rating will be about 500 VA.

Copper loss requires knowledge about both currents and wire resistance's. With mean wire length of 200 mm per turn, the primary wire length should be about 60 m, while the secondary wire should be 10 m long. Since #14 wire has about 8.45 mΩ/m resistance, the primary winding will have R_1 = 0.507 Ω. The secondary's #6 wire has resistance of 1.32 mΩ/m, so R_2 = 0.0132 Ω. With primary current of 4.16 A, the copper loss on the primary side will be 8.77 W. The secondary current should be 25 A, based on the turns ratio. This current produces loss of 8.25 W. Notice that the two windings have losses that match.

The magnetic loss reflects the total variation in flux. In this case, the flux varies from -1.5 T to + 1.5 T as the voltage swings between negative and positive peaks. The RMS flux density is

$1.51\sqrt{2}$ = 1.06 T. Substituting into the core loss equation the power loss per cubic centimeter will be

$$P = 7\text{x}10^{-6}\, 1.06^2 60^2 \text{ W/cm}^3 = 0.028 \text{ W/cm}^3$$

With total volume of 500 cm^3, the core loss should be about 14.2 W. At full load, this transformer can handle 500 W and exhibits total losses of 31.2 W. This suggests an efficiency of 94% - a fairly typical value for a transformer of this power rating.

Core losses estimated along the lines of this section are only approximate, and are strongly dependent on the properties of the specific core material. It is very important to recognize the strong effects of frequency and flux variation on core loss. In a transformer, flux variation is typically large to take full advantage of saturation limits. In an inductor, flux variation might be relatively low. **Note that high frequency operation with its increased loss constants tends to offset the benefits of the low flux variation that is associated with high frequency**. Losses and saturation limits make magnetic design a significant challenge. The need for smaller, more efficient cores continues to drive the development of new materials and geometry's.

PLEASE READ CHAPTER 5 OF ERICKSON