LECTURE 33
Inductor Design

A. Overview of Copper versus Core Loss in Inductors
1. Core Material Limitations
2. Core Materials Compared
3."Filter" Inductor Design via Erickson's Four Step Design Rules

4. Ten Commandments For Inductor Design

5. Summary

B. Inclusion of Core Losses and Relation to Wire Winding Losses

1. Sinusoidal Flux Density B(wt) Driving the Core

2. $B_{peak} \sim \dfrac{\int v dt}{N A_c (\text{core})} = \dfrac{l}{N A_c}$

THIS ALSO SAYS N~ 1/B that is as B decreases the number of turns will increase

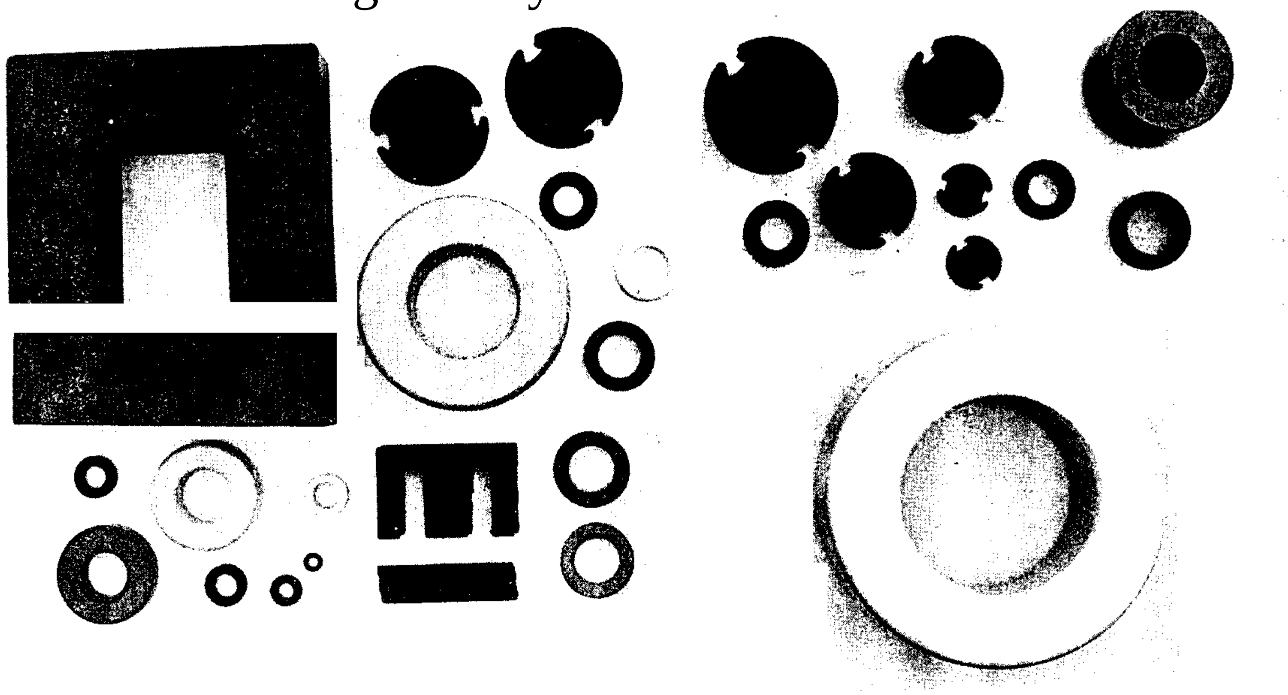
3. Tailoring B_{opt} for Minimizing Total Losses:

   a. Trading Fe vs Cu to Achieve Minimum Total Loss

b. N↑ makes for larger I^2R(wire) losses

c. N↓ makes for higher B_{peak} and more core losses

The various core geometry's are shown below



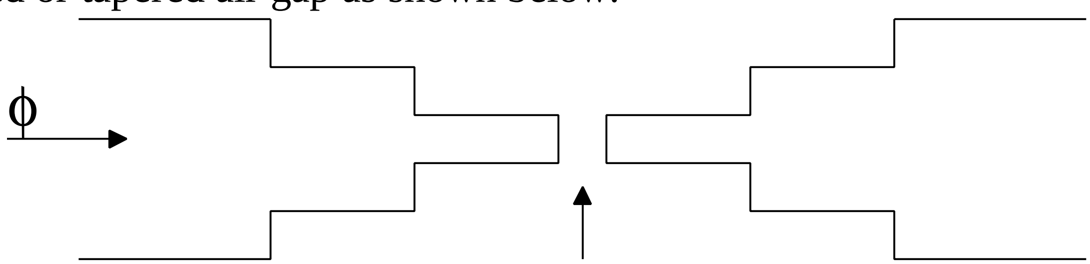
For a term paper on Integrated Inductor Design See as a starting point the article "LAYOUT OF INTEGRATED RF SPIRAL INDUCTOR" in Circuits and Devices March 1998 pgs 9-12

LECTURE 33
Inductor Design Methodology

A. Overview of Copper versus Core Losses

For HW#1 do problems 14.1(easy) and 14.5(harder)

An inductor is a device whose purpose is to store and release energy. A filter inductor uses this capability to smooth the current through it and a two-turn flyback inductor employs this energy storage in the flyback converter in-between the pulsed current inputs. The high μ core allows us to achieve a large value of $L = \mu N^2 A_c/l_c$ with small A_c and l_c so large L values are achieved in small volumes. However, high μ will limit the maximum energy storage in the core with no air gap. Since the magnetic core material itself is incapable of storing significant energy, energy storage is accomplished in a non-magnetic air gap(s) in series with the core. These gaps minimize the inductor variations caused by changes in core properties and help avoid core saturation. If non-linear L(i) is desired, as it is in magnetic amplifers, it can also be achieved with a stepped or tapered air gap as shown below:



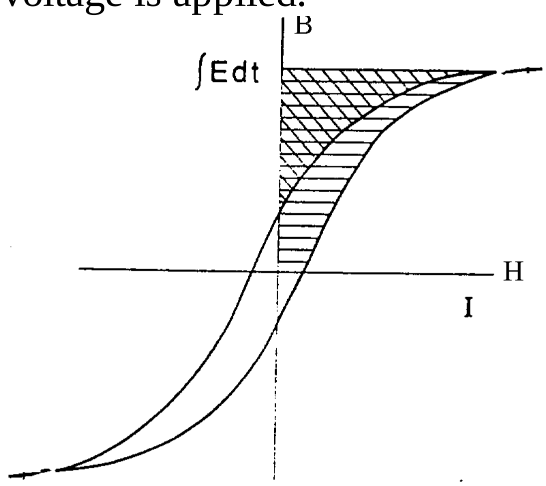
Both core and wire winding losses as well as saturation effects establish design rules for those who "wind their own inductors" as shown below. Limitations of magnetic cores are crucial to good inductor design

1. **Core Material Limitations:** In dc applications, inductors are primarily thought of as current operated devices. Too large a current, will saturate the magnetic core via ampere-turn limits set by $ni < B_{SAT}A_C\,\Re$, this means $ni < B_{SAT}/\mu$. This limit on I_{MAX} will in turn limit the maximum energy that can be stored in an ungappped core to

$B^2(SAT)$ *l(core)*A(core) /2μ.

A convoluted inter-dependence of the wire current and the magnetic field in a transformer core is as follows. The volt-sec limit says that B(core)= $l_p = \int V_L\, dt / N A_C$. Hence, we find that N~1/B and later we will employ $N^2 \sim 1/B^2$ in relations for the copper loss to show $P_{Cu} \sim 1/B^2$.

As regards the inductor voltage applied without considering the equivalent series resistance r_L, i_L on a DC basis will continue to increase with a DC V_L ,since all of the applied voltage appears across v_L. With r_L the inductor current increases until $i_L r_L$ equals the input voltage. Again the B-H curve of the inductor core looks like a $\int E_L dt$ versus I_L curve as shown below. At high ∫Edt,I $\rightarrow \infty$ where the core saturates and L $\rightarrow$ 0. For a flux or B change to occur a time interval is required over which the applied inductor voltage is applied.



In high frequency PWM applications, the major magnetic core material limitations are: 1. Flux Density Saturation and 2. Core losses, both of which depend upon flux swing and applied frequency of the flux. In these applications inductor windings are usually driven with rectangular voltage waveforms derived from low impedance sources. Since the voltage, pulse width, and number of turns are quite accurately known, it is easy to apply Faraday's Law to determine the maximum flux swing and appropriately limit it. . In the case of a sinusoidal voltage, Faraday's law gives $V = N\, d\varphi / dt = N\omega A_C B_{SAT}$. Hence $V/\omega N A_c$ must be less than B_{SAT}.

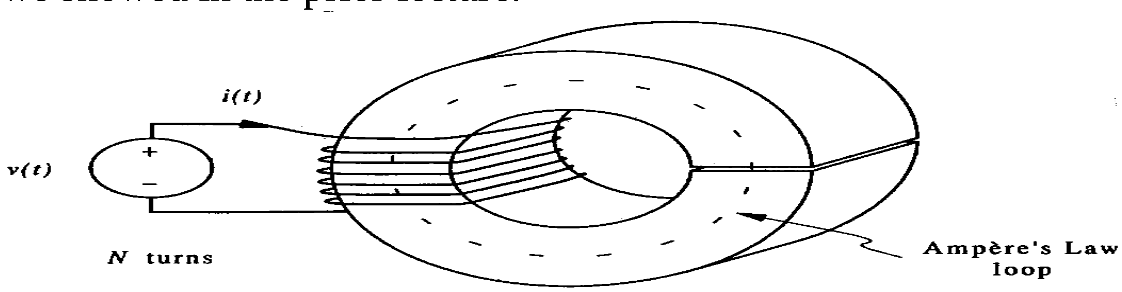
A flyback transformer is actually an inductor with multiple windings. It stores energy taken from the input in its mutual inductance during one portion of the switching period, then delivers energy to the output during a subsequent interval. In a flyback transformer, the magnetizing current is virtually important, because it represents the energy storage required by the application. In this case, the magnetizing current can be calculated quite accurately using Ampere's Law, because it depends on the very predictable characteristics of the gap in series with the core, and the uncertain core contribution to energy storage is negligible. Magnetizing inductance is an essential element in a flyback transformer.

2. Core Materials Compared

There are three major core materials of interest.

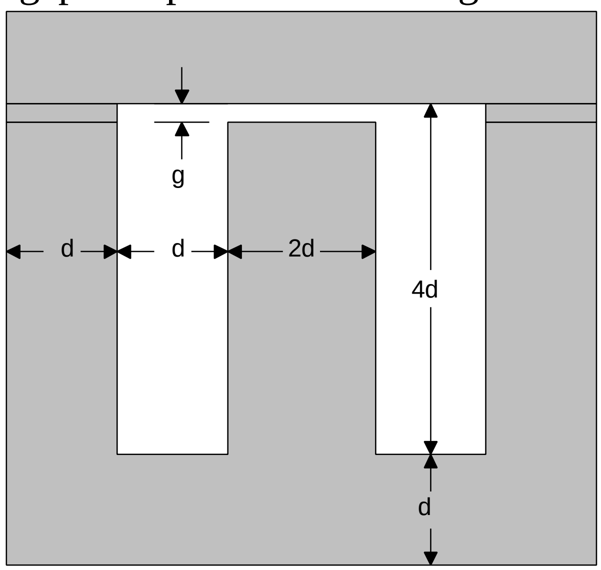
Metal Alloy Tape	Powdered Metal	Ferrites
1. B_{sat} = 1 Tesla	1. B_{sat} = ½ Tesla	1. B_{sat} = 0.2 Tesla
2. μ_r = 60,000	2. μ_r = 100	2. μ_r = 1000
3. ρ_c is low, high eddy current losses	3. Highest Loss Core	3. ρ_c is high, low eddy current losses
4. Sharp Saturation suddenly $L \rightarrow 0$	4. Soft Saturation L(1/4 tesla) = ½L(B=0) Hence I_m increaes gradually	4. Sharp Saturation suddenly $L \rightarrow 0$

For a slotted core inductor the " winding your own L magnetic core" looks like that below. Most energy storage occurs in the air gap region as we showed in the prior lecture.

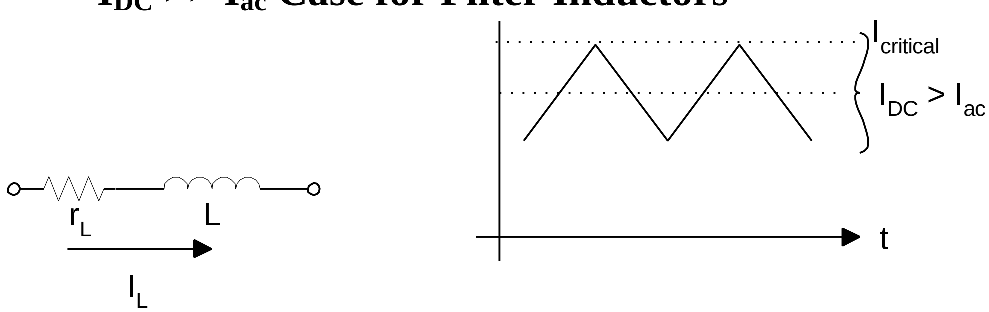


$$W(air) = \tfrac{1}{2}B^2 l_g/A_g\mu_o >> W(core) = \tfrac{1}{2}B^2 l_c/A_c\mu_c$$

The above air gap allows lots of leakage flux to impinge on the wire windings. For a filter inductor this is not an issue. But for inductors with very high AC current components this leakage flux can cause very substantial increases in the AC wire resistance, via previously discussed proximity effects. We can reduce the level of leakage flux by a different core design as show below. The air gap in an "E type" core is inside the core enclosure and if the g/d ratio is chosen small, the short center leg providing the air gap will produce little fringe flux as shown below:



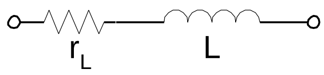
I_{DC} >> I_{ac} Case for Filter Inductors

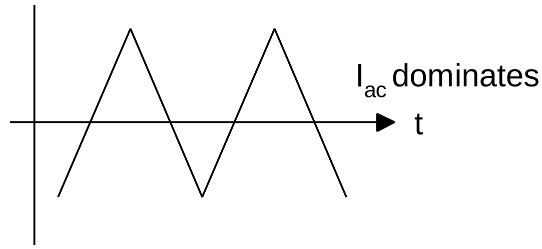


P_{cu}(loss) > P_{core}(loss) since B_{AC} is negligable

$I_L^2(\text{rms})\, r_L$ > core loss

I_{ac} >> I_{DC} Case for Circuit Inductors in PWM Converter Circuits: I_{ac} << I(crititical)



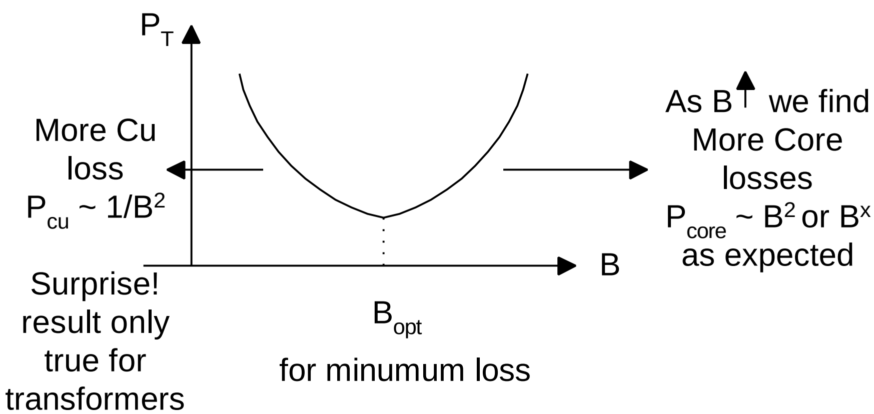


$P_{total\ loss} = P_{cu}(loss) + P_{core}(loss)$

Herein we will learn to trade Fe vs Cu in order to minimize the total loss. That is we can find an optimum number of turns of wire that provide minimum total loss. If N is too high I^2R wire losses dominate. While if N is too low $B_{peak} = \int E dt / NA_c$ will rise and core losses dominate.

The question is now how do we rationally trade off copper losses and core losses to achieve the desired inductor for the specific circuit use.

While the graph below illustrates the balance needed to achieve minimum total loss, **we will not fully understand this plot till lectures 34 –35**.



3. "Filter" Inductor Design via Ericksons Four Step Design Rules

We tailor the inductor design to the switched mode converter circuit topology. Design considerations for inductors include:

- core size and permeability
- effects of air gaps on reluctance and L linearity as well as the sensitivity of L to changes in the core
- core loss limits
- saturation values of flux density in the chosen material
- core window size (for allowing all needed wire windings)
- current density allowed in wire windings

- maximum values of ac voltage, dc volt-sec, and dc current
- operating frequency

1st Select core via the emperical K_g factor

Both hysteresis and eddy current core loss effects in inductors are caused by time-varying flux. If an inductor carries a constant dc current below its saturation limit, the core flux will be constant, and the hysteresis and eddy current losses of the core will be zero. In practice any switch mode converter has large current ripple in the inductors, at the switching frequency. The flux variation will be follow the AC current. Magnetic material can handle only a limited loss per unit volume without getting too hot. Losses of 1 W/cm^3 are usually considered high for ferrites, and values in the range of 0.2 W/cm^3 are more common in well designed cores. At any rate we can translate circuit variations into a single parameter which helps to specify the core needed as shown in Lecture 32

$$K_g \geq \frac{L^2 I_{max}^2 \; r(\text{wire})}{B_{pk}^2 \; R \; K_u} * 10^8 \left[\text{cm}^5\right]$$

Get L, R, I_{max} from <u>PWM converter electrical specs</u>

Get B_{pk}, K_u from manufacturers core and wire specs respectively. The core geometry gives us both A_c(core) and W_A(wire winding area) as well as the mean length of turn wound on the core.

2nd Spec the air gap to be cut in the inductor core

$$\ell_g = \frac{m_o \; L \; I_{max}^2}{B_{pk} \; A_c} \; [\text{m}]$$

As an alternative to the gap size, l_g, we can specify the A_L, or specific inductance in mH per turn^2, factor for an inductor core. N_w is the number of wire turns and $\Re_c$(core) is core reluctance including the airgap.

$A_L \equiv \frac{L}{N^2_w} = \frac{1}{\Re_c} = \frac{1}{\Re_c}\left[\frac{\text{mH}}{\text{turn}}\right]$ - We arbitrarily fix N @ 1000 turns as core manufacturers do so in their A_L specifications.

$1 \leq A_L \leq 1000$; A_L is typically 100 for N = 1000 turns.

You spec A_L - Core Manufacturer will cut l_g to size to meet your spec's

3rd Spec # of wire turns. No fractional turns are allowed making this an iterative procedure

$$N_w = \frac{L \; I_{max}}{B_{pk} \; A_c}$$

4th Choose wire insulation, wire type and current carrying area or AWG# size

$$\text{Area Wire} \le \frac{K_u (\text{window area})}{N}$$

All the wire wound on a given core must fit into its winding *window,* the opening for wire turns. The window also must hold insulation and any structure on which the wire is mounted. In practice, only about 50% of the window area can actually carry active conductor, as the rest is voids or wire insulation. This fraction is called *fill factor.* In a two-winding transformer, this means that each winding can fill not more than 25% of the total window area.

That is given the wire winding area, W_A, and the required number of turns we set the required wire size. We usually use AWG #'s instead of wire current area to specify a wire. This choice of wire size will be modified by high frequency skin effects as we will show later. Wire size is an important aspect of the inductor design since a given wire can handle only a limited current density to avoid excessive power loss. The wire-winding window of a given core must have enough area so that copper wire of a given diameter can be used and all the required number of turns fit. We do this to avoid excessive Ohmic heating of the wire so wire loss influences key details in magnetic core geometry choices. For an inductor design, core saturation primarily limits the amp-turn values possible but the current density limit in the wire also represents an amp-turn limit. A trade-off rule results.

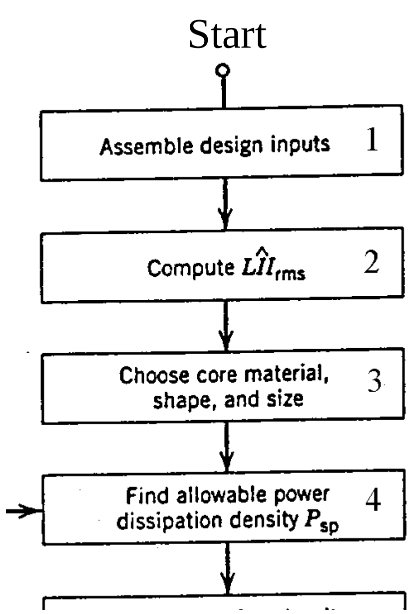
Consider what happens if the rule is violated. If the core window is too small, the wire size chosen could be too small for the required current and the wire will overheat before the saturation amp-tum limit of the magnetic core is reached. If the wire-winding window is too large, then core saturation will be reached prematurely and the copper winding might be underutilized. Core size sets the mean length of turn required to encircle the core and hence the length per turn of the wire.

R_{wire} = (resistance per meter of the wire) x (length per turn) X N

Or alternatively $R=\rho N\ (MLT)/A_W(wire)$. MLT depends on core geometry. Cores made of sectioned structures tend to be easier to wind with automated wire winding machines than toroids. For toroid cores, windings are often designed to form a single layer of copper material around the inside of the core. This keeps the device small and minimizes flux leakage. For other core shapes, windings often use the largest wire that will fit conveniently into the window. This minimizes losses, and maximizes the power rating.

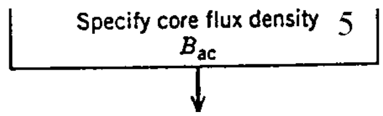
3. General Inductor Design via Ten Design Rules(SKIP THIS SECTION IF YOU ARE AN UNDERGRADUATE)

a. Overview of ten step L design sequence

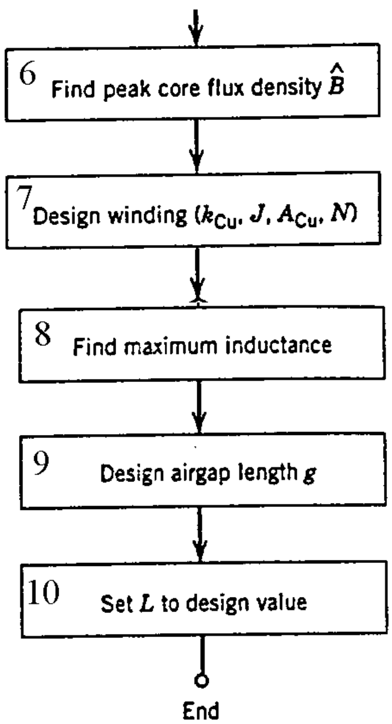


L I_{rms} and f required by PWM converter circuit.

1. T(core surface) }In steps 1-2 the
   }Energy and heat
2. Stored energy required}flow requirements
   ~1/2 Li^2 }help chooses both
   }material type and
3. From tables of cores vs}geometric shape
   1/2 Li^2 }of the required
   }core to wind wire
4. Thermal Analysis }upon.
   T(surface), T(ambient)



Steps 5 and 6 quantify:
B_{ac} waveform in the core from core data specifies eddy current core loss



Peak B is crucial to hysteresis loss
B(peak) < B_{sat}

Winding parameters for copper wire

Maximum
$L_{max} = N^2/\Re(\text{core})$

}$L_{desired} < L_{max}$

}$L = N^2/(\Re(\text{core}) + \Re(\text{gap}))$

}8-10 set
}the "L"
}value by
}μ(core)
}and
}l_g(gap)

b. Detailed Approach for Each of 10 Steps

For HW#1 verify the specific quantities in the example below on the left hand side.

<u>1</u>

Assemble Inductor design inputs

<u>STEP 1</u>

<u>Six Design Inputs</u>	<u>Specific</u>
(1) L values desired by circuit:	(1) L = 300 μH with 4A rms
(2-3) I_{rms} and I_{peak}:	I_{peak} = found from

depends on the **Current waveshape** — $\sqrt{2}\,I_{rms}$ only for a sine wave. eg. $I_p = \sqrt{2}\ 4 = 5.6$

(4) Switch Frequency — We choose 100 KHz operation

(5) Max T of core surface — $T_s = 100^oC$

(6) Max ambient temperature — $T_a = 40^oC$

The above <u>Six</u> parameters enter into the inductor design inputs of step one. Key is the design product: $L\,I_{rms}\hat{I}_p \approx$ stored energy. We get both I_{rms} and I_p(peak) from I_L vs time waveforms. This sets the both <u>required</u> core material and geometric core shape required to dissipate the heat as shown below.

STEP 2

Compute $L\hat{I}\,I_{rms}$ in units of H-A^2

<u>General</u>

Instead of

$$e_L = \frac{1}{2} L\,I_{rms}^2$$

Use $LI_{rms}I_{max}$

↓↓

Better accounts for odd waveforms found in PWM converter <u>circuits</u>

<u>Specific</u>

$LI_{rms}I_{peak}$ for a <u>sinusoid</u> with the chosen Lis .007 H-A^2 for this case

μ and B_{SAT} alone doesn't fully specify the core as we must also consider shape and size of the core to accommodate windings and heat flow

core material - Choice of switching f is also crucial

	-	to match to the core
core size	-	$B_{max} < B_{sat}$
core shape	-	ease of putting N windings of wire
	-	"on the iron"

(core cost as well as wire cost) is also an issue in some low cost commodity circuits.

STEP 3

Choose core material, shape and size

We need a manufacturers core data base to guide our possible core choices. Below we list just the pertinent data for this 100 kHz example

Table 30-3 Database of Core Characteristics Needed for Inductor Design

Core No.	Material	$AP = A_w A_{core}$	R_θ at $\Delta T = 60°C$	P_{sp} at $\Delta T = 60°C$	J_{rms} at $\Delta T = 60°C$ and P_{sp}	B_{ac} at $\Delta T = 60°C$ and 100 kHz:	$k_{Cu} J_{rms} \hat{B} A_w A_{core}$
•	•	•	•	•	•	•	•
8[a]	3F3	2.1 cm^4	9.8°C/W	237 mW/cm^3	$3.3/\sqrt{k_{Cu}}$	170 mT	$0.0125\sqrt{k_{Cu}}$
•	•	•	•	•	•	•	•

[a]Core number 8 is the same as listed in Table 30-1 with $a = 1$ cm.

$N\phi = Li$ = flux linkage	|	
Where $N_w = K_{cu} \dfrac{\text{Awindow}}{A_{cu}}$	|	$J_{rms} = \dfrac{I_{RMS}}{A_{cu}}$
$\phi = B\ A_{core}$	|	$A_{copper} = A_{cu}$

K_{Cu} can vary from 0.3(Litz wire) to 0.9 for foil. **We will need a larger core for the choice of Litz wire ands a smaller core for foil wire. Wire type with a copper fill factor will need to be balanced with core size.** Now we reconcile electrical circuit specs and the wire and core specifications.

<u>Circuit Spec's</u>		<u>Core and Winding Specifications</u>
$LI_{rms}I(peak)$	←→	Five Parameters K_{cu}, J_{rms}, B_{pk}, A_w, A_c
Design specs		Two Materials Issues: J_{rms}, B_{pk}

from circuit Three Geometry Choices: A_w, A_c, K_{cu}

K_{cu} is derived from wire size,the number of turns and the core wire winding area and is not an independent variable. It varies by a factor of 3 from Litz wire $K_{cu} \approx 0.3$ to Foil $K_{cu} \approx 0.9$

STEP 4 Thermal Resistance, R_θ, and Power loss

Assume the total inductor power: $P_L \equiv P_{core} + P_{winding}$. As a first guess assume that that core and wiring contribute equal parts.

Realize that $P_L(\max)$ is limited via thermal heat: $\dfrac{T_s - T_a}{R(\text{core total})}$

We cannot allow the core to reach 100°C from the balance of heat versus heat taken away.

$LI_{rms}I_{peak} \sim K_{cu}[J_{rms}B_{pk}A_w, A_c]$

Note that the type of wire must be chosen at his point but not wire size.

A specific choice is Litz wire with $K_{cu} = 0.3$.

Note the trends set in motion by K_{cu} choice.

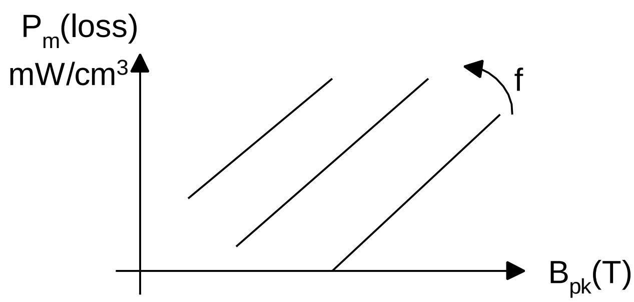
$K_{cu} \uparrow$ then required core size $\downarrow$

$\downarrow$ then required core size $\uparrow$

This is classic Trading “core” for “copper” in the inductor design.

General	**Specific**
$P \equiv \dfrac{T_s - T_a}{R_q(\text{core})}$	From core data base or by calculating R_θ from geometry
$P = P_m V_T$	Typically for a core
V = Core windings plus Core Volume,	$R_\theta \sim 10^o/W$
$V_T = V_c + V_w$	

In <u>General</u> Core loss P_m varies with the flux density to some power and P_m increases with frequency as shown below.



From P_M vs B_{pk} vs f or from the chosen core data base we find B and then P_m. Where P_m is Specific Power density. Take as illustrative for 3F3 core under the above conditions ≈ 250 mW/cm^3.

Next we do two steps together: specifing B(core) and B_{peak}

5

Specify core flux density B_{ac}

6

Find allowable maximum core flux density $\hat{b}$

This depends on our choice for core material and upon the core temperature. For fixed (T_s-T_A) and given frequency of applied current we find from the loss equation

$P_M \sim k\,(B_{ac})^2/ f^2$

We find B_{ac} by employing a maximum core temperature of 100 degrees and knowing the core thermal impedance. **The allowed B_{ac} (max)= 170 mT sinusoidal for the case we are discussing here**.

$B(wt) = B_{ac}\text{Sinwt}\ B_{ac}$,while $1.414 x B_{ac} \equiv B_{pk}$. Here we have to insure that B_{max} does not exceed B_{SAT} for the chosen core material. Otherwise we have to iterate with a new core or new wire or both as shown below in step 7. The flux density in the core is proportional to the inductor current.

STEP 7

Design wire winding (k_{cu}, J,

A_{cu}, N)

General | **Specific**

1. $K_{cu} = N \frac{A_{cu}}{A_{window}}$

1. Each type of wire has

We use W_A for core wire window and K_{cu} is set from choice of wire type. We previously chose 0.3 Litz wire

$K_{cu} = 0.3$ for Litz Wire

This allows us to set W_A for the core when the number of wire turns is known

2. A_{cu} from $\frac{I_{rms}}{J_{rms}}$ wire database

I_{rms} comes from circuit waveforms

J_{rms} comes from wire data base and

2. $A_{cu} = \frac{4A}{J_{rms}}$

J_{rms} from wire data sheet

4A from circuit spec.

We utilize:

K_{Cu} =0.3, and find first J_{RMS} and then the required wire area at the given current level, 4A. A_{cu} as determined form 4A/ A_{cu} = 6A/mm^2

This is shown on the next page.

General **Specific**

The wire winding losses in Watts per cm^3 is next sought

3. $P_M = P_{windings} = 22\, k_{cu}\, J^2_{rms} \left[\frac{mW}{cm^3}\right]$

3. $J_{rms} = \frac{3.3}{\sqrt{k_{cu}}} =$ $6A/mm^2$

Given P_m we find J_{rms}. This sets the wire area, A_{Cu}=4A/6A/mm^2=0.7mm^2 We also know from the core data A(window) =140 mm^2 so we know

$$N = \frac{K_{cu}\ A_{window}}{A_{cu}}$$

$$N = \frac{.3140}{0.7} = 60^+$$

Note the importance of A_{window} = 140 mm^2 from core data base for the number of turns of copper wire.

Required # of copper wire turns results!

8

Find maximum inductance of selected magnetic core

General

$$L_{max} \equiv \frac{N\ A_c\ B_{pk}}{I_{peak}}$$

L_{max} must > L_{spec} to insure low i_{ripple}

But not too much bigger because bigger core is more costly.

Specific

A_c=1.5 * 10^{-4}m^2 from chosen magnetic core data base

N, B_{pk} and I_p are all given

$$L_{max} = \frac{60^+\ 1.5*10^{-4}\ 170}{4\sqrt{2}}$$

= 290 μH

General

$$L \sim \frac{N^2}{R_c}, \quad R_c \sim \frac{l_{core}}{\mu \, A_{core}}$$

If $L_{max} < L_{spec}$ then
Choose a bigger core $R_c \uparrow$
But if $L_{max} >> L_{spec}$ we have chosen too big a core. Save $ by reducing core size.

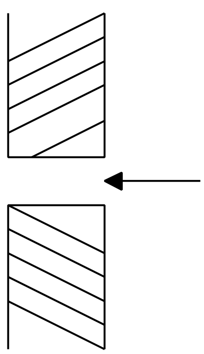
Specific

L_{spec} is 300μH so we are under target. What to do? So close just use no air gap with core or choose smaller core size with tailored gap.

9

Design airgap length "g"

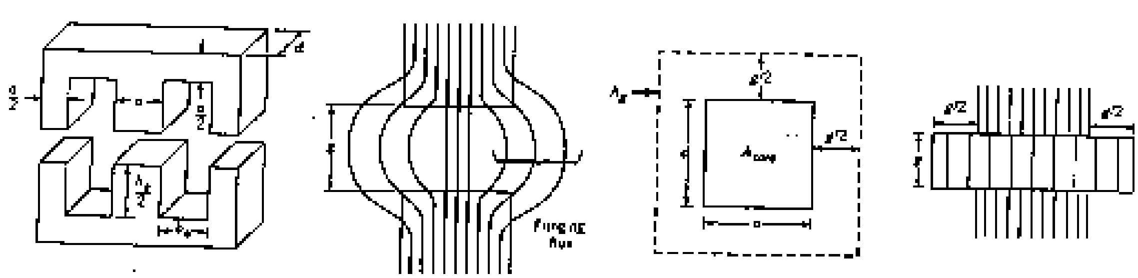
Set the core air gap "g" so $B_{pk} < B_{sat}$ (for chosen core) B_{sat} varies only 0.1 → 2 Tesla as we choose various core materials. If $L_{max} > L_{spec}$ then we can tailor L_{spec} from L_{max} by adding precision air gaps. The gap reluctance neglecting fringing is:



$$\Re_{gap} = \frac{l_g}{\mu_o \, A_g}$$

l_g usually < 1 mm

Precision mylar sheet @ 1/2 mil is available when we clamp two pieces together to form a gap.
Notebook paper is 3 mil / 3 thou



Double E core has three distributed air gaps -3 air gaps $\Rightarrow$ are better than one Why?

Pbm of one big gap is that B(fringe) from the core goes out further into copper windings increasing skin effect and eddy current loss in the wires.

The effective air gap with an E core geometry is made in a cross-section and neglecting fringing.

Σ gaps = 3 * E core spacing
$\downarrow$
Make 1/3 of size for less fringing
gap size g << d,a - core sizes

The area of the gap A_g =
A_g (a+g)(d+g)
if flux extends out additional g/2 from core edge

$A_g \approx ad + g(a+d) + 0$ second order terms

$$\Re_{total} = \underset{\displaystyle \frac{\ell_c}{m_c\ A_c}}{\underset{\downarrow}{\Re_c(\text{core})}} + \underset{\displaystyle \frac{3g}{m_o\ A_g}}{\underset{\downarrow}{\Re_{gaps}}}$$

Usually assume $\Re_c$ is small compared to $\Re$(air gap).
We tailor 3 g to give desired B_{pk}(core) when I_{pk} is applied so that we do not exceed B(critical).
$NI = \Re_g \phi$

$$\frac{NI_{pk}}{A_c\ B_{pk}} = \frac{3g}{m_o\ A_g}$$

Tune g for desired B_{pk} so $B_{pk} < B_{sat}$ (core)

10

Set L to design value

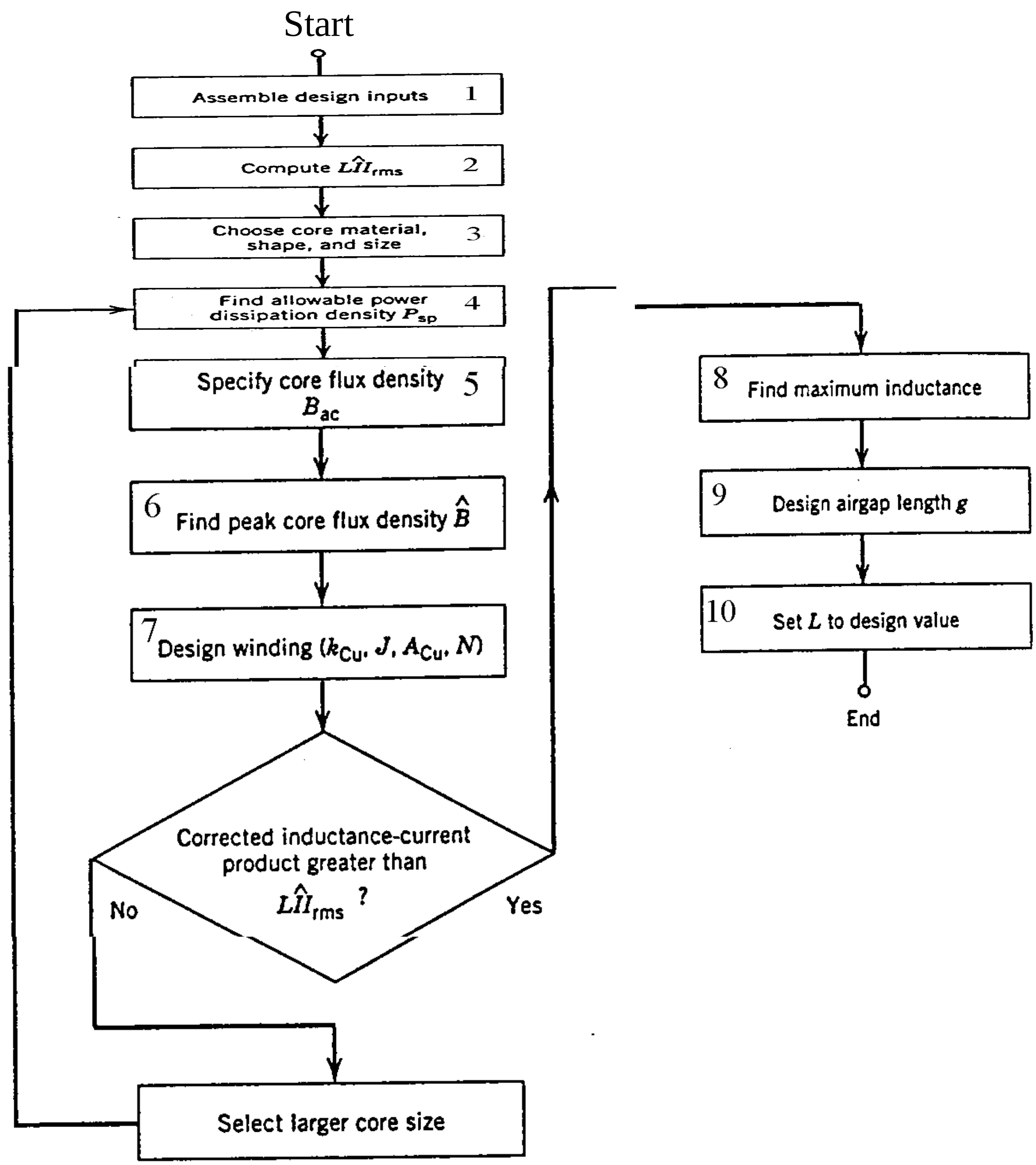
if $L_{spec} < L_{max}$ We have two routes to reduce L_{max} to reach L_{spec}.

$$L = \frac{N^2}{\Re_g}$$

We could increase $\Re_g$ alone but this both reduces B_{pk} and B_{ac} which is good only for core loss. Since $\Re_g = \dfrac{3g}{m_o \ A_g}$ by reducing A_g we could use a smaller core, thereby reducing L and lowering core cost.

If we reduce N^2 to reach L_{spec} from L_{max}, then we save copper. But lower N increases B_{pk} and hence core loss again we trade: "Cu" for "iron".

Summary



B. Calculating Magnetic Core Loss

1. For Assumed Sinusoidal B(wt) Excitation

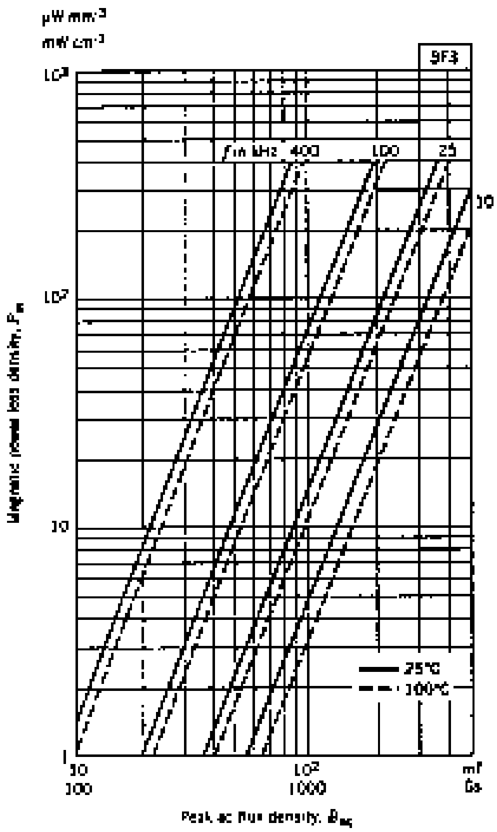
The total power loss including both hysteresis and eddy currents.

$$P_m \left[\frac{\text{W per}}{\text{cm}^3}\right] = k_{fe}\ (B_{pk})^x\ f^y\ V_{core}.$$ Where $V_{core} = A_c l_c$

The exponent "x" depends on how wide the hysteresis loop expands horizontally (AH) as well as vertically (AB).

$1.5 \le X \le 3.0$; 2 is typical of practical core materials

$1 \le Y \le 2$; depending on core material



$$0 \le P_m \le 250\ \frac{\text{mW}}{\text{cm}^3}$$

B_{max} is the key parameter

$P_T = P_m * V_{core} = P_m A_c l_c$

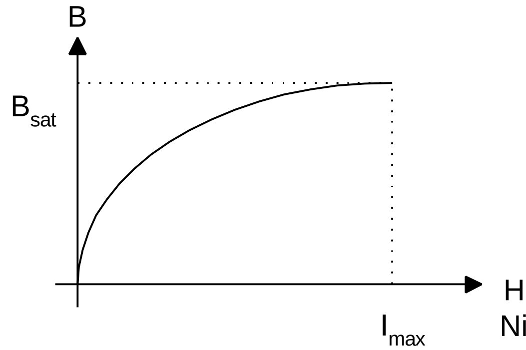
Most PWM converter i_L waveforms are square or triangle waves not sinusoids. These signals contain DC components. B(dc) causes no core losses only core saturation! Harmonics of the switch frequency also occur for many converter waveforms ⇒ Higher core losses due to the f^2 dependence of eddy current losses.

2. Restraining B_{peak} Values to below B(critical)

We now consider voltages applied across the inductor, rather than currents driven through the inductor. Volt-sec balance will be constrained by B_{SAT} of the chosen core material as shown earlier.

a. A soft B-H curve

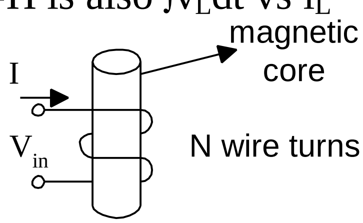
With soft B-H curves L will decrease slowly with current and not change precipitously. Often $L(B_{sat}/2) = L((B{=}0)/2)$ for soft saturation cores



However above B_{sat} $\mu \rightarrow \mu_o$ and $L \rightarrow$ short circuit or very small values

Then for $B > B_{SAT}$ we find $L = \dfrac{m_o A_c N^2}{\ell_c}$ where we replace μ(core) by μ(air) and L is 1000 times smaller or so

b. B-H is also $\int v_L dt$ vs i_L

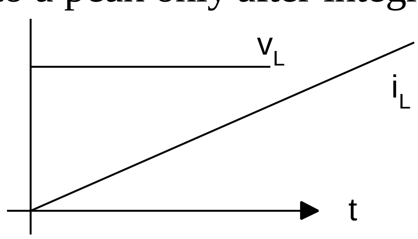


$$V_L = L \frac{di_L}{dt}$$

$$i_L = \frac{\int V_L dt}{L}$$

$$N i_L = f_c \Re_c, \quad f_c = B_c A_c$$

For V_{in} to the inductor being a sinusoid or any waveform the ϕ increases to a peak only after integrating V_L. A step of V_L is easiest to visualize.



$$f = \frac{Ni}{\Re_c} = \frac{N \int v_L dt}{\dfrac{N^2}{\dfrac{m_c l_c}{A_c}}}$$

Where $\Re_c = \dfrac{\ell_c}{m_c A_c N^2}$

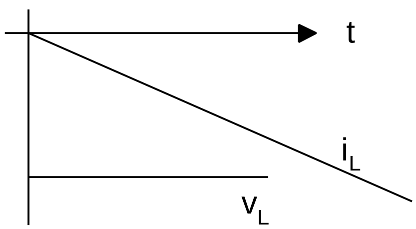
The flux varies as $\dfrac{\int v_L dt}{m_c \, N \, A_c} \ell_c$

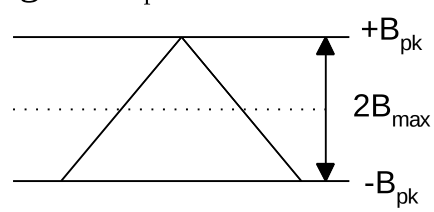
$f \approx \dfrac{\int v_L dt}{N} = B A_c$. Now B_{pk} occurs when i_L and ϕ peaks.

$$B_{pk} = \frac{\int v dt}{N \; A_c} = \frac{l_1}{N \, A_c} \text{(volt-seconds)}$$

The maximum B_{pk} occurs for when V_L is either $\pm V_L$ for a long period of time and the inductor current is ramping up to high values.

The V_L and $\pm V_L$ square wave excitation vs. time causes a triangular B_{pk} waveform vs time.





$$B_{max} \equiv \frac{l(v - \sec)}{2 \; N \; A_c}$$

This integral dependence of B on V_L says:

1. $B_{max} \sim \dfrac{1}{N}$ This relates the choice of wire turns to core issues.

It says for many wire turns $\underline{B_{max} \downarrow}$ while for one wire turn we achieve maximum B which must not exceed B_{SAT}. **Hence, it sets a minimum number of wire turns required**.

2. $B_{max} \sim \dfrac{1}{A_c}$ This is sets up the need for larger size cores.

There are relations between the copper loss and core loss.

3. $N\uparrow$ $B_{max} \downarrow$ reduces iron loss } Cu-Fe loss trading occurs

$N \uparrow$ increases I^2R Cu loss } trade again!

We then have to distinguish the cases of current through an inductor, **amp-turn limits**, and the case of voltage impressed across an inductor, termed **volt-sec limits**. Each sets limits to inductor performance and in some cases they act in concert to limit inductor operation.

Summary of Saturation Design Rules:

Rule	Interpretation
$Ni < B_{sat}\Re A$	Amp-turn limit for an inductor
$W_{max} = ½B_{sat}{}^2 l_{core}A_{core}/\mu$	Maximum energy that can be stored in a given core.
$W_{max} = ½Bsat^2V_{gap}/\mu_o$	Maximum energy is determined by air gap volume if the core has high μ.
$V_o/N < wB_{sat}A$	Maximum volts per turn (for a transformer) at frequency w.
$\int vdt < NB_{sat}A$	Maximum volt-seconds for an inductor or transformer.