LECTURE 27
Basic Magnetic's Issues in Transformers

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A. Overview

1. General Comments on Transformer Inductance's L_m and L_l(leakage) as well as Transformer resistance's

The magnetic's issues in transformers involve one core and several sets of current carrying coils wound around the core. The generation of the magnetic flux inside the core by one set of current carrying coils and the transmission of the flux throughout the core is the first step in transformer action. The magnetization flux is created via **a magnetizing inductance, L_m,** which is placed in the primary of the transformer. Placement of auxiliary coils on the same core, that are subsequently subjected to the same flux or a portion of it generated by the primary coils, will generate induced voltages in the auxiliary coils.

The core is fully specified by the manufacturers in terms of its magnetic properties. The permeability value will reveal how much of the core flux is in the interior of the core and how much flux leaks into the region outside the core. This leakage flux will have several effects:

- The creation of leakage inductance's, L_l(leakage), that lie in the electrical circuits of both the primary and the secondary of the transformer
- The increase in the wire winding resistance's due to the leakage flux which penetrates the wires causing PROXIMITY effects as will be described in Lecture 28

3. Core Materials and Their Available Geometric Shapes

b. Transformer core materials

On the following page we review the choice of core materials choices as well as the operating conditions of the cores. In general ferrites and permalloy(see sectionC) are core materials.

Common Materials choices and Manufacturers are listed below.

Core Materials

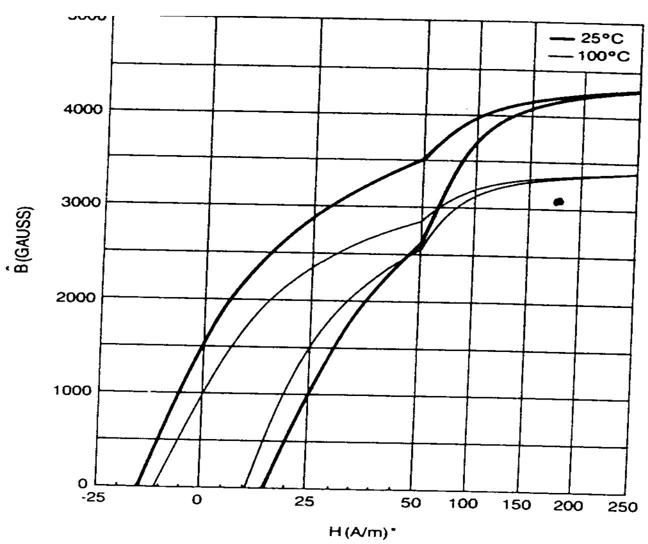
Manufacturer	Core Material (Ferrites) <100 kHz	Core Material (Ferrites) <1 MHz
Magnetics, Inc.	F, T, P	F, K, N
TDK	P_7C_4	P_7C_{40}
Ferroxcube	3C8	3C85
Siemens	N27	N67

Next we list practical guides to the actual B_{SAT} limits in cores versus the frequency of the current in the coils wound around the core. Note that in practice the full B-H curve cannot be employed at higher frequencies(above 1MHz) due to excessive losses.

Flux Density Limits vs. Frequency

Frequency	Maximum Operational Flux Density (B_{max})
<50 kHz	$0.5B_{sat}$
<100 kHz	$0.4B_{sat}$
<500 kHz	$0.25B_{sat}$
<1 MHz	$0.1B_{sat}$

Elevated core temperatures due to heating effects will lower B_{SAT}



* 100 A/m is 1.25 oersted

degradation of B_{sat} with core temperature (3C8 material shown). (Courtesy of Philips Components.)

b. Commercially Available Core Shapes

Core shapes are crucial to insuring we can easily wind the primary and auxiliary coil wires around the core. Each coil will have a specified current flowing which sets the desired diameter of the chosen wire for that winding. We wish all coils to dissipate the same I^2R losses so that no one coil is getting hotter than another is. Also all coils must fit on the same core.

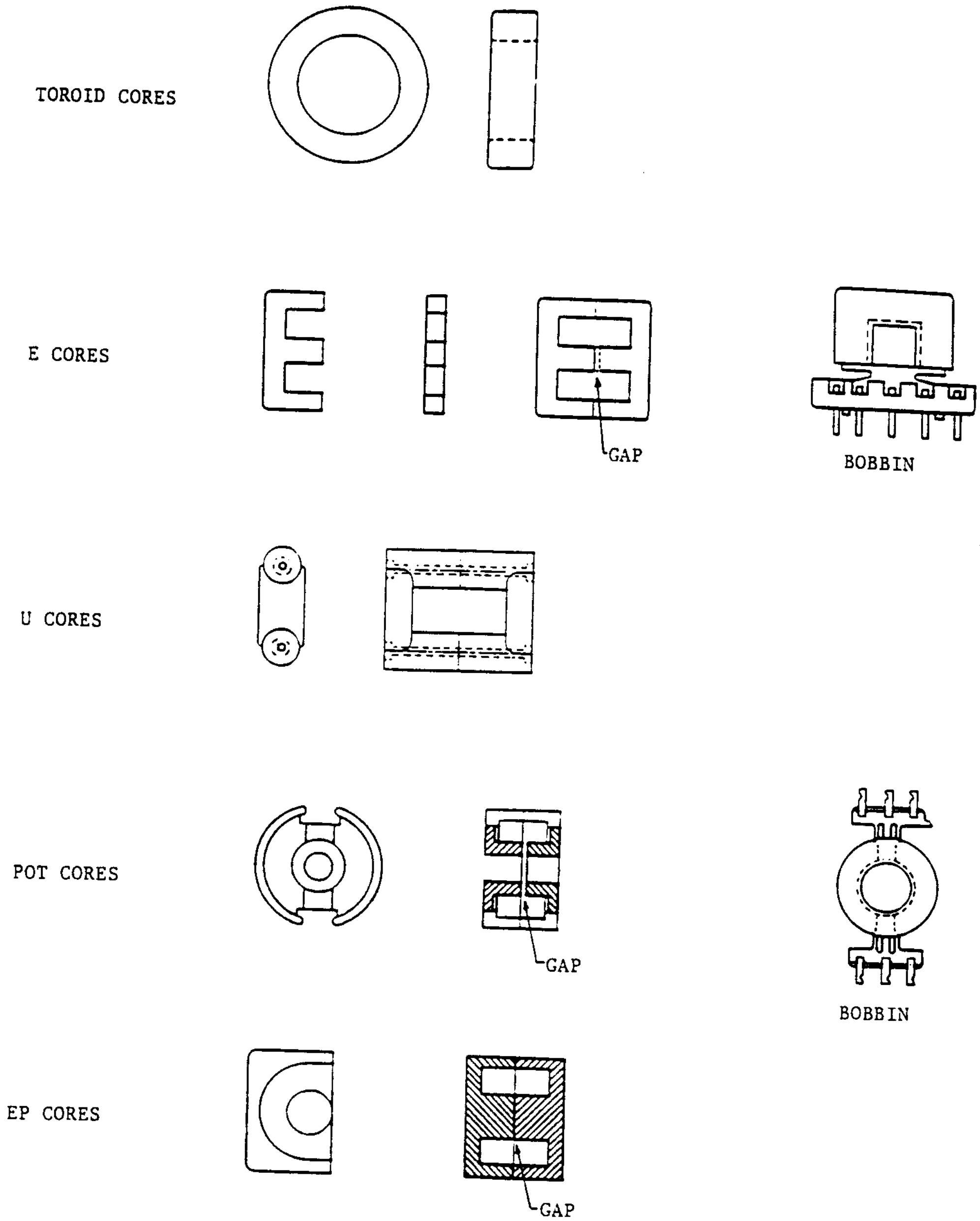
All wire coils wound around the core must fit in the winding windows provided by the core manufacturer. Each core shape will have unique wire winding area and unique cooling factors dependent on the core geometry. The cooling factors set the total amount of power loss(wire loss plus core loss) that the core can handle without heating up to above 100 degrees Celsius where wire insulation degrades and core saturation flux is lowered from the room temperature values. Moreover, excessive core temperatures will heat up near-by solid state switches and degrade their performance as well.

The two major core shapes are toroidal and closed loop rectangular-like. The later has various forms and shapes while the toroidal cores differ only in outer diameter and core cross-section. Below we list some possibilities. We compare the wire magnetic shielding, core cost and cost to wire the core(manufacturing cost).

Core Styles and Their Merits

Basic Core Style	Core Material		Gapped	Winding Shielding	Core Cost	Manufacturing Cost
	Permalloy	Ferrite				
Toroid	×	×	Yes/No	No	Low	High
E-Core		×	Yes	No	Low	Low
U-Core		×	Yes	No	Low	Low
Pot-Core		×	Yes	Yes	High	Medium
EP-Core		×	Yes	No	Low	Medium

Wiring of toroids is costly as there are no machines to do so. They must be hand wound whereas the other cores can be wound by machine. Only pot cores provide magnetic shielding. In the figure below we illustrate the various core geometry's. We emphasize that **wire winding area determines core size**.



The common styles of magnetic cores.

Pot cores while offering good magnetic shielding do not have good thermal cooling due to the enclosed windings inside the core structure. Trade–offs in economic cost, thermal cooling capability, wire winding cost and leakage flux from the various cores must be made in any magnetics choice for a transformer core. Paramount of all considerations is the goal to choose a core that does not under any circumstance saturate. $B_{max} < B_{sat}$ for all conditions. The applied voltage ,V, to a coil of N turns will set the maximum B field in the core via Faraday's law. We find:

$$B\max = \frac{V}{kNA}\frac{1}{f}$$

Where :

K is 4.44 for Sine waves($2^{-1/2}$x2π) and 4 for squarewaves,etc.

N is the number of wire turns

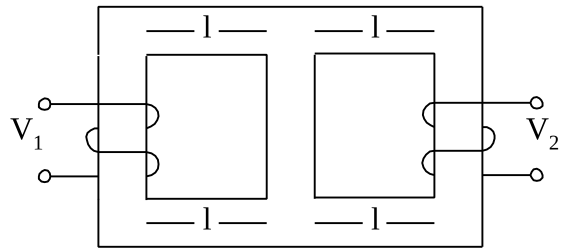
A is the cross-sectional area of the core material

f is the applied frequency

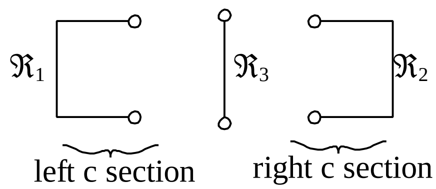
The point is that expected B_{max} as expected form operating conditions sets the required core area ,A, that we need to provide to avoid core saturation. Given the size of the core cross-section and the wire sizes employed in each winding the open area core requirements are set in order to fit all of the required wire windings around the core. Hence, the overall core size is set by both magnetic and wire winding requirements acting together. If the total power dissipated in the closely coupled core and wires is excessive then the core temperature will rise and B_{sat} will be lowered. Likewise if the operating frequency is raised the B_{sat} will also be lowered, so care must be taken in magnetics design at all times.

We next solve an illustrative problem for the flux levels in a specific core structure of the open rectangle shape with two windows for wire windings. That is we have two opposing C shaped sections facing a middle section of core as shown on page 7.

B. Flux Paths in Cores:Erickson Pbm 12.3

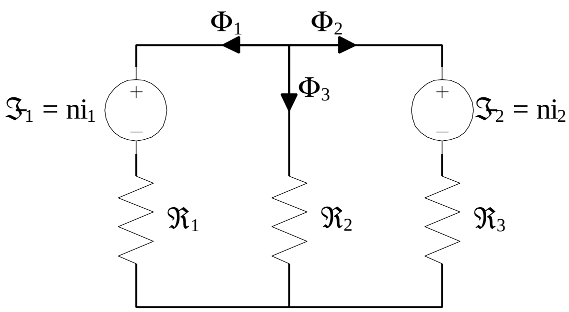


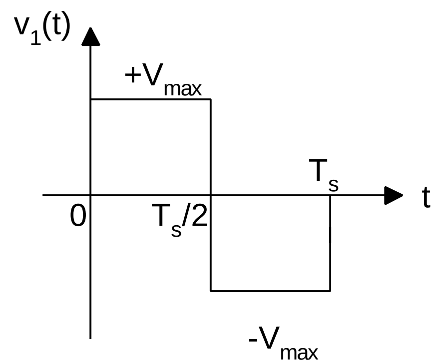
Two C Sections and an inner section in-between



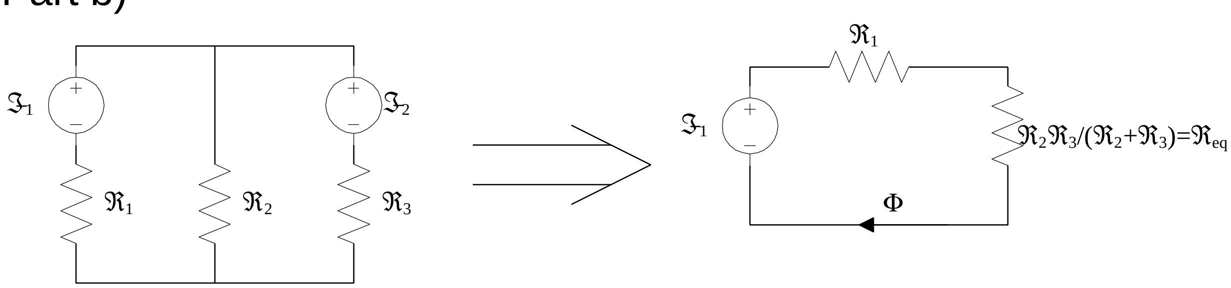
Part a) $\Re_1 = \Re_2 = 3\Re_3 = 3\Re \Rightarrow \Re = \dfrac{\ell_e}{m A_c}$

Where A_c = core Area and l_e is core length for one section.





Part b)



$$\Re_T = \Re_1 + \Re_{eq} = \frac{3l}{m A_c} + \frac{3}{4}\,\frac{l}{m A_c} = \frac{15}{4}\,\frac{l}{m A_c} = \frac{15}{4}\Re$$

$\Re = l/\mu_c A_c$, $\Re_{EQ} = \frac{(3\Re)\Re}{4\Re}$, $\Re_1 = 3\Re$

$\Re_{total} = \Re_1 + \Re_{eq} = 3\Re + \frac{3}{4}\Re$

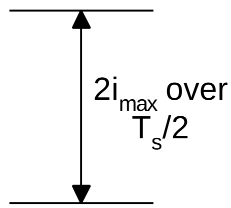
$F_1 = \Phi \Re_T \Rightarrow n i_1(t) = \frac{15}{4} \frac{l}{\mu A_c} \Phi_1 \Rightarrow \underline{\Phi_1 = \frac{4}{15} \frac{n i_1 \mu A_c}{l}} \Rightarrow$

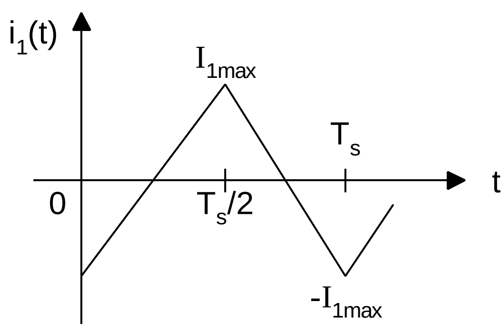
$= 15\,\Re/4$

OR: $\underline{\Phi_1 = \frac{4}{15} \frac{n i_1}{\Re}} \Rightarrow v_1(t) = n \frac{d_2 \Phi_1}{dt} = \frac{4}{15} \frac{n^2}{\Re} \frac{d i_2(t)}{dt} \Rightarrow$

$\underline{i_1(t) = \int \frac{d i_1(t)}{dt} = \frac{15}{4} \frac{\Re}{n^2} \int_0^{T_s} v_1(t)\,dt} \Rightarrow \underline{L_{max} = \frac{15}{16} \frac{\Re}{n^2} T_s V_{max}}$

$2 i_{max} = \frac{15}{4} \frac{\Re}{n^2} \int_0^{T_{s/2}} V_{max}\,dt$

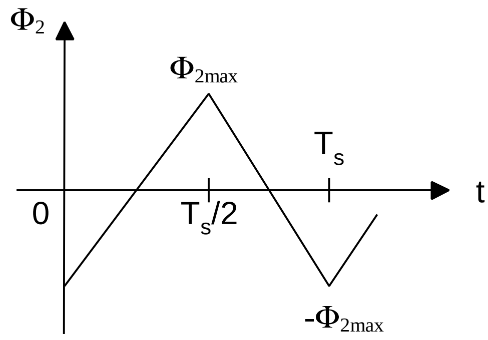




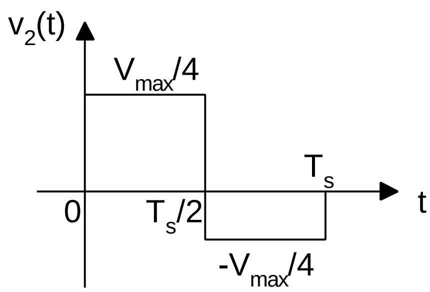
$\Phi_2 = \frac{\Re_3}{\Re_3 + \Re_2} f_1$ flux divider same as for currents

$f_1 = \frac{4}{15} \frac{n i_1 \mu A_c}{l} = \frac{4}{15} \frac{ni}{\Re}$

Part c) $\underline{\Phi_2(t) = \left(\frac{1}{3+1}\right) \Phi_1(t) = \frac{\Phi_1(t)}{4}} - \underline{\Phi_{2max} = \frac{n I_{1max}}{15\Re}}$



Part d) $$v_2 = n\frac{d\Phi_2(t)}{dt} = \frac{n^2}{15\Re}\frac{di_1}{dt} = \frac{n^2}{15\Re}\frac{15\Re}{4n^2}v_1(t) = \frac{v_1}{4}$$



$n_1 = n_2$ but $f_2 = \frac{f_1}{4}$

C. Electrical Models of Transformers

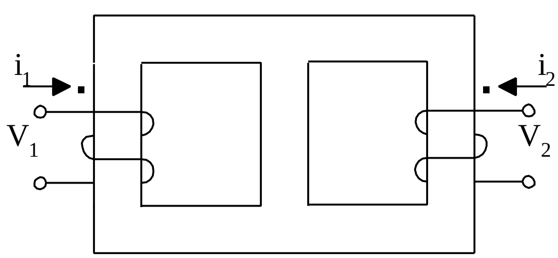
1. Overview of the Model Elements

A transformer is a magnetic device with multiple windings whose purpose is not to store energy, but to transfer energy instantaneously from input winding to output winding(s). In addition, the wire windings are often electrically insulated to provide high voltage dc isolation between input and output. The turns ratio of the magnetically coupled windings can be adjusted to obtain the desired relationship between input and output voltages.

A practical transformer does store energy in one large (magnetizing) inductance, L_m, which is placed in parallel with the input voltage. This draws current even when the secondary is open circuited. There are also two much smaller leakage inductance's, L_l, which are in placed in-between the input voltage and the input to the transformer. High current flows here and even low values of leakage inductance will degrade the transformer circuits performance in several important respects. Leakage inductance's are normally considered undesirable parasitics, whose minimization is one of the important goals of transformer design. On the other hand the magnetizing inductance is required to activate the magnetic fields and its value is normally desired to be as large as possible so as to draw as small a current as possible. **The goal of transformer design is to simultaneously achieve high L_m and low values of L_l(leakage) as described below**.

Series resistance's will arise due to DC wire resistivity and due high frequency skin effects in individual wires which increases wire resistance above DC levels. Finally, due to cooperative magnetic field effects one can inadvertently make the resistivity of certain turns of wire in a coil greater than that the resistivity of other turns located in different spatial positions in the same coil as explained by cooperative magnetic field effects in Lecture 28.

2. **Ideal Transformer versus Real Transfo**rmers with L_l(leakage) and L_m (magnetizing) Effects via understanding MMF Sources

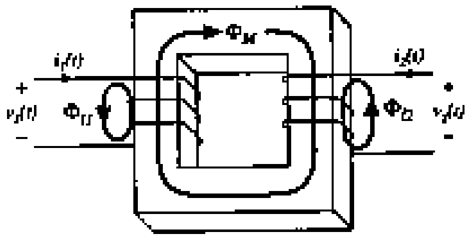


$F_{total} = n_1 i_1 + n_2 i_2 \rightarrow$ sign convention of current into dots
$\phi \Re_{total}$ } core loop

Dot convention +ni into the dot $\Re = \dfrac{\ell_c}{m_c A_c}$ (for no air gaps)

a. Magnetizing Inductance:

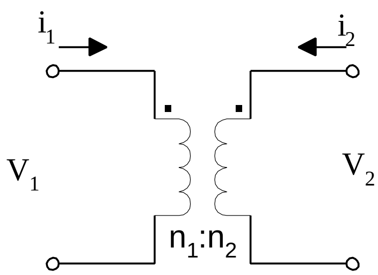
Current will flow at i_1 even if $i_2 = 0$ due to magnetizing inductance.

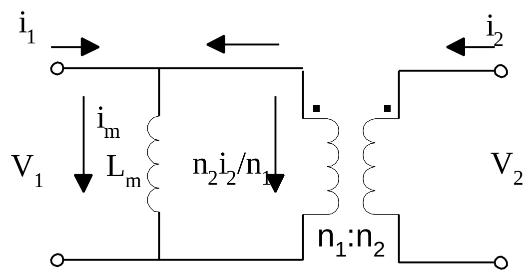


L_{11} for n_1 coil $\equiv \dfrac{N_1^2}{R_1}$

L_{22} for n_2 coil $\equiv \dfrac{N_2^2}{R_2}$

$n_1 i_1 + n_2 i_2 = \Re\phi$





$\Re = 0$

ideal trf.

$\Re\phi = n_1 i_1 + n_2 i_2$

$\Re \neq 0$

non-ideal trf.

$\phi\Re = n_1 i_1 + n_2 i_2$ (only for the core)

$\Re \neq 0$ since $\mu_r \neq \infty$

ϕ flows only through the core that both n_2 and n_1 windings enclose

$$f = \frac{n_1 i_1 + n_2 i_2}{\Re} + \phi(\text{leakage}) + \phi(\text{core})$$

$\Re \rightarrow 0$ for $\mu_r \rightarrow \infty$

$$= \frac{n_1}{\Re}\left[i_1 + \frac{n_2}{n_1} i_2\right] \phi(\text{leakage}) + \phi(\text{core})$$

$0 = n_1 i_1 + n_2 i_2$

Ideal

$$V_1 = n_1 \frac{d f}{dt}$$

f(core effects)

The transformer magnetizing current is then:

$$V_2 = n_2 \frac{d f}{dt}$$

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

$$\frac{i_1}{i_2} = \frac{n_2}{n_1}$$

$P_1 = P_2$

$i_1 V_1 = i_2 V_2$

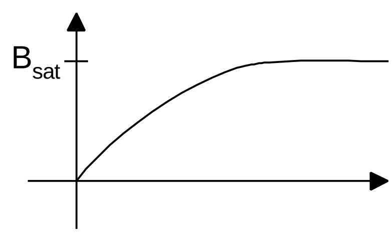
$$i_m = \frac{1}{L_m} \int V_1 dt,$$

using $Li_1 = N_1\phi; \phi = BA$ we find:

$$B = \frac{1}{N_1 A} \int V_1 dt \quad \rightarrow \quad B \sim \frac{1}{N}$$

$\int V_1 dt$ is in volt-sec.

V-sec balance is required or
$i \rightarrow \infty$
$B \rightarrow \infty$
This is prevented when the core of transformer saturates



$B > B_{sat}$
We want to avoid saturation

$$B \sim \frac{1}{NA} \int V_1 dt \leq B_{sat}$$

Increase NA to avoid B_{sat}

Note: Large i_1 and i_2 flowing don't saturate the transformer when $i_1 n_1 + n_2 i_2 = 0$ since $i_m \equiv 0$. Rather large i_m occurs via the route $\frac{\int V_1 dt}{NA}$.

In high frequency PWM converter applications containing transformers, the major core material limitations are:

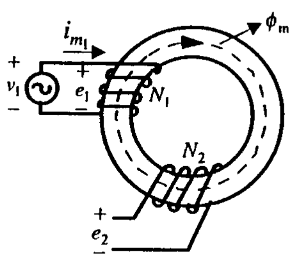
1. Saturation causing catastrophic core failure
2. Excessive Core losses causing elevated core temperature
3. Leakage inductance causing series voltage drops

1 and 2 of which depend upon the absolute value of the flux swing while 3 depends on the absolute value of the core μ. In these applications, transformer and inductor windings are usually driven with rectangular voltage waveforms derived from low impedance sources. Since the voltage, pulse width, and number of turns are quite accurately known, it is easy to apply Faraday's and Ampere's Law to determine the flux swing and appropriately limit it.

In summary we find for an ideal two turn transformer with zero magnetic reluctance in the core or infinite permeability the magnetizing inductance is so large as to draw no current:

Transformers

Assumptions:
No leakage flux
winding resistance zero



Ideal case

$\mu_m \to \infty$

$\Re_m \to 0$

$L_m = \frac{N_1^2}{\Re_m} = \infty$

$i_{m_1} = 0$

Faraday's law:

$$v_1 = e_1 = N_1 \frac{d\phi_m}{dt}$$

$$\phi_m(t) = \frac{1}{N_1} \int v_1 \, dt$$

$$e_2 = N_2 \frac{d\phi_m}{dt}$$

$$\frac{e_1}{N_1} = \frac{e_2}{N_2} = \frac{d\phi_m}{dt} \quad \to \text{equal volts/turn}$$

$$or \; \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

Ampere's law

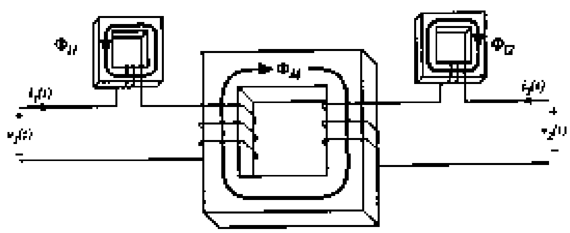
$$\text{Amp} - \text{turns} = N_1 \, i_{m_1}(t) = \Re_m \, \phi_m(t)$$

$$\therefore i_{m_1}(t) = \frac{\Re_m}{N_1} \phi_m(t)$$

Since I_m is zero we have no problems with core saturation occurring under any external load conditions of transformer operation. That is no matter how high the current in the turns we will not experience any build up of core flux due to equal and opposite MMF's generated by the two coils.

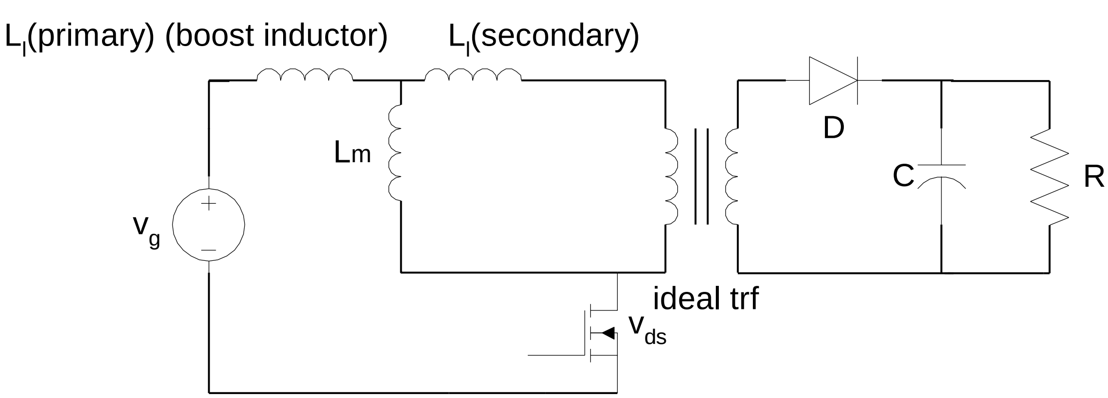
b. Physical Origins of **Leakage Inductance's**

There are leakage inductances on both the primary and secondary due to flux leakage from the core. For μ_r = 5000 nearly 0.02% of the flux is outside the core. Leakage Inductance's L_{l1} and L_{l2} arise because even with a core and windings there is flux that goes into the winding window and not into the core. This flux will enter into the wires lying just outside the core and cause inductance. Recall the permeability of copper wire is the same as free space or air.

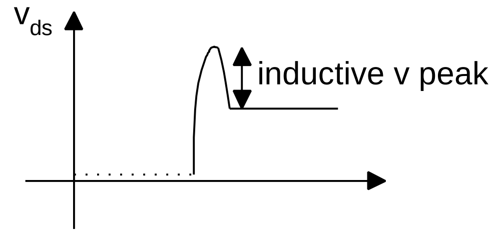


$L_{l1} = N_1^2/\Re_1$(leakage). Where field solvers can find $\Re_1$(leakage)
$L_{l2} = N_2^2/\Re_2$(leakage). Where field solvers can find $\Re_2$(leakage)
Later we show in a well designed transformer that has near unity coupling between the N_1 and N_2 coils and equal leakage paths:
$L_{l1}/L_{l2} = (N_1^2/N_2^2)$

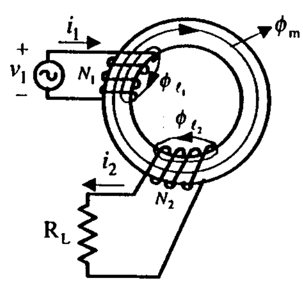
Now consider how the boost converter is effected both by "leakage inductance" and L_m besides the L_1 (boost or choke inductor). L_m plays a crucial role in transferring energy from the primary to the secondary as we saw previously.



Upon turn-off of the transistor leakage inductance causes a big spike on the "standoff voltage"



⇒ωε want minimum values of $L_{leakage}$ to protect solid state switches from large transients. In summary, the full electrical model of a transformer with leakage flux paths included and the resistivity of the wire coils fully accounted for as well as the magnetization inductance is given below:

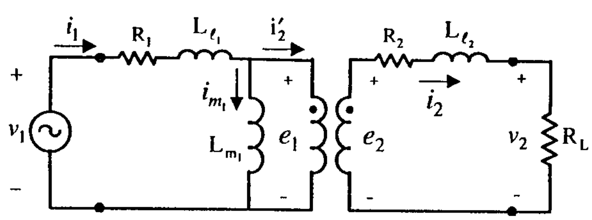


$$\phi_1 = \phi_m + \phi_{l_1}$$

$$\phi_2 = \phi_m - \phi_{l_2}$$

$$\therefore e_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_m}{dt} + \underbrace{N_1 \frac{d\phi_{l_1}}{dt}}_{L_{l_1}\frac{di_1}{dt}}$$

$$e_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_m}{dt} - \underbrace{N_2 \frac{d\phi_{l_2}}{dt}}_{L_{l_2}\frac{di_2}{dt}}$$

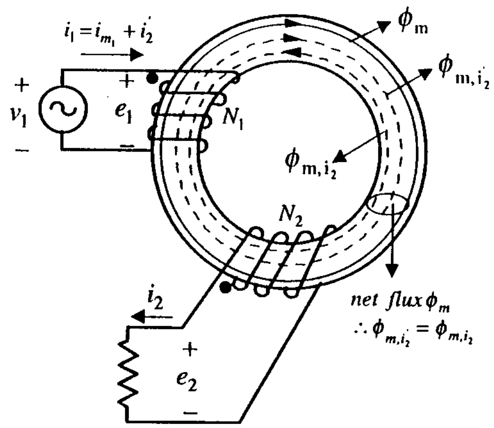


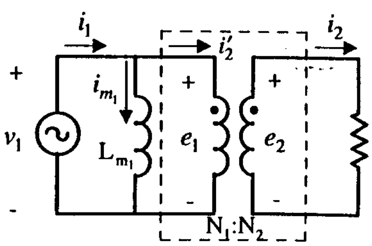
$$v_1 = R_1 i_1 + \underbrace{N_1 \frac{d\phi_m}{dt}}_{e_1} + \underbrace{N_1 \frac{d\phi_{l_1}}{dt}}_{L_{l_1}\frac{di_1}{dt}}$$

$$v_2 = \underbrace{N_2 \frac{d\phi_m}{dt}}_{} - \underbrace{N_2 \frac{d\phi_{l_2}}{dt}}_{} - R_2 i_2$$

We have now reduced the magnetic effects in a transformer to equivalent electrical elements that form a simple two loop model. Load effects are next and they act to underline the importance of high magnetizing inductance values and associated low values of the magnetizing currents as compared to the currents in the secondary and primary coils as shown on the top of page 16. In particular note the relative magnitudes of the currents forming i_1.

Transformer with secondary loaded





$e_1 = v_1$

i_2 flows due to e_2 and the load

ϕ_m is unchanged

$\therefore \phi_{m,i_2'} = \phi_{m,i_2}$

$$\Downarrow$$

$N_1 i_2' = N_2 i_2$ or $i_2' = \frac{N_2}{N_1} i_1$

$$\therefore i_1 = i_{m_1} + \underbrace{\frac{N_2}{N_1} i_2}_{i_2'(t)}$$

Idealized case:

$i_{m_1} = 0$

$i_1 = \frac{N_2}{N_1} i_2$ or $N_1 i_1 = N_2 i_2$

in general, $N_1 i_1 = N_2 i_2 + N_3 i_3$

We have eliminated leakage inductances for circuit clarity above

c. Minimizing L(leakage)

Leakage inductance arises because only a large fraction, but not all of the flux created by the primary winding travels in the high permeability, but not infinite permeability core. A small but measurable amount of flux travels in the air of the core windows and also passes through the wires wound around the core. For $\mu = 5000$ about 0.02% of the flux travels in the air. Thus, roughly 0.01% of the flux leaks out for the primary winding and L_{l1} is roughly 10,000 times smaller than the core magnetizing inductance. However, even a small leakage inductance, placed in series in-between the input voltage and the primary winding, will cause substantial series voltage drops. Is there any clever way to reduce L_l(leakage) without resorting to higher μ cores??

L(leakage) can be minimized by “winding your own wire turns

properly in the core window." This involves extra expense as you must use interleaved primary/secondary windows so that the peak mmf has a minimum value rather than a maximum value as shown just below for non-interleaved windings.

NON_INTERLEAVED WIRE COILS

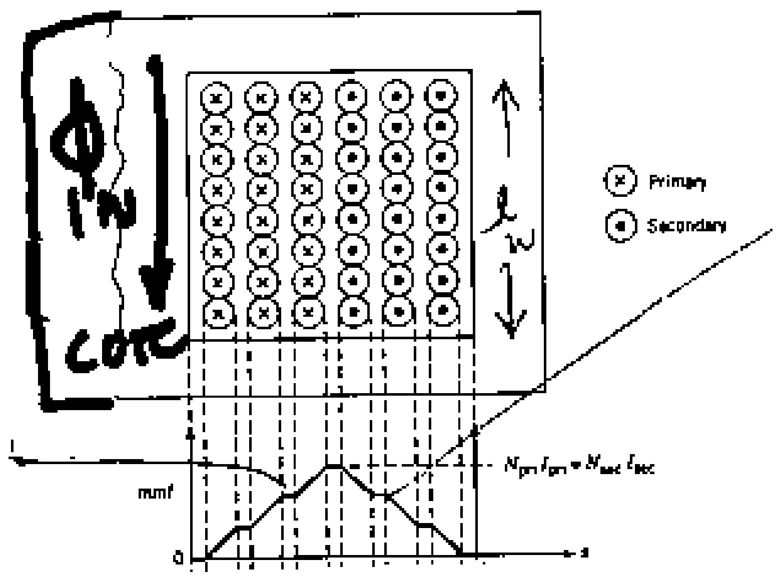
Other half of windings

$\int H \bullet dl = I$

allows

$(m_p - m_s)i = F(x)$

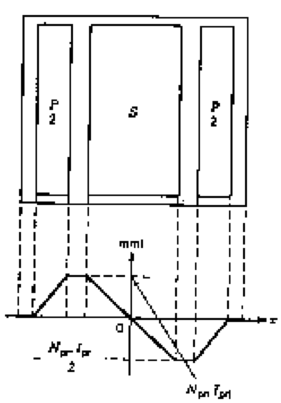
$$H = \frac{¶(x)}{ln}$$

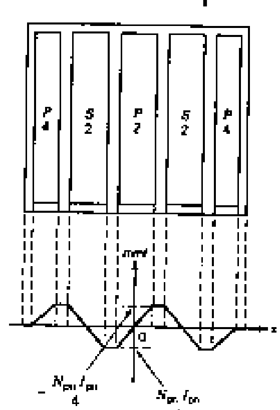


Note: $\vec{B}$ is parallel to winding.

Flux leakage into the core window occurs and intercepts the wires.

We can reduce the maximum mmf achieved in the air window ,where copper winding resides, by **carefully alternating the mmf generated by the primary and secondary windings as shown below.** Keep the same number of turns and the same volume of turns but better partition windings to cause the spatial flux distributions to have lower peak values as shown below.

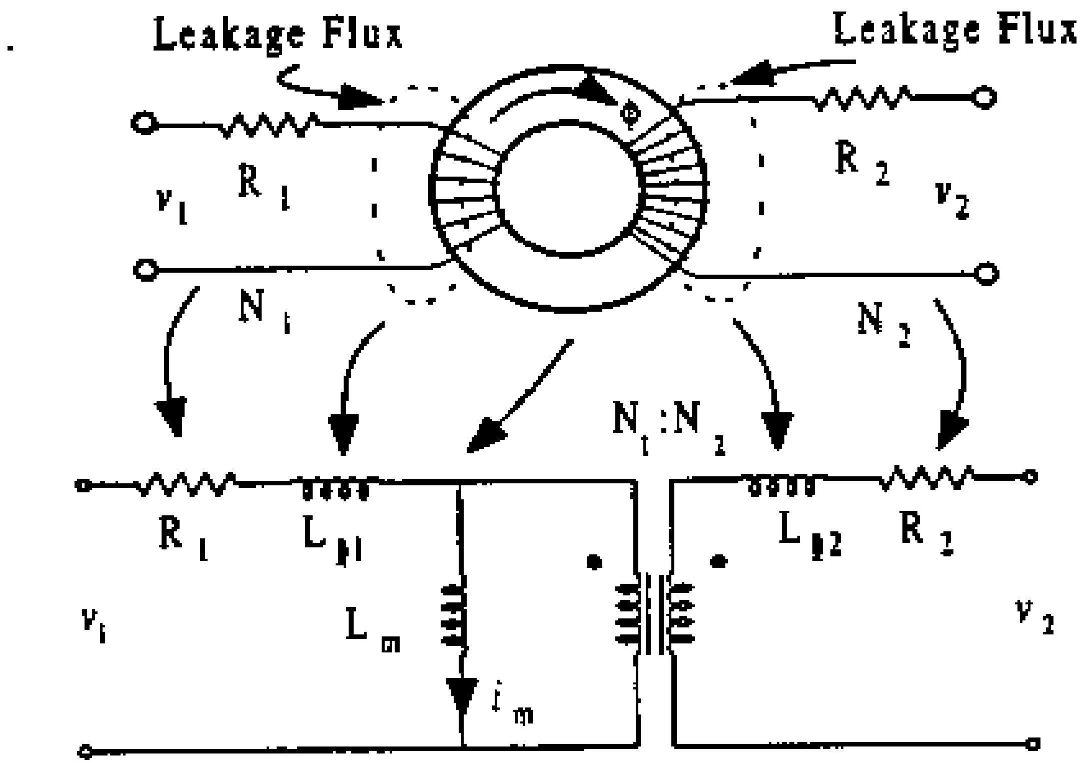




2 partitions:
ϕ_{max} = 1/2 f(above)

4 partitions:
ϕ_{max} = 1/4 f(above)

We will show in later lectures these lower leakage flux values will cause lower leakage inductance and also lower resistivity in wire coils due to reduced proximity effects in wires.
Summary L_l(leakage) and L_m(magnetizing):



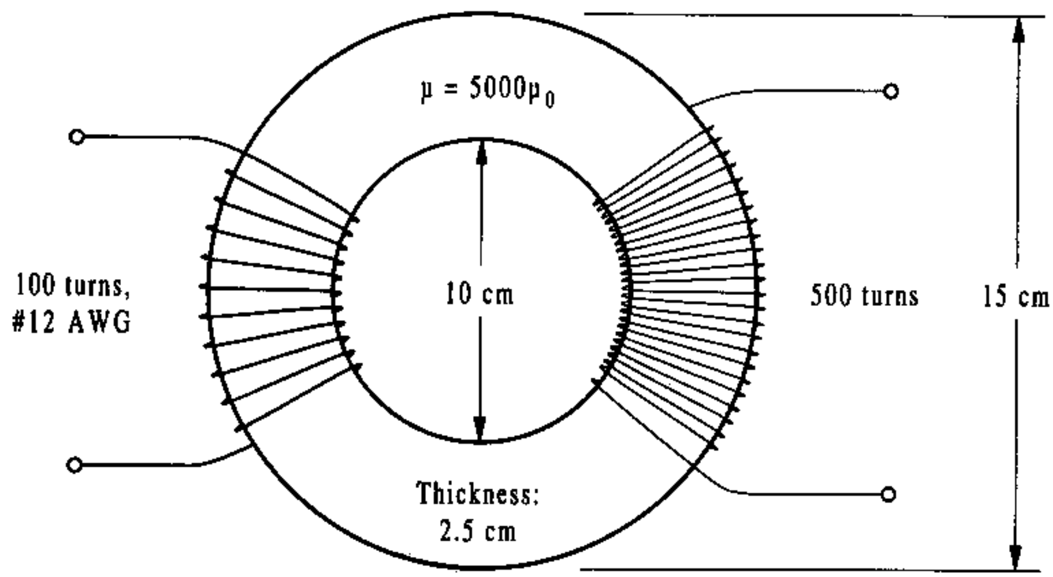
In a well designed transformer Both leakage inductance and wire resistances go as:

$L_{l\,2} = L_{l1}*(N_2/N_1)^2$

$\Re_2 = \Re_1*(N_2/N_1)^2$

This is shown below by a numerical example for a toroid core:
A toroidal core with outside diameter of 15 cm and inside diameter of 10 cm, which makes the core 2.5 cm thick forming a 2.5 x 2.5 cm cross-section. See page 19. It is made of ferrite material with $\mu = \mathit{5000}\ \mu_o$. The core has two windings. The primary is wound with 100 turns of #12 AWG wire and 500 turns of a different wire size are wound on the secondary. The windings are "physically

separated," meaning that they are confined to opposite sections of core rather than overlapped (coil separation permits higher electrical isolation between input and output coils, at the expense of higher leakage flux). Suggest a wire size for the secondary, and compare the two windings. Develop a circuit mode for the complete transformer, based on an estimate



Toroidal transformer

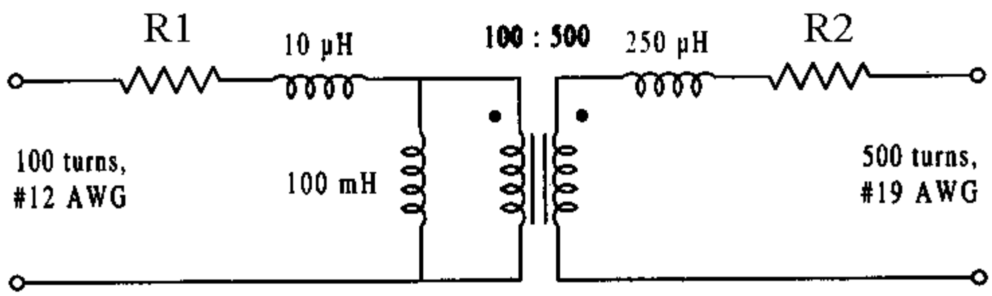
that because μ_r= *5000*, about 0.02% of the flux follows a leakage path.

If the transformer is used with an input converter waveform that is approximately a 120 V_{rms} sine wave at 2500 Hz, what will the input current be with no load? Suggest a load rating.

Given low leakage and good coil coupling, the secondary voltage should be 500/100 = 5 times the primary voltage but the current at the secondary should be 1/5 of that at the primary. For an equal current density in both the primary and secondary windings to achieve equal heat dissipation per unit volume , the secondary wire size should be about 1/5 that of the primary. Since the #12 AWG wire on the primary has an area of about 3.3 mm^2 the secondary wire should be about 0.66 mm^2, which corresponds to just about #19 AWG wire (area .653 mm^2). If smaller wire were used, it would have higher current density than the primary, and would get hotter than the primary. If larger wire is used, the primary will run hotter than the secondary. In either case, mismatch in the current densities will make a spatial portion of the whole transformer reach the thermal limit before the rest. In short heat is generated more uniformly if the current densities in primary and secondary are matched.

The total area of the primary winding will be given by 100(3.309 mm^2) = 331 mm^2, while that of the secondary will be 500(0.653 mm^2) = 326 mm^2. The coils should both fit onto the core-one on each side.

Next we estimate the magnetizing inductance, L_m, of the transformer. The core reluctance is needed to determine L_m. The toroidal core length is approximately the circumference of the corel = 0.125π. The magnetic core area is 6.25 cm^2. The core reluctance is therefore $\Re = l/(\mu A) = 10^5\ H^{-1}$. From the primary coil side, the magnetizing inductance is $N_1^2/\Re$ = 100 mH. Total leakage flux of 0.02% suggests 0.01% of L_m for each leakage coil or about $N_1^2/\Re = 10^4/(10^9\ H^{-1})$ = 10 μH. The secondary side value of the leakage inductance is $(5)^2$ x 10μH) = 250 μH.



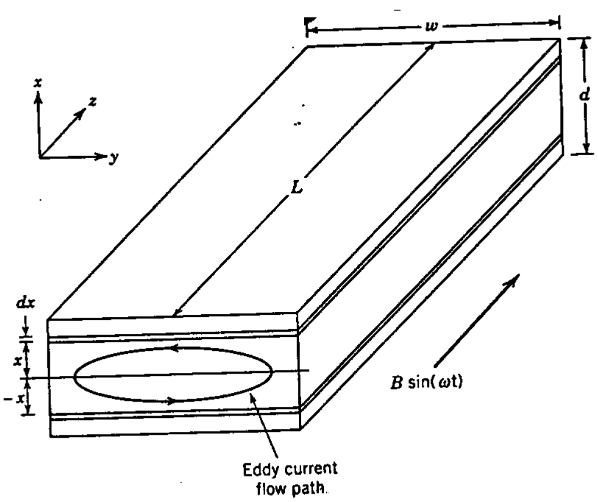
For HW#5 find R_1 and R_2 for the transformer and the total resistive wire loss in Watts for a 400W load with the current density in all wires limited to 100A/cm^2.

D. Details of Eddy Currents in Other Cores

ρ(core) for ferrites has a range: $200 \leq \rho_c \leq 2000$ Ω-cm . Hence ferrite core electrical resistivity is usually high even in large size cores. This high resistivity avoids major problems with either eddy currents or non-uniform B profiles in ferrites.

Although ferrite cores are the most common we also employ in power electronics on occasion permalloy or mopermalloy cores. Mopermalloy is a mix of ferrite and molybdenum that acts as a distributed air gap as metal has the same permeability as air. The distributed air gap is useful for soft saturation and for cores with high B_{SAT} as encountered for inductors with high DC levels or uni-polar applications.

1. Effects of a More Electrical Conductive Magnetic Core

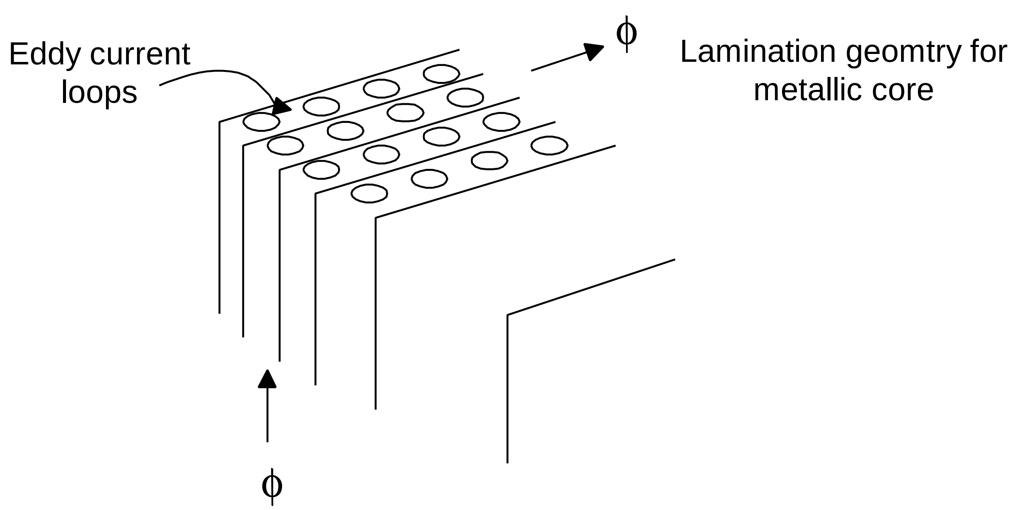


Consider a time-varying flux field inside a core with stacked laminations to break-up eddy currents. Flux is created by a separate N*I(ωt) = F(ωt) drive = ℜϕ. The area of the lamination to flux is the product wdx. B times wdx will give the flux traveling through the lamination

If the electrical resistivity ρ of the core magnetic material is low B(wt) passing through a core surface area,wdx, induces eddy currents flowing in the area ,Ldx, which try to screen the interior of the core from the applied B. That is the direction of the eddy current loop induced is such as to create an opposing current to reduce the applied field. **We emphasize that the eddy currents**

flow through the cross-sectional area Ldx while the flux flows through the area wdx. Metal alloy cores have $50 < \rho < 150$ microns Ω-cm which is about 10^5 lower in resistivity than ferrite cores. If the core was solid it would act as a shorted turn. Hence tape thin regions of core material are wound together to form bulk metal alloy cores with insulating laminations between layers of tape. As we show below in such cores eddy current loss is small.

In the core lamination itself V(eddy) = $\omega\phi$,so we can only reduce V(eddy) for fixed flux and ω(applied) levels by laminating the core into small areas capturing less less flux. By reducing the flux area A the flux level (BxA) captured in each lamination is made less. Hence V(eddy),and also V^2/R eddy current loss decreases in that lamination. Note that the thin lamination also acts to increase the effective electrical resistivity of the core as shown below. Hence V^2(eddy)/$\Re$ decreases even more dramatically than from V(eddy) effects alone.



$$\text{R(lamination)} = \frac{r\text{l(core)}}{\text{area}} = \frac{rl}{wdx}$$

See above diagram for the flux path of area wdx and total length L. Inside of each lamination of eddy **current flows across the cross-sectional area Ldx and** we find the electrical resistivity varies as:.

$$P(\text{eddy current loss}) \sim \frac{V^2}{R} \sim \frac{dx^2\, w^2 B^2 f^2}{r\,(\text{core})}$$

Eddy current loss in a lamination varies as the lamination thickness squared, dx^2. Thinner laminations have smaller losses than thicker ones. So low resistivity metal cores can approach ferrite resistivity only if highly laminated. Typically, the core thickness must be divided into laminations 1/1000 of the bulk core dimensions.

2. Spatial B Profiles and Skin Effects in Magnetic Cores

To date we have assumed that the B profile inside the core is uniform and constant. Is this true?? Non-uniform B results when flux concentrates. We found for current flowing in wires that the skin depth equation at a frequency f in a medium of ρ and μ_r is:

$$d = \sqrt{\frac{r}{p m_o m_r f}}$$

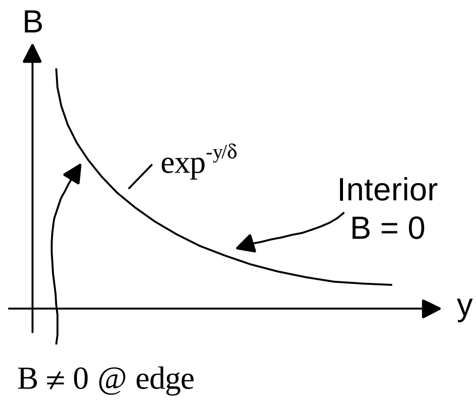
This causes current density profiles to be non-uniform. We note μ_r = 1 for Cu and Magnetic Fields penetrating copper wire do experience a skin depth.

$\rho = 2.3 \times 10^{-8}$ Ω-m and $\mu_r = 1.0$

δ(100 kHz) = 0.24 mm or ¼ mm

Does a similar effect occur for magnetic field profiles in various cores? If so what is the effect on the transformer?

Unfortunately, due to the core electrical diamagnetic properties the B profile does vary due to magnetic skin effects as well. B(y) shown below decreases spatially from the edge of the core toward the interior in a highly conductive iron core operating at mains(50-60Hz) frequency:



$$d \equiv \sqrt{\frac{2}{w\,m s}}$$

δ for pure iron at 60 Hz is <u>1 mm</u>
δ for Fe-Si (3%) is <u>3 mm</u> due to lower σ for Fe-Si. Hence adding Si to steel is good for making better transformer iron as it makes B more uniform versus y. However, if Si > 3%, B_{sat} drops more dramatically, as compared to the desired reduction in σ. Once B > B_{sat}, B doesn't preferentially flow in high μ material as compared to the external air magnetic circuit and we lose the role of the core - a low reluctance path for flux that couples remote Cu coils with 100% efficiency. Hence, any worries about skin effects are useless in this case.
If we do not reach core saturation, we need to make y $\leq$ δ and B is back to flowing in the core interior in a uniform manner. In the magnetic core portion of the lamination sandwich stack we choose thickness of the iron-Si alloy core sheet t = 0.3 mm is less than the skin depth for Fe-Si ,δ = 3mm so that the B field remains uniform across the core and we do not suffer non-uniform B.

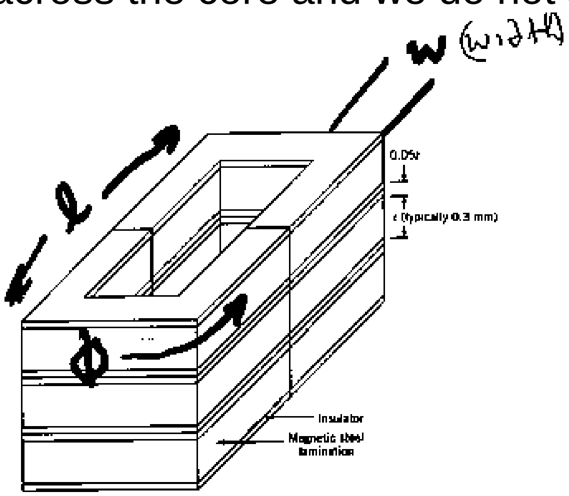


Figure 30-5 Magnetic core for a transformer or inductor made from a stack of magnetic steel laminations separated by insulators.

ϕ = tw B   tw is the cross-sectional flux flow area
V generated in the core to drive eddy currents is proportional to
dϕ/dt ~ wt f B
where f is the ac frequency.
That is by shrinking tw we shrink ϕ and hence dϕ/dt.

We better achieve this by laminating the core by adding insulator sheets to insure y $\leq$ δ.

Laminated versus ferrite cores are contrasted below:

•Permalloy Tape Core

$\rho = 55 * 10^{-4}\ \Omega$-m and $\mu_r = 30{,}000$

δ(100 kHz) = 0.007 mm

To avoid skin effects tape thickness from which we make the core must be 12 microns or less (0.001 mm).

•Ferrite

$\rho = 20\ \Omega$-m and $\mu_r = 1500$

δ(100 kHz) = 18 cm

Hence ferrite skin effects on B can be ignored completely even for big cross-section cores. Lamination is unnecessary for uniform B profiles.

b. Final Comments on Ferrite Cores for use in fabricating both Inductors“L” and Transformers up to MHz Operating Frequency.

Cores are wound with Cu wires to create flux paths that efficiently couple several distinct Cu coils with the exact same flux. Although for current and voltage scaling between the two coils only the total number od turns matters, precisely how these wire coils are spatially wound on the core strongly effects the leakage flux distribution in the core window. This flux intercepts the Cu wire and in turn alters both leakage inductance and resistive loss via PROXIMITY effects. This is subtle but important.

As given in lecture 28, we will also see that the choice of a **long thin core window is preferred** over a small square core window. This core geometry choice and it’s effects on H or B fields both within and outside the core is clear when we realize that n wire turns will fit in a long length single coil or in a multi-layer stack of n turns total. The former choice reduces the H field effects as compared to a stack of n turns in several layers with a

smaller core length, *l*, via the simple relationship,.$Hl = Ni$. **We prefer large core length** *l in the window area to achieve LOWER H.* This in turn reduces both leakage inductance and PROXIMITY EFFECTS on wire resisitivity.
Ferrite Core usually has an associated bobbin or coil former (for Cu wire that surrounds the core). Usually the cores are physically split to allow for machine based remote winding of copper wire coils, which are later inserted around the core of interest as shown below.

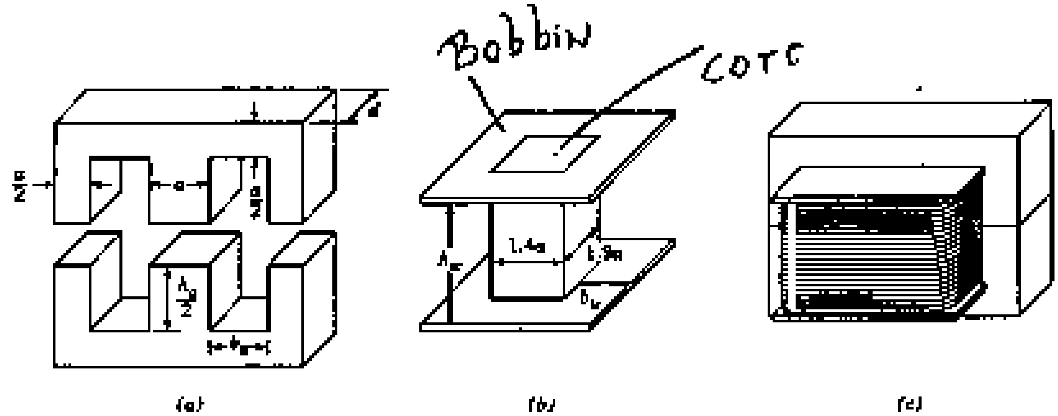


Figure 30-6 Dimensioned diagram of (a) a double-E core (b) bobbin, and (c) assembled core with winding.

In summary **choice of optimum dimensions for magnetic cores** employed involve several apparently disparate factors:

- Value of required magnetizing inductance
- Cost of core material at core geometry specified
- Cost of making Cu Windings
- Required thermal dissipation of heat generated in cores and surrounding wires as set by the core geometry via conductive, convective and radiative means
- Desired values of leakage inductance
- Desired reduction of proximity effects in wire resistance