LECTURE 12

Lossy Converters (Static State Losses but Neglecting Switching Losses)

HW #2 Erickson Chapter 3 Problems 6 & 7

A. General Effects Expected:
   V(load)↓ as I(load)↑

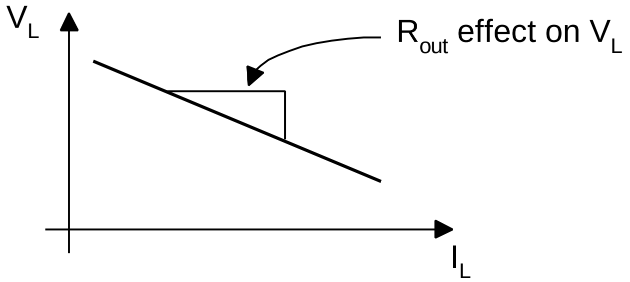
B. Switch and Resistive Losses Change M(D)
   Modified DC Voltage Transfer
   Functions M(D) with loss $\neq$ M(D) for no loss

   1. General aspects of the origin of M(D) changes in static state loss terms: R_L,V_d, R_d, and R_{on}
      - a. V_{max} / V_{min}
      - b. V/L Slopes
   2. General solution for lossy Boost Converter
      - a. R_L only: M(D, R_L), $\eta \neq 1$
      - b. Static State Device DC losses: M(D, V_d, R_{on}, R_D)
   3. Buck Converter
      - a. Problem 3.3 of Erickson
   4. General solution for lossy Buck-Boost Converter
      - a. M(D, V_d, R_{on}, R_L), Problem 3.4 of Erickson
   5. Compare for a specific task:
      - a. Buck vs. Buck-Boost Circuits, Problem 3.5 of Erickson

A. GENERAL EFFECTS EXPECTED ON M(D)

Lossy Converters: Consider the static state DC loss only, no dynamic switch loss will be considered here

Generally one expects, due to finite R_{out}, from any power supply for V_L to fall as I_L increases. In converters R_{out} is much more complex, especially with feedback.



For fixed P_{out} and V_{in} for a converter the input current increases proportional to the losses so that $P_{in}=P_{out}$+losses. The losses we will consider will also include device losses but only in the static states due to finite R_{on} and V_{on} values.

B. **Effects on voltage DC transfer function M(D) due to resistive and device voltage drops.** Besides decreasing converter efficiency static state losses will also effect dc transfer functions. M(D) becomes a function of R_L, V_{on}, R_{on}, etc. when losses are included as shown below.

1. V_{max}/V_{min} changes as seen by any converter inductor will cause di/dt slope changes.

⇒ Δi_L = V/L (in units of A/sec) slopes change, affecting D and D' themselves just by including resistance and device voltage drops. Hence, the value of M(D) changes.

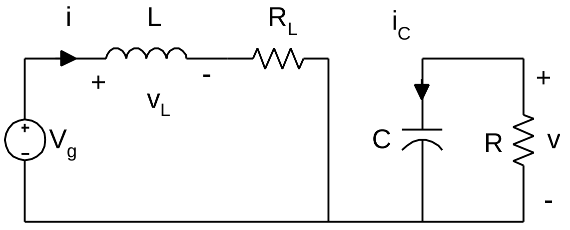
2. Including inductor resistance R_L, device R_{on}, as well as V_{on} changes M(D), from M(D) with these effects ignored. Lets consider the inductor

resistance effects.

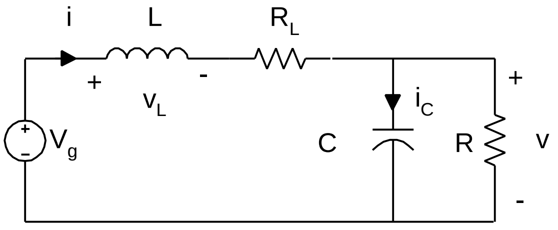
a. Illustrative **R_L** (inductor series equivalent resistance) effects in boost topology

M will change from M(D) to M(D, R_L) as shown below. R_L is finite not only because of simple copper resistive losses. Wound inductors at high frequencies (0.1 - 1 MHz) have additional core and proximity effect losses. To a first approximation: **$R_{ac/}\,R_{dc}$ =0.5Õ (wire dia. in cm)X $f^{1/2}$ x100**
Larger diameter wire has more effect than a small diameter
Below we carefully distinguish between R_L with the dominant R for the load.

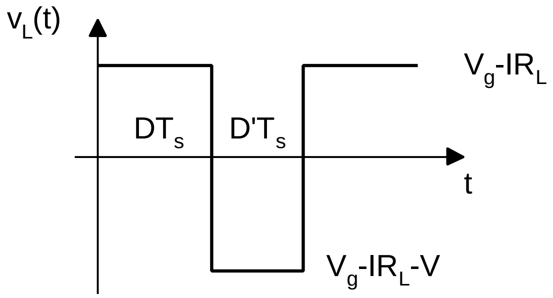
Boost with R_L during D



Boost with R_L during D'



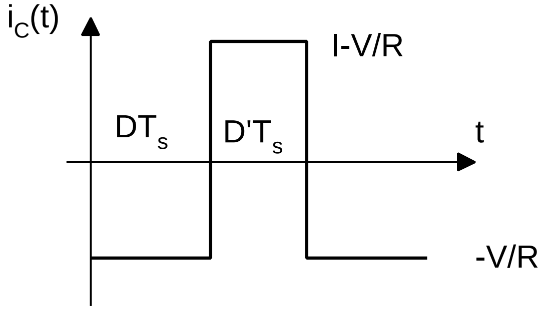
1. V-sec balance across V_L



$$D[V_g - IR_L] + D'[V_g - IR_L - V] = 0$$

$$\Rightarrow V_g - IR_L - D'V = 0$$

2. Current-time balance across the capacitor



$$D[-V/R] + D'[I - V/R] = 0$$

$$\Rightarrow D'I - V/R = 0$$

$$\frac{V_o}{V_g} = \frac{1}{D'}\begin{bmatrix}\text{Usual}\\ \text{Boost}\\ \text{lossless}\\ \text{term}\end{bmatrix}\left[\frac{1}{1+\frac{R_L}{D'^2 R}}\right]$$

Second term has all effects of including finite R_L.
← $R_L > D'^2 R$ big effect
Also big effects occur for $D' \to 0$ or for $D \to 1.0$

Note some extreme converter operating conditions will amplify finite R_L effects.

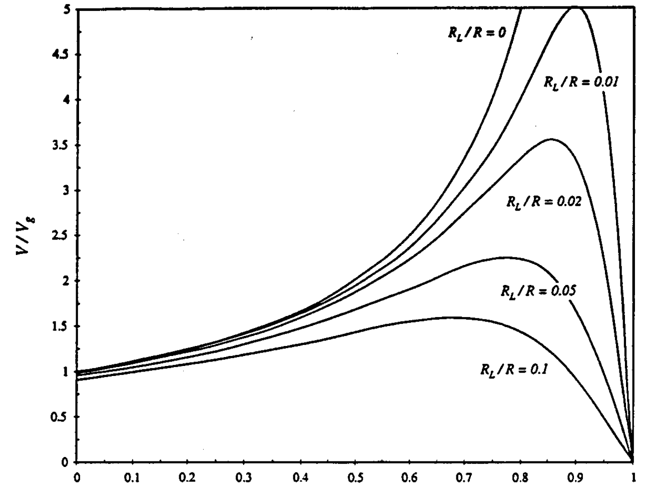


Fig. 3.9. Output voltage vs. duty cycle, boost converter with inductor copper loss.

Note well:
(1) R is the load resistor
(2) R_L = inductor losses

For $D \to 1.0$
R_L/R has a big effect on M(D)
As $R_L/R \to 0$ no effect on M(D)

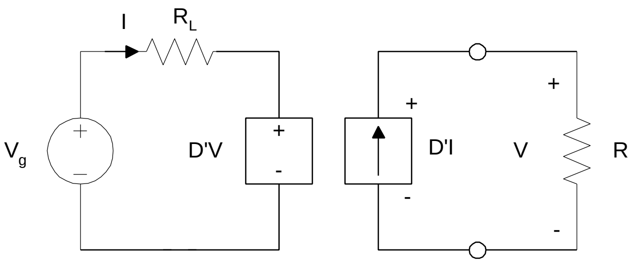
Note also as **D ® 1.0, M(D) has big "rollover"** if R_L is even 1% of the load resistance. Note also in DC gain at high D that **for $D \leq 0.2$, R_L effects are minimal**.

Summary

The operating duty cycle, D value, amplifies equivalent series resistance ,R_L, effects.

$D > 0.5$ and R_L/R even 1% you see big changes to M(D).

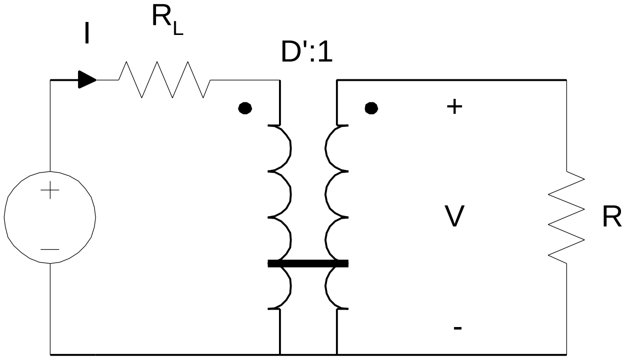
$D < 0.5$ you see no effects.



Equivalent Circuit with “DC Transformer”:

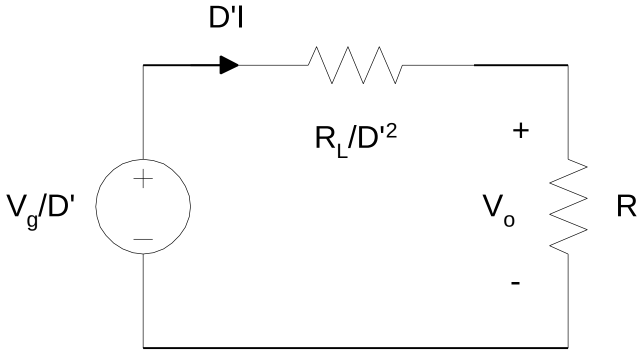
Use to get the operating efficiency

$\eta = P_{out} / P_{in}$



Equivalent circuit model of the boost converter, including a D’:1 dc transformer and the inductor winding resistance R_L.

Referring to the secondary to get one loop equation.



Clearly want $R_L << D'^2R$

Easy if D’ = 1.0, D = 0

Hard to achieve if $D' \to 0$

$D \to 1.0$

$$V_o = V_g \underset{\overset{\uparrow}{1/D'}}{\begin{bmatrix}\text{Ideal}\\ \text{Case}\end{bmatrix}}\left(1+\frac{R_L}{D'^2 R}\right)^{-1}$$

$$I(\text{input}) = \frac{V_g}{D'^2}\begin{bmatrix}\text{Ideal}\\ \text{Case}\end{bmatrix}\left(1+\frac{R_L}{D'^2 R}\right)^{-1}$$

$$h = \frac{P_{out}}{P_{in}} = \frac{V_o}{V_g}D' = \left(1+\frac{R_L}{D'^2 R}\right)^{-1}$$

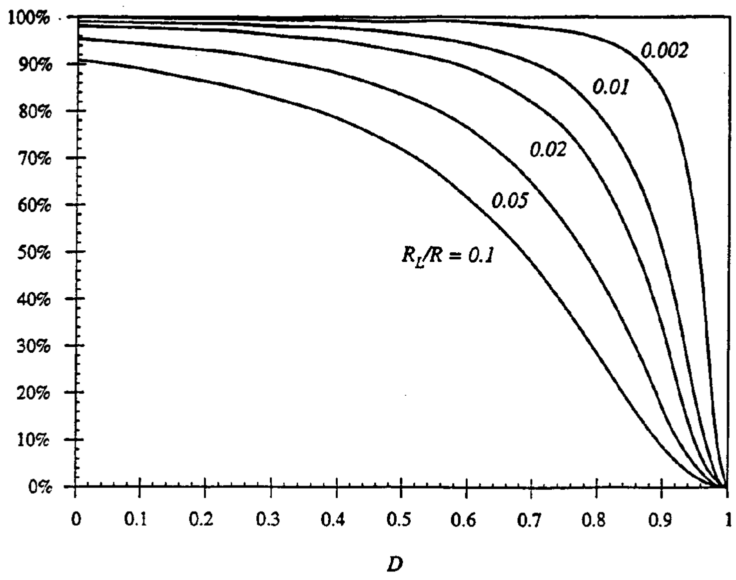


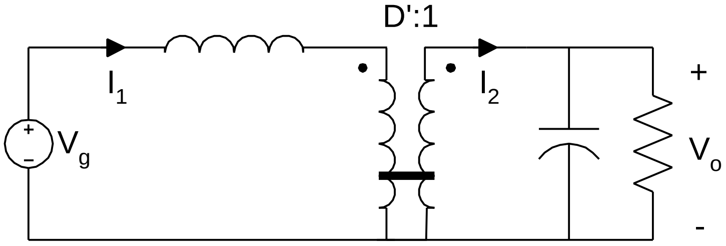
Fig. 3.15. Efficiency vs. duty cycle, boost converter with inductor copper loss.

$\eta(D, R_L/R)$

1) As the on duty cycle D→0 one gets high η but efficiency is little effected by R_L

2) D→1 and D'→0 $\eta \to 0$ still we get much lower η for R_L not negligible and much greater sensitivity to R_L changes.

Compare the above to the <u>Lossless Boost:</u>



$I_2 = D'I_1$ $\quad\quad$ $\eta = D'V_o/V_g = P_o/P_{in}$

$V_o = V_g/D'$ $\quad\quad$ $= 1.0$ (lossless)

Next we include static device losses to the system loss.

b. Static State Device Loss Effects in the Boost Topology.

In our static states of the switching devices we only consider R_{on} from the transistor but both R_D and V_D for the diode when on, because $V_{on}(TR) \approx 0$. We expect to obtain $M(D, R_L, V_{on}, R_D, V_D)$ and $\eta(D, R_L, R_{on}, R_D$ and $V_D)$.

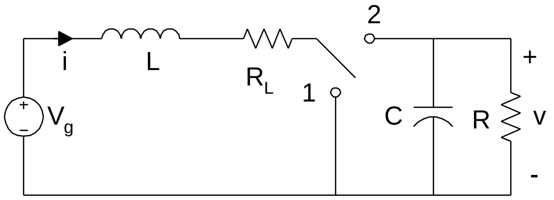


Fig. 3.6. Boost converter circuit, including inductor resistance R_L.

SW position 1:

SW position 2:

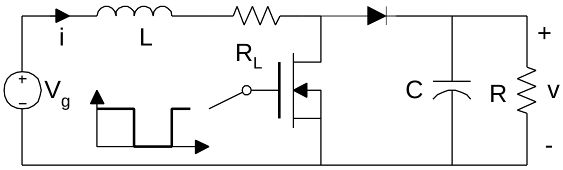


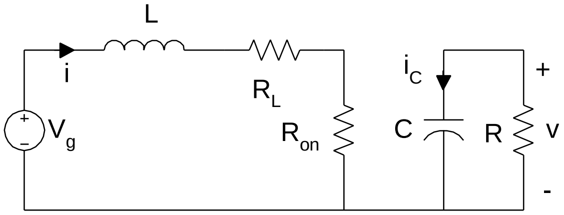
Fig. 3.22. Boost converter example.

Transistor is on case: $R_{on}(I)$

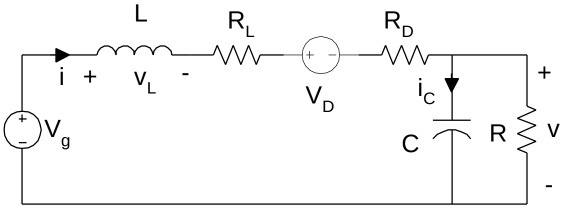
Diode on case: V_D and R_D

Effects of R_{on}, V_D and R_D follow from revised circuits which change values of V_L and hence circuit D and D' periods as well.

Period DT_s Circuit Connections

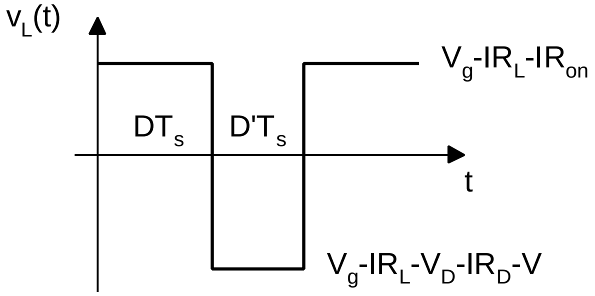


Period $D'T_s$ Circuit Connections



Next, we invoke both volt-sec and current-sec balance on the inductor voltage ,$\mathbf{V_L}$, and capacitor current ,$\mathbf{I_C}$, respectively.

(1) Volt-sec balance on V_L vs. time

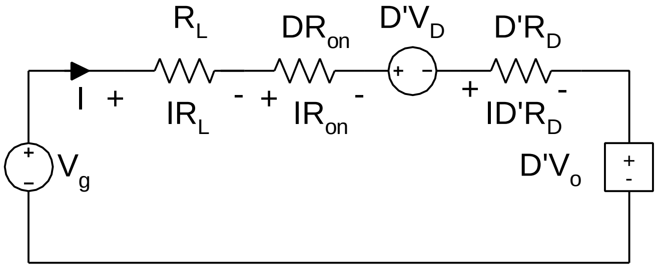


$$D[V_g - IR_L - IR_{on}] + D'[V_g - IR_L - V_D - IR_D - V_o]$$

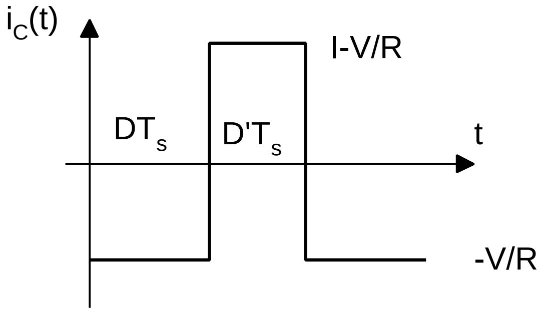
Input Conditions = 0 yields:

$$V_g - IR_L - IDR_{on} - D'V_D - ID'R_D - D'V_o = 0$$

This is the input circuit loop for the above equation:

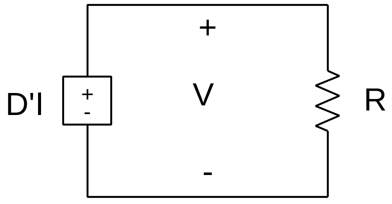


(2) Current-sec balance on I_C vs. time



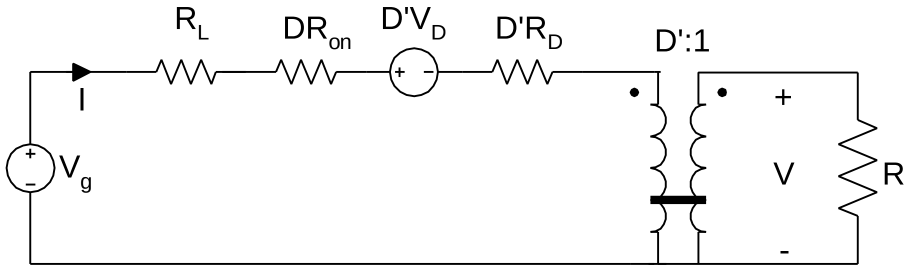
$$D[-V/R] + [I - V/R] = 0$$

Output conditions yield the simple equivalent output circuit result:



$$D'I - V/R = 0$$

Combining Input/Output with a DC transformer:



$$\frac{V_o}{V_g}=\frac{1}{D'}\left[1-\frac{D'V_D}{V_g}\right]\frac{1}{1+\dfrac{R_L+DR_{ON}+D'R_D}{D'^2R}}$$ =M(D, R_L, V_{on}, R_D, V_D)

↑ lossless ideal

↑ V_D factor

↑ <u>Resistive Effects</u>
Only for R → ∞ do
we get full output

For the extreme D = 0 ⇒ $DR_{on} = 0$, $D'R_D = R_D$ and resistive factor is slightly simplified

$$h=\frac{P_o}{P_{in}}=\frac{VD'I}{V_gI}=D'\frac{V}{V_g}$$, but V/V_g differs with losses

$$h=\left[1-\frac{D'V_D}{V_g}\right]\Big/\left[1+\frac{R_L+DR_{on}+D'R_D}{D'^2R}\right]$$ = η(D, R_L, R_{on}, R_D, V_D)

↑ Diode on Voltage

↑ Resistive Effects

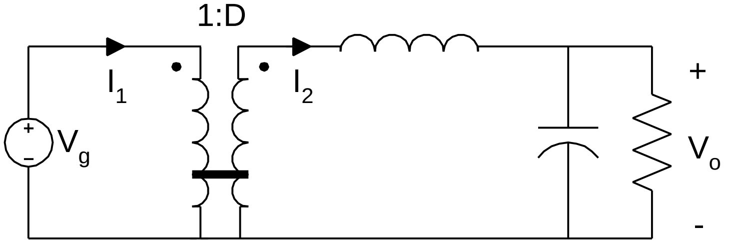
(3) <u>The following is problem 3.3 of Chapter 3 Erickson</u>

Buck Converter with:

- Input L_1-C_1 filter L_1 has R_{L1} winding resistance due to copper wire

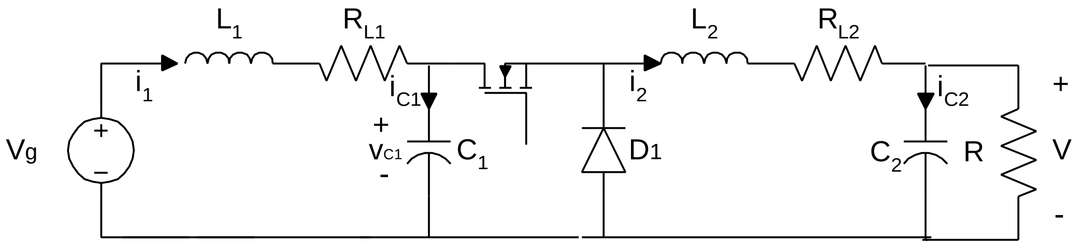
- Transistor with R_{on}
- Diode with R_D and V_D

Lossless Buck:

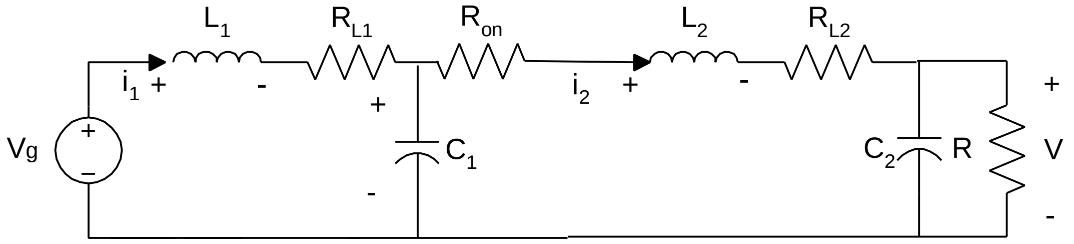


$V_o = DV_g$ $\eta = V_o/(DV_g) = P_o/P_{in}$

$I_1 = I_2/D$ $= 1.0$ (lossless)



a) Q(on)



$V_{L1} = V_g - V_{C1} - I_1R_{L1}$

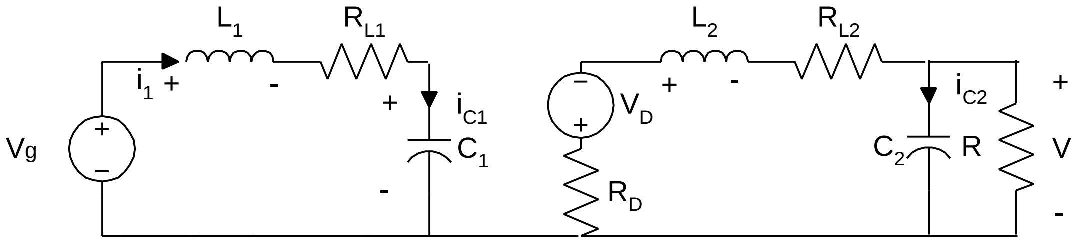
$V_{L2} = -I_2R_{on} + V_{C1} - V - I_2R_{L2}$

$I_{C1} = I_1 - I_2$

$I_{C2} = I_2 - V/R$

Next we will consider the switch case when the transistor is off and the diode is on.

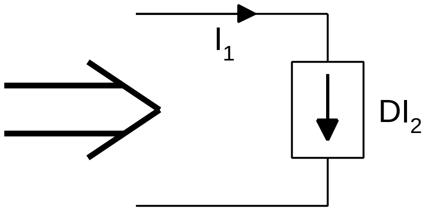
Q(off)



$V_{L1} = V_g - V_{C1} - I_1R_{L1}$
$V_{L2} = -V_D - I_2R_D - V - I_2R_{L2}$
$I_{C1} = I_1$
$I_{C2} = I_2 - V/R$

Do Volt-second balance on L’s / Charge balance on C’s

1. L_1: $D(V_g - V_{C1} - I_1R_{L1}) + D'(V_g - V_{C1} - I_1R_{L1}) = 0$
   $\therefore\ V_g = V_{C1}+I_1R_{L1}$
2. L_2: $D(-I_2R_{on} + V_{C1} - V - I_2R_{L2}) + D'(-V_D - I_2R_D - V - I_2R_{L2}) = 0$
   $\therefore\ -DI_2R_{on} + DV_{C1} - D'V_D - D'I_2R_D - V - I_2R_{L2} = 0$
3. C_1: $I_1 - DI_2 = 0 \qquad \therefore\ I_1 = DI_2$
4. C_2: $I_2 - V/R = 0 \qquad \therefore\ I_2 = V/R$



Input Circuit

From 1, 3, and 4 $\quad V_{C1}=V_g - I_1R_{L1}=V_g - DI_2R_{L1}=V_g - DR_{L1}\dfrac{V}{R}$

$$\therefore\ V_{C1} = V_g - DR_{L1}\frac{V}{R}$$

Output Circuit:
Then:

$$(2)\quad -DR_{on}\frac{V}{R} + DV_g - D^2R_{L1}\frac{V}{R} - D'V_D - D'R_D\frac{V}{R} - V - R_{L2}\frac{V}{R} = 0$$

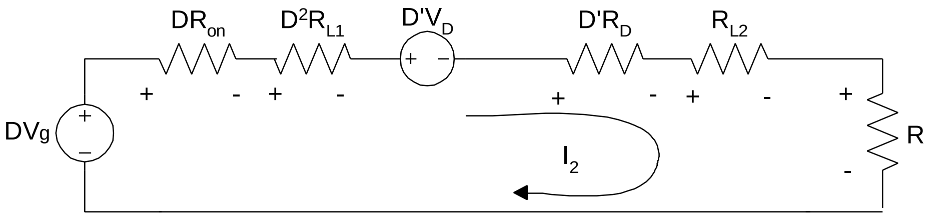


Fig. 1

DC Trf Model:

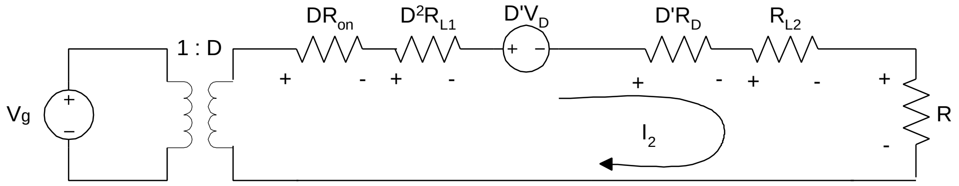


Fig. 2

b) From Fig. 1

$$V = -DR_{on}\frac{V}{R} + DV_g - D^2R_{L1}\frac{V}{R} - D'V_D - D'R_D\frac{V}{R} - R_{L2}\frac{V}{R}$$

c) Using the Voltage Divider:

$$V = \frac{R}{R + DR_{on} + D^2R_{L1} + D'R_D + R_{L2}}(DV_g - D'V_D)$$

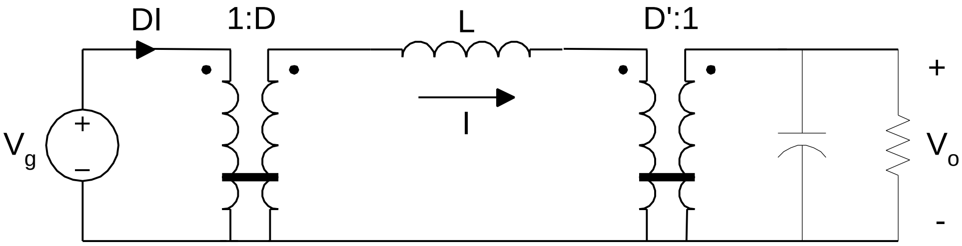
Filtered Buck

$$\frac{V}{V_g} = (D - D'\frac{V_D}{V_g})\frac{R}{R + DR_{on} + D^2R_{L1} + D'R_D + R_{L2}}$$

$$\eta = \frac{V}{DV_g} = (1 - \frac{D'V_D}{DV_g})\frac{R}{R + DR_{on} + D^2R_{L1} + D'R_D + R_{L2}}$$

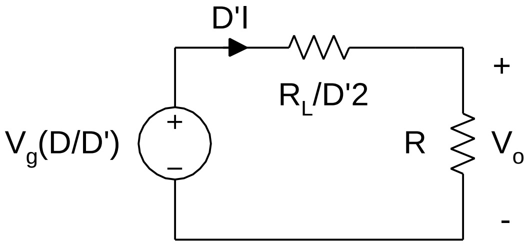
Next we solve the general equations for the Buck-Boost

4. Buck-Boost solved in general



(a) Equivalent Series Resistance ,$R_{L,}$ losses only

- Assume lossless except for R_L
- Reflect all to load side. For DC L → short and C → open.



For DC conditions:

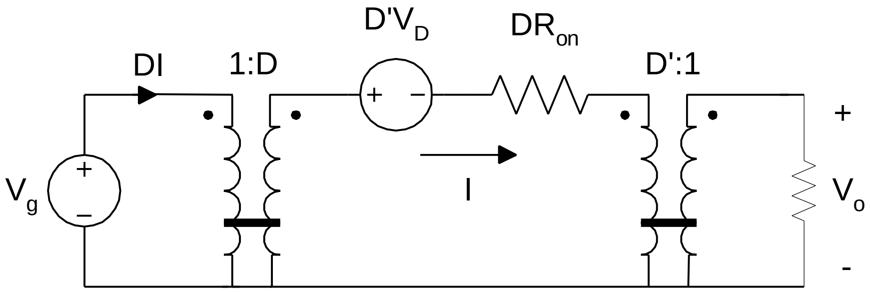
$$V_o = \frac{R}{\frac{R_L}{D'^2} + R} V_g \frac{D}{D'}$$

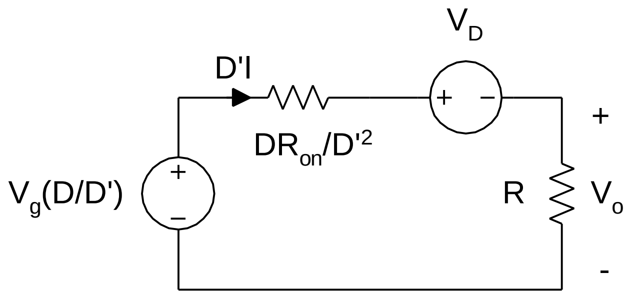
(b) Device losses only

Buck-Boost with device loss ($R_L \equiv 0$)

Transistor: R_{on}

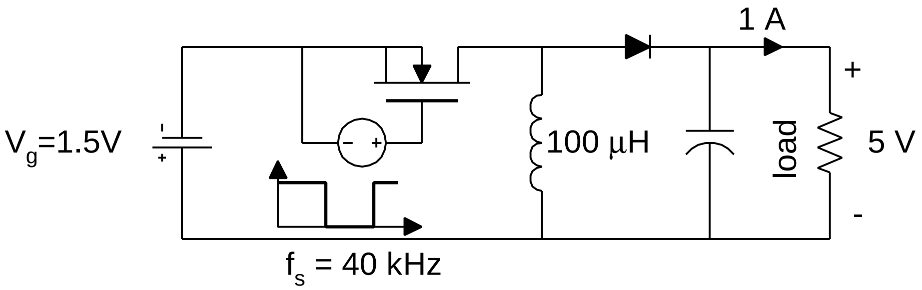
Diode: V_D





$$V_o = \left[V_g \frac{D}{D'} - V_D\right] \frac{R}{R + DR_{on}/D'^2}$$

(c) Buck-Boost Converter including all static losses. This is problem 3.4 from Erickson



V_{out} required is +5 V and V_g = -1.5 V

Diode: V_D = 0.5 V Schottky

R_D = 0 (approx.)

Transistor: R_{on} = 35 mΩ

<u>Ideal</u> Buck-Boost:

$$\frac{V_o}{V_g} = \frac{-D}{1-D} = \frac{-D}{D'}$$

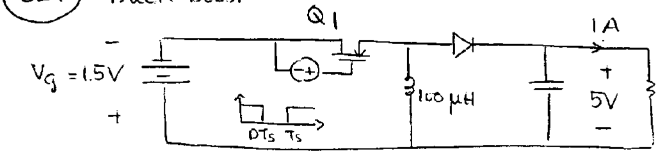
<u>Real</u> Buck-Boost: we will find in the next few pages

$$V_o = \underset{\substack{\uparrow \\ \textbf{Diode} \\ \textbf{Effect}}}{\left[\frac{D}{D'} V_g - V_D\right]} \underset{\substack{\uparrow \\ \textbf{Resistive} \\ \textbf{Effect}}}{\frac{R}{R + \text{other}(R_{on}, R_L)}}$$

Below we work out Erickson problem 3.4 for illustration of how to do the HW for Chapter 3

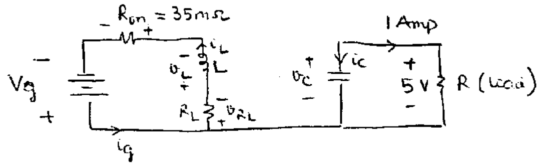
HW # 2 will be Erickson Chapter 3 Pbms.6 and 7. This will be due next week along with any questions asked in the lectures

3-4 Buck boost

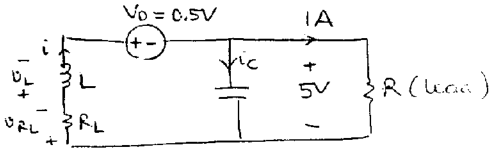


Load = R = $\frac{5V}{1A}$ = 5Ω.

(a) when MOSFET conducts



when diode conducts



$$\begin{cases} v_L(t) = v_g - v_{R_L} - v_{R_{on}} \approx V_g - IR_L - IR_{on} \\ i_C(t) = -\frac{v}{R} \approx -\frac{V}{R} \end{cases}$$

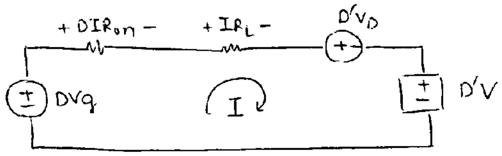
$$\begin{cases} v_L(t) = -v_D - iR_L - v \approx -V_D - IR_L - V \\ i_C(t) = i - \frac{v}{R} \approx I - \frac{V}{R} \end{cases}$$

DC component of inductor voltage is given by

$$\langle v_L \rangle = D\left(V_g - IR_{on} - IR_L\right) + D'\left(-V_D - IR_L - V\right) = 0$$

$$DV_g - DIR_{on} - DIR_L - D'V_D - D'IR_L - D'V = 0$$

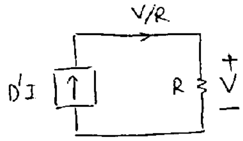
since $D + D' = 1 \Rightarrow \boxed{DV_g - DIR_{on} - IR_L - D'V_D - D'V = 0}$



$$\text{and } \langle i_C \rangle = D\left(-\frac{V}{R}\right) + D'\left(I - \frac{V}{R}\right) = 0$$

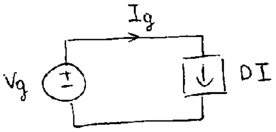
$$-D\frac{V}{R} + D'I - D'\frac{V}{R} = 0$$

$$\boxed{D'I - \frac{V}{R} = 0}$$

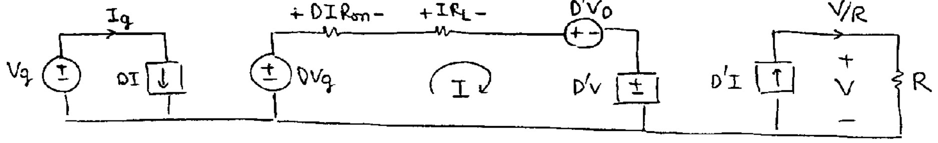


also $\langle i_g \rangle = I_g = DI$

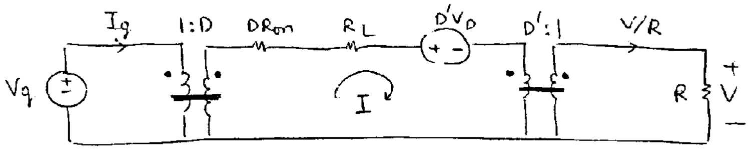
$$\boxed{I_g = DI}$$



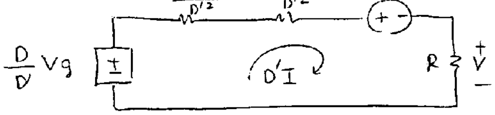
Put 3 ccts together:



The completed dc equivalent cct model (Buck Boost)



(b) Reflecting everything in the middle cct, we get



Vg = 1.5Volts

V = 5 Volts

R = 5Ω

Ron = 35 mΩ

$$V = \left(\frac{D}{D'} V_g - V_D\right)\left(\frac{R}{R + \frac{D}{D'^2} R_{on} + \frac{R_L}{D'^2}}\right)$$ using voltage divider

$$\Leftrightarrow \frac{V}{V_g} = \left(\frac{D}{D'} - \frac{V_D}{V_g}\right)\left(\frac{RD'^2}{RD'^2 + DR_{on} + R_L}\right) = \frac{D}{D'} \underbrace{\left(1 - \frac{D'}{D}\frac{V_D}{V_g}\right)\left(\frac{1}{1 + \frac{D}{D'^2}\frac{R_{on}}{R} + \frac{R_L}{D'^2}}\right)}_{\text{Efficiency } \eta}$$

with $\frac{D}{D'} \equiv$ dc conversion ratio in the ideal case
$(R_L = R_{on} = V_D = 0)$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{D'IV}{DIV_g} = \frac{D'}{D} \cdot \frac{V}{V_g} \quad \Leftrightarrow \quad 0.7 = \frac{1-D}{D} \cdot \frac{5}{1.5}$$

$$\Leftrightarrow \boxed{D = 0.83} \quad \text{and} \quad D' = 1 - 0.83 = \boxed{0.1}$$

$$\text{So } 0.7 = \left(1 - \frac{0.17}{0.83}\,\frac{0.5}{1.5}\right)\left(\frac{1}{1 + \frac{0.83}{0.17^2} \cdot \frac{35m}{5} + \frac{R_L}{0.17^2(5)}}\right) \Rightarrow \boxed{R_L \approx 20 m\Omega}$$

$$I = \frac{1}{D'} I_2 = \frac{1\text{Amp}}{D'} = \frac{1}{0.17} = 5.88\text{Amp}$$

(c) Compute power loss:

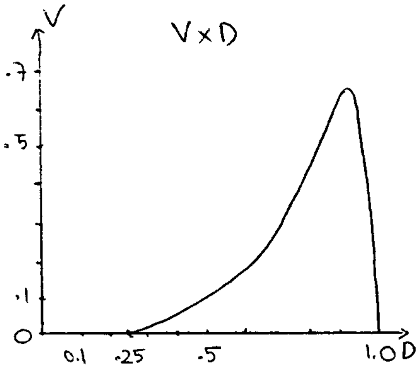
$$\begin{cases} P_{R_L} = I^2 R_L = (5.88)^2 (20m) = \boxed{0.69W} \\ P_{R_{on}} = I^2 R_{on} D = (5.88)^2 (35m)(0.83) = \boxed{1W} \\ P_{diode} = V_D I = (0.5)(1) = \boxed{0.5W} \end{cases}$$

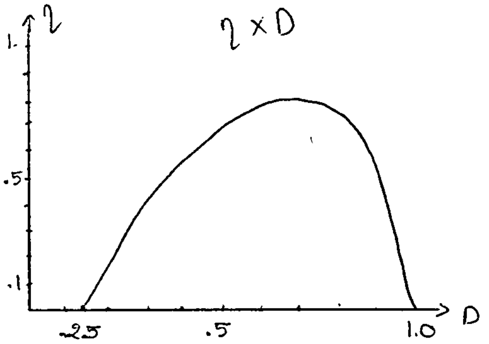
$$P_{in} = 0.69 + 1 + 0.5 + 5 = \boxed{7.19W}$$

$$P_{out} = 5(1)^2 = \boxed{5W}$$

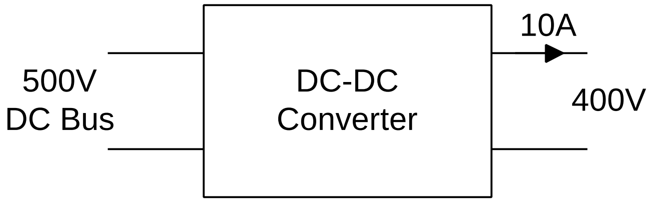
Check : $\eta = \frac{P_{out}}{P_{in}} = \frac{5}{7.19} = 0.69 \approx 0.7 \Rightarrow$ OK!

(d) Plots :





(5) Compare Buck vs. Buck-Boost Topology for DC Conditions Only. This is problem 3.5 of Erickson.



Transistor: $R_{on} = 0.5\ \Omega$ $R_D \to 0$

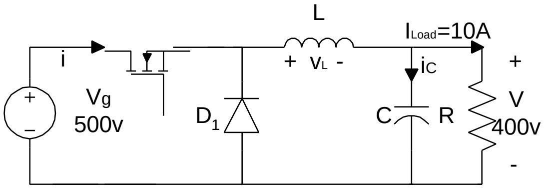
Diode: $V_D \to 0$ Inductor: $R_L \to 0$

Find D and η for placing different topologies inside the DC-DC converter. Then compare why one topology might be favored over the other for the specified conditions.

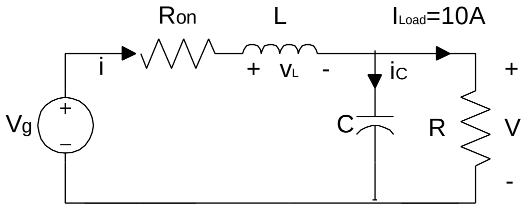
Prob. 3.5

Only loss is Mosfet with Ron = 0.5Ω

Buck



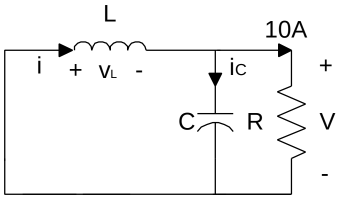
Mosfet on and Diode off.



$v_L(t) = V_g - i R_{on} - V \approx V_g - I R_{on} \quad - V$

$i_c(t) = i - 10 \approx I - 10$

Mosfet off and Diode on.



$v_L(t) = -V$

$i_c(t) = i - 10 \approx I - 10$

Thus averaging we get:

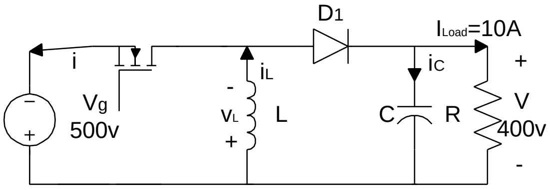
$\langle v_L \rangle = (V_g - I R_{on} - V)D + (-V)D' = 0$

$\langle i_c \rangle = D(I - 10) + D'(I - 10) = 0$

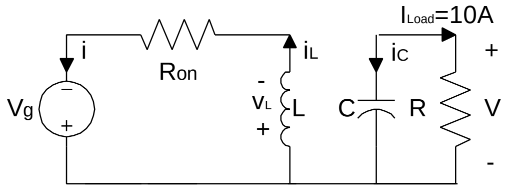
So $\Rightarrow$ $DV_g - DIR_{on} - V = 0$

$I - 10 = 0 \;\Rightarrow\; I = 10$

Buck-Boost



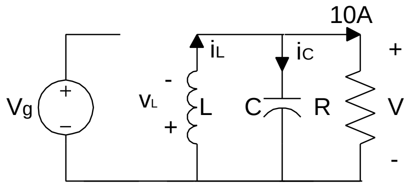
Mosfet on and Diode off.



$v_L(t) = V_g - i R_{on} \approx V_g - I R_{on}$

$i_c(t) = -10$

Mosfet off and Diode on.



$v_L(t) = -V$

$i_c(t) = i - 10 \approx I - 10$

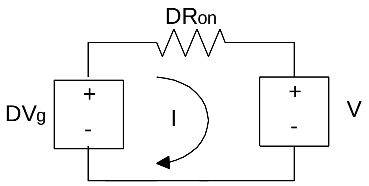
Thus averaging we get:

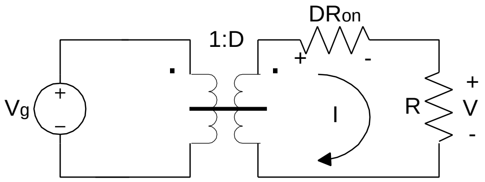
$\langle v_L \rangle = (V_g - I R_{on})D + (-V)D' = 0$

$\langle i_c \rangle = D(-10) + D'(I - 10) = 0$

So $\Rightarrow$ $DV_g - I R_{on} D - V + VD = 0$

$ID' - 10 = 0 \;\Rightarrow\; I = 10/D'$





First we need D:

DVg - 10RonD - V = 0

D = V/(Vg - IRon)

D = 0.81; D' = 0.19

Solving for efficiency: **Ig = DI**

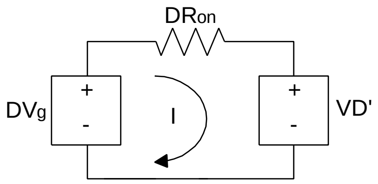
η = P_{out}/P_{in} = V I_{load}/VgDI

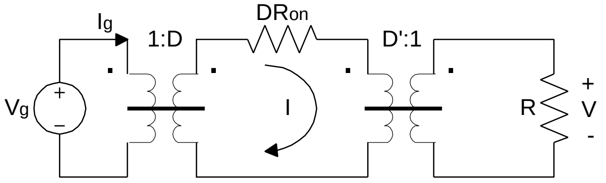
= (4000)/[DVg(10)]

η = 0.99 = **99%**

$P_{loss} = P_{in} - P_{out}$

P_{loss} = 4050 - 4000 = **50 Watts**





DVg - 10RonD/D' - VD' = 0

DVg - VgD^2 - 10RonD - V + 2VD -VD^2 = 0

D^2(-Vg - V) +D(2V + Vg - 10Ron) -V = 0

Using the quadratic equation for D:

D = 0.45; D' = 0.55

Solving for efficiency: **Ig = ID**

η = P_{out}/P_{in} = V I_{load}/VgID

=(4000)/[VgD(10/D')]

η = 0.978 = **97.8%**

$P_{loss} = P_{in} - P_{out}$

P_{loss} = 4090 -4000 = **90 Watts**

The BUCK is better for this application!

Another way to calculate the Power loss is to use the voltage divider and find Vg with Respect to V as shown below:

$$V=\frac{RDVg}{R+DRon}$$

Knowing that:

V = DVg for the lossless Buck:

we can see that $\frac{R}{R+DRon}=1$

for the lossless Buck case thus:

$$\eta = \frac{R}{R+DRon}$$

η = 0.99 = **99%**

since η = Pout/Pin:

Pin = Pout/η = 4000/.99 = 4040.4 W

Ploss = 4040.4 - 4000 = **40.4 W**

$$V=\frac{RDVg}{D'(R+\frac{DRon}{D'^2})}$$

V = DVg/D' for the Buck-Boost:

$$\eta = \frac{R}{(R+\frac{DRon}{D'^2})}$$

η = 0.98 = **98%**

Pin = Pout/η = 4000/.98 = 4081.63 W

Ploss = 4081.63 - 4000 = **81.63 W**

The BUCK is better for this application!