ECE 555: Exam (Spring 2016)

Due Tuesday, April 5th, 2016

Problem 1: Using the given U, V, $\dot{x}$, and $\dot{\theta}$:

$$
\begin{aligned}
U &= \tfrac{1}{2889}\begin{bmatrix} 1292 & -2584 \\ 2584 & 1292 \end{bmatrix} & \dot{x} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
V &= \tfrac{1}{3}\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{bmatrix} & \dot{\theta} &= \begin{bmatrix} 0.825 \\ -0.34 \\ 1.01 \end{bmatrix}
\end{aligned}
$$

perform the following steps:

(a) Find the rotation angle, ψ, by which $\hat{u}_1$ is rotated in relation to $\dot{x}_1$.

(b) Solve for the non-zero singular values, namely σ_i for $i = 1, 2, \ldots, \min(m, n)$. Recall $\dot{x} = J\dot{\theta}$ and use the provided matrix values.

(c) Determine the condition number, k, for the matrix D obtained from the singular value decomposition process.

(d) Solve for the Jacobain using the following two equations:

$$
\begin{aligned}
J &= UDV^T \\
J &= \textstyle\sum_{i=1}^{\min(m,n)} \sigma_i \hat{u}_i \hat{v}_i^T
\end{aligned}
$$

Show they produce the same matrix.

(e) Solve for the pseudo-inverse Jacobian, $J^{\dagger}$, using the following two equations:

$$
\begin{aligned}
J^{\dagger} &= J^T(JJ^T)^{-1} \\
J^{\dagger} &= \textstyle\sum_{i=1}^{\min(m,n)} \frac{1}{\sigma_i} \hat{v}_i \hat{u}_i^T
\end{aligned}
$$

Show they produce the same matrix.

Problem 2: Consider a planar, 2R robotic manipulator. The Jacobian transforms a unit circle in the joint space, $\|\dot{\theta}\|_2 = \sqrt{\dot{\theta}_1^2 + \dot{\theta}_2^2} = 1$ into an ellipse in the workspace. Actually, different norms can be used to describe the unit ball in the joint space.

(a) If the maximum velocity of each joint is 1, what norm should be used to describe the unit ball in the joint space. Justify your answer.

(b) What is the shape of the unit ball in the joint space using the norm chosen in (1)? Draw your answer.

(c) Choose several joint velocities in the unit ball defined in part (1). Calculate the end-effector velocities when $J = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} \\ \sqrt{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$. Plot these end-effector velocities in the workspace to see the resultant shape. Use MATLAB and include the generated plot.

Problem 3: Consider the 3R robotic manipulator shown in Fig. 1 with link lengths shown below each link.

(a) Calculate the full Jacobian, J^0.

(b) Find the singular value decomposition of the Jacobian using MATLAB and write down the matrix of singular values. From these, is it possible to compute the condition number in a standard way? Justify your answer and include the requirements for calculating the condition number.

(c) Assume it is desired to have an end-effector tracking error of less than 0.2mm. Determine an appropriate damping factor and condition number for the damped least squares method.

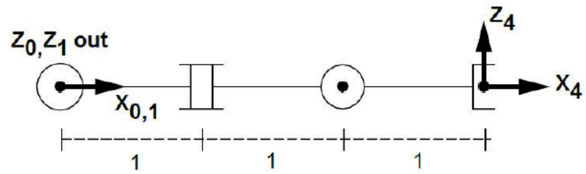

Figure 1: 3R robotic manipulator with unit link lenghts.

Problem 4:

(a) Given the DLS equation of J^{λ}, prove the maximum joint-velocity is $\frac{1}{2\lambda}$.

(b) Assume the Jacobian has two singular values, $\sigma_1 > \sigma_2$. If priority is to be given to end-effector tracking error, to which value of σ should λ be closer?

(c) Assuming $J = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$, calculate the different values of λ when the constraint is on:

  (i) Joint angle velocity (assume $\dot{\theta}_{\text{max}} = 0.4\text{rad/s}$).

  (ii) End effector tracking error (assume $\Delta R = 0.5$).

  (iii) Conditioning of equations (assume $\kappa_{\text{max}} = 1.8$).

Problem 5: This questions uses paper 6, 'Path Planning and the Topology of Configuration Space'.

(a) Why is it important to know object connectivity in configuration space?

(b) Consider a two link manipulator with $L_1 = L_2$ and the arm pointed along the standard, positive x_1 axis when $\theta_1 = \theta_2 = 0$. Determine if the two point obstacles, defined in cartesian space, connect in configuration space for each scenario. If so, in what configuration?

  (i) Obstacle 1: $[1, 0]^T$, Obstacle 2: $[0.13397, 0.5]^T$

  (ii) Obstacle 1: $[0.8, 0.7071]^T$, Obstacle 2: $[0.13397, 0.5]^T$

Problem 6:

(a) Explain each term in the following equation:

$$\dot{\theta} = (J_e^{\dagger}\dot{x}_e) + [J_0(I - J_e^{\dagger}J_e)]^{\dagger}(\dot{x}_0 - (J_0 J_e^{\dagger}\dot{x}_e))$$

Why is each term really needed? What is the significance of the bracket in the equation?

(b) Explain how the rank of a matrix and its pseudo-inverse are related. That is, consider the nature of the resulting pseudo-inverse for a full rank matrix vs. those of a singular matrix. What happens to the pseudo-inverse when a matrix transitions from full rank to singular?

(c) Explain one method for minimizing the effect of a rank change. What are the limitations of this method?