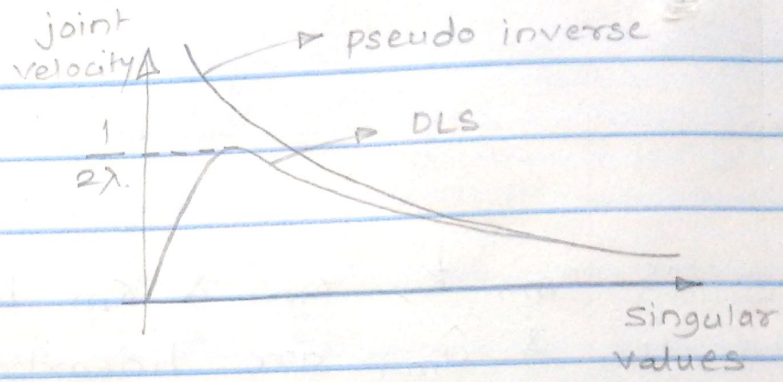
1- Find $\lambda(\theta_{max})$ where σ_{min}= 0.3 and θ'_{max}=0.8 rad/s

2- Find σ_{max} for this Jacobian J = $\begin{bmatrix} 1 & 0.25 & 0 \\ 0.5 & 0.25 & 1 \end{bmatrix}$ by using the iterative method AV=V':

a. After 1 iterations and write all steps required. use arbitrary Vector V=$[0.5\ 0.5\ 0.5]^T$

b. After 2 and 5 iterations use Vector V= $[0.1\quad 0.1\quad -0.5]^T$as arbitrary Vector. (use Matlab or any other tool)

c. Find σ_{max} by using SVD and compare with results from a and b.



Q (i) Given the DLS equation of $J^{(\lambda)}$, prove that the maximum value is $\frac{1}{2\lambda}$.

(ii) Assume that the jacobian has two singular values ($\sigma_1 > \sigma_2$). If priority is to be given to end-effector tracking error, to which value of σ should λ be closer to.

(iii) Calculate the different values of λ when the constraint is on (a) joint angle velocity (assume $\dot{\theta}_{max} = 0.4$ rad/sec), (b) end effector tracking error (assume $\Delta R = 0.5$), and (c) conditioning of equations (assume $k_{max} = 1.8$).

for $J^{(\lambda)} = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$

Question:

Consider a robot as below, all links' length is equal to 1m

(a) Find Jacobian

(b) If the constraint of the angle velocity is 0.2 r/s, find the appropriate damping factor (λ)

(c) If the $\dot{X}_D = [4,1]^T$, use the damped least-squares (DLS) method and the λ in (b) to find $\dot{\theta}$. What is the actual velocity $\dot{X}_A$?

i) Find the truncated SVD and the resulting A matrix for the following matrix. Use rank K=3. Use MATLAB if required.

$$A = \begin{bmatrix} 2 & 0 & 8 & 6 & 0 \\ 1 & 6 & 0 & 1 & 7 \\ 5 & 0 & 7 & 4 & 0 \\ 7 & 0 & 8 & 5 & 0 \\ 0 & 10 & 0 & 0 & 7 \end{bmatrix}$$

What happens if we choose a higher K or smaller K?

ii) What is the problem with using SVD in real-time applications?

a) Why we use truncated SVD rather than normal SVD?
b) Briefly describe how truncated SVD works.

This question uses Paper 4, The Singular Value Decomposition: Computation and Applications to Robotics

Q. 1: ..... 1 point
For a $6x6$ matrix, why is the maximum number of rotations per sweep 15? What situations would need less than 15?

Q. 2: ..... 1 point
Why would using a previous iteration SVD with a small added pertubation require less sweeps than an arbitrary matrix?

Q: Consider the Jacobian matrix J given as :

$$J = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

a) Find the minimum eigen value and corresponding eigenvector for J. [USE power method without MATLAB]

b) Find $\lambda(\dot{\theta}_{max})$ using the minimum eigenvalue (σ_{min}) found above in part (a). Given: $\underline{\dot{\theta}_{max} = 0.25 \text{ rad/sec}}$.

1. Determining minimum singular value is vital in estimating damping coefficient. One of the proposed technique is to use power method. Compare the computation time of using power method vs explicitly determining minimum singular value using SVD. Give the ratio of computation time: SVD/Power method. Simulate for 100 cycles.
   [Procedure: Generate a 6x6 random matrix, perform SVD in one case and matrix-vector multiplication (simulating simple power method) in other. Measure computation time using tic-toc functions available in Matlab.]

Homework 4

- What are Given Rotations? State the property of Given Rotations.
- If $a_i = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ & $a_j = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ [Do not used MATLAB for matrix multiplicat]

Calculate a_i' & a_j' (Consider $q \geq 0$)

- State the constraint on orthogonality & for the above columns, determine if the rotation is performed (if threshold is 0.5).

1) Using the standard eigenvalue/eigenvector equation $Ax = \lambda x$ and given matrix A and the matrix of eigenvectors V belonging to A, derive a way to determine the eigenvalues, λ_i, of A and then calculate them. Do not use the typical method of solving $(A - \lambda I)x = 0$. Please check your answer using the $eig(A)$ Matlab command. (Hint: Break A into rows a_1, a_2, a_3, a_4 and multiply $a_i * x$. Take that result and divide by the x_i component.)

$$A = \begin{bmatrix} 1 & 2 & 5 & 6 \\ 2 & 4 & 3 & 6 \\ 0 & 3 & 9 & 8 \\ 3 & 3 & 3 & 3 \end{bmatrix} \qquad V = \begin{bmatrix} 0.4501 & 0.4691 & -0.1956 & -0.5474 \\ 0.4505 & 0.3312 & 0.7617 & 0.8078 \\ 0.6723 & 0.4159 & -0.5895 & -0.1798 \\ 0.3774 & -0.7052 & 0.1846 & -0.1243 \end{bmatrix}$$

2) Using the Power method for calculating eigenvectors shown in class, use the A matrix from above and the random vector x below to calculate the normalized eigenvector associated with the largest eigenvalue for each ε value below. Find the largest eigenvalue using your method of choice for each ε and compare it to the actual eigenvalue found in part 1. If you use a program such as Matlab please include your code, otherwise show all work.

$$x = \begin{bmatrix} 14 \\ .03 \\ 147 \\ 6 \end{bmatrix} \qquad \varepsilon = 1,\ 0.1,\ 0.01,\ 0.001,\ 0.0001$$

In robotics, the relationship between acceleration and torque can be represent as :$\ddot{\theta} = J^{+}\left(\ddot{x} - \dot{J}\dot{\theta}\right) - [H(I - J^{+}J)]^{+}\tilde{\tau}$. Actually, the torque need to be constrained to physical limitation, the homogeneous solution part will effect the magnitude of torque.
When consider the case that an acceleration along the homogeneous solution is allowed.

(1) In which ways that the $\ddot{\theta}_H$ can affect the torque requirement.
(2) In which method could the homogeneous acceleration term minimize the instantaneous torque requirement
(3) Once the the homogeneous acceleration method minimized the instantaneous torque, how could we get the information about the affect to the homogeneous velocity?

(Hint: Please read the Article 7 at the part of Kinetic effects of a homogeneous solution)

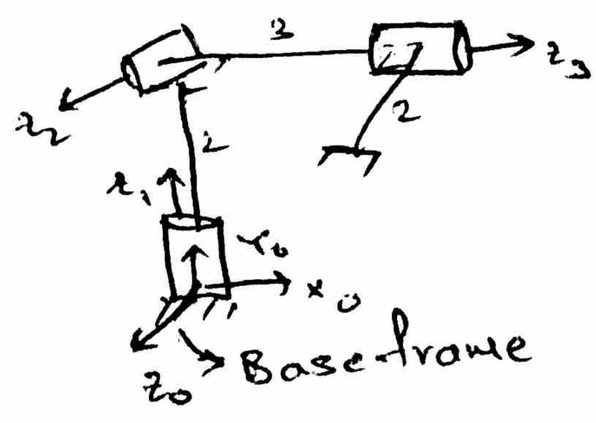
a* What is the disadvantage of damped-least square approach for solving the inverse kinematic equations?

b* find damping factor in all the 3 kinds of simulation of

(a) Constant damping factor (maintain the actual joint velocity norm to within 0.05 radians per computational interval)

(b) Variable damping factor → Assume $\dot{\theta}_{max}$ to be 0.03.

(c) Numerical filtering. for the ~~following~~ following, whose tool tip velocity is $3\hat{x} + 2\hat{y} + 2\hat{z}$.



consider σ_{eff} to be 1.814

c* Also explian the principle involved in Numerical filtering.

$$J = \begin{bmatrix} 1 & -3 & -2 \\ 2 & -1 & 5 \end{bmatrix}$$

Applying the method introduced in class, estimate this Jacobian's maximal singular value σ_{max}, maximal input singular vector v1 and maximal output singular vector u1.

(1) Given an arbitrary initial v1 vector, how to update the v1 vector in each iteration (write the equations)? What is an appropriate termination condition?

(2) According to your answers to the first question, write the matlab code and calculate the estimated σ_{max}, v1 and u1. (give your matlab code)

(3) Using svd algorithm in Matlab to calculate the U, D, V matrix of this Jacobian. Compare the σ_{max}, v1 and u1 calculated by svd algorithm in matlab and the results calculated by your algorithm to prove the accuracy your algorithm.

Homework problem 4:
For a redundant manipulator (single degree of redundancy):
(1). How do we get the equation shown below :

$$\ddot{\theta} = J^{+}\left(\ddot{x} - \dot{J}\dot{\theta}\right) + (I - J^{+}J)\ddot{\phi} \quad \text{(Equation (6) in Article 7)}$$

Please provide the derivation process.
(2). For the equation:

$$\ddot{\theta} = -J^{+}\dot{J}\dot{\theta}_{H}$$

What is the relationship between the joint angle acceleration and the homogeneous solution curvature.
(3). What will happen when the manipulator (shown in Fig.1 and Fig.2 in Article 7) is approaching the internal singular configuration.

Now consider an error tracking problem, you have a manipulator welding a beautiful line with a desired velocity of 2m/s. At a certain time, the end effector has a desired position of 7m, but the measured distance is only 5.5m. Assume the new updates occur every 0.25 seconds, what value of k_p should be used as a driven proportional control in order to reduce the accumulated error to a range between 1dm and 2dm in one update time?

Find the joint velocity $\dot{\theta^{(k)}}$ using the truncated SVD. Here k=1 and $J = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix}$

$\dot{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(The geometric definition of singular value) Calculate the singular value of A=$\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$. How does the matrix A change a unit circle S = $x^2 + y^2 = 1$?