EE507 – Plasma Physics and Applications

Solutions to homework #3

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1) a) @ 100 eV $\sigma_{i\,He} = 3.66\times10^{-17}\ cm^2$

$$N_{He} = \frac{1/760\ atm \quad 6.02\times10^{23}\ 1/mol}{0.082\ \frac{atm\ \ell}{K\,mol} \quad 298\ K} = 3.24\times10^{19}\ \frac{1}{\ell} = 3.24\times10^{16}\ \frac{1}{cm^3}$$

$$\lambda = \frac{1}{N\sigma} = 0.84\ cm$$

b) $$\nu = v\,\sigma N = \frac{v}{\lambda} = \frac{5.9\times10^{8}\ cm/s}{0.84\ cm} = 7.1\times10^{6}\ \frac{1}{sec}$$

c) $$N_e = \frac{J}{v} = \frac{6.25\times10^{18}\ e/cm^2\,sec}{5.9\times10^{8}\ cm/sec} \simeq 10^{10}\ \frac{1}{cm^3}$$

d) $$R = N_{He}\,N_e\,\sigma v = N_e\,\nu = 7.4\times10^{18}\ \frac{1}{cm^3\,sec}$$

e) $$\sigma_{i\,He} = 3.66\times10^{-17}\ cm^2$$
$$\sigma_{e\,He} = 0.107\times10^{-18}\ cm^2 \qquad \frac{\sigma_i}{\sigma_e} = 342$$

f) $$N_i = R\tau = 7.4\times10^{18}\ \frac{1}{cm^3\,sec}\ 1\cdot10^{-6}\ sec = 7.4\times10^{12}\ \frac{1}{cm^3}$$

2) $$N_{coll} = \frac{100\ eV}{24.5\ eV} \simeq 4 \qquad L = 4\lambda = 3.36\ cm$$

3) $$\sigma_{i\,Hg} \simeq 6.5\times10^{-16}\ cm^2 = 17.8\ \sigma_{i\,He}$$

$$\lambda_{Hg} = \frac{\lambda_{He}}{17.8} = 0.05\ cm$$

$$R_{Hg} = 17.8\,R_{He} = 1.3\times10^{20}\ \frac{1}{cm^3\,sec}$$

4) $$N_{He} \simeq 10^{17}\ \frac{1}{cm^3} \qquad N_e = 10^{12}\ cm^{-3} \qquad T_e = 2\ eV$$

$$R = N_{He} N_e \int_0^\infty F(E)\, \sigma(E) \sqrt{\frac{2E}{m_e}}\, dE$$

$$F(E)\, dE = 4\pi v^2 f(v)\, dv$$

$$\text{so} \quad F(E) = 4\pi v^2 f(v) \frac{dv}{dE} = \frac{1}{kT_e} \frac{2}{\sqrt{\pi}} \sqrt{\frac{E}{kT_e}}\, e^{-E/kT_e}$$

$$R = N_{He} N_e \int_0^\infty \frac{1}{kT_e} \frac{2}{\sqrt{\pi}} \sqrt{\frac{E}{kT_e}} \sqrt{\frac{2E}{m_e}}\, \sigma(E)\, e^{-E/kT_e}\, dE =$$

$$= N_{He} N_e \underbrace{2\sqrt{\frac{2kT_e}{\pi m_e}}}_{\langle v \rangle} \int_0^\infty \frac{E}{kT_e}\, \sigma(E)\, e^{-E/kT_e}\, d\left(\frac{E}{kT_e}\right)$$

SINCE T_e IS SMALL ($T_e \ll E_{th}$) THE ONLY PART OF $\sigma(E)$ RELEVANT TO THE INTEGRAL IS THE LINEAR RAMP

$$\sigma(E) \cong \sigma_0 (E - E_{th}) = kT_e \sigma_0 \left(\frac{E - E_{th}}{kT_e}\right)$$

$$R \cong N_{He} N_e\, 2\sigma_0 \sqrt{\frac{(2kT_e)^3}{\pi m_e}} \int_{E_{th}}^\infty \frac{E}{kT_e} \left(\frac{E - E_{th}}{kT_e}\right) e^{-E/kT_e}\, d\left(\frac{E}{kT_e}\right) =$$

$$= N_{He} N_e \sigma_0 \langle v \rangle \left[kT_e \int_{E_{th}}^\infty \left(\frac{E}{kT_e}\right)^2 e^{-E/kT_e}\, d\left(\frac{E}{kT_e}\right) - E_{th} \int_{E_{th}}^\infty \frac{E}{kT_e}\, e^{-E/kT_e}\, d\left(\frac{E}{kT_e}\right) \right]$$

$$\sigma_0 \cong 0.1 \times 10^{-17} \frac{cm^2}{eV}$$

JUST INTEGRATE AND PLUG NUMBERS!

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7)

TRANSITION	λ (Å)	A (s^{-1})	E_u (eV)	I (a.u.)
$8p\ ^3P - 2s\ ^3S$	2723.19	7.8×10^5	24.3712	89
9p	2696.12	5.5×10^5	24.4169	40
10p	2677.135	4.04×10^5	24.4495	20

$E_\ell = 19.8196$ eV

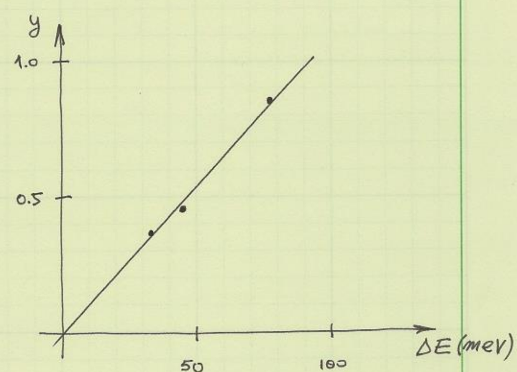
FOR TWO LINES BELONGING TO THE SAME ATOMIC SPECIES

$$\frac{I_1}{I_2} = \frac{A_1 g_1 \lambda_2 U_2 n_1}{A_2 g_2 \lambda_1 U_1 n_2} e^{-\frac{E_1 - E_2}{kT}}$$

IF THE LINES BELONG TO THE SAME IONIZATION STAGE AND HAVE THE SAME WEIGHT FOR THE UPPER LEVELS THEN

$$\underbrace{-\ln\left(\frac{I_1 A_2 \lambda_1}{I_2 A_1 \lambda_2}\right)}_{y} = \frac{E_1 - E_2}{kT}$$

LINE 1	LINE 2	ΔE	y
2723.19	2677.135	-0.0783	0.852
2723.19	2696.12	-0.0457	0.460
2696.12	2677.135	-0.0326	0.392



$kT \cong 0.093$ eV $\cong 0.1$ eV OR (BETTER YET?) $kT = 0.095 \pm 0.05$ eV BUT CAREFUL!

8) FOR RADIATIVE DECAYS TO CAUSE LESS THAN 10% DEPARTURE FROM LTE IN TRANSITIONS WITH $\Delta E \sim 4.7$ eV THE DENSITY MUST BE

$$n_e \geq 1.6 \times 10^{12}\, T_e^{1/2}\, \Delta E^3 = 5.7 \times 10^{15}\ cm^{-3}$$