EE507 – Plasma Physics and Applications

Solutions to homework #2

HWK #2 EE580

1

1) $\bar{v}_g = \frac{m}{q}\frac{\bar{g}\times\bar{B}}{B^2}$

$m_e = 9.1\times10^{-31}$ kg

$m_p = 1.67\times10^{-27}$ kg

$q_e = 1.6\times10^{-19}$ C

$B = 3\times10^{-5}$ T

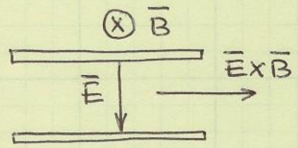
$g = 9.8\ \frac{m}{s^2}$

ASSUME $\bar{g}\perp\bar{B}$

$$v_{ge} = \frac{m_e}{q_e}\frac{g}{B} = 1.8\times10^{-6}\ \frac{m}{s}$$

$$v_{gp} = \frac{m_p}{q_e}\frac{g}{B} = 3.4\times10^{-3}\ \frac{m}{s}$$

2)



$$\bar{v}_E = \frac{\bar{E}\times\bar{B}}{B^2}$$

$$v_E = \frac{E}{B} \Rightarrow E = v_E B = 1\times10^{6}\ \frac{V}{m}$$

$v_E = 1\times10^{6}\ \frac{m}{s}$

$B = 1$ T

3) $$n = n_0\, e^{e^{-r^2/a^2}-1} = n_0\, e^{e\phi/kT_e} \quad (1)$$

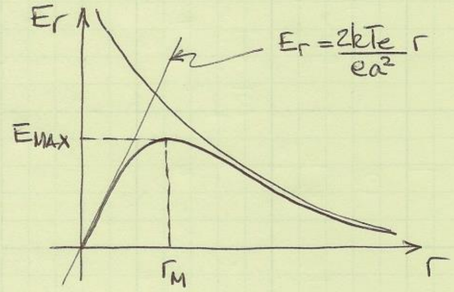
a) WE NEED TO FIND THE EXPRESSION FOR THE ELECTRIC FIELD TO CALCULATE THE $\bar{E}\times\bar{B}$ DRIFT.

FROM (1) $$\frac{e\phi(r)}{kT_e} = e^{-r^2/a^2} - 1 \Rightarrow \phi(r) = \frac{kT_e}{e}\left(e^{-r^2/a^2} - 1\right)$$

$$E = -\nabla\phi = -\frac{\partial\phi}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} - \frac{\partial\phi}{\partial z}\hat{z} = -\frac{\partial\phi}{\partial r}\hat{r}$$

$$E_r(r) = -\frac{\partial\phi}{\partial r} = \frac{kT_e}{e}\left(\frac{2r}{a^2}\right)e^{-r^2/a^2}$$

HWK #2 EE580



$e = 1.6\times10^{-19}\,c$

$a = 0.01\ m$

$kT_e = kT_i = 0.2\ eV$

$$\left.\frac{dE_r}{dr}\right|_{r_M} = \frac{2kT_e}{ea^2}\left(1 - \frac{2r_M^2}{a^2}\right)e^{-r_M^2/a^2} = 0 \Rightarrow \frac{r_M^2}{a^2} = \frac{1}{2}$$

$$E_{MAX} = E(r_M) = \frac{2kT_e}{a^2 e}\sqrt{\frac{a^2}{2}}\, e^{-1/2} = 17.2\ \frac{J}{cm} = 17.2\ \frac{V}{m}$$

$$\bar{v}_E = \frac{\bar{E}\times\bar{B}}{B^2} \qquad v_{E_{MAX}} = \frac{E_{MAX}}{B} = 86\ \frac{m}{s}$$

b) $F_e = qE = 1.6\times10^{-19}\,\cancel{c}\ 17.2\ \frac{J}{m\cancel{c}} = 2.75\times10^{-18}\,N$

FOR POTASSIUM

$$F_g = Mg = 39\ 1.67\times10^{-27}\,kg\ 9.8\ \frac{m}{s^2} = 6.38\times10^{-25}\,N$$

$$\frac{F_e}{F_g} = \left(2.3\times10^{-7}\right)^{-1} = 4.3\times10^{6}$$

c) $r_L = \frac{M v_\perp}{eB} = 0.01\ m$

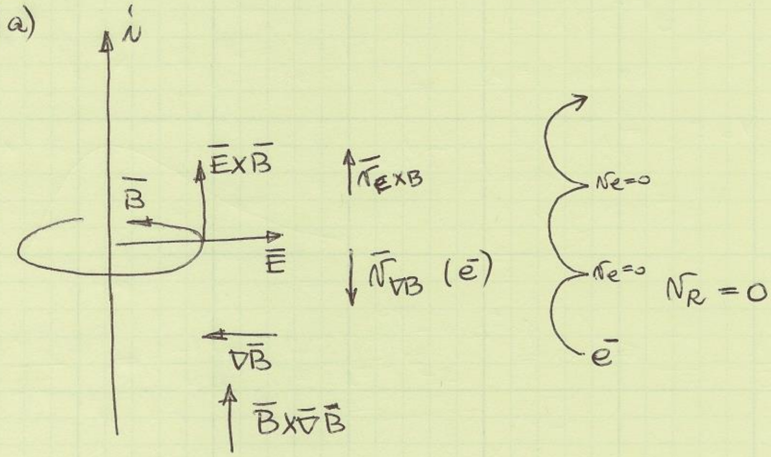
THERMAL VELOCITY $v = \sqrt{\frac{2kT}{M}} = 9.91\times10^{2}\ \frac{m}{s}$

$$\Rightarrow B = \frac{M v_\perp}{e r_L} = 0.04\ T$$

HWK #2 EE580

3

4) a)



b) FOR AN INFINITE WIRE $E \propto \frac{1}{r}$ so $\phi = -\int E\,dr = $
$= C_1 \ln\left(\frac{r}{r_W}\right) + C_2$

@ $r = r_W = 1\times10^{-3}\,m$ $\phi = 460\,V$

@ $r = 0.1\,m$ $\phi = 0$

$C_1 \ln(1) + C_2 = 460V \Rightarrow C_2 = 460V$

$C_1 \ln(100) + C_2 = 0V \Rightarrow C_1 = -100\,V$

$\phi(r) = 460V - 100V \ln\left(\frac{r}{r_W}\right)$

$E(r) = -\frac{\partial\phi}{\partial r} = \frac{100}{r}\frac{V}{m}$

$E(r = 1cm) = 1\times10^4\,V/m$

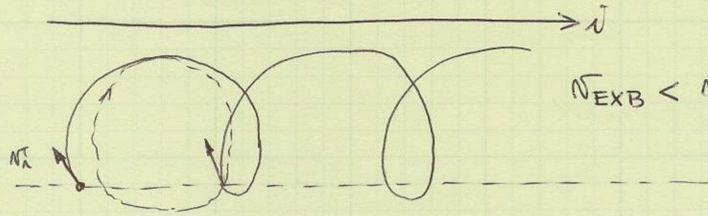
$B = \frac{\mu_0 i}{2\pi r}$ $\quad B(r=1cm) = \frac{4\pi\times10^{-7}\,H/m\ \ 500\,A}{2\pi\ (0.01m)} = 0.01\,T$

$v_E = \frac{E}{B} = 1\times10^6\,\frac{m}{s}$

$$\bar{v}_{\nabla B} = -\frac{1}{2} v_\perp r_L \frac{\bar{B} \times \bar{\nabla} B}{B^2} \Rightarrow v_{\nabla B} = \frac{1}{2} \frac{v_\perp^2}{\omega_c} \left| \frac{\nabla B}{B} \right|$$

HOW DO WE ESTIMATE $v_\perp$?

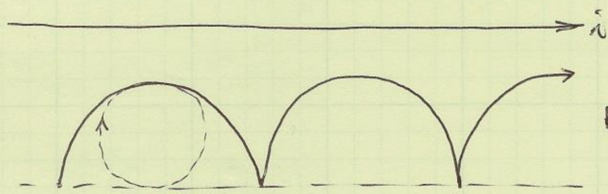
IF v_i FOR THE ELECTRON AT $t=0$ WERE $\neq 0$



$v_{E\times B} < v_\perp$

(UNDISTURBED)

BUT THE ELECTRON IS AT REST!



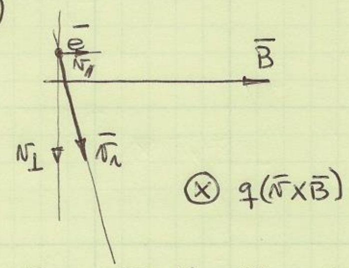
$v_{E\times B} = v_\perp$

$$\omega_c = \frac{eB}{m} = 1.76 \times 10^9 \ \frac{1}{s}$$

$$\frac{dB}{dr} = -\frac{\mu_0 i}{2\pi r^2} = -\frac{B}{r} \quad ; \quad \left| \frac{\nabla B}{B} \right|_{r=0.01} = 100 \ \frac{1}{m}$$

$$v_{\nabla B} = \frac{v_E^2}{2\omega_c} \left| \frac{\nabla B}{B} \right| = \frac{10^{12}\, m^2/s^2 \ 100 \ 1/m}{2 \ 1.76 \times 10^9 \ \frac{1}{s}} = 2.84 \times 10^4 \ \frac{m}{s}$$

5)



$$E = 2\,keV = \frac{1}{2} m v_i^2$$

$$v_i = \sqrt{\frac{2E}{m}} = 2.65 \times 10^7 \frac{m}{s}$$

$$v_\perp = v_i \sin 87 = 2.646 \times 10^7\, m/s$$

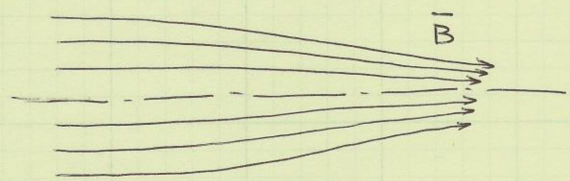
$$v_\parallel = v_i \cos 87 = 1.38 \times 10^6\, m/s$$

$$r_L = \frac{m_e v_\perp}{e B} = \frac{0.9 \times 10^{-30}\, kg \;\; 2.646 \times 10^7\, m/s}{1.6 \times 10^{-19}\, C \;\; 0.001\, T} = 0.149\, m$$

$$\omega_e = \frac{qB}{m_e} = 1.78 \times 10^8 \frac{1}{s}$$

$$d = \text{distance between turns} = v_\parallel \frac{2\pi}{\omega} = 0.05\, m$$

6)



$$\bar{F} = q\, \bar{v} \times \bar{B} = q \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ \cancel{v_r}^{\,0} & v_\theta & v_z \\ B_r & \cancel{B_\theta}^{\,0} & B_z \end{vmatrix} = q\left(v_\theta B_z \hat{r} + B_r v_z \hat{\theta} - v_\theta B_r \hat{z}\right)$$

F_z IS THE FORCE THAT STOPS THE PARTICLE.

WE CAN OBTAIN B_r FROM $\bar{\nabla} \cdot \bar{B} = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$$

$$r B_r = - \int_0^r r' \left(\frac{\partial B_z}{\partial z}\right) dr' \cong -\frac{1}{2} r^2 \left[\frac{\partial B_z}{\partial z}\right]$$

ASSUMING THAT $\frac{\partial B_z}{\partial z}$ DOES NOT CHANGE MUCH WITH r

SO $$B_r = -\frac{1}{2} r \frac{\partial B_z}{\partial z}$$

AND THE STOPPING FORCE IS $F_z = \frac{1}{2} e r_L v_\theta \left(\frac{\partial B_z}{\partial z}\right)$

THE EQUATIONS OF MOTION IN z WILL BE

$$x(t) = x_0 + v_0 t - \frac{1}{2} a t^2 \qquad a = \frac{F_z}{m}$$

$$v(t) = v_0 - at \qquad v_0 = v_z$$

AT STOPPING TIME t_s $v(t) = 0$ $\qquad x_0 = 0$

SO $t_s = \frac{m v_z}{F_z}$ AND

$$d = v_z \frac{m v_z}{F_z} - \frac{1}{2}\frac{F_z}{m}\left(\frac{m v_z}{F_z}\right)^2 =$$

$$= \frac{1}{2}\frac{m v_z^2}{F_z} = \frac{\cancel{1}}{\cancel{2}} \frac{m v_z^2}{\frac{\cancel{1}}{\cancel{2}} e r_L v_\theta \left(\frac{\partial B_z}{\partial z}\right)} =$$

$$= \frac{m v_z^2}{e r_L v_\theta \left(\frac{\partial B_z}{\partial z}\right)}$$

7) $F_z = \frac{1}{2} e r_L v_\theta \left(\frac{\partial B}{\partial z}\right) =$

$= \frac{1}{2} \, 1.6\times10^{-19}\,C \;\; 100m \;\; 10^{6}\frac{m}{s} \;\; 10^{-13}\frac{T}{m} = 8\times10^{-25}\,N$

$t = \frac{m v_z}{F_z} = \frac{0.9\times10^{-30}\,kg \;\; 4\times10^{5}\,m/s}{8\times10^{-25}\,N} = 0.45\,s$

$d = \frac{1}{2}\frac{m v_z^2}{F_z} = 9\times10^{4}\,m$