EE507 – Plasma Physics and Applications

Solutions to homework #1

1. $$\lambda_D = \left(\frac{\varepsilon_0 k T_e}{n_e e^2}\right)^{1/2}$$

$$N = \frac{4}{3}\pi \lambda_D^3 n_e = \frac{4}{3}\pi \left(\frac{\varepsilon_0 k T_e}{e^2}\right)^{3/2} \frac{1}{\sqrt{n_e}}$$

	$\sim T_e$ (eV)	$\sim n_e$ (cm^{-3})	λ_D (cm)	N
EARTH IONOSPHERE	0.02	10^5	0.33	1.5×10^4
GAS NEBULA	1	10^4	7.4	1.7×10^7
SOLAR WIND	10	10	744	1.7×10^{10}
FLAMES	0.1	10^8	0.023	5.4×10^3
GLOW DISCHARGE	3	10^9	0.041	2.8×10^5
SOLAR CORONA	100	10^9	0.23	5.4×10^7
ALKALI METAL PLASMA	0.3	10^{12}	4×10^{-4}	2.8×10^2
LOW PRESSURE ARC	3	10^{13}	4×10^{-4}	2.8×10^3
FUSION EXPERIMENTS	1000	10^{13}	7.4×10^{-3}	2×10^7
HIGH PRESSURE ARCS	1	10^{18}	7.4×10^{-7}	2
MAGNETIC FUSION REACTOR	20000	10^{15}	3.3×10^{-3}	1.5×10^8
LASER PLASMA	100	10^{21}	2.35×10^{-7}	54
Z - PINCH	300	10^{21}	4×10^{-7}	2.8×10^2

2. $$U = \frac{e^2 n_e^{1/3}}{4\pi\varepsilon_0} \qquad K = \frac{3}{2} k T_e$$

ASSUME $U \ll K$ THEN

$$\frac{e^2 n_e^{1/3}}{4\pi\varepsilon_0} \ll \frac{3}{2} k T_e \ ;$$

$$1 \ll \left(\frac{3}{2} k T_e \frac{4\pi\varepsilon_0}{e^2 n_e^{1/3}}\right)^{1/2} ;$$

$$1 \ll \left(\frac{\varepsilon_0 k T_e}{e^2 n_e}\right)^{1/2} n_e^{1/3} \sqrt{6\pi} \; ;$$

$$\frac{1}{(6\pi)^{3/2}} \ll \lambda_D^3 \, n_e$$

SO IF $\lambda_D^3 n_e \gg 1$ FOR SURE $U \ll K$.

3\. a) $\varepsilon_0 \dfrac{d^2\phi}{dx^2} = -qn$

$$\frac{d\phi}{dx} = \int \frac{d^2\phi}{dx^2} = -\frac{qn}{\varepsilon_0} x + C_1$$

$$\phi = \int \frac{d\phi}{dx} = -\frac{qn}{2\varepsilon_0} x^2 + C_1 x + C_2$$

BOUNDARY CONDITIONS $\quad \phi(d) = \phi(-d) = 0$

$$\text{SO} \quad C_1 = 0 \;, \; C_2 = \frac{qnd^2}{2\varepsilon_0}$$

$$\text{AND} \quad \phi = \frac{qn}{2\varepsilon_0}(d^2 - x^2)$$

b) FOR $d = \lambda_D$

$$U = q\phi(x=0) = q^2 \frac{n\lambda_D^2}{2\varepsilon_0} = \frac{\cancel{q^2}\cancel{n}}{2\cancel{\varepsilon_0}} \frac{\cancel{\varepsilon_0} kT_e}{\cancel{n}\cancel{q^2}} = \frac{kT_e}{2} = K$$

SO FOR $d > \lambda_D$, $U > K$.

4\. $$n_e = n_\infty \, e^{\frac{e\phi}{kT_e}}$$

$$\text{IF} \quad \frac{e\phi}{kT_e} \ll 1 \;\Rightarrow\; n_e \cong n_\infty \left(1 + \frac{e\phi}{kT_e}\right)$$

IN SPHERICAL COORDINATES

$$\varepsilon_0 \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = -e(n_i - n_e)$$

FOR $r \to \infty$ $\quad n_i = n_e = n_\infty$

$$n_e \simeq n_\infty \left(1 + \frac{e\phi}{kT_e}\right)$$

$$\varepsilon_0 \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = e n_\infty \frac{e\phi}{kT_e} \qquad \text{WHEN } \frac{e\phi}{kT_e} \ll 1$$

LET'S TRY $\phi = C_1 e^{-kr}/r$

$$\frac{d\phi}{dr} = -kC_1 \frac{e^{-kr}}{r} - C_1 \frac{e^{-kr}}{r^2}$$

$$r^2 \frac{d\phi}{dr} = -kC_1 r e^{-kr} - C_1 e^{-kr}$$

$$\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = \cancel{-kC_1 e^{-kr}} + k^2 C_1 r e^{-kr} + \cancel{k C_1 e^{-kr}}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = k^2 C_1 \frac{r e^{-kr}}{r^2} = \frac{e^2 n_\infty C_1}{\varepsilon_0} \frac{e^{-kr}}{r} \frac{1}{kT_e}$$

$$\Rightarrow k^2 = \frac{e^2 n_\infty}{\varepsilon_0 k T_e} \qquad \Rightarrow k = \frac{1}{\lambda_D}$$

@ $r = a$ $\quad \phi = \phi_0$ $\quad$ SO $\quad C_1 = \dfrac{a\phi_0}{e^{-a/\lambda_D}}$

AND THE EXPRESSION FOR THE POTENTIAL $\phi(r)$ IS THEN

$$\phi = \frac{a\phi_0}{e^{-a/\lambda_D}} \frac{e^{-r/\lambda_D}}{r}$$

• $v_{RMS} = \sqrt{\int_0^\infty v^2 g(v)\,dv}$

$$\int_0^\infty v^2 g(v)\,dv = C_1 \int_0^\infty v^4 e^{-C_2 v^2} dv = C_1 \frac{3}{8} \pi^{1/2} C_2^{-5/2} =$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3}{8} \pi^{1/2} \left(\frac{2kT}{m}\right)^{5/2} =$$

$$= \frac{3kT}{m}$$

SO $\quad v_{RMS} = \sqrt{\frac{3kT}{m}}$

ALSO $\quad \left(\frac{1}{2} m v^2\right)_{RMS} = \left(\frac{1}{2} m v_x^2\right)_{RMS} + \left(\frac{1}{2} m v_y^2\right)_{RMS} + \left(\frac{1}{2} m v_z^2\right)_{RMS} =$

$$= \frac{3}{2} kT$$

SO $\quad v_{RMS} = \sqrt{\frac{3kT}{m}}$