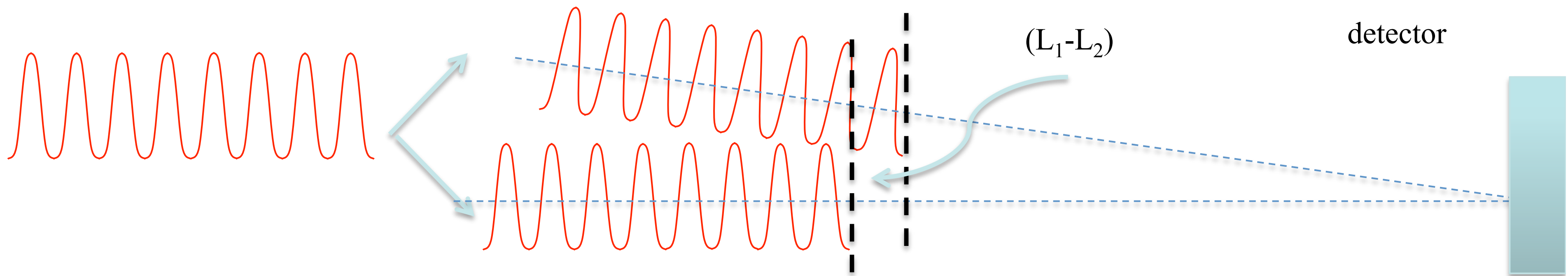
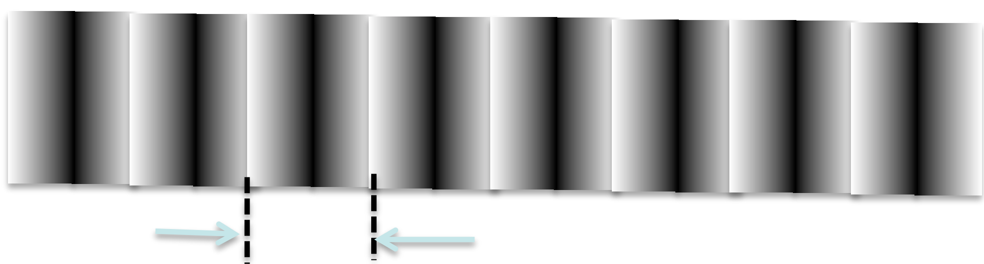
Temporal (longitudinal) coherence

Perfectly coherent field = the phase can be determined in time and space



Suppose a field that is separated into two branches by an interferometer (any). The two fields (beams) are delayed by a distance (L_1-L_2) and re-combined with a certain small angle Θ to produce interference fringes



$$\Delta x = \frac{\lambda}{2\sin\theta} \cong \frac{\lambda}{2\theta}$$

$$E_1 = \frac{E_0}{\sqrt{2}}\exp\left[-j\left(k\cos\frac{\theta}{2}z + k\sin\frac{\theta}{2}x\right)\right]\exp\left(-j2kL_1\right)\exp\left(-j\Delta\phi\right)$$

Depends only of t, phase jump during the time that the signal is delayed

$$E_2 = \frac{E_0}{\sqrt{2}}\exp\left[-j\left(k\cos\frac{\theta}{2}z - k\sin\frac{\theta}{2}x\right)\right]\exp\left(-j2kL_2\right)$$

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$$E_1 = \frac{E_0}{\sqrt{2}} \exp\left[-j\left(k\cos\frac{\theta}{2}z + k\sin\frac{\theta}{2}x\right)\right]\exp\left(-j2kL_1\right)\exp\left(-j\Delta\varphi\right)$$

$$E_2 = \frac{E_0}{\sqrt{2}} \exp\left[-j\left(k\cos\frac{\theta}{2}z - k\sin\frac{\theta}{2}x\right)\right]\exp\left(-j2kL_2\right)$$

The intensity in the detector plane is obtained combining these two fields

$$I(x,y) = \left(E_1 + E_2\right)\bullet\left(E_1 + E_2\right)* = \left(\frac{E_0^2}{\eta_0}\right)\cos^2\left[\left(\underbrace{k\left(L_2 - L_1\right) + k\frac{\theta}{2}x}_{\text{Depends on the alignment}} + \frac{\Delta\phi}{2}\right)\right]$$

Random phase

The random phase will make the position of the black and bright fringes wander. The effect is that the fringes will be blurred (because the typical time for the phase change is much more smaller than the typical integration time of the detector). The "blurring" of the fringes is evaluated with the "visibility" function defined as

$$V = \frac{\langle I_{\max}\rangle - \langle I_{\min}\rangle}{\langle I_{\max}\rangle + \langle I_{\min}\rangle}$$

The visibility varies from 0 to 1. It measures how deep the field is modulated in the interference pattern

Temporal (longitudinal) coherence

The fringes will blur completely when the phase change due to the random term is π (the bright fringes will fall in the dark fringes positions and vice versa)
For example if we assume that the change in the phase is

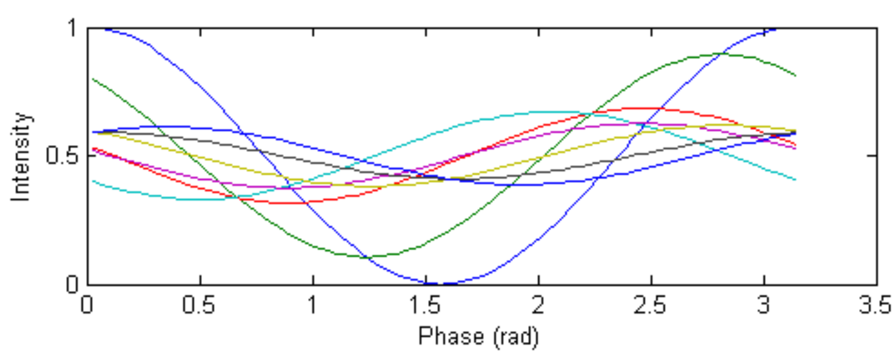
$$\frac{d\phi}{dt} \cong 10^{-5}\,\omega \quad \rightarrow \quad \Delta\phi = \frac{d\phi}{dt}\Delta t = \pi \quad \Rightarrow \quad 10^{-5}\,\frac{2\pi c}{\lambda}\,\frac{2\left(L_2 - L_1\right)}{c} = \pi$$

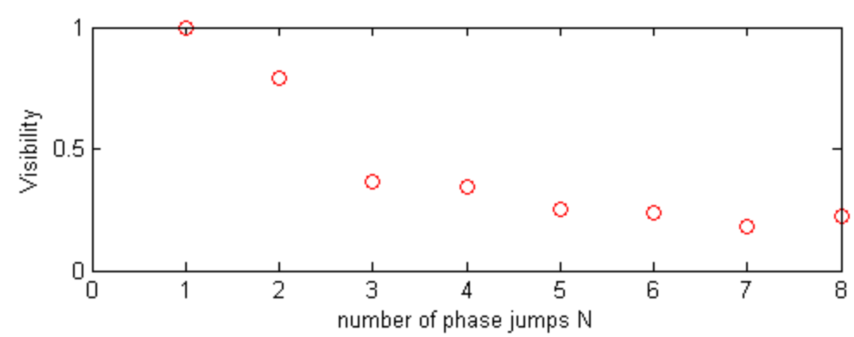
That makes $\left(L_2 - L_1\right) = 10^5\,\dfrac{\lambda}{4}$ that for a wavelength of 5000Å (0.5 μm) yields 2.5 cm (1 in.)

Example: Superpose two waves with the same amplitude, polarization and frequency, but let us allow the phase to randomly jump every T seconds. In the short exposure the integrated intensity yields perfect fringes. For longer exposures (NT seconds) the integrated intensity will yield:

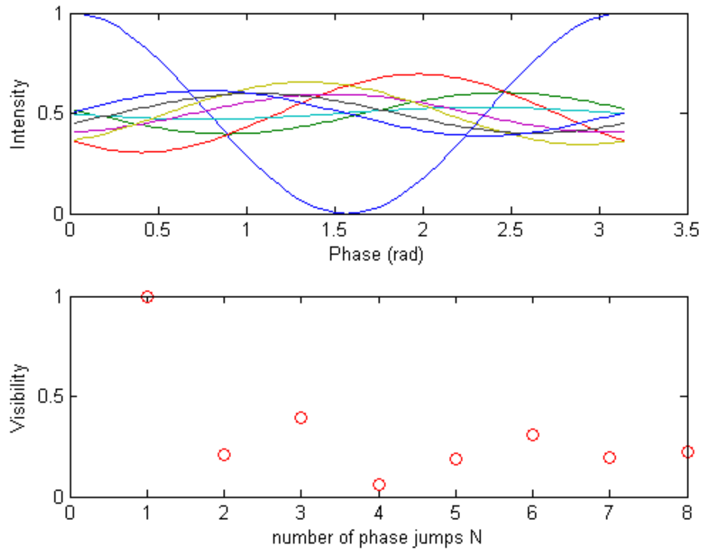
$$D_N = \frac{1}{NT}\left[\mathrm{I}_{\max}\,T\cos^2\left(\frac{k\,\theta_x}{2} + \frac{\phi_0}{2}\right) + \mathrm{I}_{\max}\,T\cos^2\left(\frac{k\,\theta_x}{2} + \frac{\phi_0 + \phi_1}{2}\right) + \mathrm{I}_{\max}\,T\cos^2\left(\frac{k\,\theta_x}{2} + \frac{\phi_0 + \phi_1 + \phi_2}{2}\right) + \ldots + \mathrm{I}_{\max}\,T\cos^2\left(\frac{k\,\theta_x}{2} + \frac{\phi_0 + \phi_1 + \ldots + \phi_{N-1}}{2}\right)\right]$$

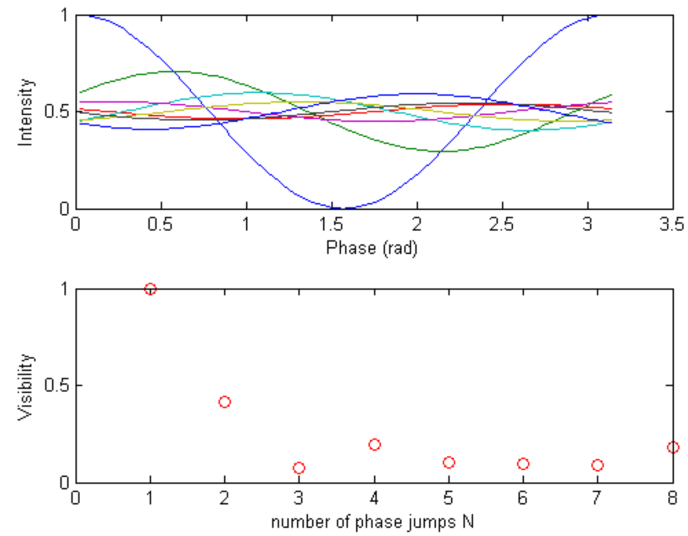
$$V = \frac{\langle I_{\max}\rangle - \langle I_{\min}\rangle}{\langle I_{\max}\rangle + \langle I_{\min}\rangle}$$

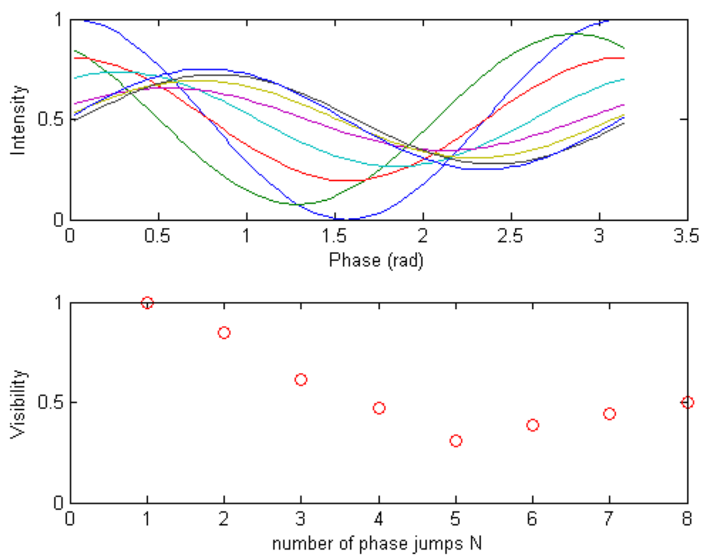


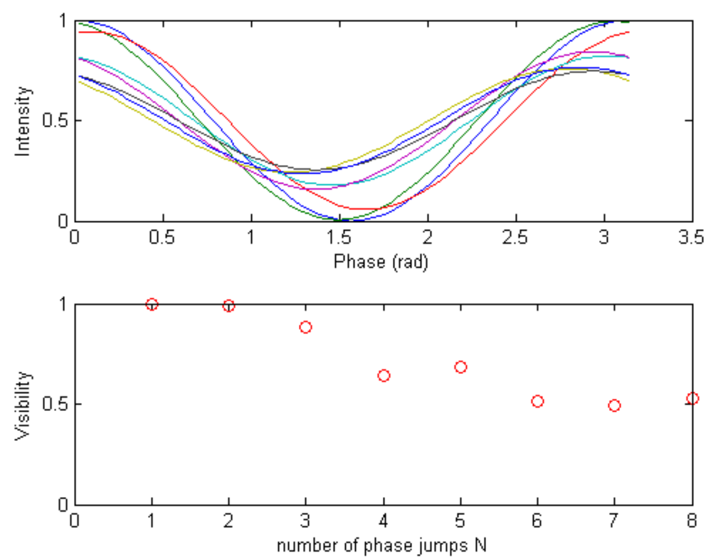


Temporal (longitudinal) coherence



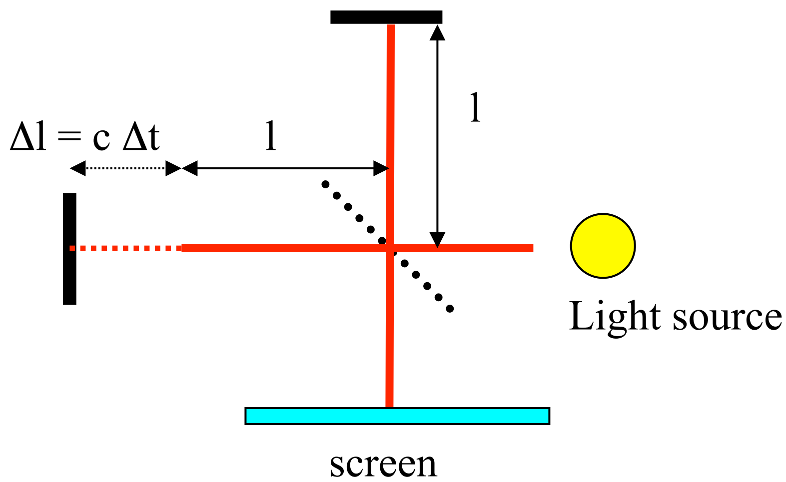




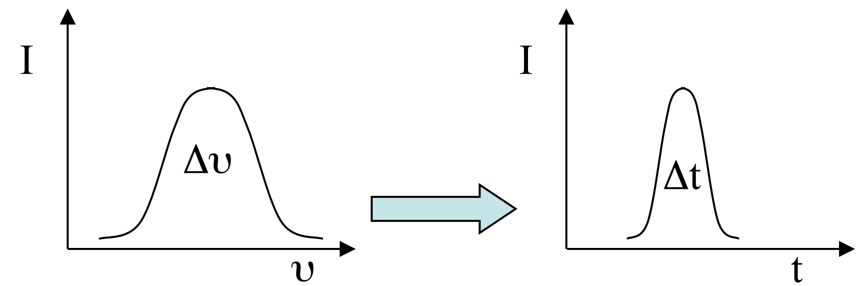


Temporal (longitudinal) coherence

To evaluate coherence we use an interferometer



$\Delta\omega\, \Delta t \geq \frac{1}{2} \rightarrow \Delta t \approx 1 / \Delta\upsilon$



If **c Δ t** > **Δ l** light pulses will not overlap and <u>interference will not be visible</u>

Coherence length **Δ l** = **c Δ t ≈ c / Δυ** → $\mathbf{\Delta l = \lambda^2 / \Delta\lambda}$

λ Mean value for the wavelength
Δλ << **λ**

Basic idea:
* If the source is perfectly monochromatic there will be a single fringe system
* If the source is not monochromatic, each λ will generate its own fringe system with different period. Then the superposition will blur the fringes

Some numbers:

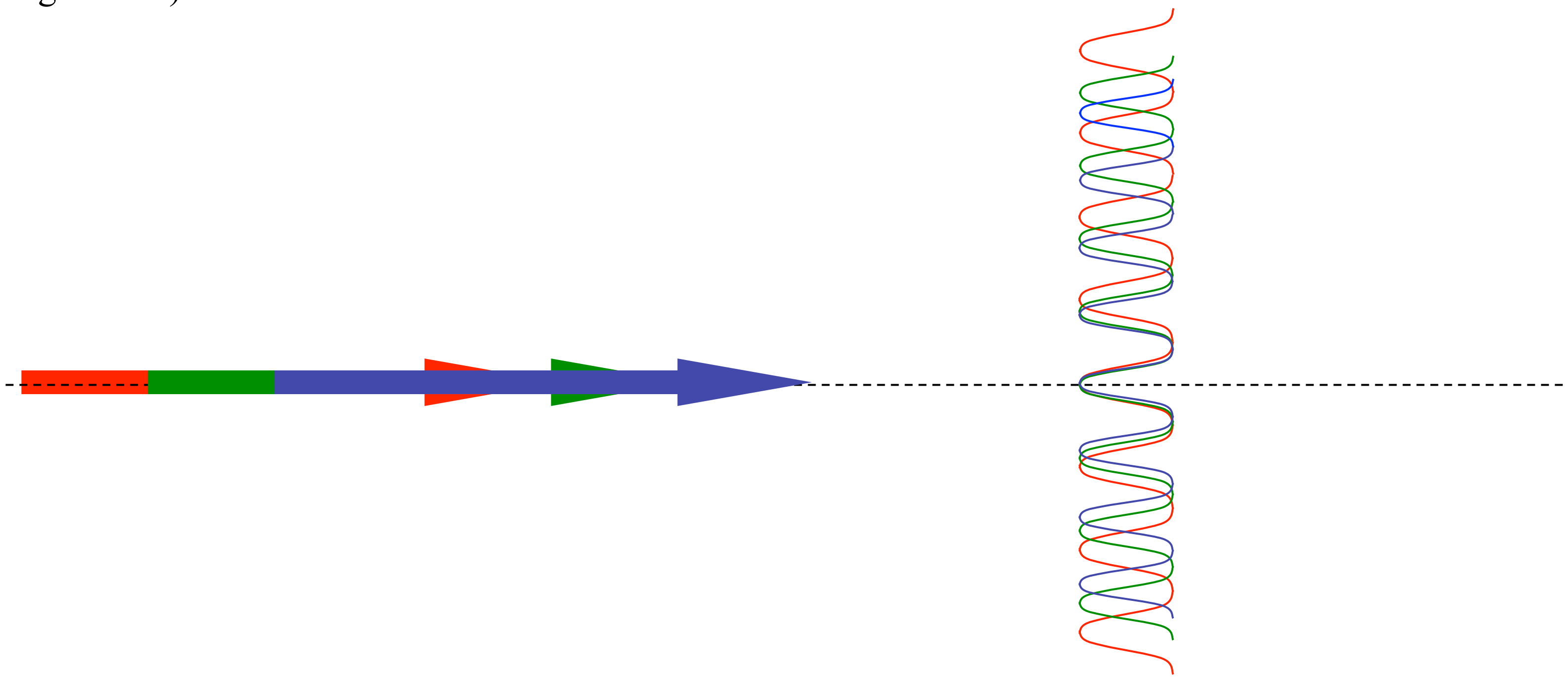
Na spectral lamp

$\Delta\upsilon \approx 10^8$ 1/s → Δ l = 3 m

Stabilized laser

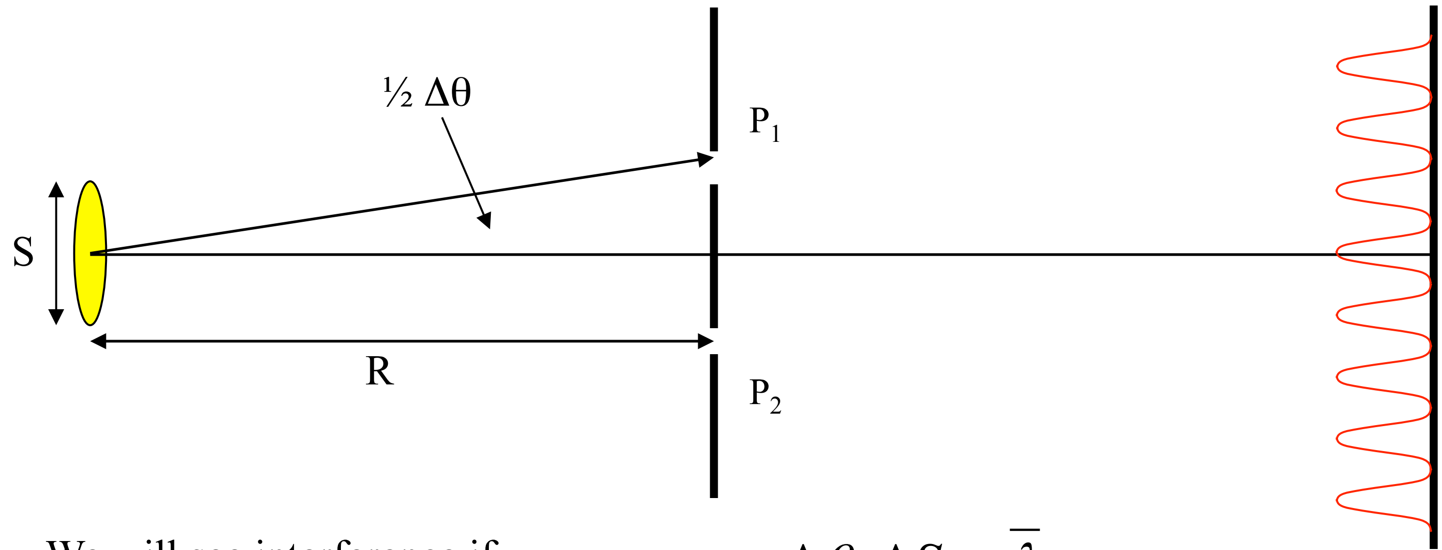
$\Delta\upsilon \approx 10^4$ 1/s → **Δ l = 30 km**

Temporal (longitudinal) coherence



Spatial (transverse) coherence

We use a double slit Young experiment



We will see interference if $\Delta\theta \;\; \Delta S \leq \bar{\lambda}$

Hence the interference will be visible if the holes are in an area $\Delta A \approx (R \;\, \Delta\theta)^2 \approx \dfrac{R^2 \;\, \lambda^2}{S}$

Coherence area

Source area

$\sqrt{\Delta A}$: transversal coherence length

The coherence length increases with R. What defines the coherence is actually the solid angle

$$\frac{\Delta A}{R^2}$$

Spatial (transverse) coherence

$$\Delta\Omega = \frac{\Delta A}{R^2} \cong \frac{\lambda^2}{S} \qquad \text{replacing} \qquad S = R^2\,\Delta\Omega'$$

Solid angle of the source calculated from the holes

$$\boxed{\Delta A \cong \frac{\lambda^2}{\Delta\Omega'}}$$

•Basic idea: two points in the source produces fringes shifted. If we increase the distance between the holes, the fringe separation is reduced. If we superpose these two fringe patterns the interference fringes get blurred

Example : Thermal source

$\Delta S = 1\ \ mm\ \ (source\ \ size)$

$\bar{\lambda} = 500\ \ nm$

$R = 2\ \ m$

$$\Delta A = \frac{R^2}{S}\lambda^2 = \frac{4\ \ (500\ \ 10^{-9})^2\ m^2}{(10^{-3})^2} = 1\,mm^2$$

Sun (filtering λ = 500 nm)

Sun's angular extension α = 4,65 mrad

$$\Delta\Omega' \cong \pi\,\alpha^2 \cong 3.14\,(4.65\,10^{-3})^2\ srad =$$

$$= 6.81\,10^{-5}\ srad$$

$$\Delta A = \frac{\lambda^2}{\Delta\Omega'} = \frac{(5\,10^{-5})^2\ cm^2}{6.81\,10^{-5}} \cong 3.6\,10^{-3}\ mm^2$$

→coherence length ≈ √ΔA = 61 μm

α-Orion (angular extension α = 2.3 10^{-7} rad)

$$\Delta\Omega' \cong \pi\,\alpha^2 \cong 4.15\,10^{-14}\ srad$$

$$\Delta A = \frac{\lambda^2}{\Delta\Omega'} = \frac{(5\,10^{-5})^2\ cm^2}{4.15\,10^{-14}} \cong 6\ m^2$$

→coherence length ≈ √ΔA = 2.45 m