

**PER UNIT CALCULATIONS****(A Decimal method of using percent)**

The per unit system is a method of normalizing system and equipment impedances so that circuit or network problems can be solved in a straightforward manner for voltages, currents and watts and vars without having to convert values whenever there is a change in voltage across a transformer or when a difference in equipment size is encountered. The per unit system can be applied to either single or three phase systems.

A system KVA or MVA base is chosen first. For quick applications a base of 1, 10 or 100 MVA is most convenient. This will be the 100% or 1 per unit value. Next, the line to line voltage (KV) or voltages are selected. These will usually be the nominal ratings on the major transformer at the substation or at the customer location on distribution systems. Again, each voltage value selected becomes the 100% or 1 per unit value. For the KVA and KV selected as base there will be a unique value of per unit base (or 100%) amperes and base ohms to match the base KVA selected.

To find the base values we first select base KVA and base KV (line to line). Then

$$\text{Base KVA} = \sqrt{3} \text{ Base KV}(\text{line to line}) \times I^*(\text{line})$$

$$\text{Base current } I^*(\text{line}) = \text{Base KVA} / \sqrt{3} \text{ Base KV}(\text{line to line})$$

As with any balanced three phase circuit, we can represent it on a line to neutral basis and consider an equivalent wye system. Remember that 100% or 1 per unit line to line voltage is also 100% or 1 per unit line to neutral voltage.

$$\text{Base ohms} = \text{Base V}(\text{line to neutral}) / \text{Base current}$$

$$= \frac{\text{Base KV}[\text{line to line}]}{K * \sqrt{3}} * \frac{\sqrt{3} \text{ Base KV}[\text{line to line}]}{\text{Base KVA}}$$

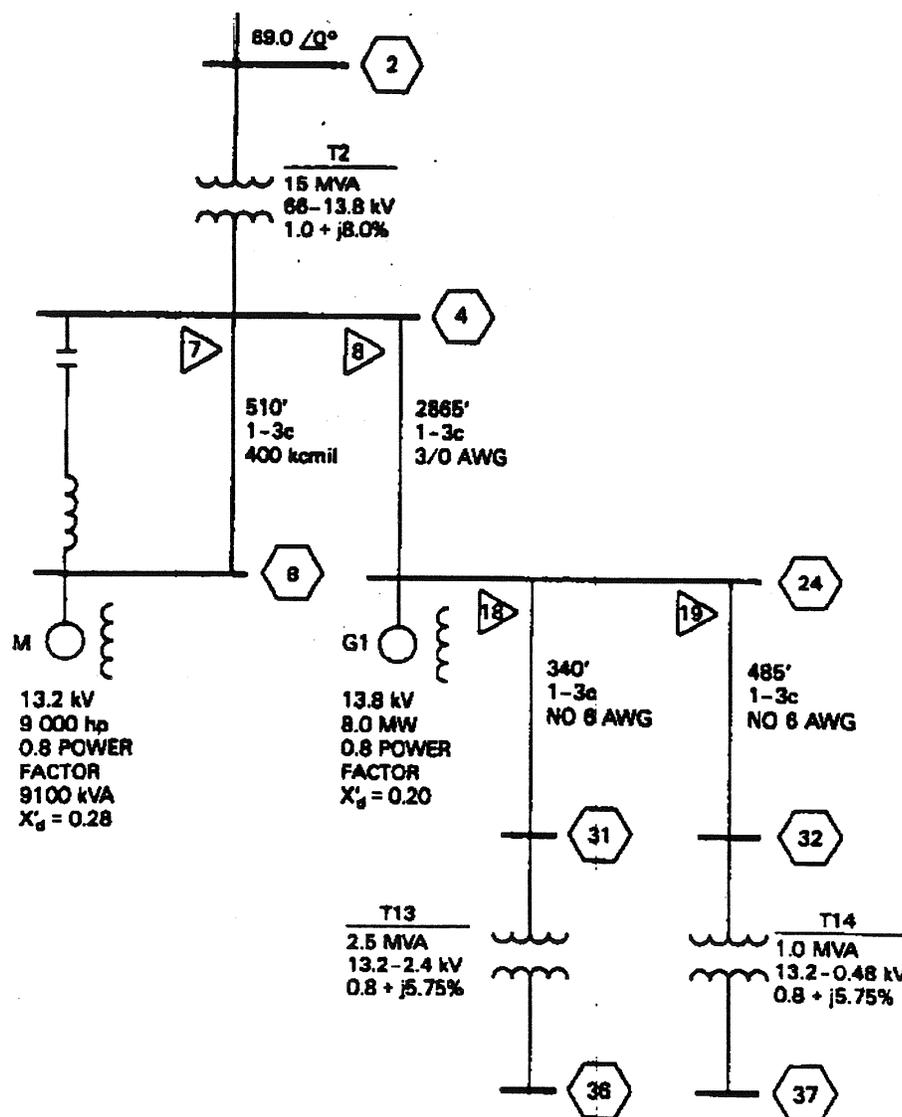
$$= \frac{\text{Base KV}^2[\text{line to line}]}{\text{Base MVA}}$$

To convert a generator or transformer to the base you selected, it is only necessary to remember that it requires a straight ratio.

$$\frac{\text{Nameplate ohms}}{\text{Nameplate KVA}} = \frac{\text{Per unit ohms}}{\text{Base KVA}}$$

### Problem #1

The steps in reducing the raw data to per unit are as follows:



Impedance Diagram Raw Data

- (1) Select base power:  $S = 10,000$  KVA
- (2) Determine base voltages
  - (a) Select bus 2 nominal voltage of 69 KV as base
  - (b) Calculate base voltages at other system levels

$$\begin{aligned} \text{Bus 4: KV} &= 69.0 * (13.8 / 66) && \text{where } 13.8 / 66 \text{ equals} \\ &= 14.427 \text{ KV} && \text{turns ratio of T2} \end{aligned}$$

$$\begin{aligned} \text{Bus 36: KV} &= 14.427 * (2.4 / 13.2) && \text{turns ratio of T13} \\ &= 2.623 \text{ KV} \end{aligned}$$

$$\begin{aligned} \text{Bus 37: KV} &= 14.427 * (0.48 / 13.2) && \text{turns ratio of T14} \\ &= 0.525 \text{ KV} \end{aligned}$$

(3) Calculate base impedances =  $(\text{base KV})^2 / \text{base MVA}$

$$\begin{aligned} \text{(a) 69 KV system:} \\ Z &= (69^2 * 10^3) / (10,000) \\ &= 476.1 \Omega \end{aligned}$$

$$\begin{aligned} \text{(b) 13.8 KV system:} \\ Z &= (14.427^2 * 10^3) / (10,000) \\ &= 20.82 \Omega \end{aligned}$$

$$\begin{aligned} \text{(c) 2.4 KV system:} \\ Z &= (2.623^2 * 10^3) / (10,000) \\ &= 0.688 \Omega \end{aligned}$$

$$\begin{aligned} \text{(d) 0.48 KV system:} \\ Z &= (0.525^2 * 10^3) / (10,000) \\ &= 0.02756 \Omega \end{aligned}$$

(4) Calculate base currents =  $\text{base KVA} / (\sqrt{3} * \text{base KV})$

$$\begin{aligned} \text{(a) 69 KV system:} \\ I &= 10,000 / (1.732 * 69.0) \\ &= 83.67 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{(b) 13.8 KV system:} \\ I &= 10,000 / (1.732 * 14.427) \\ &= 400.2 \text{ A} \end{aligned}$$

(c) 2.4 KV system:

$$I = 10,000 / (1.732 * 2.623)$$

$$= 2201 \text{ A}$$

(d) 0.48 KV system:

$$I = 10,000 / (1.732 * 0.525)$$

$$= 11,000 \text{ A}$$

(5) Summarize the base data in table

**System Base Values  
(Base Power 10 000 kVA)**

Bus	Base kV	Base Z	Base I
2	69.00	476.1	83.67
4	14.427	20.82	400.2
8	14.427	20.82	400.2
24	14.427	20.82	400.2
31	14.427	20.82	400.2
32	14.427	20.82	400.2
36	2.623	0.688	2201.0
37	0.525	0.027 56	11 000.0

(6) Convert transformer impedances to per unit using

$$Z_2 = Z_1 * ((\text{base KV}_1)^2 / (\text{base KV}_2)^2) * (\text{base KVA}_2) / (\text{base kVA}_1)$$

(a) T2:

$$Z = ((1.0 + j8.0) / 100) * (66^2 / 69^2) * (10 / 15)$$

$$= 0.00610 + j0.04880$$

(b) T13:

$$Z = ((0.8 + j5.75) / 100) * (2.4^2 / 2.623^2) * (10 / 15)$$

$$= 0.02679 + j0.19255$$

(c) T14:

$$Z = ((0.8 + j5.75) / 100) * (13.2^2 / 14.427^2) * (10 / 1)$$

$$= 0.06697 + j0.48135$$

(7) Calculate line impedance in ohms:  $Z = \text{actual ohms} / \text{base ohms}$   
 Lines 7, 8, 18, and 19 are 3/C, copper cables, paper insulated, shielded conductors; dielectric constant: 3.7

(a) Line 7:

$$\begin{aligned} Z &= (0.01515 + j0.01887) / 20.82 \\ &= 0.000728 + j0.000906 \text{ per unit} \end{aligned}$$

(b) Line 8:

$$\begin{aligned} Z &= (0.19138 + j0.12119) / 20.82 \\ &= 0.0009192 + j0.005821 \text{ per unit} \end{aligned}$$

$$\begin{aligned} Y &= Z_C / -jX_C = 20.82 / -j8630 \\ &= 0 + j0.002413 \text{ per unit} \end{aligned}$$

(c) Line 18:

$$\begin{aligned} Z &= (0.16864 + j0.02074) / 20.82 \\ &= 0.00810 + j0.001 \text{ per unit} \end{aligned}$$

(d) Line 19:

$$\begin{aligned} Z &= (0.24056 + j0.02959) / 20.82 \\ &= 0.01155 + j0.00142 \text{ per unit} \end{aligned}$$

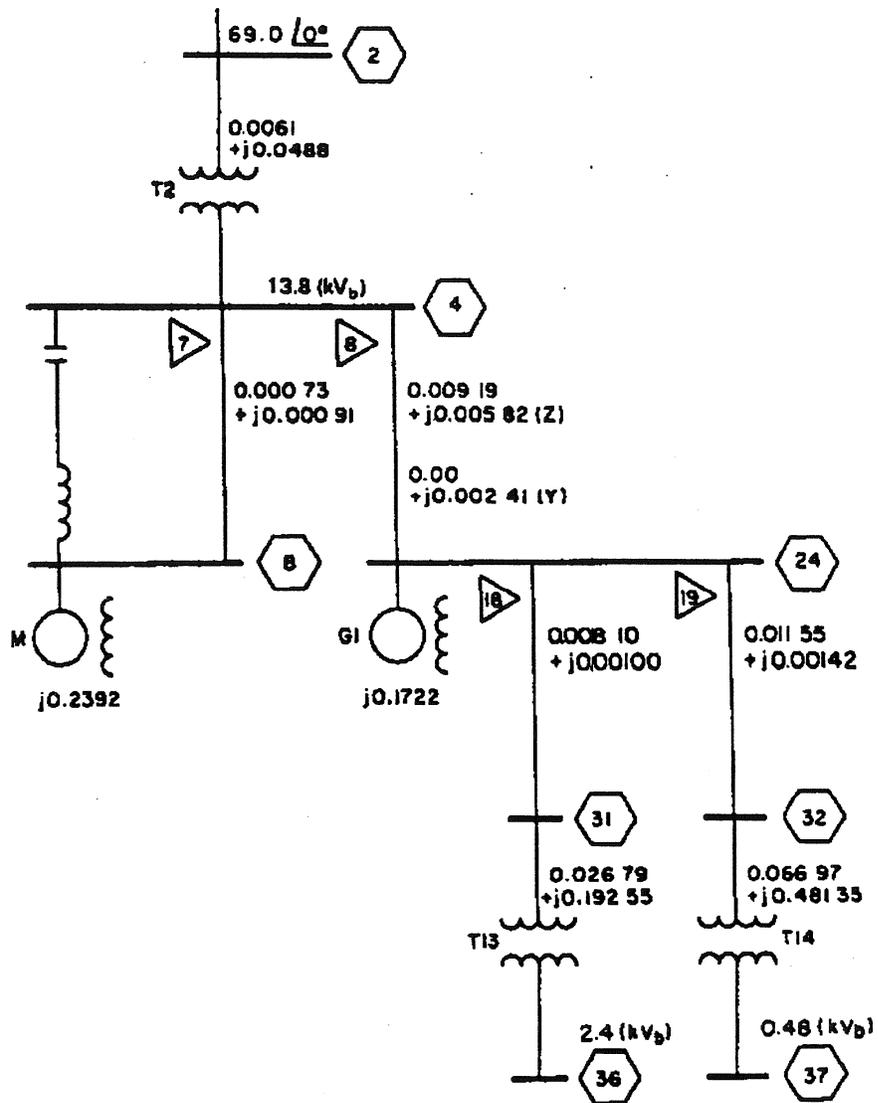
(8) Calculate  $X_d'$  of two synchronous machines in per unit using equation from (6)

(a) Synchronous motor on bus 8:

$$\begin{aligned} X_d' &= j0.28 * ((13.2^2) / (14.427^2)) * (10,000 / 9,800) \\ &= j0.2392 \end{aligned}$$

(b) Generator G1:

$$\begin{aligned} X_d' &= j0.20 * ((13.8^2) / (14.427^2)) * (10,000 / (8500/0.8)) \\ &= j0.1722 \end{aligned}$$



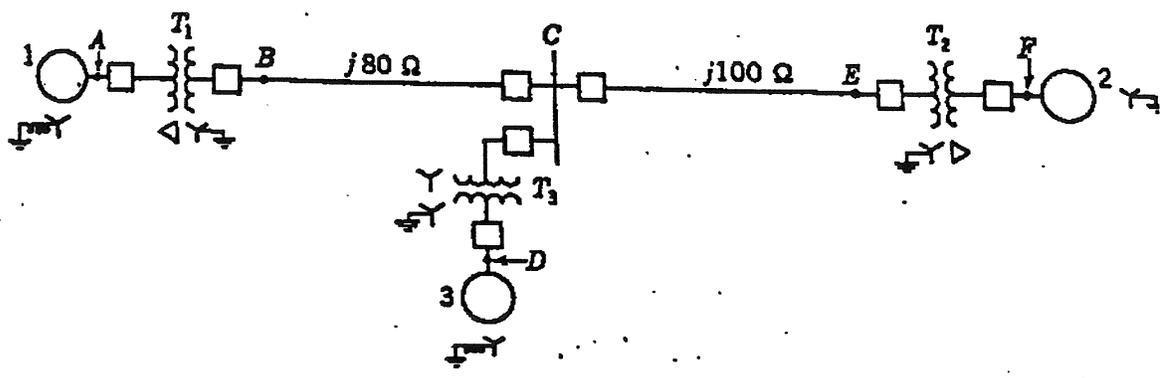
**Impedance Diagram Per Unit Data**  
**(Base MVA = 10)**

### Problem #2

The one-line diagram of an unloaded power system is shown below. Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are rated as follows:

- Generator 1: 20 MVA, 13.8 KV,  $X'' = 0.20$  per unit
- Generator 2: 30 MVA, 18 KV,  $X'' = 0.20$  per unit
- Generator 3: 30 MVA, 20 KV,  $X'' = 0.20$  per unit
- Transformer  $T_1$ : 25 MVA, 220Y/13.8 KV  $\Delta$ ,  $X = 10\%$
- Transformer  $T_2$ : Single phase units each rated 10 MVA, 127/18 KV,  $X = 10\%$
- Transformer  $T_3$ : 35 MVA, 220Y/22 KV Y,  $X = 10\%$

Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. Choose a base of 50 MVA, 13.8 KV in the circuit of generator 1.



Solution:

Gen. 1  $X'' = 0.2 \times \frac{50}{20} = 0.50 \text{ pu}$   
 3- $\phi$  rating  $T_2 = 220/10 \text{ kV}, 30 \text{ MVA}$   
 Base in trans. line:  $220 \text{ kV}, 50 \text{ MVA}$   
 Base for Gen. 2 is  $18 \text{ kV}$   
 Gen. 2  $X'' = 0.2 \times \frac{50}{30} = 0.333 \text{ pu}$   
 Base for Gen. 3 is  $22 \text{ kV}$   
 Gen. 3  $X'' = 0.2 \left( \frac{20}{22} \right) \times \frac{50}{30} = 0.275 \text{ pu}$   
 $T_1 - X = 0.1 \times \frac{50}{25} = 0.20 \text{ pu}$   
 $T_2 - X = 0.1 \times \frac{50}{30} = 0.167 \text{ pu}$   
 $T_3 - X = 0.1 \times \frac{50}{35} = 0.143 \text{ pu}$

Transmission Lines

$$\text{Base } Z = \frac{(220)^2}{50} = 968 \Omega$$

$$\frac{80}{968} = 0.0826 \quad \left. \vphantom{\frac{80}{968}} \right\} \frac{100}{968} = 0.1033$$

