Thus, the inductor should have an inductance of 0.24 H. If a larger inductance were used, the current ripple would be smaller, but the dc voltages and currents would remain the same.

21.39 Impedance transformation

So far, we have assumed that the converter feeds power to an active load E_0. However, it may also be used to connect a higher-voltage dc source E_S to a lower-voltage load resistor R_0 (Fig. 21.62). Equations 21.20 and 21.21 still apply, but we now have the additional relationship $E_0 = I_0R_0$. Furthermore, the apparent resistance R_S across the terminals of the source is given by

$$R_S = E_S/I_S$$

We can therefore write

$$R_S = E_S/I_S = \frac{E_0/D}{I_0 D} = \frac{E_0}{I_0 D^2}$$

$$\therefore R_S = \frac{R_0}{D^2} \qquad (21.22)$$

where

R_S = apparent resistance across the source [Ω]
R_0 = actual load resistance [Ω]
D = duty cycle

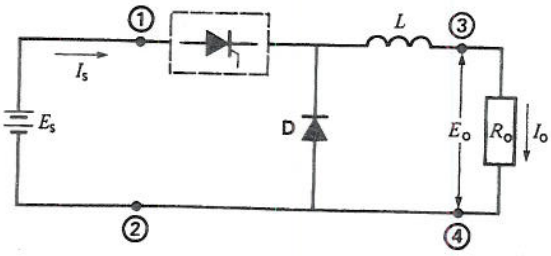


Figure 21.62
A chopper can make a fixed resistor R_o appear as a variable resistance between terminals 1-2.

The converter, therefore, has the ability to transform the resistance of a fixed resistor to a higher value whose magnitude depends upon the duty cycle. Thus, the chopper again behaves like a dc transformer in which the turns ratio is D.

Example 21-12

The chopper in Fig. 21.62 operates at a frequency of 4 kHz and the *on* time is 20 μs. Calculate the apparent resistance across the source, knowing that $R_0 = 12\ \Omega$.

Solution

The duty cycle is

$$D = T_a/T = T_a f = 20 \times 10^{-6} \times 4000 = 0.08$$

Applying Eq. 21.22, we have

$$R_S = R_0/D^2 = 12/(0.08)^2 = 1875\ \Omega$$

This example shows that the actual value of a resistor can be increased many times by using a chopper. Although a chopper can be compared to a transformer, there is an important difference between the two. The reason is that a transformer permits power flow in both directions—from the high-voltage side to the low-voltage side or vice versa. The step-down chopper we have just studied can transfer power only from the high-voltage side to the low-voltage side. Because power flow in both directions is often required, we now examine a dc-to-dc converter that achieves this result.

21.40 Basic 2-quadrant dc-to-dc converter

Consider Fig. 21.63a in which two *mechanical* switches S1 to S2 are connected across a dc voltage source E_H. The switches open and close alternately in such a way that when S1 is closed, S2 is open and vice versa. The time of one cycle is T, and S1 is closed for a period T_a. It follows that the duty cycle of S1 is $D = T_a/T$, while that of S2 is $(1 - D)$. When S1 is closed, terminal 1 is at the level of point 3 and

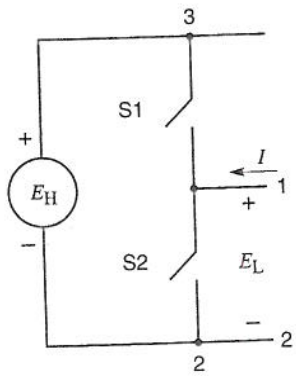


Figure 21.63a
Two-quadrant dc-to-dc converter.

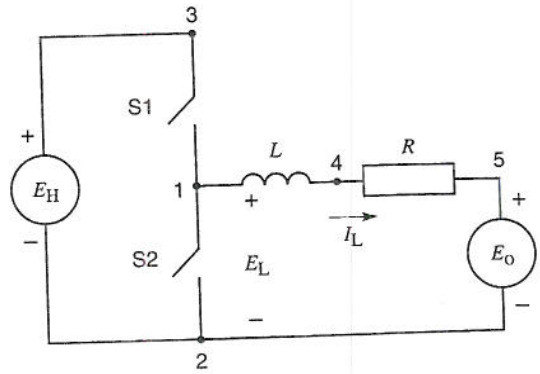


Figure 21.64
Power can flow from E_H to E_O and vice versa.

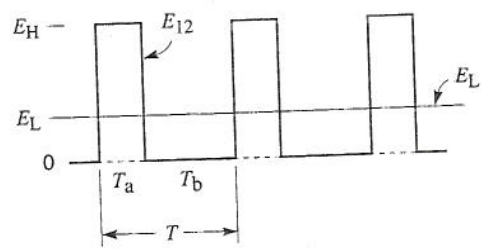


Figure 21.63b
Waveshape of E_{12} and average value E_L.

so the output voltage is $E_{12} = E_H$ for a period T_a. Then, when S1 is open, S2 is closed and so $E_{12} = 0$ for a period T_b. The output voltage oscillates, therefore, between E_H and zero (Fig. 21.63b) and its *average* dc value E_1 is given by Eq. 21.21

$$E_L = DE_H \tag{21.21}$$

By varying D from zero to 1, we can vary the magnitude of E_L from zero to E_H.

In examining Fig. 21.63a, it is apparent that the circuit on the left-hand side of terminals 1, 2 is never open. For example, if current I happens to flow into terminal 1, it can find its way back to terminal 2 either via S2 (if S2 is closed) or via S1 and source E_H if S2 is open. Because one of the switches is always closed, it is evident that current I can always circulate, no matter what its direction happens to be. This is a crucially important feature of the converter. It is called a *two-quadrant converter* because current I can flow in either direction, but the polarity of the dc voltage E_L remains fixed: Terminal 1 is always (+) with respect to terminal 2.

Suppose we want to transfer dc power from terminals E_{12} to a load such as a battery, whose dc voltage E_{52} has a value E_0 (Fig. 21.64). Knowing that E_{12} is fluctuating while E_0 is constant, it is essential to place a buffer between the two, otherwise short-circuit currents will result. We could place a resistor between points **1** and **5**, but that would involve I^2R losses which would reduce the efficiency of the converter. The best solution is to use an inductance L as shown in Fig. 21.64. It has the advantage of opposing ac current flow while offering no opposition to dc. We assume that the load has a small internal resistance R.

Suppose that both the voltage source E_H and the duty cycle D are fixed. Consequently, the dc component E_L between points **1** and **2** is constant. If E_0 is exactly equal to E_L, no dc current will flow and no dc power exchange will take place. But if E_0 is less than E_L, a dc current I_L will flow from terminal 1 into terminal 5. Its magnitude is given by

$$I_L = (E_L - E_0)/R \tag{21.23}$$

Power equal to $E_L I_L$ will, therefore, flow from terminals 1, 2 toward the battery. This dc power can

only come from the higher voltage source E_H. In this mode of operation, with E_0 less than E_L, the converter acts like the step-down (buck) chopper we covered in Section 21.37.

On the other hand, if E_0 is *greater* than E_L, a dc current I_L will flow out of terminal 5 and into terminal 1. Its magnitude is $I_L = (E_0 - E_L)/R$. Power now flows from the low-voltage battery side E_0 to the higher voltage side E_H. In this mode of operation, with E_0 greater than E_L, the converter acts like a step-up (boost) chopper.

The mechanical switching system of Fig. 21.63a is therefore able to transfer dc power in both directions—from high-voltage side to low-voltage side or vice versa. Again, because the current can reverse while the polarity of E_L remains the same, this buck/boost converter operates in two quadrants.

Let us now examine the behavior of the converter more closely, by means of an example.

$\frac{1}{20}$ms = 50 μs

Example 21-13

The following data is given on a buck/boost converter (Fig. 21.65):

$E_H = 100$ V $E_0 = 30$ V $R = 2\ \Omega$ $L = 10$ mH

switching frequency = 20 kHz with a duty cycle D of 0.2 for S1.

10μs

Determine the following:

a. The value and direction of the dc current I_L
b. The peak-to-peak ripple superposed on the dc current

Solution

Referring to Fig. 21.65, the value of $E_L = DE_H = 0.2 \times 100$ V = 20 V.

Because the battery voltage is greater than E_L, current I_L flows out of terminal 5 and into terminal 1. Its average value is

$$I_L = (30\text{ V} - 20\text{ V})/2\ \Omega = 5\text{ A}$$

The duration of one cycle is

$$T = 1/f = 1/20\,000 = 50\ \mu\text{s}$$

Thus, S1 is closed for a time $T_a = 0.2 \times 50\ \mu\text{s} = 10\ \mu\text{s}$ and S2 is closed for 40 μs.

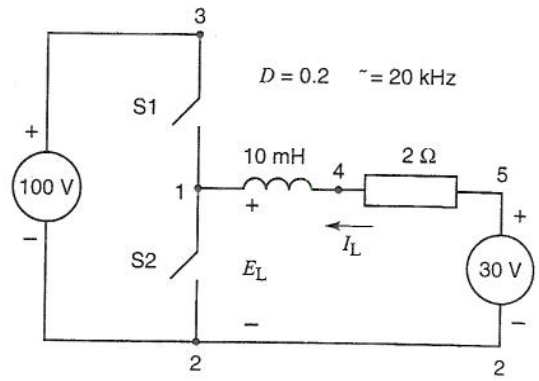


Figure 21.65
Circuit of Example 21-13.

start

To determine the peak-to-peak ripple, let us examine the situation when S2 is closed (Fig. 21.66). Assuming the current i is momentarily equal to its dc value of 5 A, the voltage E_{41} across the inductor is equal to the battery voltage minus the IR drop in the resistor: 30 V − 5 A × 2 Ω = 20 V. Knowing that I_L is flowing into terminal 4 and that terminal 4 is (+) with respect to terminal 1, it follows that I_L must be increasing. The inductor accumulates volt-seconds and during the 40 μs that S2 is closed, the magnetic "charge" totals 20 V × 40 μs = 800 V·μs. Therefore the current increases by an amount ΔI = 800 V·μs/10 mH = 0.08 A.

Let us now see what happens when S1 is closed and i is again momentarily 5 A (Fig. 21.67). The

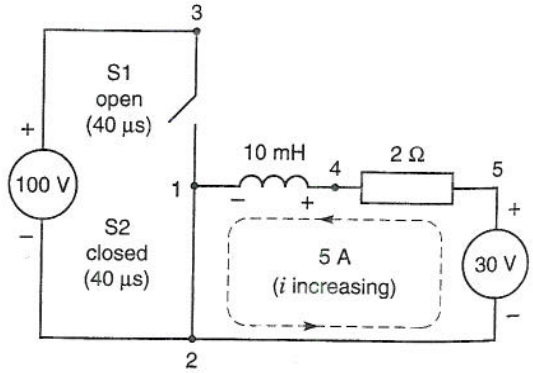


Figure 21.66
See Example 21-13.

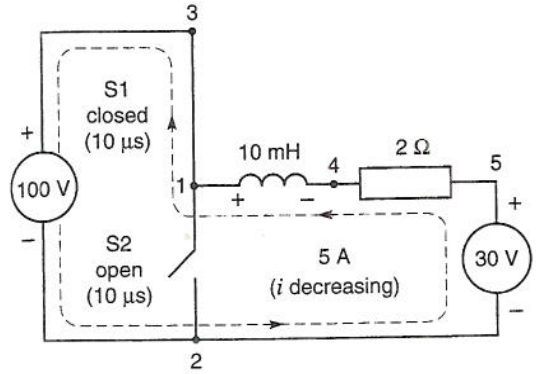


Figure 21.67
See Example 21-13.

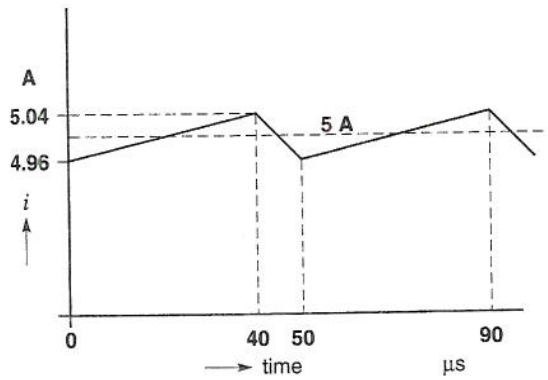


Figure 21.68

voltage across the inductor is now 100 V − (30 − 10) V = 80 V, but terminal 4 is *negative* with respect to terminal 1. The current i is therefore decreasing. The volt-seconds discharged during the 10 μs interval is 80 V × 10 μs = 800 V·μs. The change in current is ΔI = 800 V·μs/10 mH = 0.08 A.

We observe that the decrease in current when S1 is closed is the same as the previous increase when S1 was open. Consequently, the peak-to-peak ripple is 0.08 A. The dc current fluctuates between 5.04 A and 4.96 A (Fig. 21.68). Direct-current power flows from the lower voltage battery toward the higher voltage source. The converter is said to function in the boost mode.

Note that if the duty cycle were raised to 0.45, the value of E_L would increase to 100 × 0.45 = 45 V. The current flowing into the battery would reverse and its value would become

$$(45\text{ V} - 30\text{ V})/2\ \Omega = 7.5\text{ A}$$

Direct-current power now flows from the 100 V source toward the battery, causing the latter to charge up. Under these conditions, the converter is said to operate in the *buck* mode. Thus, the transition from boost to buck can be effected very smoothly by simply varying the duty cycle. Fig. 21.65 can be considered to be the mechanical equivalent of a buck/boost chopper.

21.41 Two-quadrant electronic converter

Figs. 21.66 and 21.67 show the direction of current flow in the case of a boost converter. If the converter operated in the buck mode, the currents would follow the same paths but in the opposite direction. With mechanical switches this creates no problem because they can carry current in either direction. But in the real world we have to deal with electronic switches, which inherently carry current in only one direction. Therefore, in order to get bidirectionality, diodes have to be placed in antiparallel with the respective semiconductor switches Q1 and Q2 (Fig. 21.69). The switch contacts are shown with an arrowhead to indicate the allowed direction of current flow. For example, when current flows into terminal 1, it can continue on to terminal 2 either by way of diode D1 and source E_H or by way of Q2, provided Q2 is closed.

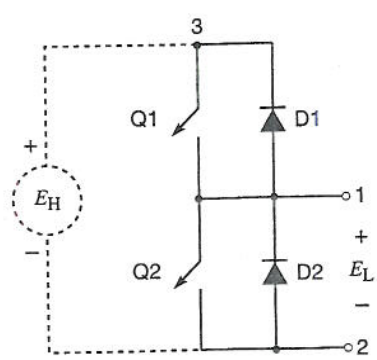


Figure 21.69
Two-quadrant electronic converter

Similarly if current flows out of terminal 1, it can take the path through diode D2 or the path through Q1 and E_H, provided that Q1 is closed.

In this figure, Q1 and D1 together perform the same way as the mechanical switch S1. Similarly, Q2 and D2 together perform the same way as mechanical switch S2. Fig. 21.69 therefore represents the essence of a 2-quadrant electronic converter. If a dc voltage E_H is applied between terminals 3 and 2, the converter generates a dc voltage E_L between terminals 1 and 2 and the relationship is again $E_L = DE_H$, where D is the duty cycle of Q1.

It is important to note that Q1 and Q2 cannot be closed at the same time, otherwise a short-circuit will result across source E_H. Thus, for a very brief period, called *dead time,* both switches must be open. The current is carried by one of the two diodes during this instant.

Power can be made to flow from the higher voltage side to the lower voltage side or vice versa. The power transported in one direction or the other depends upon the respective voltages and the duty cycle. The 2-quadrant converter of Fig. 21.69 is the basic building block for most switch-mode converters.

21.42 Four-quadrant dc-to-dc converter

limitation

The 2-quadrant converter we have studied can only be used with a load whose voltage has a specific polarity. Thus, in Fig. 21.69, given the polarity of E_H, terminal 1 can only be (+) with respect to terminal 2. We can overcome this restriction by means of a *4-quadrant converter.* It consists of two identical 2-quadrant converters arranged as shown in Fig. 21.70. Switches Q1, Q2 in converter arm A open and close alternately, as do switches Q3, Q4 in converter arm B. The switching frequency (assumed to be 100 kHz) is the same for both. The switching sequence is such that Q1 and Q4 open and close simultaneously. Similarly, Q2 and Q3 open and close simultaneously. Consequently, if the duty cycle for Q1 is D, it will also be D for Q4. It follows that the duty cycle for Q2 and Q3 is $(1 - D)$.

The dc voltage E_A appearing between terminals A, 2 is given by

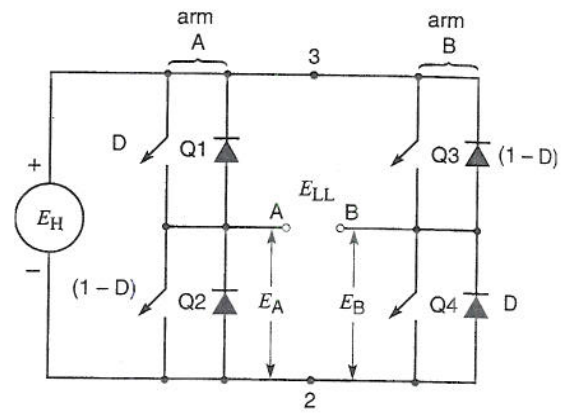


Figure 21.70
Four-quadrant dc-to-dc converter.

$$E_A = DE_H$$

The dc voltage E_B between terminals B, 2 is

$$E_B = (1 - D)E_H$$

The dc voltage E_{LL} between terminals A and B is the difference between E_A and E_B:

$$E_{LL} = E_A - E_B$$
$$= DE_H - (1 - D)E_H$$

thus

$$E_{LL} = E_H(2D - 1) \qquad (21.24)$$

Equation 21.24 indicates that the dc voltage is zero when $D = 0.5$. Furthermore, the voltage changes linearly with D, becoming $+E_H$ when $D = 1$, and $-E_H$ when $D = 0$. The polarity of the output voltage can therefore be either positive or negative. Moreover, if a device is connected between terminals A, B, the direction of dc current flow can be either from A to B or from B to A. Consequently, the converter of Fig. 21.70 can function in all four quadrants.

The *instantaneous* voltages E_{A2} and E_{B2} oscillate constantly between zero and $+E_H$. Fig. 21.71 shows the respective waveshapes when $D = 0.5$. Similarly, Fig. 21.72 shows the waveshapes when $D = 0.8$. Note that the instantaneous voltage E_{AB} between the output terminals A, B oscillates between $+E_H$ and $-E_H$. In practice, the alternating

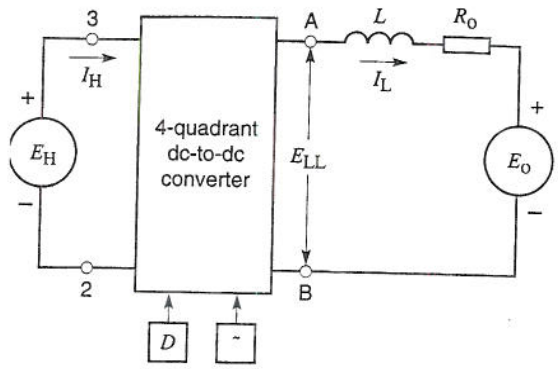


Figure 21.74
Four-quadrant dc-to-dc converter feeding an active dc source/sink E_o.

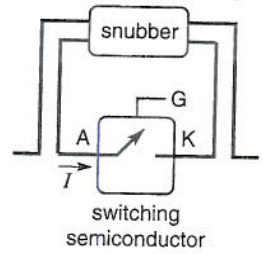
voltage level (high or low) and release it at another voltage level (low or high). And it performs this duty automatically, in response to the electronic switches and their duty cycle.

21.43 Switching losses

All semiconductor switches such as GTOs, MOSFETs, and IGBTs have losses that affect their temperature rise and switching efficiency. The switches all function essentially the same way, but to focus our analysis we assume the switching device is a GTO. The switching operation involves four brief intervals:

1. *Turn-on time* T_1: The current in the GTO is rapidly increasing while the voltage across it is rapidly decreasing.
2. *On-state time* T_2: The current has reached a stable value I_T, and the voltage V_T across the GTO is about 2 to 3 volts.
3. *Turn-off time* T_3: The current in the GTO is rapidly falling while the voltage across it is rapidly increasing.
4. *Off-state time* T_4: The current in the GTO is zero while the forward voltage is relatively high.

The sum of $T_1 + T_2 + T_3 + T_4$ is equal to the period T of one cycle which, in turn, is equal to $1/f_c$ where f_c is the switching frequency.



T_1 = turn-on time
T_2 = on-state time
T_3 = turn-off time
T_4 = off-state time
D = duty cycle = $\frac{T_2}{T}$

Figure 21.75a
Switching semiconductor and snubber.

During each interval the instantaneous power dissipated in the GTO is equal to the product of the instantaneous voltage across it times the instantaneous current that flows through it. However, we are mainly interested in the *average* power loss during one cycle because it determines the temperature rise of the GTO. The average power is equal to the energy dissipated in the GTO during one complete cycle, divided by T.

It so happens that the voltage across the GTO is substantial during the turn-on and turn-off periods, being far greater than the on-state voltage drop of 2 to 3 volts. As a result, the instantaneous power loss during these intervals can be very high.

Fig. 21.75a shows a GTO with its anode, cathode, and gate. In addition to the circuit that is being switched (not shown), a *snubber* is connected to the GTO. A snubber is an auxiliary circuit composed of R, L, C components (usually including semiconductor devices) that control the magnitude and rate of rise of the anode voltage E_{AK} as well as the anode current I. The purpose of a snubber is to aid commutation and to reduce the losses in the GTO.

Fig. 21.75b indicates the voltage, current, power, and energy associated with each of the four intervals of the switching operation. For example, during stage T_1 the instantaneous voltage across the GTO is V_1, the instantaneous current is I_1, and the average power dissipated is P_1. Consequently, the energy dissipated during this stage is P_1T_1 joules. On the other hand, the energy dissipated during interval T_4 is zero because the current is nil.

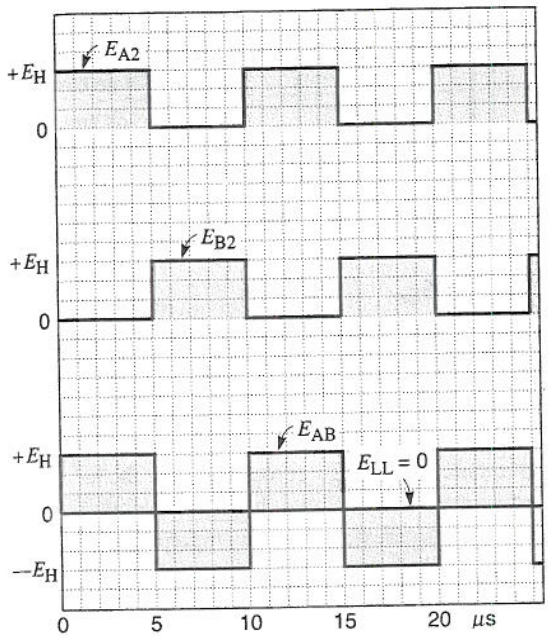


Figure 21.71
Voltage output when $D = 0.5$. The average voltage is zero.

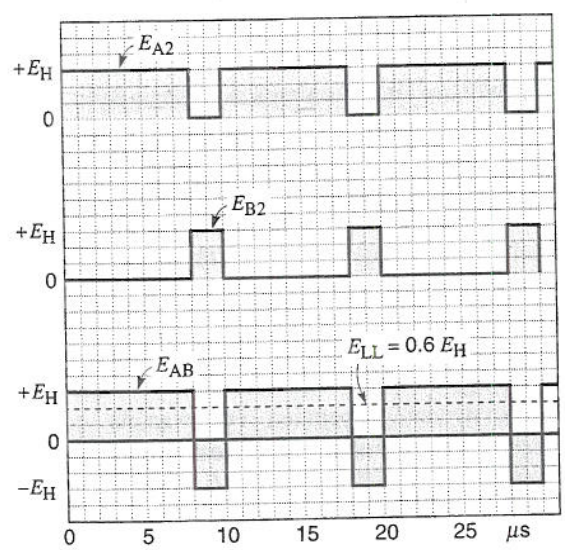


Figure 21.72
Voltage output when $D = 0.8$. The average voltage E_{LL} is 0.6 E_H.

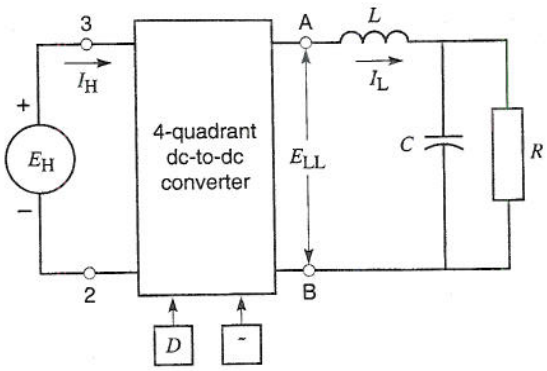


Figure 21.73
Four-quadrant dc-to-dc converter feeding a passive dc load R.

components that appear between terminals A, B are filtered out. Consequently, only the dc component E_{LL} remains as the active driving emf across the external device connected to terminals A, B.

Consider, for example, the block diagram of a converter feeding dc power to a passive load R (Fig. 21.73). The power is provided by source E_H. As we have seen, the magnitude and polarity of E_{LL} can be varied by changing the duty cycle D. The switching frequency f of several kilohertz is assumed to be constant. Inductor L and capacitor C act as filters so that the dc current flowing in the resistance has negligible ripple. Because the switching frequency is high, the inductance and capacitance can be small, thus making for inexpensive filter components.

The dc currents and voltages are related by the power-balance equation $E_H I_H = E_{LL} I_L$. We neglect the switching losses and the small control power associated with the D and f input signals.

Fig. 21.74 shows the converter connected to an active device E_0, which could be either a source or a load. If need be, the polarity of E_0 could be the reverse of that shown.

In all these applications we can force power to flow from E_H to E_0, or vice versa, by simply adjusting the duty cycle D. This 4-quadrant dc-to-dc converter is therefore an extremely versatile device.

The inductor L is a crucially important part of the converter. It alone is able to absorb energy at one