

Understanding Mixed Delta-Wye Transformers

461 Ch 12
Supplement
vP

I Four Cases

Δ -Y Step down

Y-D Step up

II Review of 1 ϕ Transformer Ckt. Model

III Y-D or Δ -Y Transformation Amplitude and Phase Effects

A. Phase

$$L_{\text{sec}} = L_{\text{prim}} - 30 \quad \text{Step-down}$$

$$L_{\text{sec}} = L_{\text{prim}} + 30 \quad \text{Step-up}$$

B Amplitude $\frac{\sqrt{3}}{N} \{ \}$

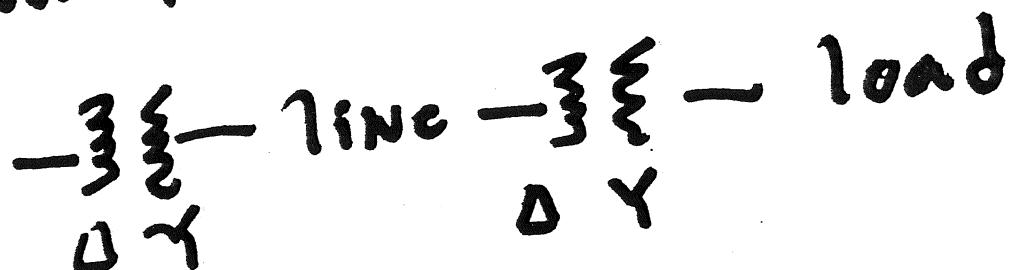
$$V_{\text{sec}} = \frac{V_{\text{prim}} N}{\sqrt{3}} \text{ for } Y-D$$

$$V_{\text{sec}} = V_{\text{prim}} N \sqrt{3} \text{ for } \Delta-Y$$

IV Primary ($\frac{\Delta}{Y}$) - Secondary ($\frac{\Delta}{Y}$)

(A) Load ($\frac{\Delta}{Y}$) : 8 Cases

IV (B) $\Delta - \gamma$: Series of two
step-down transformers & a load



V Generation - Transm - Distrib
Systems

A. Example #1

B Example #2

VI Transformers: Add. Info.

Understanding Mixed Δ-γ and γ-Δ Transformers

I

Ideal Three-Phase Transformer Connections

The ideal three-phase transformer is a bank of three ideal single-phase transformers. The primary and secondary of the ideal three-phase transformer is connected either delta or wye. The purpose of this section is to present the per-phase equivalent circuit models for various ideal three-phase transformer connections. There are six possible ideal transformer connections of interest in Table 5.1. By definition, the primary winding is the one connected to the electrical source and the secondary winding is the output winding. A step-up or unit transformer has a higher secondary voltage than the primary; and a step-down or power transformer has a lower secondary voltage than the primary.

Table 5.1 - Ideal Three-Phase Transformer Connections

<u>Primary</u>	<u>Secondary</u>	<u>Comment</u>	<u>Figure</u>
1. wye	wye	step-up or step-down	
2. delta	delta	step-up or step-down	
3. delta (low-voltage)	wye (high-voltage)	step-up	Fig. 5.2
4. delta (high-voltage)	wye (low-voltage)	step-down	Fig. 5.3
5. wye (high-voltage)	delta (low-voltage)	step-down	Fig. 5.4
6. wye (low-voltage)	delta (high-voltage)	step-up	Fig. 5.5

At this point, it is necessary to describe the notation that was used to derive the per-unit equivalent circuits of the ideal three-phase transformer connections in Figs. 5.2 through 5.5. \mathbb{V}_{ab} denotes the terminal line (i.e., phase a - to - phase b) voltage phasor of the primary. The subscript, "ab", refers to the reference polarity of phase a with respect to phase b. \mathbb{V}_{an} is the phase (i.e., phase a - to - neutral) voltage phasor of the primary. Note from Unit 4 that the line and phase voltage phasors are related by $\mathbb{V}_{ab} = \sqrt{3} \mathbb{V}_{an} \angle 30^\circ$ for a balanced, three-phase system. Corresponding secondary line and phase voltage phasors are distinguished with primes on the subscripts, namely $\mathbb{V}'_{a'b'}$ and $\mathbb{V}'_{a'n'}$, respectively.

\mathbb{I}_a and \mathbb{I}_{an} both denote the primary side line current phasor of phase a, where the former notation is reserved for a delta-connected primary and the latter notation is reserved for a wye-connected primary. Note that the assumed current reference arrows for primary side line current phasors are directed into the transformer. \mathbb{I}'_a and $\mathbb{I}'_{n'a'}$ both denote the corresponding secondary side line current phasor of phase a, where the former notation is reserved for a delta-connected secondary and the latter notation is reserved for a wye-connected secondary. Note that the assumed current reference arrows for secondary side line current phasors are directed out of the transformer.

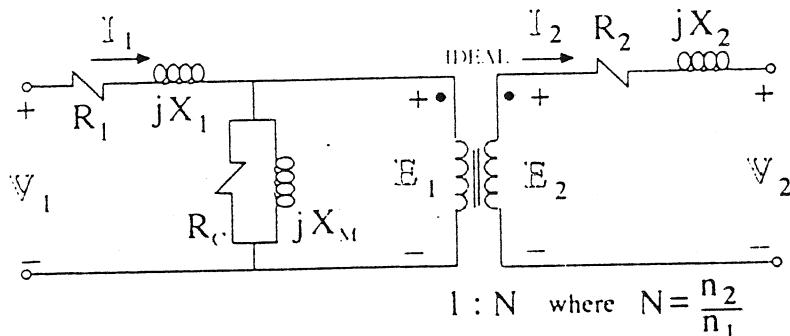
Next, the per-unit equivalent circuits of the ideal three-phase transformer connections are presented. Table 5.2 summarizes the results from Figs. 5.2 through 5.5 for the three-phase transformer connections of Table 5.1. In general, the per-unit equivalent circuit of an ideal three-phase transformer is depicted at the bottom of Figs. 5.2 through 5.5 as a box which contains an ideal single-phase transformer and a phase-shifting block. As a result, the box has an effective turns ratio, K, which is generally a complex number with a magnitude and phase angle. The magnitude of the complex turns ratio, $|K|$, is the turns ratio of the ideal single-phase transformer; and the phase angle of the complex turns ratio, $\angle K$, is the angle in the phase-shifting block.

Key

II Review of 1Φ Transformer Model

Figure 5.6 - Per Phase Equivalent Circuits for Single Phase Transformer

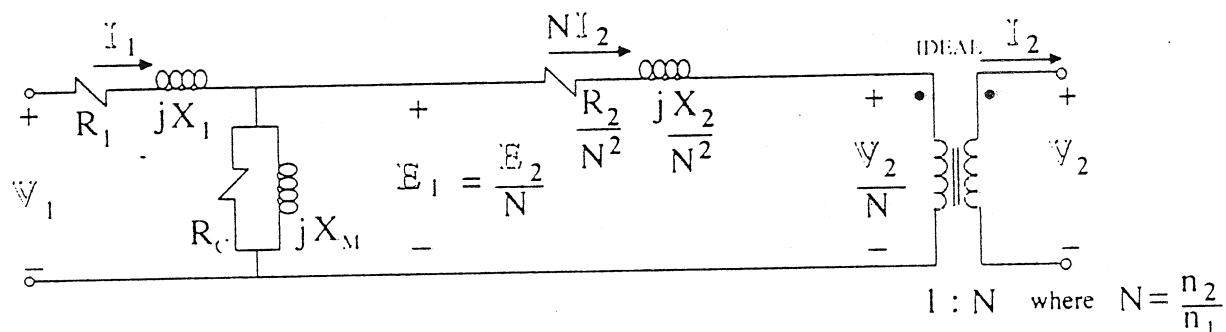
"Exact" Equivalent Circuit



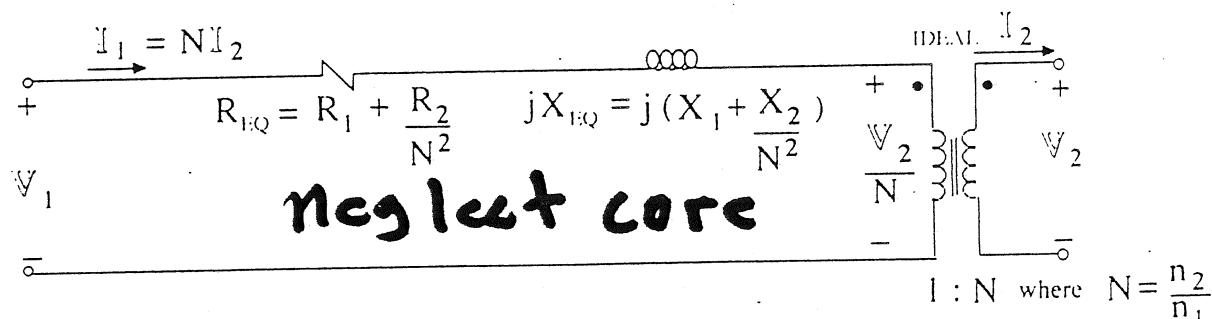
$\frac{V_1}{V_2} = N$ Step UP
 $N > 1$

$\frac{V_2}{V_1} = \frac{1}{N}$ Step DOWN
 $N < 1$

"Exact" Equivalent Circuit Referred to Primary



Final Approximate Equivalent Circuit Referred to Primary



- R_1 and R_2 are series resistances which account for conductor losses of the primary and secondary windings, respectively.
- X_1 and X_2 are series reactances which account for leakages fluxes at the primary and secondary sides.
- X_M is a shunt reactance which accounts for the small primary magnetization current which must flow to sustain the magnetic field.
- R_C is a shunt resistance to account for magnetic core losses due to the effects of hysteresis and eddy currents.

The exact equivalent circuit models are shown at the top of Fig. 5.6. The second model has the secondary electrical quantities referred to the primary, based on Eqs. (5.2), (5.4) and (5.5). The third approximate equivalent circuit model neglects core losses and primary magnetization current and is adequate for the purpose of this

III Four Cases of Mixed $\Delta - \gamma$

We consider 3

A. Phase Effects

$\gamma - \Delta$
 Prim sec
 for
 abc

Step-Down
 $L_{sec} = L_{prim} - 30$
 Step-Up
 $L_{sec} = L_{prim} + 30$

$\Delta - \gamma$ Step Down
 Prim sec $L_{sec} = L_{prim} - 30$
 abc Step Up
 $L_{sec} = L_{prim} + 30$

B Amplitude Effects

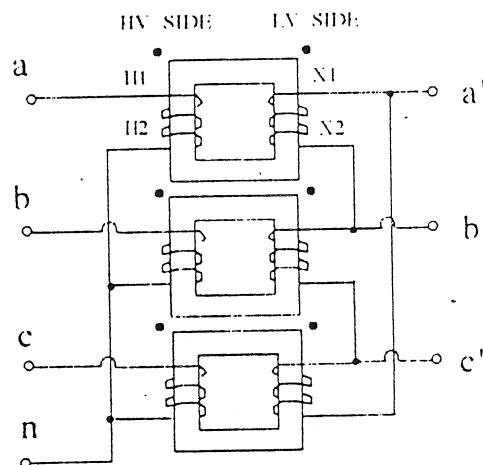
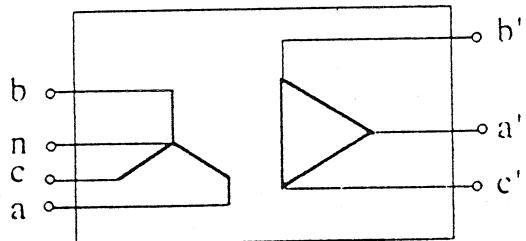
$\gamma - \Delta$ Step-up
 or
 Step-down $V_{sec} = \frac{V_p N}{\sqrt{3}}$

$\Delta - \gamma$ Step-up
 Step-down $V_{sec} = V_p N \sqrt{3}$

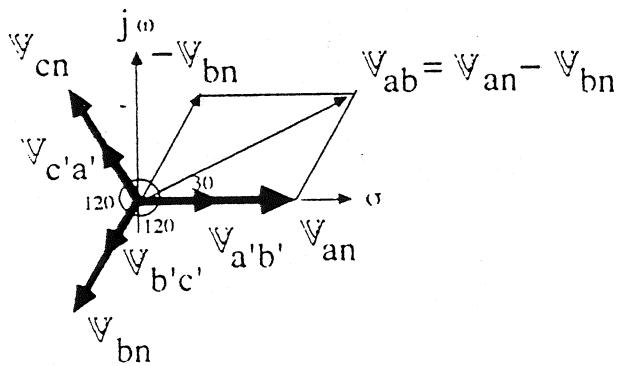
Step-down Y - Δ 41

Figure 5.4 - Per-Phase Circuit for Ideal Primary Wye (HV) - Secondary Delta (LV)

Primary Y (HV) - Δ (LV) Secondary

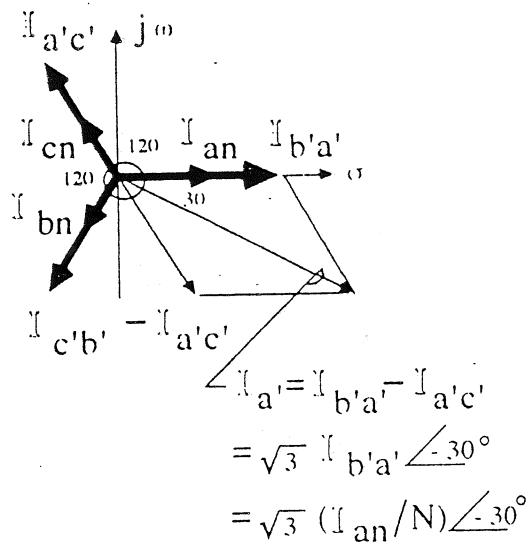


$$\text{Turns Ratio } 1 : N = \frac{n_2}{n_1} \quad (n_1 > n_2)$$



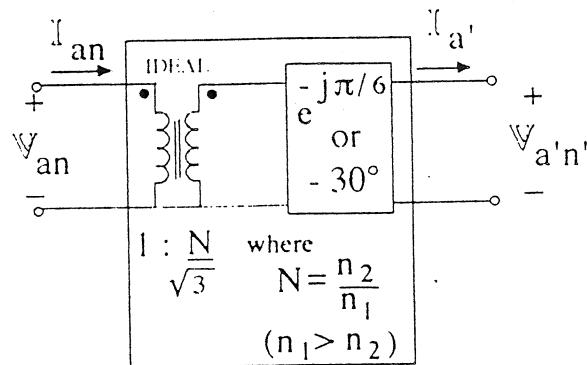
$$\begin{aligned} V_{ab} &= \sqrt{3} V_{an} \angle 30^\circ \\ &= \sqrt{3} (V_{a'b'} / N) \angle 30^\circ \end{aligned}$$

$$\text{or } V_{a'b'} = K_5 V_{ab}$$



$$\text{or } I_{a'} = \frac{1}{K_5^*} I_{an}$$

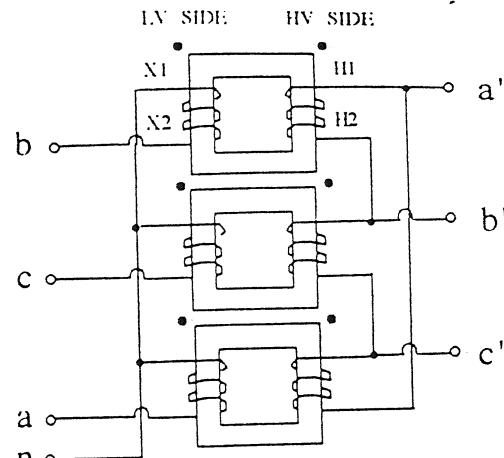
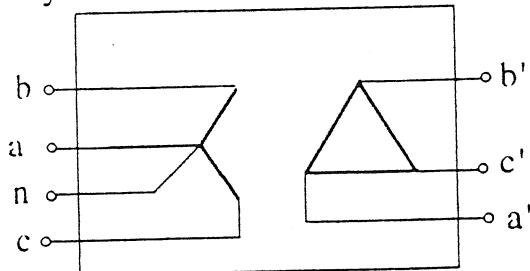
$$\text{where } K_5 = (N / \sqrt{3}) \angle -30^\circ$$



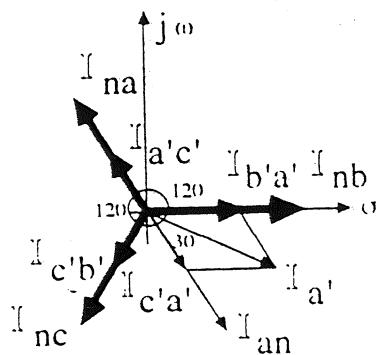
Step-up Y - Δ 5/

Figure 5.5 - Per-Phase Circuit for Ideal Primary Wye (LV) - Secondary Delta (HV)

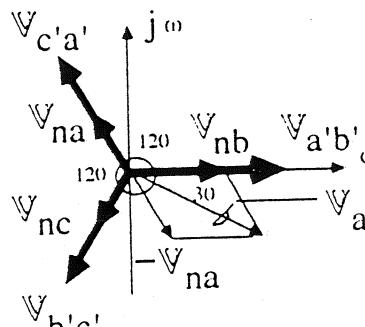
Primary Y (LV) - Δ (HV) Secondary



$$\text{Turns Ratio } 1 : N = \frac{n_2}{n_1} \quad (n_1 < n_2)$$



$$\begin{aligned} I_{a'} &= I_{b'a} - I_{a'c'} \\ &= I_{b'a} + I_{c'a'} \\ &= \sqrt{3} I_{c'a'} \angle 30^\circ \\ &= \sqrt{3} (I_{an}/N) \angle 30^\circ \end{aligned}$$

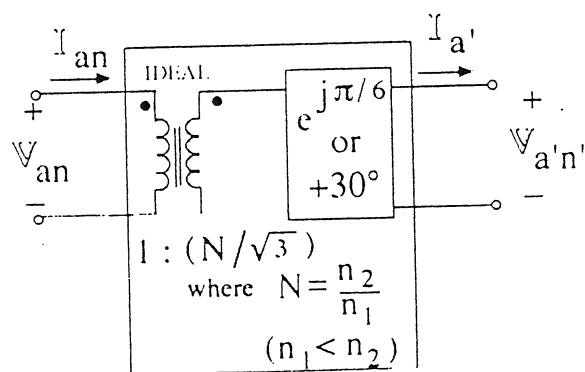


$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ &= V_{nb} - V_{na} \\ &= \sqrt{3} V_{nb} \angle -30^\circ \\ &= \sqrt{3} (V_{a'b'}/N) \angle -30^\circ \end{aligned}$$

$$\text{or } I_{a'} = \frac{1}{K_6^*} I_{an}$$

$$\text{or } V_{a'b'} = K_6 V_{ab}$$

$$\text{where } K_6 = (N / \sqrt{3}) \angle +30^\circ$$

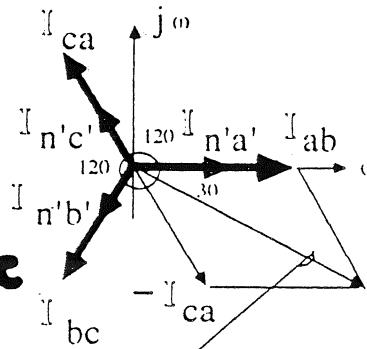
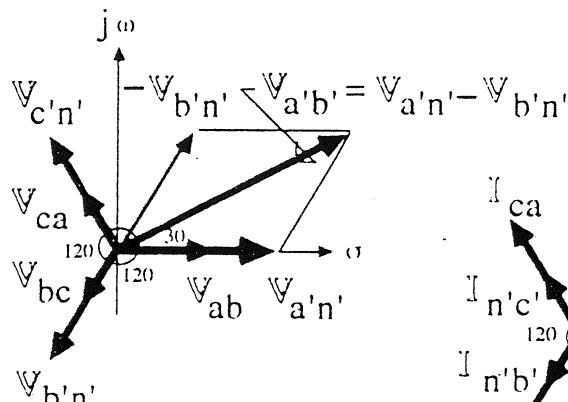
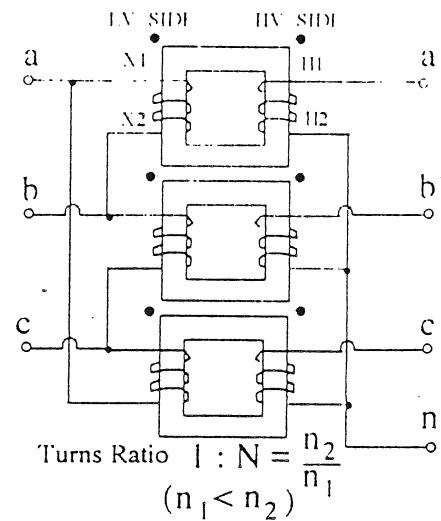
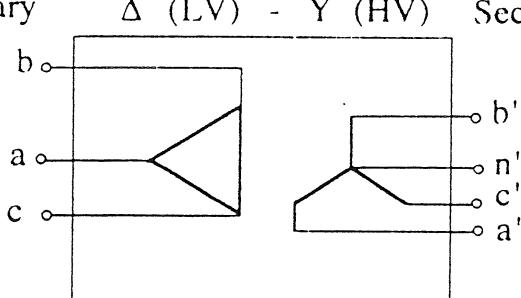


Step-up $\Delta - \gamma$ 6

Figure 5.2 - Per-Phase Circuit for Ideal Primary Delta (LV) - Secondary Wye (HV)

Step-up

Primary Δ (LV) - γ (HV) Secondary



a b c sequence

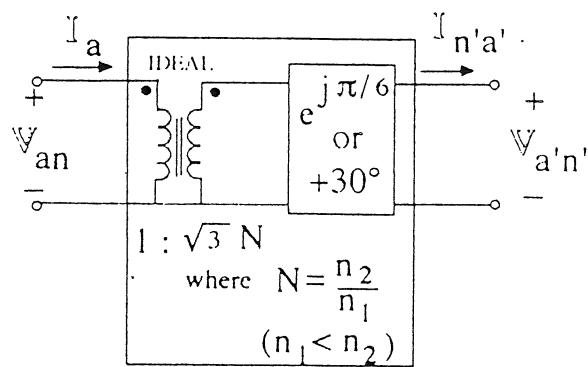
$$\begin{aligned} V_{a'b'} &= \sqrt{3} V_{a'n'} \angle 30^\circ \\ &= \sqrt{3} N V_{ab} \angle 30^\circ \end{aligned}$$

$$\begin{aligned} I_a &= I_{ab} - I_{ca} \\ &= \sqrt{3} I_{ab} \angle -30^\circ \\ &= \sqrt{3} N I_{n'a'} \angle -30^\circ \end{aligned}$$

or $V_{a'b'} = K_3 V_{ab}$

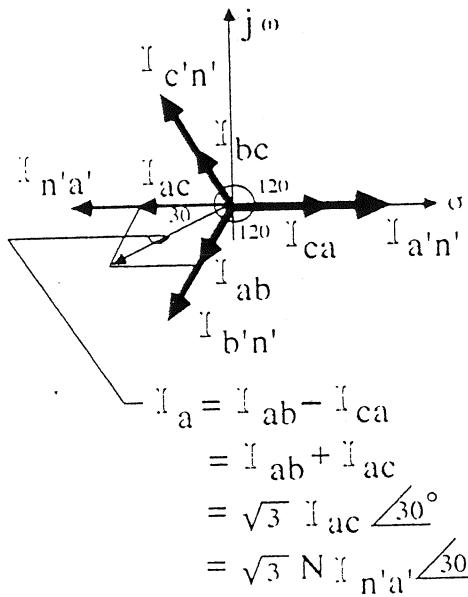
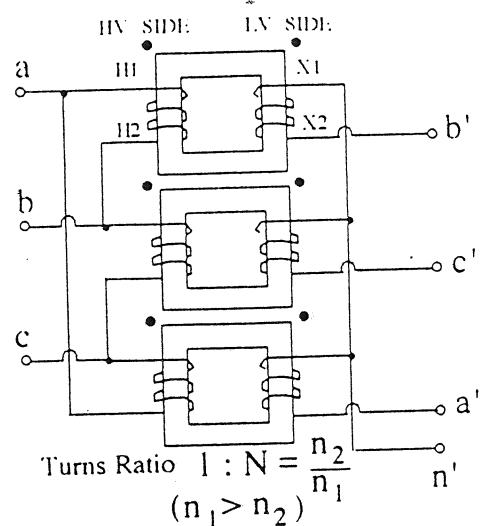
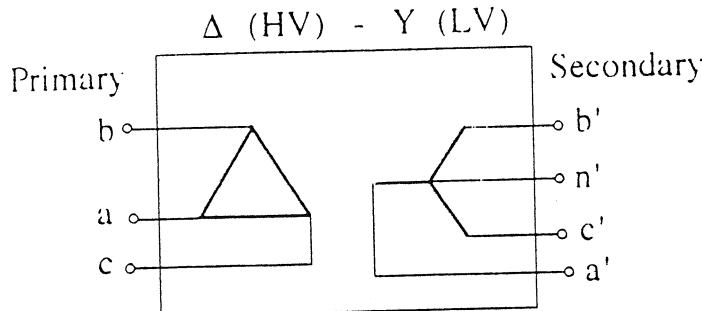
or $I_{n'a'} = \frac{1}{K_3} I_a$

where $K_3 = \sqrt{3} N \angle 30^\circ$



Step-Down Δ - Y 71

Figure 5.3 - Per-Phase Circuit for Ideal Primary Delta (HV) - Secondary Wye (LV)

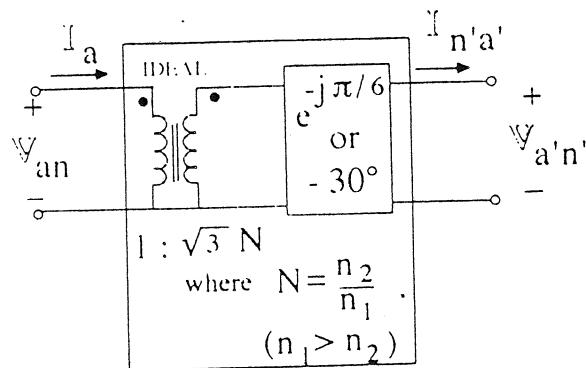


or $I_{n'a'} = \frac{1}{K_4} I_a$

$V_{a'b'} = V_{a'n'} - V_{b'n'} = V_{n'b'} - V_{n'a'} = \sqrt{3} V_{n'b'} \angle -30^\circ = \sqrt{3} N V_{ab} \angle -30^\circ = K_4 V_{ab}$

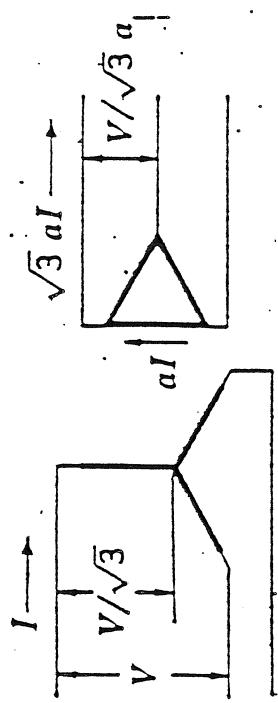
or $V_{a'b'} = K_4 V_{ab}$

where $K_4 = \sqrt{3} N \angle -30^\circ$

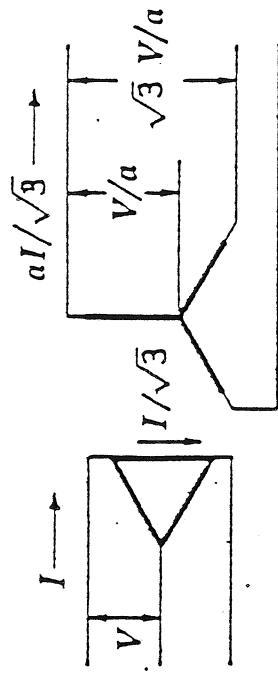


Step-down { transformer} and Step-up { Transformer}

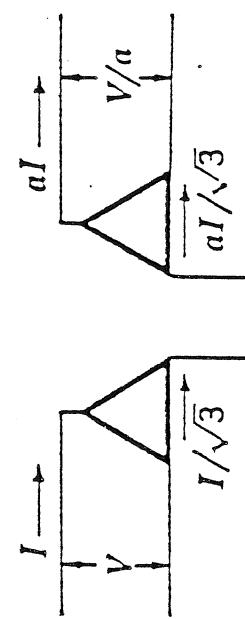
Turns ratio for
a 3-phase



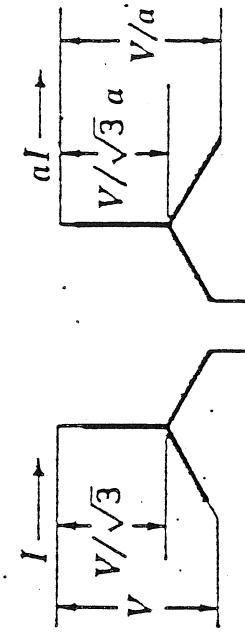
(a) Y- Δ connection



(b) Δ -Y connection



(c) Δ - Δ connection



(d) Y-Y connection
Common 3-phase transformer connections; the transformer windings are indicated by the heavy lines.

100

Summary: 3 Σ for transf.
 $\Delta - \Delta$ NO Phase issues 91
 $\gamma - \gamma$

$\Delta - \gamma$ Always an extra
 $\gamma - \Delta$ 30° phase shift
 occurs

$\Delta - \gamma$ Step-up +30 } Both
 Step-down -30 } $V_s = V_p N$
 $\gamma - \Delta$ Step up +30 } Both
 Step down -30 } $V_s = N V_p / \sqrt{3}$

Table 5.2 - Complex Turns Ratio, K, and Relationships Between Primary and Secondary Electrical Quantities for Ideal Three-Phase Transformers [11]

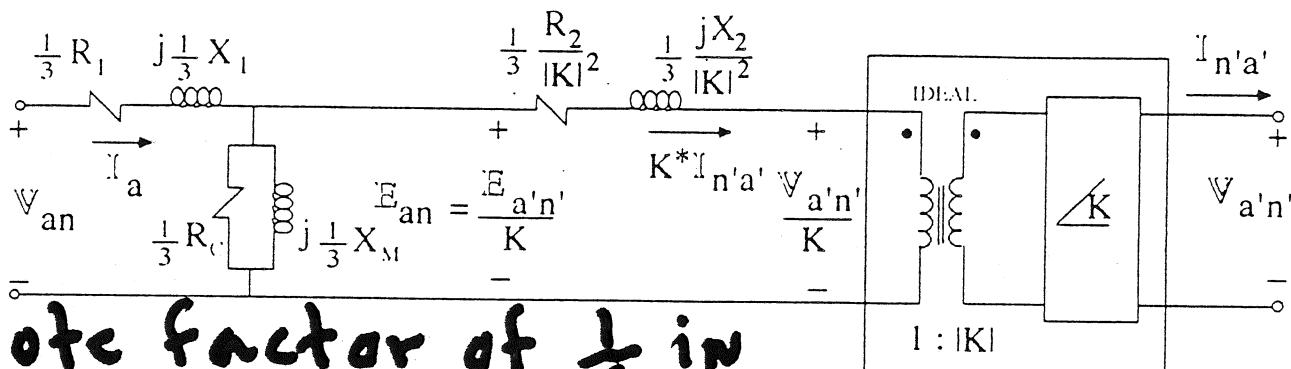
Connection	Line Voltages	Line Currents	K
1. wye-wye	$\mathbb{V}_{a'b'} = K_1 \mathbb{V}_{ab}$	$I_{n'a'} = \frac{1}{K_1} I_a$	$K_1 = N = \frac{n_2}{n_1}$
2. delta-delta	$\mathbb{V}_{a'b'} = K_2 \mathbb{V}_{ab}$	$I_{a'} = \frac{1}{K_2} I_a$	$K_2 = N = \frac{n_2}{n_1}$
3. delta (LV) - wye (HV)	$\mathbb{V}_{a'b'} = K_3 \mathbb{V}_{ab}$	$I_{n'a'} = \frac{1}{K_3} I_a$	$K_3 = \sqrt{3} N \angle +30^\circ$
4. delta (HV) - wye (LV)	$\mathbb{V}_{a'b'} = K_4 \mathbb{V}_{ab}$	$I_{n'a'} = \frac{1}{K_4} I_a$	$K_4 = \sqrt{3} N \angle -30^\circ$
5. wye (HV) - delta (LV)	$\mathbb{V}_{a'b'} = K_5 \mathbb{V}_{ab}$	$I_{a'} = \frac{1}{K_5} I_a$	$K_5 = \frac{N}{\sqrt{3}} \angle -30^\circ$
6. wye (LV) - delta (HV)	$\mathbb{V}_{a'b'} = K_6 \mathbb{V}_{ab}$	$I_{a'} = \frac{1}{K_6} I_a$	$K_6 = \frac{N}{\sqrt{3}} \angle +30^\circ$

Per Phase Circuit Diagram of Three Phase Trf.

Figure 5.7: Per-Phase Equivalent Circuits for Three-Phase Transformers

Below $K = \frac{1}{\sqrt{3}}$ at prior models

DELTA - CONNECTED PRIMARIES



Note factor of $\frac{1}{\sqrt{3}}$ in primary

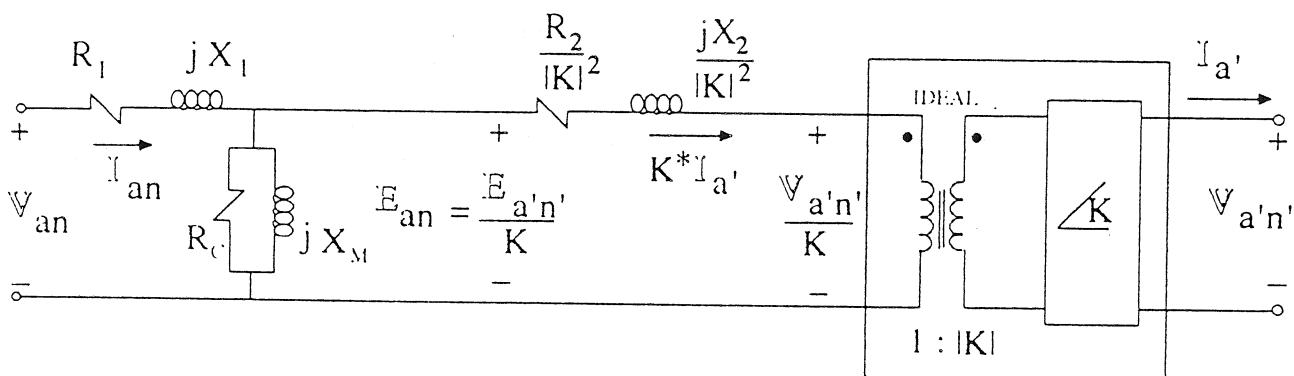
$$\begin{aligned} \text{where } K &= |K| \angle K = K_2 = N \\ &= K_3 = \sqrt{3} N \angle +30^\circ \\ \text{and } N &= \frac{n_2}{n_1} \quad = K_4 = \sqrt{3} N \angle -30^\circ \end{aligned}$$

Primary Delta - Secondary Delta

Primary Delta (LV) - Secondary Wye (HV)

Primary Delta (HV) - Secondary Wye (LV)

WYE - CONNECTED PRIMARIES



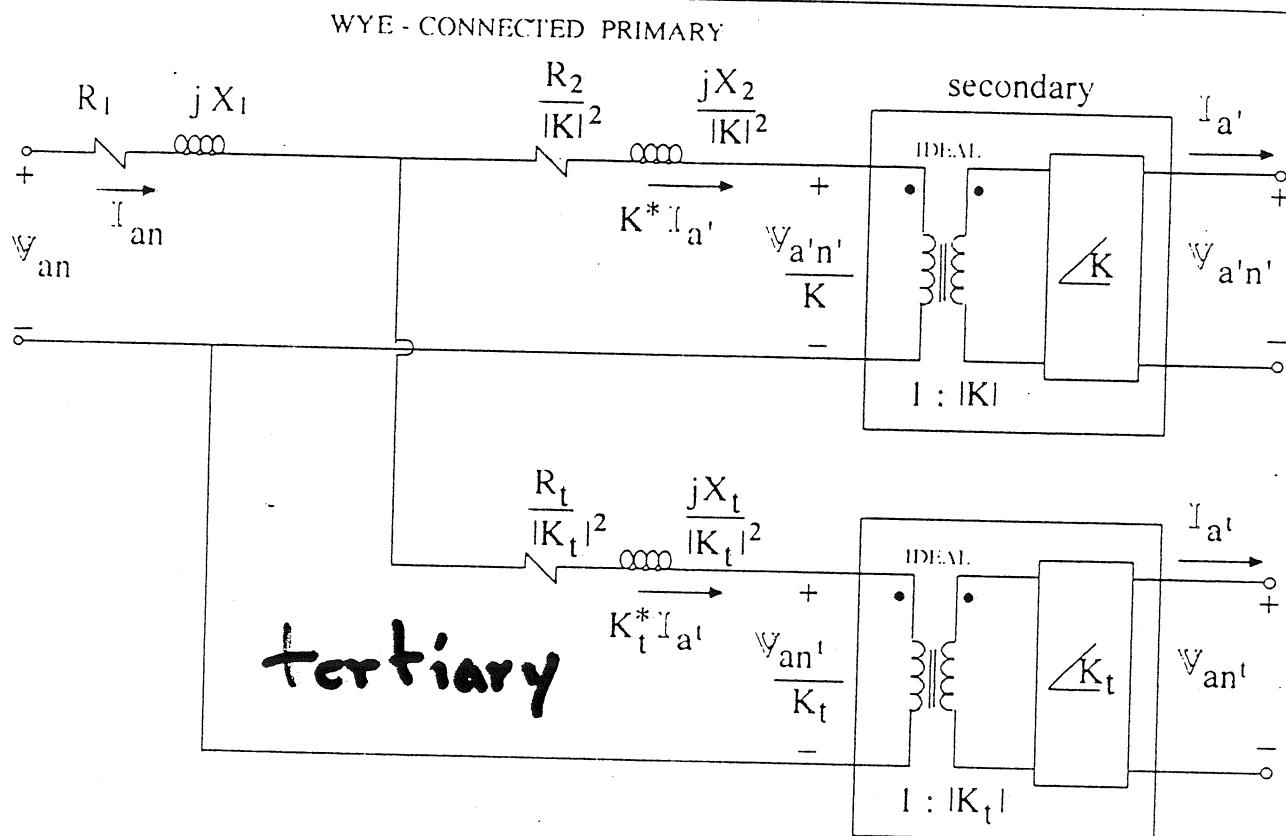
$$\begin{aligned} \text{where } K &= |K| \angle K = K_1 = N \\ &= K_5 = \frac{N}{\sqrt{3}} \angle -30^\circ \\ \text{and } N &= \frac{n_2}{n_1} \quad = K_6 = \frac{N}{\sqrt{3}} \angle +30^\circ \end{aligned}$$

Primary Wye - Secondary Wye

Primary Wye (HV) - Secondary Delta (LV)

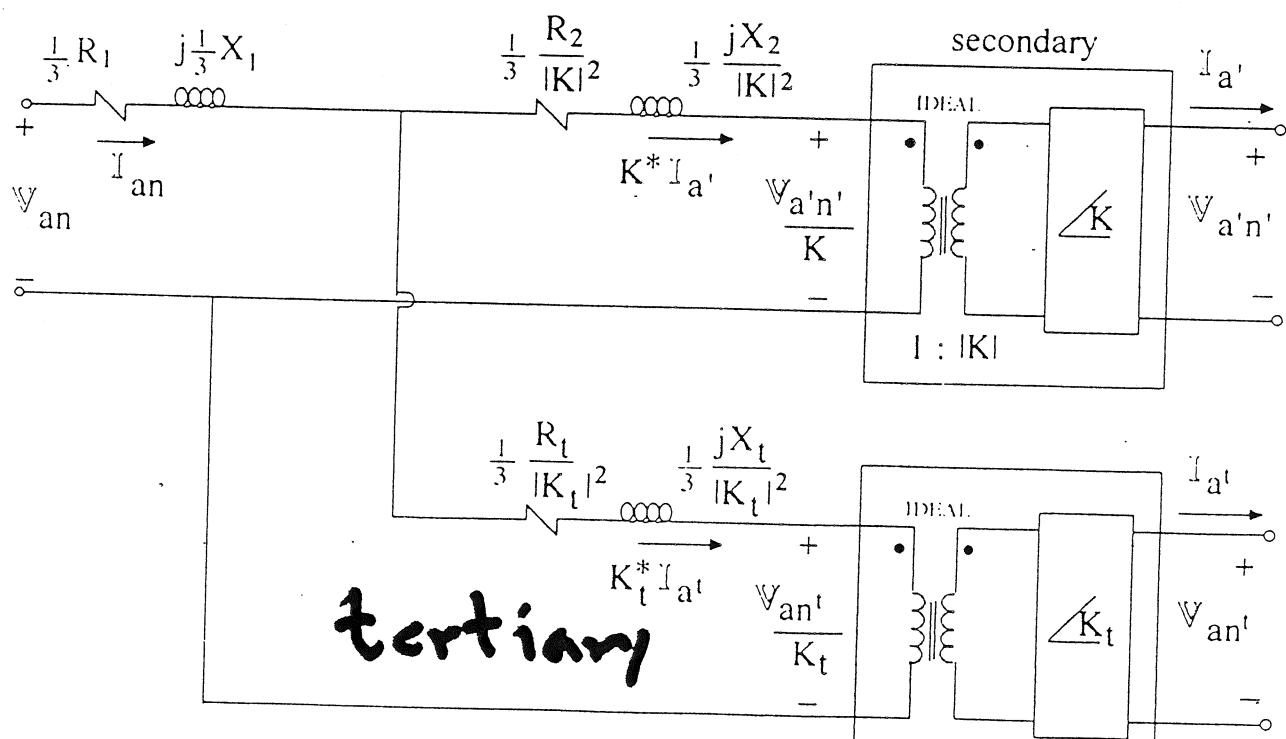
Primary Wye (LV) - Secondary Delta (HV)

Fig. 5.8: Per-Phase Equivalent Circuits for Three-Winding, Three-Phase Transformers



tertiary

DELTA - CONNECTED PRIMARY



tertiary

A. 8 Possible Windings 12/

Three 5 MVA single-phase transformers, each rated 8:1.39 kV, have leakage impedance of 6%. These can be connected in a number of different ways to supply three identical ⁿ 5 ohm resistive loads. Various transformer and load connections are outlined in the following table. Complete the table columns. Use a three-phase base of 15 MVA.

Case No.	Transformer Connection		Load Connection to Secondary	Line-to-Line base KV		Load R in per Unit	Total Z as Viewed from the high side	
	Primary	Secondary		HV	LV		Per Unit	Ohms
1	WYE	WYE	WYE	13.86	2.41	12.95	$12.95 + j0.06$	$165.789 + j0.768$
2	WYE	WYE	DELTA	13.86	2.41	4.32	$4.318 + j0.06$	$55.267 + j0.768$
3	WYE	DELTA	WYE	13.86	1.39	38.82	$38.820 + j0.06$	$496.867 + j0.768$
4	WYE	DELTA	DELTA	13.86	1.39	12.94	$12.940 + j0.06$	$165.622 + j0.768$
5	DELTA	WYE	WYE	8.00	2.41	12.95	$12.953 + j0.06$	$55.266 + j0.768$
6	DELTA	WYE	DELTA	8.00	2.41	4.318	$4.318 + j0.06$	$18.423 + j0.768$
7	DELTA	DELTA	WYE	8.00	1.39	38.82	$38.82 + j0.06$	$165.632 + j0.768$
8	DELTA	DELTA	DELTA	8.00	1.39	12.94	$12.940 + j0.06$	$55.211 + j0.256$

Transformer \equiv Load

$$S = 15 \text{ MVA}$$

mm :06 Z₂(PU)

Varies
with
Wiring.

13)

Voltage Levels: HV in / LV out

$$HV_{1-1} = 8(\sqrt{3}) = 13.856 \text{ kV for Cases 1-4}$$

$$HV_{1-1} = 8 \text{ kV for Cases 5-8}$$

Δ prim



$$LV_{1-1} = 1.39(\sqrt{3}) = 2.408 \text{ kV for Cases 1 & 2,5,6}$$

Y secondary

$$LV_{1-1} = 1.39 \text{ kV for Cases 3,4,7 & 8}$$

Δ secondary

Case1:



$$\text{Load } R_{\text{Base}} = (2.408)^2 / 15 = .386$$

$$\text{Load } R_{\text{p.u.}} = R/R_{\text{Base}} = 5/.386 = 12.953_{\text{p.u.}}$$

$$Z_{\text{p.u. HV}} = 12.953 + j0.06 = Z(\text{load}) + Z(\text{trf})$$

$$Z_{\text{HV}} = (13.856)^2 (12.953 + j0.06) / 15$$

$$= 165.789 + j0.768 \text{ ohms}$$

Case2: Y ~~trf~~ $Y - \Delta$ (load)

$$\text{Load } R_{\text{p.u.}} = (5/3) / .386 = 4.318$$

Same Z (base)
as Case 1?

$$Z_{\text{p.u. HV}} = 4.318 + j0.06 = Z(\text{load}) + Z(\text{trf})$$

$$Z_{\text{HV}} = (13.856)^2 (4.318 + j0.06) / 15$$

$$= 55.267 + j0.768 \text{ ohms}$$

Explain $5/3$ in R_{load} (p.u.)?

$$\text{Case 3: } \frac{\gamma - \Delta}{13.8} - \frac{\gamma(\text{load})}{1.39} \quad 14/$$

$$R_{\text{lv Base}} = (1.39)^2 / 15 = 0.1288$$

$$\text{Load } R_{\text{p.u.}} = 5 / 0.1288 = 38.820$$

$$Z_{\text{p.u. HV}} = 38.820 + j0.06 = Z(\text{load}) + Z(\text{trf})$$

$$Z_{\text{HV}} = (13.856)^2 (38.820 + j0.06) / 15$$

$$= 496.867 + j0.768 \text{ ohms}$$

$$\text{Case 4: } \frac{\gamma - \Delta}{13.8} - \frac{\Delta(\text{load})}{1.39}$$

$$\text{Load } R_{\text{p.u.}} = (5/3) / 0.1288 = 12.94$$

Why $\frac{5}{3}$?

$$Z_{\text{p.u. HV}} = 12.94 + j0.06$$

$$Z_{\text{HV}} = (13.856)^2 (12.940 + j0.06) / 15$$

$$= 165.622 + j0.768 \text{ ohms}$$

$$\text{Case 5: } \frac{\Delta}{8.0} - \frac{\gamma}{2.4} - \frac{\gamma(\text{load})}{1.39}$$

$$\text{Load } R_{\text{p.u.}} = 5 / .386 = 12.953$$

$$Z_{\text{p.u. HV}} = 12.953 + 0.06$$

$$Z_{\text{HV}} = (8.00)^2 (12.953 + 0.06) / 15$$

$$= 55.266 + j0.256 \text{ ohms}$$

$\Delta - \gamma - \Delta(\text{load})$ '51
 Case 6: $\frac{8.0}{2.4}$

$$\text{Load } R_{p.u} = (5/3) / .386 = 4.318$$

Why $\frac{5}{3}$?

$$Z_{p.u \text{ HV}} = 4.318 + j0.06$$

$$Z_{\text{HV}} = (8.00)^2 (4.318 + j0.06) / 15$$

$$= 18.423 + j0.256 \text{ ohms}$$

Case 7: $\frac{8}{1.39} - \Delta - \gamma(\text{load})$

$$\text{Load } R_{p.u} = 5 / .1288 = 38.820$$

$$Z_{p.u \text{ HV}} = 38.820 + j0.06$$

$$Z_{\text{HV}} = (8.00)^2 (38.820 + j0.06) / 15$$

$$= 165.632 + j0.256 \text{ ohms}$$

Case 8: $\frac{8.0}{1.39} - \Delta - \Delta(\text{load})$

$$\text{Load } R_{p.u} = (5/3) / .1288 = 12.940$$

Why $\frac{5}{3}$?

$$Z_{p.u \text{ HV}} = 12.940 + j0.06$$

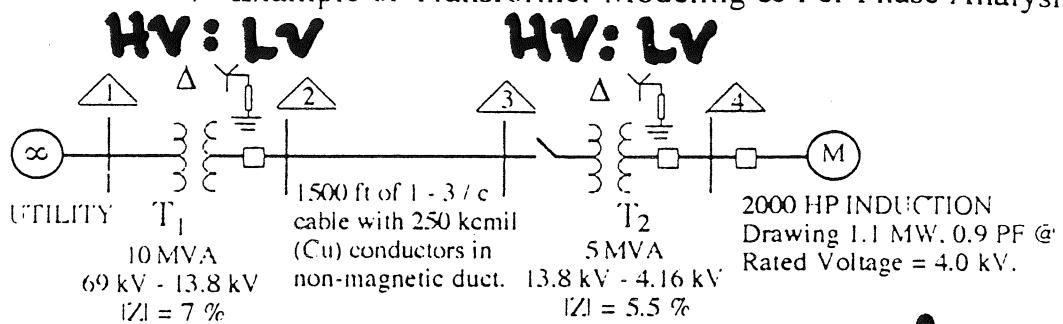
$$Z_{\text{HV}} = (8.00)^2 (12.940 + j0.06) / 15$$

$$= 55.211 + j0.256 \text{ ohms}$$

IV B. Δ - γ Transformers

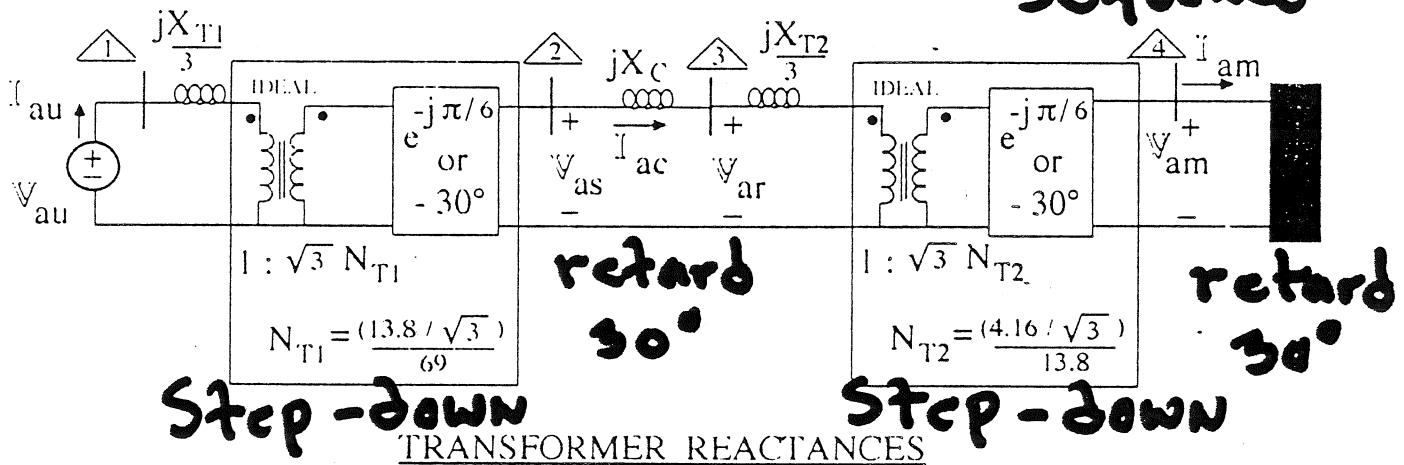
- Example of Transformer Modeling & Per Phase Analysis

'61



PER-PHASE EQUIVALENT CIRCUIT

abc Sequence



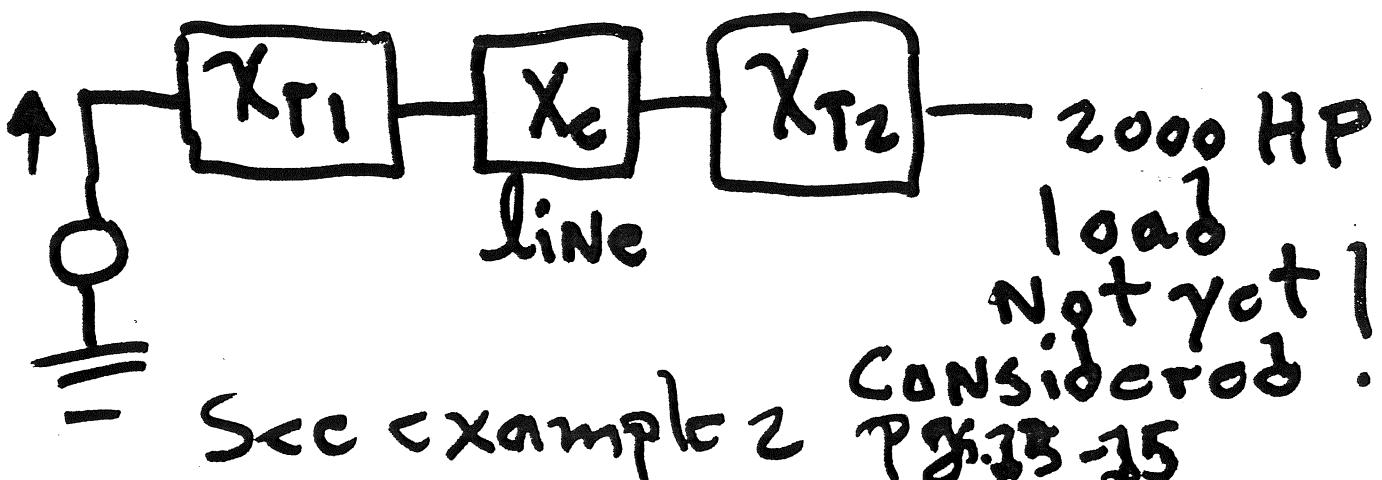
$X_{T1} \equiv |Z_{T1}| = 7\% \text{ of base series } |Z| \text{ from primary ratings}$

$$= \left[0.07 \frac{\text{rated primary kV}_{L-N}}{\text{rated primary kA}_L} \right] \frac{\text{rated primary kV}_{L-N}}{\text{rated primary kV}_{L-N}}$$

$$= 0.07 \frac{(\text{rated primary kV}_{L-N})^2}{\text{rated primary MVA}_{1\Phi}} = 0.07 \frac{(69/\sqrt{3})^2}{(10/3)} = 33.327 \Omega$$

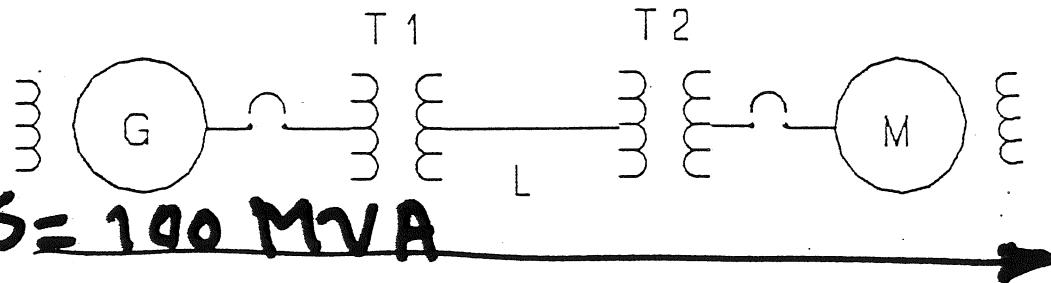
$$X_{T2} \equiv 0.055 \frac{(13.8/\sqrt{3})^2}{(5/3)} = 2.0948 \Omega \quad X_C = \underbrace{\left(\frac{0.0348 \Omega / \text{conductor}}{1000 \text{ feet}} \right) (1500 \text{ feet})}_{\text{TABLE 65 - IEEE BUFF BOOK}} = 0.0522 \Omega$$

LOAD CALCULATION



IIA #1 Example System

13/



Given Ratings and Impedance Data

Gen.

G: 30 MW, 0.9 p.f., 3 Φ 60 Hz, 13.8 kV-L-L

$x_d' = 30\% @ \text{rated MVA}$

trf.

T1: 38.3 MVA, 115-13.2 kV-L-L

$Z = j 10\% @ \text{rated MVA}$

line

L: 115 kV, $Z = 5 + j 20 \text{ ohms}$

trf.

T2: 30 MVA, 115-13.8 kV-L-L

$Z = j 10\% @ \text{rated MVA}$

Motor

M: 32,000 hp, 0.95 p.f., 3 Φ 60 Hz, $\eta = 92\%$
13.8 kV-L-L

$x_d' = 30\% @ \text{rated MVA}$

Determine all power system component impedance values in per unit on a 100 MVA base. Select 115 kV as the kV-L base value for the high voltage transmission system.

S' values for ratings

181

Old MVA Base (3Φ):

$$\text{Generator } 30 \text{ (MW) / 0.9 (p.f.)} = 33.33 \text{ MVA} = S(\text{gen.})$$

$$T_1 = 38.3 \text{ MVA (given)}$$

$$T_2 = 30 \text{ MVA (given)}$$

$$\text{Motor } (32800 \text{ (hp)} \approx 746) / (0.95 \text{ (p.f.)} * 0.92 \text{ (efficiency)})$$

$$= 28.0 \text{ MVA}$$

Remember that $S(\text{base}) = 100 \text{ MVA}$

Recall:

Old Z Base:

$$(\text{Old kV}_{\text{L-L}})^2 / \text{Old MVA Base (3Φ)}$$

Old Per Unit Z: (given) for ratings not @
operation

New MVA Base (3Φ): (given as 100)

New $\text{kV}_{\text{L-L}}$ Base:

Generator is changed to 13.2 kV to match with transformer rating. All others stay the same.

New Z Base will be for operation conditions

Same calculation as in Old Z Base except use New MVA Base and New $\text{kV}_{\text{L-L}}$ Base.

191

New Per Unit Z:

$$\begin{aligned}\text{Generator} &= ((\text{Old Per Unit Z}) * (\text{Old Z Base})) / \text{New Z Base} \\ &= ((j0.30) * (5.713)) / (1.742) = j0.9836\end{aligned}$$

All the other components follow the same process except the transmission line.

$$\begin{aligned}\text{Line} &= (\text{given impedance}) / \text{New Z Base} \\ &= (5 + j20) / 132.25 = (0.0378 + j0.1512)\end{aligned}$$

New I Base (A):

$$\begin{aligned}\text{Generator} &= \\ ((\text{New MVA Base } (3\Phi) * 10^3) &/ (\text{New kV}_{L-L} \text{ Base} * \sqrt{3})) \\ &= (100 * 10^3) / (13.2 * 1.732) = 4374\end{aligned}$$

On the next page we summarize $Z(PV)$ for each component

$Z(\text{ratings})_{PV}$ vs $Z(\text{Operation})_{PV}$

Rating of Each Component Based Per Unit 201

	G	T1	L	T2	M
Old MVA Base	33.33	38.3	—	30	28.0
Old kV_{L-L} Base	13.8	115	115	115	13.8
Old $Z \sqrt{3}$ Base	5.713	345.3	—	441	6.801
Old Per Unit Z	j0.30	j0.10	—	j.10	j.30
New MVA Base	100	100	100	100	100
New kV_{L-L} Base	13.2	115	115	115	13.8
New $Z \sqrt{3}$ Base	1.742	132.2	132.2	132.2	1.904
New Per Unit Z	j.984	j.261	.04 + j.152	j.333	j1.07
New I Base	4374	502	502	502	4184

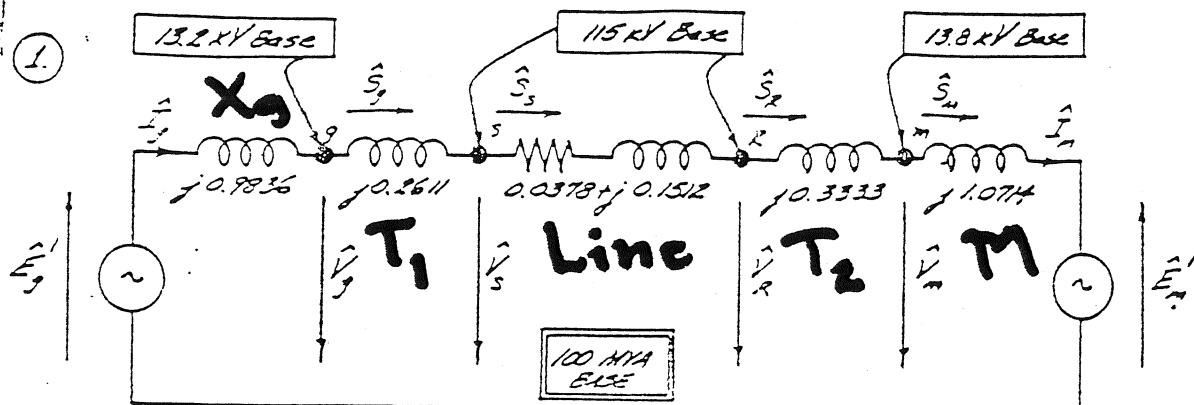
$$\text{New Per Unit } Z = \text{Old Per Unit } Z * \left[\frac{\text{Old } Z \text{ Base}}{\text{New } Z \text{ Base}} \right]$$

$$\text{New I Base (A)} = \frac{\text{New MVA Base (3Φ)} * 10^3}{\text{New } kV_{L-L} \text{ Base} * \sqrt{3}}$$

Operating Condition Based
Per Unit

Example #1 Eq. Circuit

21



Given: $\hat{V}_m = \frac{1}{10} = \frac{1+10\angle 90^\circ}{13,800 \text{ V}_L}$, $\theta_m = 10.2^\circ \text{ lag}$

$$\left. \begin{aligned} &= \frac{1+10\angle 90^\circ}{13,800 \text{ V}_L} \\ &\Rightarrow \hat{V}_m = 1172 \angle -18.2^\circ \text{ A} \end{aligned} \right\} \text{PF} = 0.95$$

$$Q_f = 78.2^\circ$$

$$\Rightarrow \hat{I}_m = \frac{28}{100} \angle -18.2^\circ = 0.28(0.95 - j0.3122) = 0.266 - j0.0874 \text{ A}$$

$$= \underline{0.28 \angle -18.2^\circ \text{ A}} \Rightarrow \underline{1172 \angle -18.2^\circ \text{ A}}$$

$$4(184)(0.125) = 1172$$

$$\hat{I}_g (\text{p.u.}) = \hat{I}_m (\text{p.u.}) = 0.28 \angle -18.2^\circ \text{ p.u.}$$

$$\hat{E}'_g = \hat{V}_m - j10.714 \hat{I}_m = 1 - j10.714(0.266 - j0.0874) = 0.9054 - j0.2350$$

$$= \underline{0.9054 \angle -19.5^\circ \text{ A}} \Rightarrow \underline{570 \angle -19.5^\circ \text{ A}} (13,800 \text{ V}_L)$$

$$\hat{V}_g = \hat{V}_m + (0.0378 + j0.1512)(0.266 - j0.0874) = 10 + 0.07524 + j0.1930$$

$$= \underline{10.928 \angle 10.28^\circ \text{ p.u.}} \Rightarrow \underline{832.8 \angle 10.28^\circ \text{ V}_L} (13,800 \text{ V}_L)$$

~~$$\frac{13.8}{S_g} = \frac{14.425}{100 \text{ MVA}} = 14.5 \text{ A}, \quad \frac{14.425}{13.8 \text{ MVA}} = 10.3 \text{ A}$$~~

$$\hat{I}_g = \underline{0.28 \angle -18.2^\circ \text{ p.u.}} \Rightarrow \underline{1225 \angle -18.2^\circ \text{ A}}$$

$$\hat{S}_g = \hat{V}_g \hat{I}_g^* = (10.928 \angle 10.28^\circ)(0.28 \angle 18.2^\circ) = 0.306 \angle 28.42^\circ \text{ p.u.}$$

$$P_g = 26.9 \text{ MW}, \quad Q_g = 14.5 \text{ MVAR}, \quad \theta_g = 22.42^\circ \text{ lag}$$

$$\hat{S}_s = \hat{V}_s \hat{I}_s^* = 0.266 + j0.0874 \text{ p.u.}, \quad P_s = 26.6 \text{ MW}, \quad Q_s = 0.0874 \text{ MVAR}$$

$$\hat{E}'_g = \hat{V}_g + j0.9836 \hat{I}_g = 10.7524 + j0.1950 + j0.9836(0.266 - j0.0874)$$

$$= 1161.2 + j0.4566 = \underline{1247.8 \angle 21.47^\circ \text{ p.u.}} \Rightarrow \underline{9.509 \angle 21.47^\circ \text{ V}_L}$$

$$(13,800 \text{ V}_L)$$

$$\hat{V}_s = \hat{V}_g + j0.9836(0.266 - j0.0874) = 10 + j0.07524 + j0.02743$$

$$= 10.07524 + j0.09266 = 10.3298 \angle 5.52^\circ \text{ p.u.} \Rightarrow 118.5 \text{ kV}_L$$

22

$$\hat{S}_e = \hat{V}_e \hat{I}_e^* = (1.03294 / 4.92^\circ) (0.28 / 18.2^\circ) = 0.28922 / 231^\circ$$

$$P_e = 26.6 \text{ MWh} \rightarrow Q_e = 11.3 \text{ MVArs}$$

$$\hat{V}_s = \hat{V}_e + (0.0378 + j 0.1512)(0.28 / -18.2^\circ)$$

$$= 1.02913 + j 0.08866 + (0.15585 / 75.8^\circ) (0.28 / -18.2^\circ)$$

$$= 1.0524 + j 0.12557 = 1.05986 / 6.83 \text{ p.u.} \Rightarrow 12.19 \text{ kV}$$

$$\hat{S}_s = \hat{V}_s \hat{I}_s^* = (1.05986 / 6.83^\circ) (0.28 / 18.2^\circ) = 0.29676 / 25.0^\circ \text{ p.u.}$$

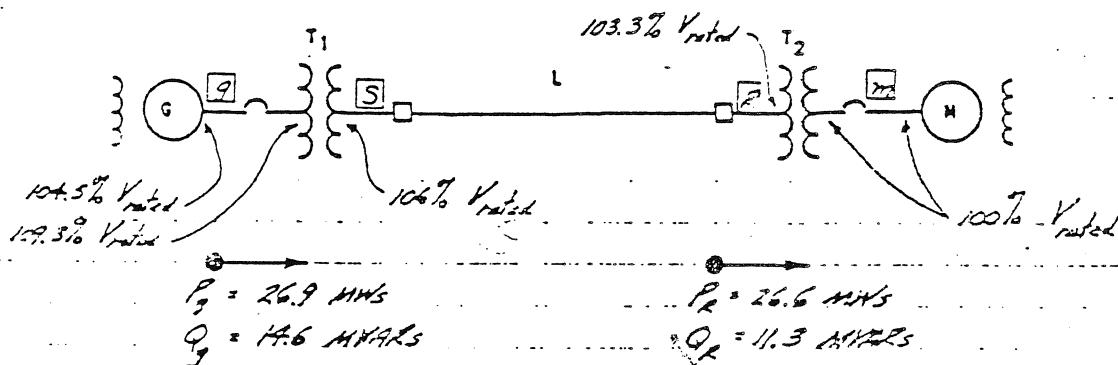
$$P_s = 26.9 \text{ MWh} \rightarrow Q_s = 12.5 \text{ MVArs}$$

$$P_g = 26.9 \text{ MWh}$$

$$Q_g = 14.6 \text{ MVArs}$$

$$P_m = 26.6 \text{ MWh}$$

$$Q_m = 8.74 \text{ MVArs}$$



$$S_g = 30.6 \text{ MVA} < 33.33 \text{ MVA rating}$$

$Q_g = 14.6 \text{ MVArs}$ may be excessive

2. For a balanced operating condition in which the motor is being operated at rated voltage, horsepower and power factor, and assuming motor terminal voltage, V_m , to be the reference voltage (assume phase a to neutral voltage), determine the following values (as indicated on Z-diagram):

Generator internal voltage, $\hat{\epsilon}_g$, in per unit and in $\text{kV}_{\text{L-N}}$

Generator terminal voltage, \hat{V}_g , in per unit and in $\text{kV}_{\text{L-N}}$

Generator phase a current, \hat{I}_g , in per unit and in amperes

Motor internal voltage, $\hat{\epsilon}_m$, in per unit and in $\text{kV}_{\text{L-N}}$

Motor terminal voltage, \hat{V}_m , in per unit and in $\text{kV}_{\text{L-N}}$

Motor phase a current, \hat{I}_m , in per unit and in amperes

Y₀ System #2 Example: Per Unit Steps 1-5

Generation - Transm - Distribution

23

Fig. 6.5 summarizes the steps to perform per-unit analysis on the example.

STEP 1: The first step of the procedure is to assign base values of line-to-line voltage and three-phase apparent power on each side of every three-phase power transformer in accordance with the selection rules of Fig. 6.4. In particular, base $MVA_{3\Phi} = 10$ was selected and is common throughout the system; and base kV_{L-L} on any side of a three-phase power transformer is simply the nominal line-to-line nameplate kV rating for that side of the transformer. The other two per-phase base quantities of current and impedance are derived from the assigned base values of line-to-line voltage and three-phase apparent power, according to the formulas of Fig. 6.4.

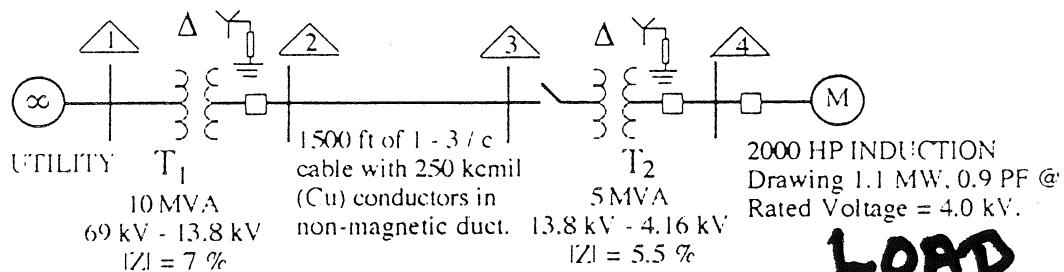
STEP 2: The next step is to draw the per-phase equivalent circuit for per-unit analysis. For this example, the per-phase equivalent circuit model of a three-phase transformer is simply a per-unit series leakage reactance (cf. Sec. 6.8). (For this example, resistances are neglected.)

STEP 3: The percentage impedances of the transformers are based on the nameplate ratings. Therefore, they must be adjusted with respect to the three-phase system base quantities, according to the formula of Fig. 6.3. Note that the unadjusted per-unit impedance of the transformer, $(X_T)^{old} \approx |Z_T|^{old}$, is found by dividing the percentage impedance by 100 %. The per-unit reactance of the distribution feeder is calculated by dividing its ohmic value by the base impedance. (The ohmic value of the distribution feeder was calculated in Fig. 5.9a.)

STEP 4: The operating line-to-line voltage magnitude of the induction motor was specified on the one-line diagram as 4.0 kV. It is divided by $\sqrt{3}$ to yield the line-to-neutral voltage magnitude. The phase angle of the line-to-neutral motor voltage is chosen as the phase angle reference for the power system, and it is arbitrarily set to 0° . Finally, the line-to-neutral phasor voltage of the induction motor in kilovolts is divided by base kV_{L-N} to yield the per-unit line-to-neutral phasor voltage. The per-unit phasor current of the induction motor is calculated by dividing the phasor current in kiloamperes by base current in kiloamperes. (The phasor current was calculated in Fig. 5.9a. from the load data of the induction motor.)

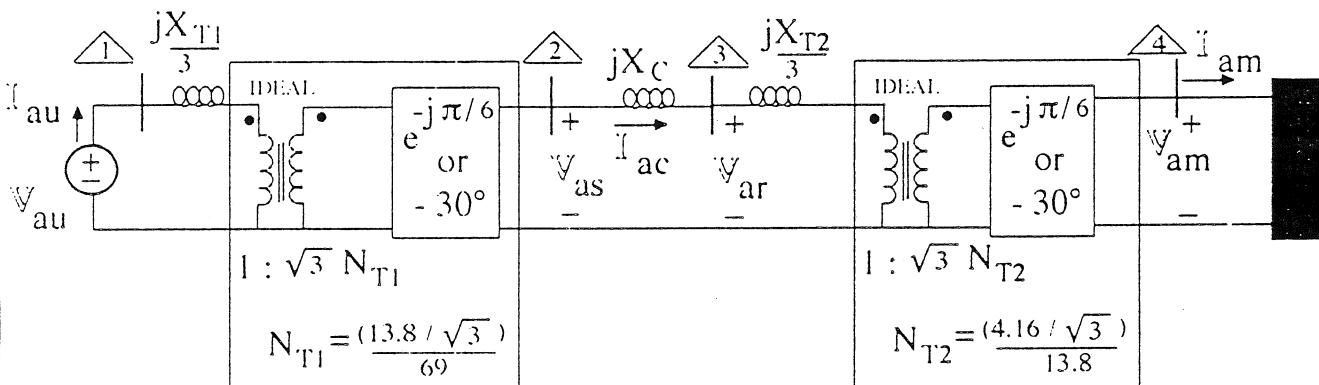
STEP 5: The utility voltage is found by applying Kirchhoff's Voltage Law to the per-phase equivalent circuit. Note that the per-unit utility phasor voltage was multiplied by base $kV_{L-N} = (69 / \sqrt{3})$ to yield the line-to-neutral utility voltage in kilovolts. Of course, the **magnitude** of the per-unit phasor voltage can be multiplied by base $kV_{L-L} = 69$ to yield the line-to-line voltage **magnitude** in kilovolts (cf. item 2 of Fig. 6.2); however, 30° must be added to the phase angle of the line-to-neutral utility voltage to yield the phase angle of the line-to-line voltage.

Figure 5.9a - Example of Transformer Modeling & Per Phase Analysis



LORD
See below

PER-PHASE EQUIVALENT CIRCUIT



TRANSFORMER REACTANCES

$$X_{T1} \equiv |Z_{T1}| = [7\% \text{ of base series } |Z| \text{ from primary ratings}]$$

$$\begin{aligned} &= \left[0.07 \frac{\text{rated primary kV}_{L-N}}{\text{rated primary kA}_L} \right] \frac{\text{rated primary kV}_{L-N}}{\text{rated primary kV}_{L-N}} \\ &= 0.07 \frac{(\text{rated primary kV}_{L-N})^2}{\text{rated primary MVA}_{1\Phi}} = 0.07 \frac{(69/\sqrt{3})^2}{(10/3)} = 33.327 \Omega \end{aligned}$$

$$X_{T2} \equiv 0.055 \frac{(13.8/\sqrt{3})^2}{(5/3)} = 2.0948 \Omega \quad X_C = \left(\frac{0.0348 \Omega / \text{conductor}}{1000 \text{ feet}} \right) (1500 \text{ feet}) = 0.0522 \Omega$$

TABLE 65 - IEEE BUFF BOOK

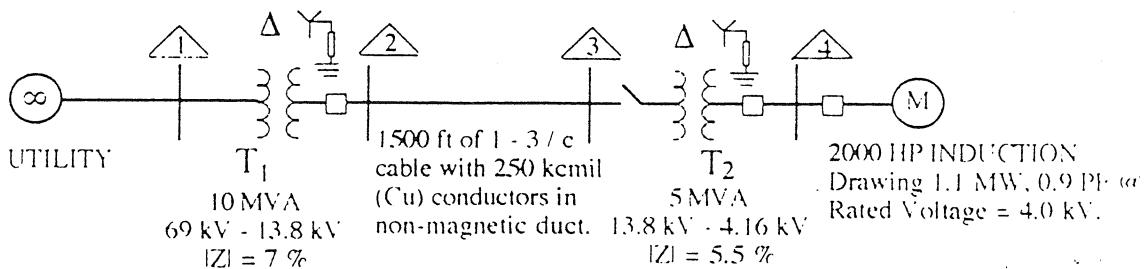
LOAD CALCULATION

$$\begin{aligned} P_{m(3\Phi)} &= 3 \operatorname{Re} \{ V_{am} I_{am}^* \} \\ &= 3 |V_{am}| |I_{am}| \cos(\theta_m) \\ &= \sqrt{3} |V_{abm}| |I_{am}| \underbrace{\cos(\theta_m)}_{\text{power factor, pf}_m} \Rightarrow |I_{am}| = \frac{(1.1 \text{ MW})}{\sqrt{3} (4.0 \text{ kV}) (0.9)} = 0.1764 \text{ kA} \end{aligned}$$

$$\begin{aligned} \theta_m &= \cos^{-1}(\text{pf}_m) = \angle V_{am} - \angle I_{am} = -\angle I_{am} \\ \text{or } \angle I_{am} &= -\cos^{-1}(0.9) = -25.84^\circ \end{aligned}$$

15

Fig. 6.5 - A Simple Example of Per - Unit Analysis
(cf. Fig. 5.9 for Reference.)

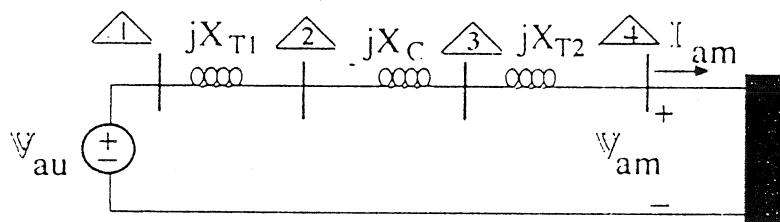


$$\begin{aligned} \text{base } kV_{L,L.} &= 69 \\ \text{base } MVA_{3\phi} &= 10 \\ \text{base } Z_L &= \frac{(\text{base } kV_{L,L.})^2}{\text{base } MVA_{3\phi}} = 476.1 \text{ ohms} \\ \text{base } kA_L &= \frac{\text{base } MVA_{3\phi}}{\sqrt{3} \text{ base } kV_{L,L.}} = 0.084 \end{aligned}$$

$$\begin{aligned} \text{base } kV_{L,L.} &= 13.8 \\ \text{base } MVA_{3\phi} &= 10 \\ \text{base } Z_L &= 19.04 \text{ ohms} \\ \text{base } kA_L &= 0.418 \end{aligned}$$

$$\begin{aligned} \text{base } kV_{L,L.} &= 4.16 \\ \text{base } MVA_{3\phi} &= 10 \\ \text{base } Z_L &= 1.73 \text{ ohms} \\ \text{base } kA_L &= 1.39 \end{aligned}$$

PER - PHASE EQUIVALENT CIRCUIT (Note: All Electrical Quantities in Per-Unit.)



DATA PREPARATION

$$\begin{aligned} X_{T1} &\equiv |Z_{T1}| = 0.07 \left(\frac{10}{10} \right) \left(\frac{69}{69} \right)^2 = 0.07 \text{ pu} \\ X_{T2} &\equiv |Z_{T2}| = 0.055 \left(\frac{10}{5} \right) \left(\frac{13.8}{13.8} \right)^2 = 0.11 \text{ pu} \\ X_C &= \frac{0.0522 \Omega}{\text{base } Z_L = 19.04 \Omega} = 0.0027 \text{ pu} \end{aligned}$$

LOAD CALCULATION

$$\begin{aligned} \mathbb{V}_{am} &= \frac{4.0}{\sqrt{3}} \angle 0^\circ \text{ kV} \\ \mathbb{V}_{am} &= \frac{\text{base } kV_{L,N}}{\sqrt{3}} = \frac{4.16}{\sqrt{3}} \text{ pu} \\ I_{am} &= \frac{0.1764 \angle -25.84^\circ \text{ kA}}{\text{base } kA_L = 1.39} = 0.127 \angle -25.84^\circ \text{ pu} \end{aligned}$$

UTILITY VOLTAGE

$$\begin{aligned} \mathbb{V}_{au} &= j(X_{T1} + X_C + X_{T2}) I_{am} + \mathbb{V}_{am} \\ &= (0.1827 \angle 90^\circ) (0.127 \angle -25.84^\circ) + 0.962 \text{ pu} \\ &= 0.9721 + j 0.0209 \text{ pu} \\ &= 0.9723 \angle 1.23^\circ \text{ pu} \\ &= 38.73 \angle 1.23^\circ \text{ kV (line-to-neutral)} \end{aligned}$$

COMPLEX POWER REQUIREMENT OF UTILITY

$$S_u = -\mathbb{V}_{au} I_{am}^* = -0.367 - j0.187 \text{ MVA (single-phase)}$$

VI Transformer's 26/ Additional Information

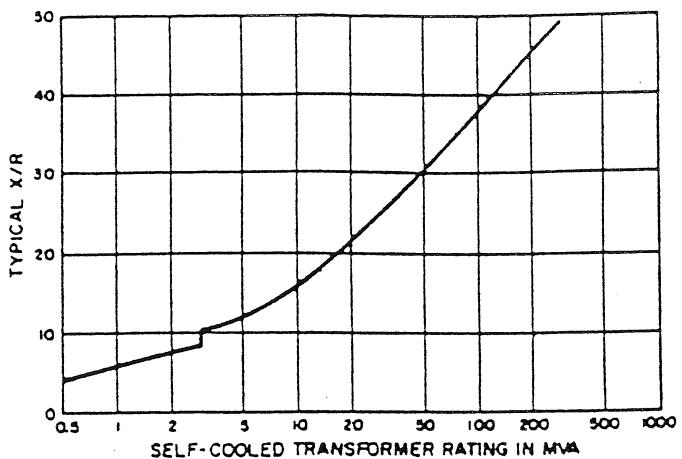


Fig N1.1
X/R Ratio of Transformers (Based on ANSI/IEEE C37.010-1979 [2])

- TRANSFORMER DATA -

- Fig. N1.1 X / R Ratio of Three-Phase Transformers (ANSI / IEEE Std 141-1986)
- Table 64a Typical Per-Unit R and X for Indoor, Open Dry-Type, 150°C Rise Three-Phase Transformers from 15 - 2500 kVA; 2.5 - 15 kV Primaries; and 208, 240, 480 and 600 volt Wye and Delta Secondaries (ANSI / IEEE Std 241 - 1990)
- Table 64b Typical Per-Unit R and X for Indoor, Open Dry-Type, 150°C Rise Single-Phase Transformers from 25 - 500 kVA; 5 and 15 kV Primaries; and 120 / 240 volt Wye or Delta Secondaries (ANSI / IEEE Std 241 - 1990)
- Table 64c Typical Range of Per-Unit Values for Indoor, Open Dry-Type, 150°C Rise Three-Phase Transformers from 15 - 500 kVA; 480 volt Primary; and 208 volt Wye Secondary (ANSI / IEEE Std 241 - 1990)
- Table 64d Typical Range of Per-Unit Values for Indoor, Open Dry-Type, 150°C Rise Single-Phase Transformers from 5 - 167 kVA; 240 x 480, 480 and 600 volt Primaries; and 120 / 240 volt Secondary (ANSI / IEEE Std 241 - 1990)
- Table 2 Data for Three-Phase Transformers with Secondaries of 2400 volts or More for 750 - 60000 kVA (ANSI / IEEE Std 242 - 1986)
- Table 14 Impedance Data for Single-Phase Transformers (ANSI / IEEE Std 242 - 1986)

Per Unit Transformer DATA

Table 64
Transformers

271

(a)

Typical Per Unit R and X Values for Indoor, Open Dry-Type
150 °C Rise Transformers Rated from 15-2500 kVA, Three-Phase,
2.5-15 kV Primaries, 208, 240, 480, 600 V Wye or Delta Secondaries

kVA	HV (kV)	LV (V)	%Z	X/R	R	X
15	2.5-15	208Y-600	3.00	0.5	0.027	0.013
30	2.5-15	208Y-600	5.00	1.0	0.036	0.035
45	2.5-15	208Y-600	5.00	1.0	0.035	0.036
75	2.5-15	208Y-600	5.50	2.0	0.025	0.049
112.5	2.5-15	208Y-600	4.50	1.5	0.025	0.037
150	2.5-15	208Y-600	4.50	2.0	0.020	0.040
225	2.5-15	208Y-600	5.00	2.5	0.019	0.046
300	2.5-15	208Y-600	5.00	2.8	0.017	0.047
500	2.5-15	208Y-600	5.00	4.0	0.012	0.049
750	2.5-15	208Y-600	5.75	2.0	0.026	0.051
1000	2.5-15	208Y-600	5.75	2.5	0.021	0.053
1000	2.5-15	480Y	8.00	3.8	0.021	0.077
1500	2.5-15	208Y-600	5.75	3.3	0.017	0.055
2000	2.5-15	208Y-600	5.75	4.0	0.014	0.056
2500	2.5-15	208Y-600	5.75	4.3	0.013	0.056

(b)

Typical Per Unit R and X Values for Indoor, Open Dry-Type
150 °C Rise Transformers Rated from 25-500 kVA, Single-Phase,
5 and 15 kV Primaries, 120/240 V Wye or Delta Secondaries

kVA	HV (kV)	LV (V)	%Z	X/R	R	X
25	5		4	2	0.018	0.036
to	to	120/240	to	to		
500	15		6	4	0.015	0.068

(c)

Typical Range of Per Unit Values for Indoor, Open Dry-Type
150 °C Rise Transformers Rated from 15-500 kVA, Three-Phase,
480 V Primary, 208 V Wye Secondary

kVA	%Z	X/R	R	X
15	4.5	0.41	0.042	0.017
to	to	to		
500	5.9	2.09	0.025	0.053

(d)

Typical Range of Per Unit R and X Values for Indoor, Open Dry-Type
150 °C Rise Transformers Rated from 5-167 kVA, Single-Phase,
240×480 V, 480 V, 600 V Primaries, 120/240 V Secondaries

kVA	HV (kV)	LV (V)	%Z	X/R	R	X
5	240×480		3	0.6	0.026	0.015
to	to	120/240	to	to		
167	600		6	2.0	0.027	0.061

281

Table 2
Data for Three-Phase Transformers With Secondaries
of 2400 V or More (750-80 000 kVA)

Primary kV	Primary kV BIL	Standard Percent Impedance (see notes)
2.4-22.9	60-150	5.5 or 6.5
- 34.4	- 200	6.0 or 7.0
- 43.8	- 250	6.5 or 7.5
- 67.0	- 350	7.0 or 8.0
- 115.0	- 450	7.5 or 8.5
- 138.0	- 550	8.0 or 9.0

NOTES: (1) Actual values are generally within $\pm 7.5\%$ of the standard values [1].
 (2) Add 0.5% for load tap changing [8].
 (3) Lower values are usually for OA 55 °C or OA 55/65 °C rise transformers.
 (4) Higher values are usually for OA 65 °C rise transformers.
 (5) X/R values are similar to those in Table 1. Consult manufacturer or use the values in [4] for transformers rated over 2500 kVA.

Table 14
Impedance Data for Single-Phase Transformers

kVA 1 ϕ	Suggested X/R Ratio for Calculation	Normal Range of Percent Impedance (%Z)*	Impedance Multipliers** for Line-to-Neutral Fault for %X for %R	
25.0	1.1	1.2-6.0	0.6	0.75
37.5	1.4	1.2-6.5	0.6	0.75
50.0	1.6	1.2-6.4	0.6	0.75
75.0	1.8	1.2-6.6	0.6	0.75
100.0	2.0	1.3-5.7	0.6	0.75
167.0	2.5	1.4-6.1	1.0	0.75
250.0	3.6	1.9-6.8	1.0	0.75
333.0	4.7	2.4-6.0	1.0	0.75
500.0	5.5	2.2-5.4	1.0	0.75

* National standards do not specify %Z for single-phase transformers. Consult manufacturer for values to use in calculation.

** Based on rated current of the winding (one-half nameplate kVA divided by secondary line-to-neutral voltage).

TRANSFORMER PROTECTION

LARGE TRANSFORMER (10 MVA OR LARGER)

- * Differential (87T)
- * Phase Overcurrent (50/51)
- * Ground Overcurrent (50/51G)
- * Sudden Pressure (63)
- * Thermal Overload (49)

SMALLER TRANSFORMER (LESS THAN 10 MVA)

- * Phase Overcurrent (50/51)
- * Ground Overcurrent (50/51N)
- * High Side Fuse
- * Thermal Overload (49)

MAGNETIZING INRUSH

Reference: "Transformer Protection Guide", Westinghouse Relay and Telecommunications Division, Industrial and commercial Power System Application Series, No. PRSC-3D, November 1985.

