

Per Unit Lecture

Topics:

- Per Unit (What and Why)
- Basics
- One Line and Per Unit

Per Unit

Ch 1
pgs 9 - 15

$$R_{pu} = \frac{R_{actual}}{R_{Base}}$$

} Units ?

$$X_{pu} = \frac{X_{actual}}{X_{Base}}$$

I_{pu} , V_{pu} , P_{pu} etc

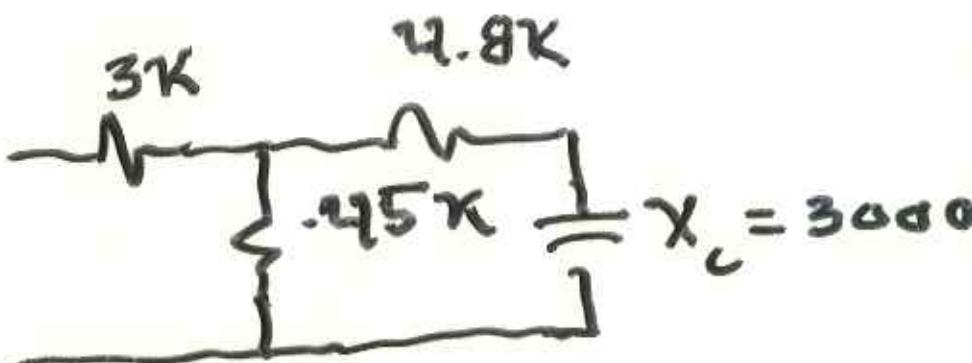
Base determination ?

Choose only V_{base} } BOGO
 S_{base} } choose
 P_{base} } z

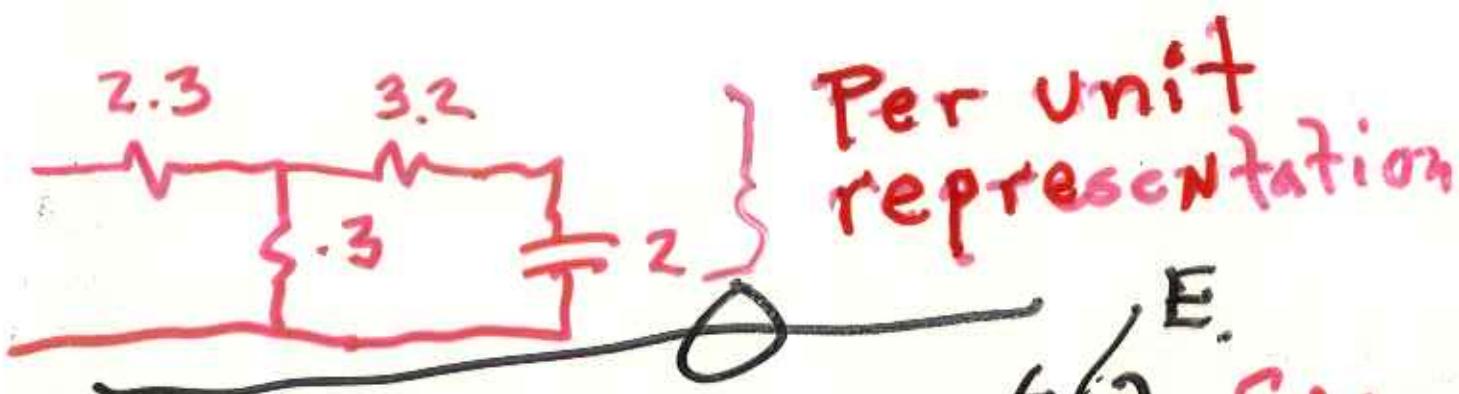
Calculate: $I_{base} = ? \frac{S_B}{V_B}$ get W

$Z_{base} = ? \frac{V_B}{I_B}$

① Specific Gravity is PU
 "Useful for insight into media"



if
 $Z_B = 1500$



② In power we prefer (V) Given (2)

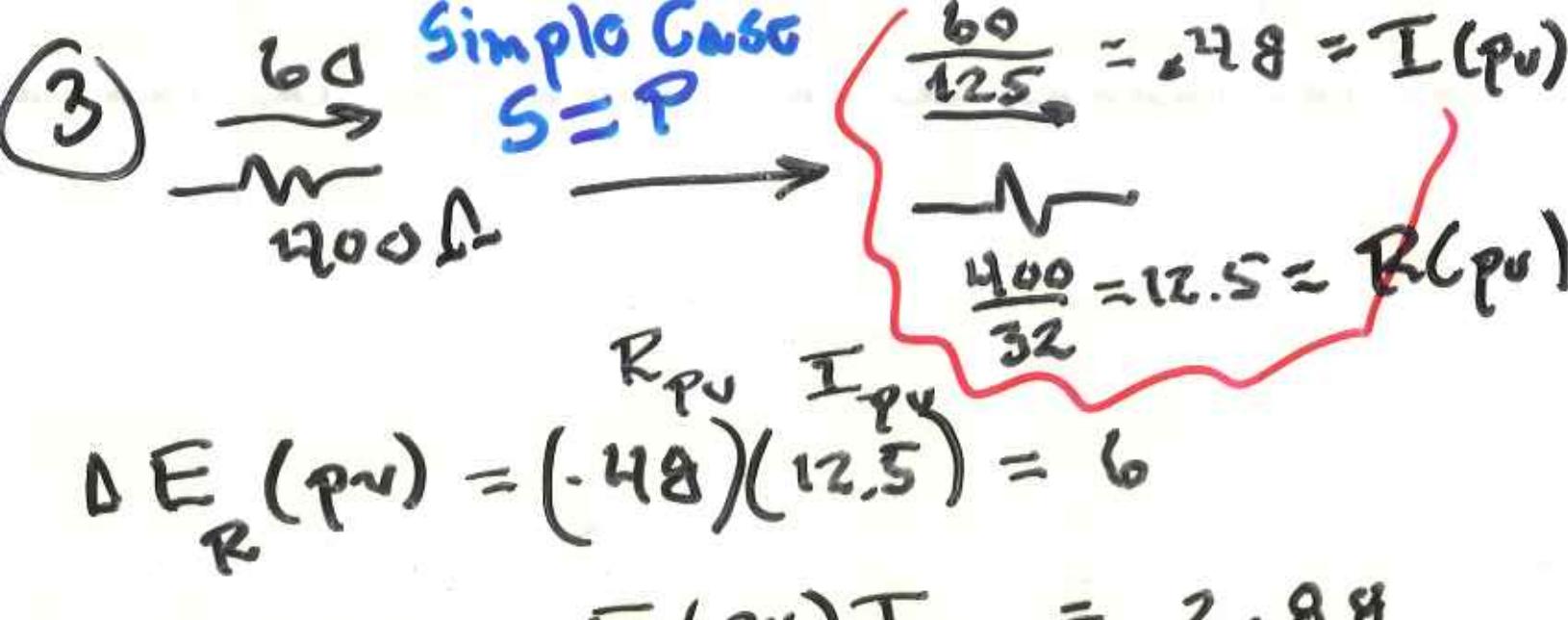
$$I_B = \frac{P_B}{E_B} \quad Z_B = \frac{E_B}{I_B}$$

True if $S_B = P_B$

S_B Base
other z derived
BOGO

Example $E_B = 4KV$ $\Rightarrow I_B = 125$

$S_B = P_B = 500kW$ $Z_B = 32 \Omega$
use this base



$$P_R(pu) = E(pu) I_{pu} = \frac{24}{6} \cdot \frac{.48}{(-.48)} = 2.88$$

for actual values:

$$E = E_p \neq E_{pu} = \frac{24kV}{4kV} 6$$

$$P = P_p \neq P_{pu} = \frac{1440kW}{500kW} 2.88$$

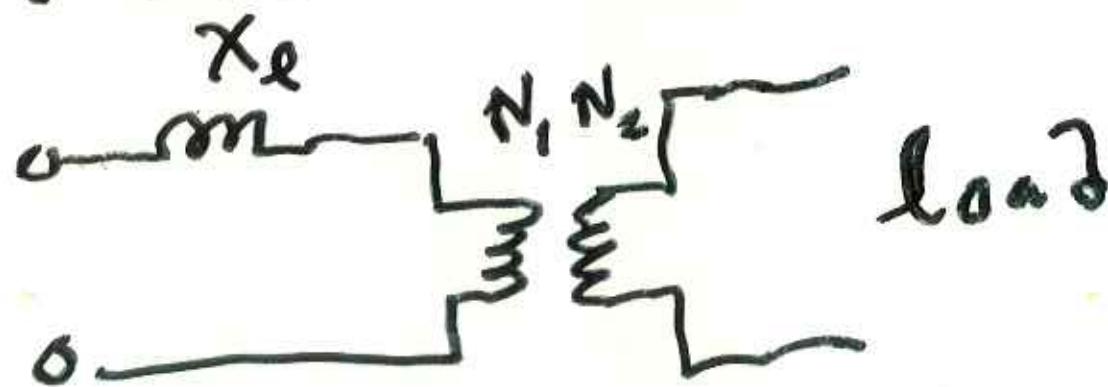
Whooa!

Pu is lots of extra work
- Why bother?

Power system is a cascade
of transformers!

Good power trf. X_m large
 X_e small
but $X_e > R_{wire}$

Trf crude model: Just X_e



Pu will be better for all
 X_e because?

Trf 1



Trf 2 Trf 3



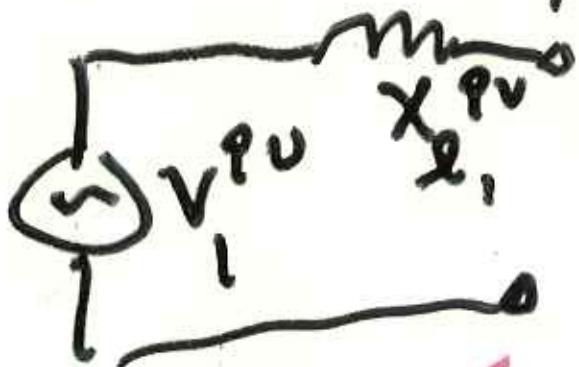
↑
Base
#1

Base #2

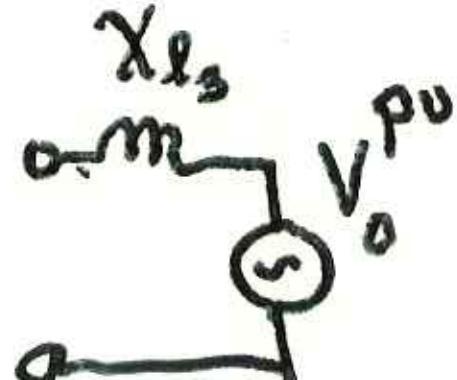
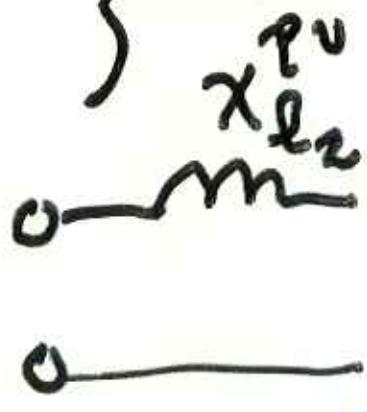
Base
#3

If in each base we

Put X_L, X_m, R in per unit
 $V_{IN}, V_{middle}, V_{out}$ too!



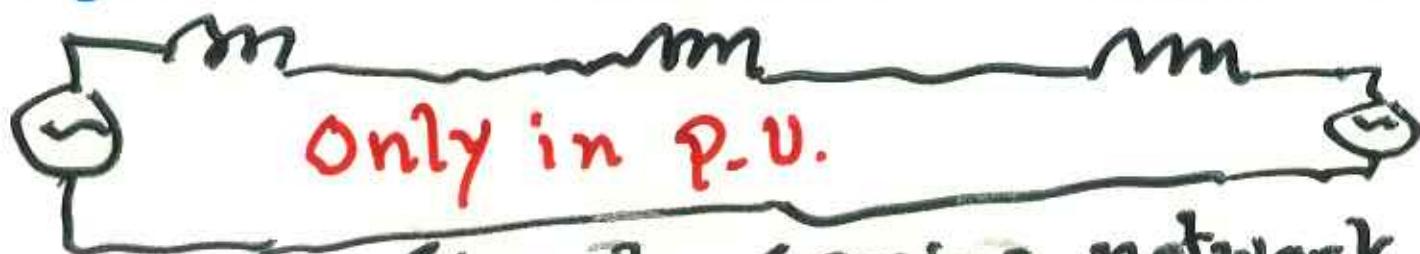
Base #1



Payoff

base #2

base #3



Only in P.U.

Simple series network

Problem #2

The one-line diagram of an unloaded power system is shown below. Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are rated as follows:

Generator 1: 20 MVA, 13.8 KV, $X'' = 0.20$ per unit

Generator 2: 30 MVA, 18 KV, $X'' = 0.20$ per unit

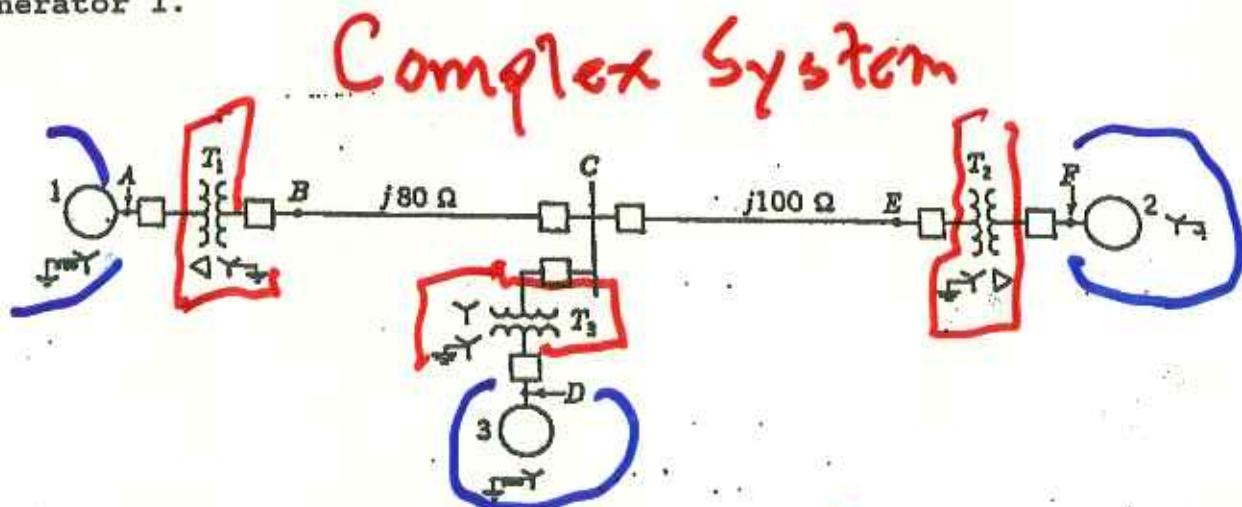
Generator 3: 30 MVA, 20 KV, $X'' = 0.20$ per unit

Transformer T_1 : 25 MVA, 220Y/13.8 KV Δ , $X = 10\%$

Transformer T_2 : Single phase units each rated 10 MVA, 127/18 KV, $X = 10\%$

Transformer T_3 : 35 MVA, 220Y/22 KV Y, $X = 10\%$

Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. Choose a base of 50 MVA, 13.8 KV in the circuit of generator 1.



3 Transformers 3&

3 Generators

2 Transmission Lines

**How
to
model?**

Solution:

$$\text{Gen. 1 } X' = 0.2 \times \frac{50}{20} = 0.50 \text{ p.u.}$$

$$3\text{-}\phi \text{ rating } T_2 = 220/10 \text{ kV, 30 MVA}$$

Base in trans. line: 220 kV, 50 MVA

Base for Gen. 2 is 10 kV.

$$\text{Gen. 2 } X'' = 0.2 \times \frac{50}{30} = 0.933 \text{ p.u.}$$

Base for Gen. 3 is 22 kV

$$\text{Gen. 3 } X''' = 0.2 \left(\frac{20}{22} \right)^2 \times \frac{50}{30} = 0.275 \text{ p.u.}$$

$$T_1 - X = 0.1 \times \frac{50}{25} = 0.20 \text{ p.u.}$$

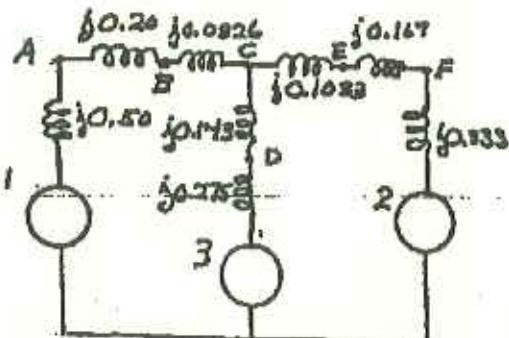
$$T_2 - X = 0.1 \times \frac{50}{30} = 0.167 \text{ p.u.}$$

$$T_3 - X = 0.1 \times \frac{50}{35} = 0.143 \text{ p.u.}$$

Transmission Lines

$$\text{Base } Z = \frac{(220)^2}{50} = 968 \text{ ohms}$$

$$\frac{80}{968} = 0.0826 \quad \left\{ \begin{array}{l} \frac{100}{968} = 0.1033 \\ \end{array} \right.$$



Simplified
via
one line
diagram

One Line Diagrams

One-line diagrams are a simplified graphical representation of a three-phase power system. These diagrams are tools employed to guide studies of the power grid. For further explanations of the following devices refer to PSS/E Lab 2 and supplements for Ch 8 and 27.

Buses

202
EAST500

1.021
510.5
-26.1

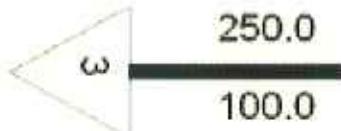
Buses as shown above are the graphical representation of the substations and taps on the transmission lines.

Branches

-473.0 478.0
89.9 -229.4

Branches are the graphical representation of the three phase transmission and distribution lines.

Loads

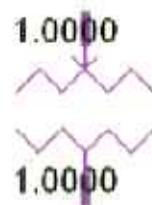


A load acts as a sink on the power system.

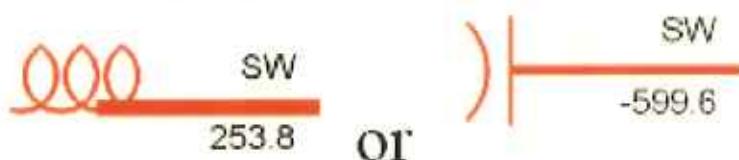
Machines



Two Winding Transformers



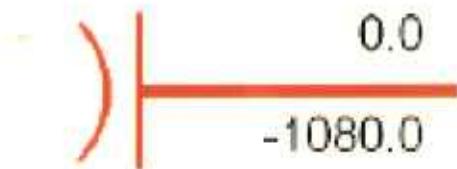
Switched Shunts



Reactor Bank

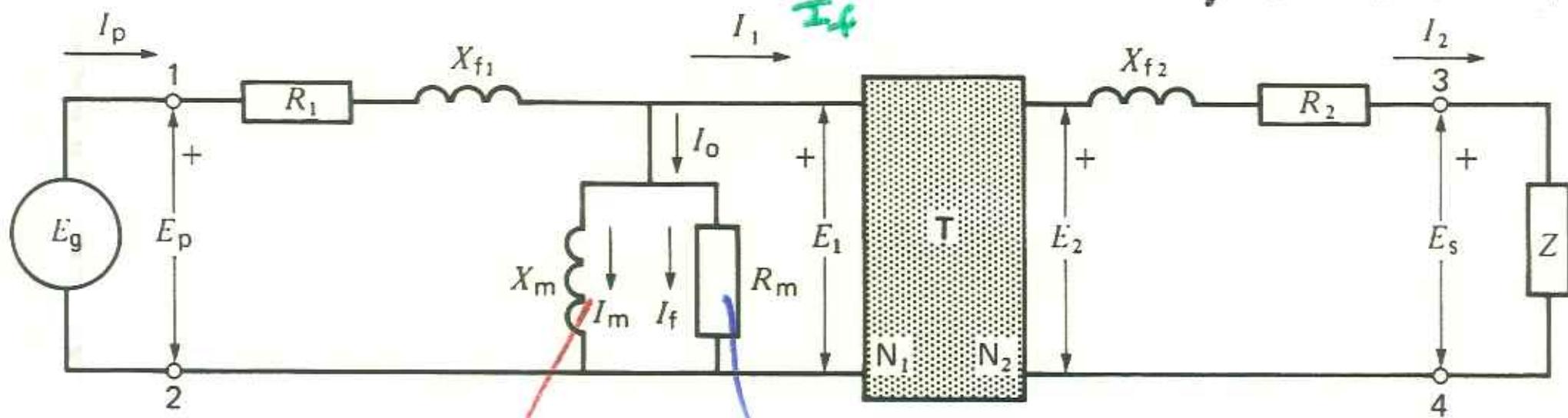


Capacitor Bank



Transformer Walk

- ① $I_2 = \frac{E_s}{Z_2}$
- ② $E_2 = E_s + I_2 [R_2 + jX_2]$
- ③ E_1, I_1 from ideal trf
- ④ $I_o = \frac{I_{Qm} + I_m}{I_f} = \frac{E_1}{[R_m + jX_m]}$

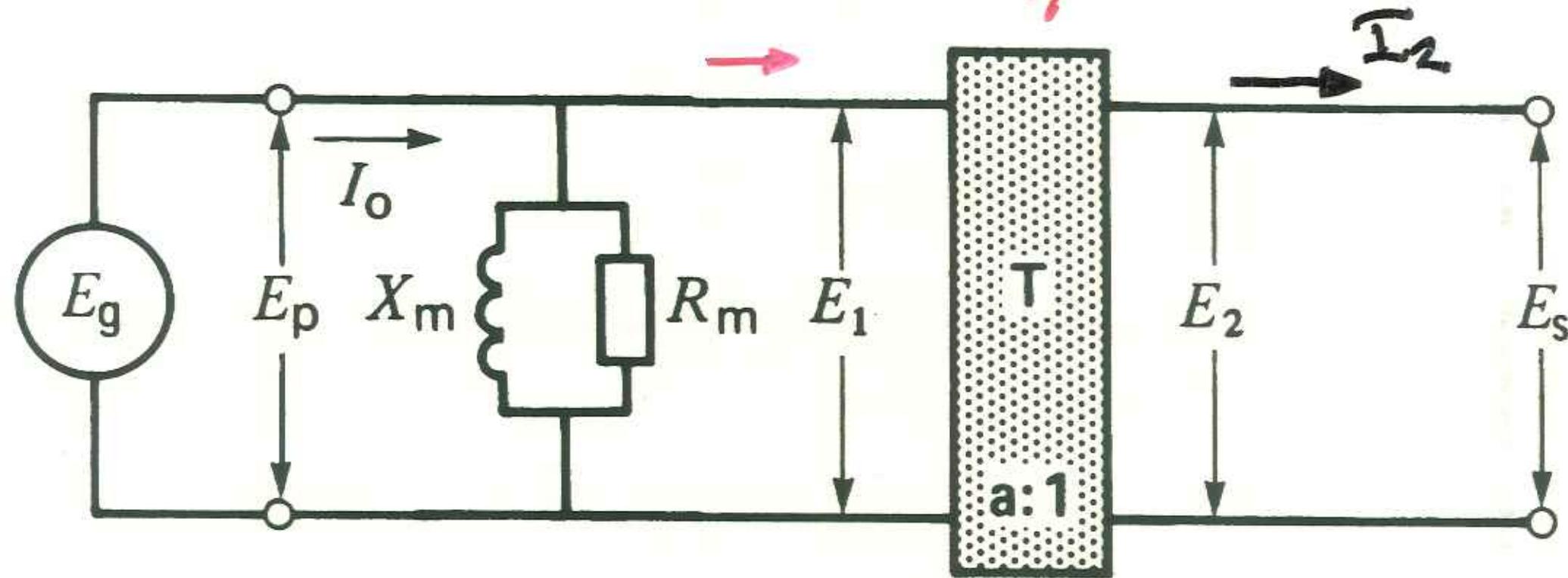


Φ_m for
core loss

- ⑤ $E_p = [I_o + I_f] [R_1 + jX_{f1}] + E_1$

Open Circuit Conditions

- ① Load z_L sets $I_2 = \frac{E_s}{z_L} = 0$
- ② ideal trf $\rightarrow E_1, I_1$



We neglect $X_{L1}, X_{L2}, R_{L1}, R_{L2}$

$$I_m = \frac{E_g}{X_m}$$

$$I_{R_m} = \frac{E_g}{R_m}$$

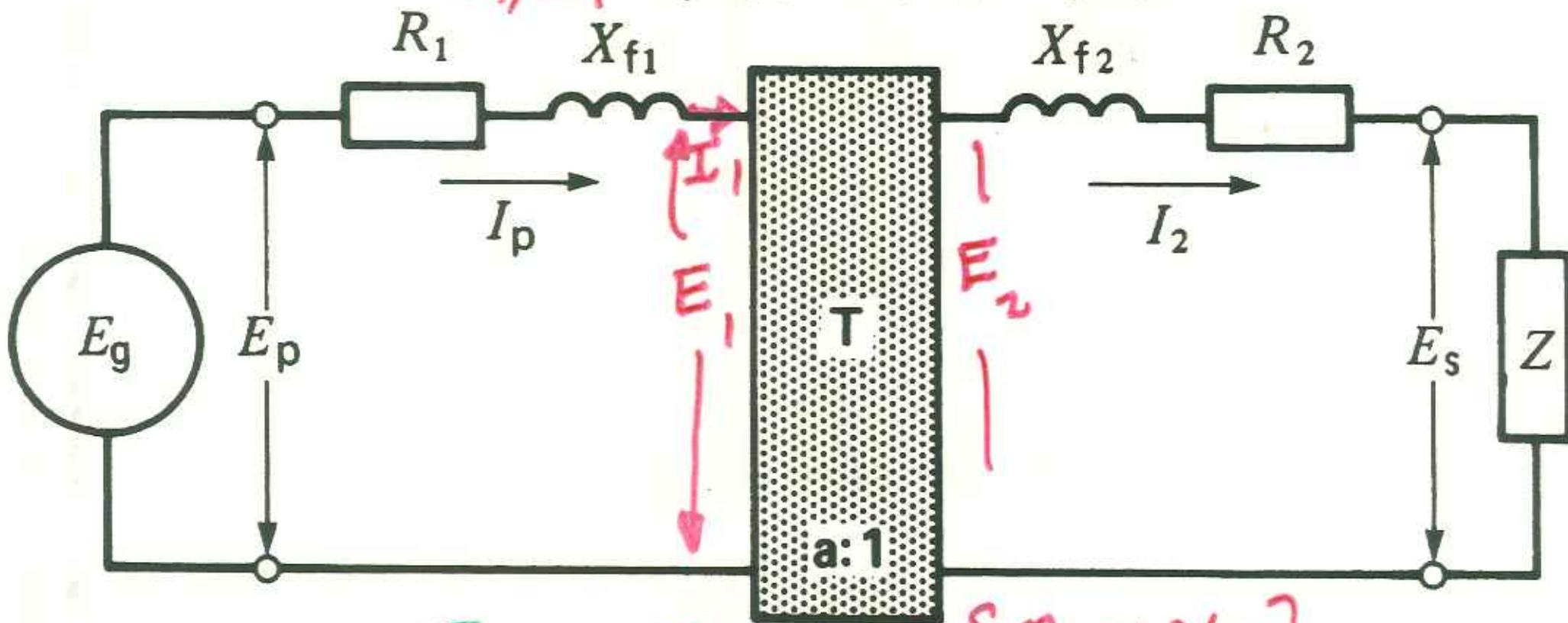
$$I_o = I_{m\pi} + I_1$$

Heavy Load $Z_L \rightarrow 0$

Load Z_L sets $I_2 = \frac{E_s}{Z_L}$

$$\bar{E}_2 = E_s - [jX_{f2} + R_2] I_2$$

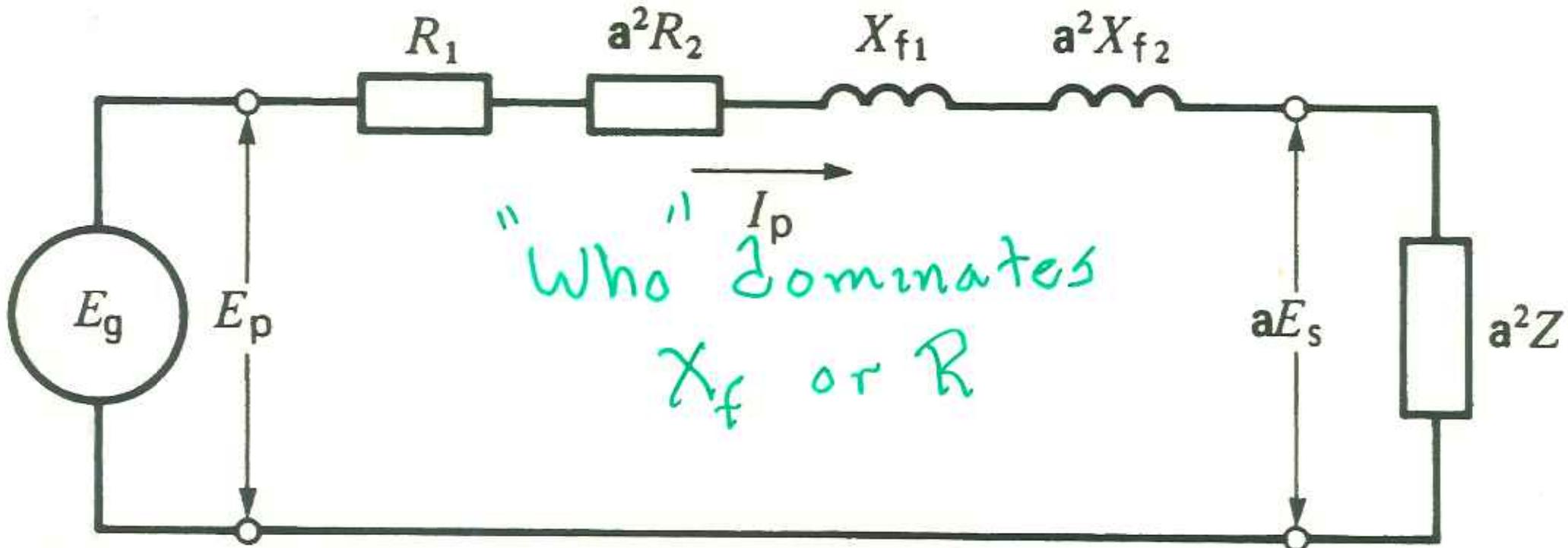
E_1, I_1 from ideal trf

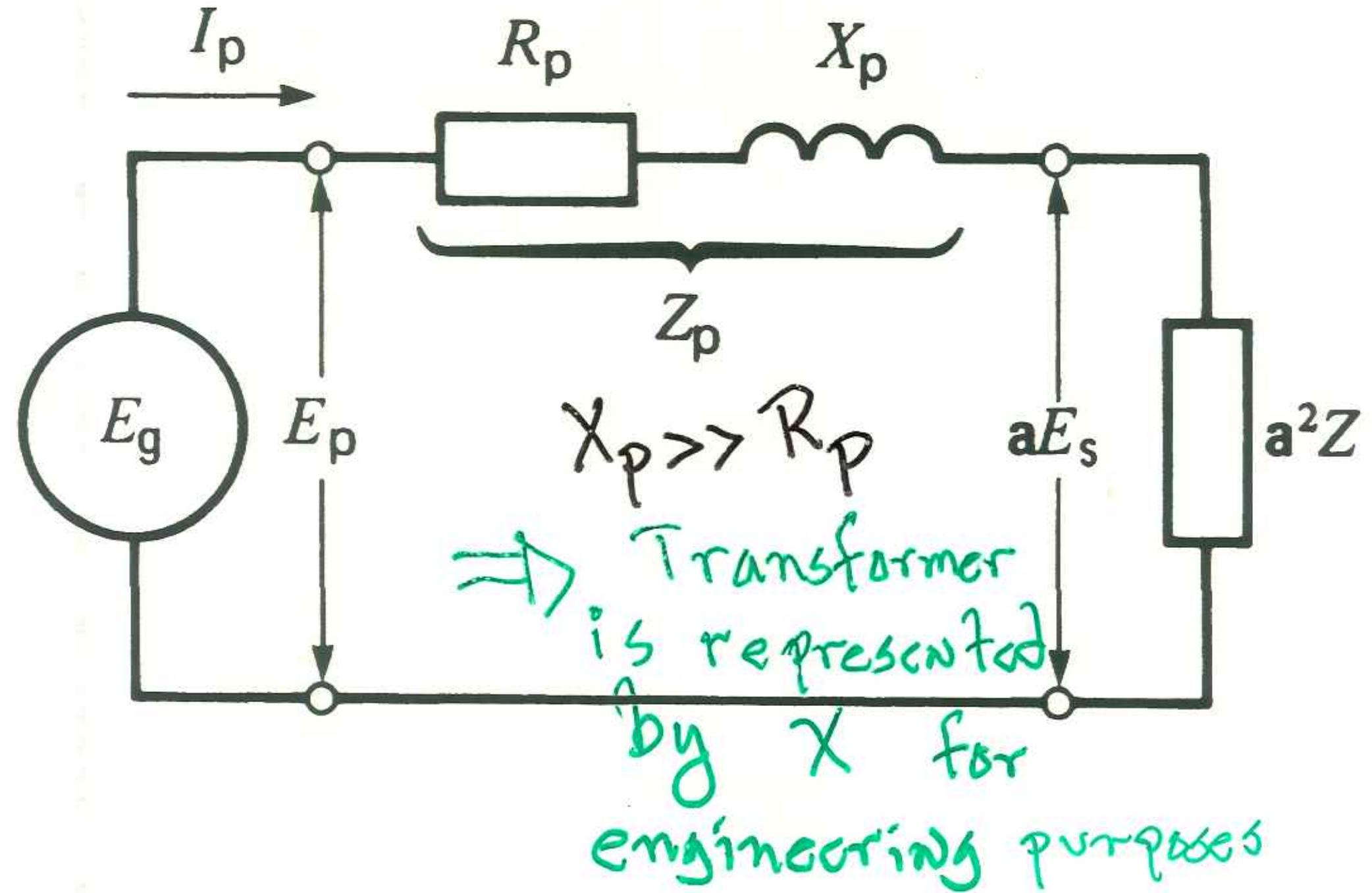


$$E_g \approx E_1 + I_1 [R_1 + jX_{f1}]$$

We assume $X_m \gg X_f$

Good Design Power Trf Neglect X_m





Per Unit and Base Values
Read Text sections p.v.

Equipment: V_{rated} , I_{rated}
 S_{rated} and Z_R

Operating Conditions: V_{op} , I_{op} , S_{op}
 $Z_{\text{op}} =$

Per Unit = $\frac{\text{Actual Value}}{\text{Base Value}} \times 100\%$

Advantages of PU.

① Trf X_e smaller range
in P.U. for
transformers

Chq next

② For a multi-level power system

Common Base System
eliminates transformer
turns ratios

Wow! to simplify 😊

\underline{Z} vs \underline{Z}_{pu} measured
↑
measured

(1) Two bases: Rated vs Operating
 Base \underline{Z} from $\frac{\text{rated values}}{V_R, I_R, S_R}$
 of equipment

(2) \underline{Z} is unchanging
 \underline{Z}_{pu} depends on if it is used
 with rated values or with
 operating values V_{op}, I_{op}, S_{op}

(3) Relation between two bases

$$\underline{Z} = \underline{Z}_{pu}^{\text{op}} \frac{\underline{Z}_{\text{base}}^{\text{op}}}{\frac{V_{op}^2}{S_{op}}} = \underline{Z}_{pu}^{\text{rated}} \frac{\underline{Z}_{\text{base}}^{\text{rated}}}{\frac{V_R^2}{S_R}}$$

$$\underline{Z}_{pu}^{\text{op}} \equiv \underline{Z}_{pu}^{\text{rated}} * \left(\frac{V_R}{V_{op}} \right)^2 \left(\frac{S_{op}}{S_R} \right)$$

Problem #2

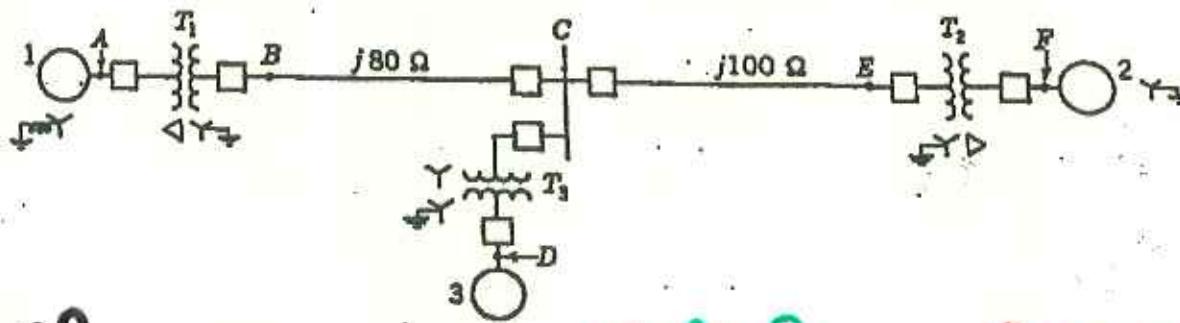
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RATINGS

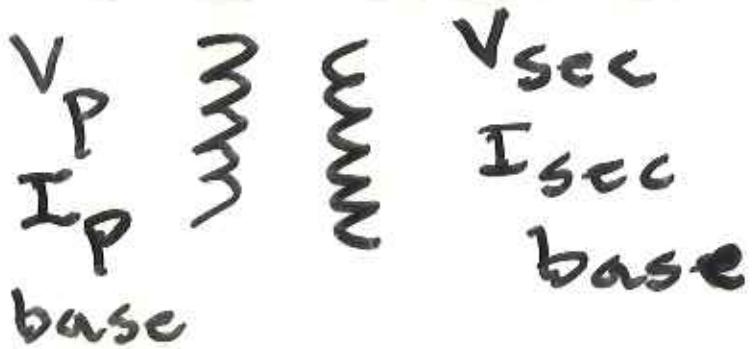
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Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. Choose a base of 50 MVA, 13.8 KV in the circuit of generator 1.

Operating



① Change all rated PU → operating PU



Transformer comes with
a rating
from manufacturer: X_{pu}^R

Engineer uses transformer
under operating conditions
find new $X_{pu}^{\text{operating}}$

Guide is $\underline{Z^{\text{rating}}} = \underline{Z^{\text{operating}}}$
its the same trf. !

$$X_{pu}^0 \frac{V_{op}^2}{S_{op}} = X_{pu}^R \frac{V_{rated}^2}{S_{rated}}$$

$$X_{pu}^0 = [X_{pu}^R] \left[\frac{V_R^2}{V_{op}^2} \right] \left[\frac{S_0}{S_R} \right]$$

$$X_{\text{new}}^{\text{pu}} = X_{\text{old}}^{\text{pu}} \times \left[\frac{S_{\text{new}}}{S_{\text{old}}} \right] \left[\frac{V_{\text{old}}}{V_{\text{new}}} \right]^2$$

8

Solution:

Gen. 1 $X' = 0.2 \times \frac{50}{20} = 0.50 \text{ pu}$
 3- ϕ rating $T_2 = 220/10 \text{ kV}, 30 \text{ MVA}$

Base in trans. line: $220 \text{ kV}, 50 \text{ MVA}$
 Base for Gen. 2 is 10 kV .

Gen. 2 $X'' = 0.2 \times \frac{50}{30} = 0.333 \text{ pu}$
 Base for Gen. 3 is 22 kV

Gen. 3 $X''' = 0.2 \left(\frac{20}{22} \right)^2 \times \frac{50}{30} = 0.275 \text{ pu}$
 $T_1 - X = 0.1 \times \frac{50}{25} = 0.20 \text{ pu}$

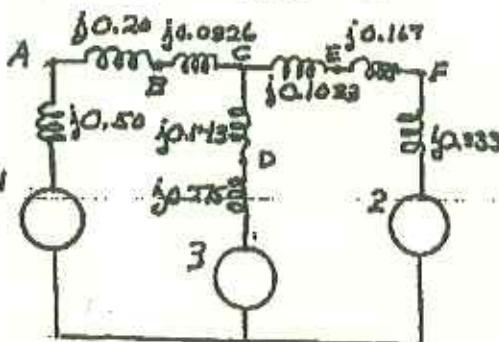
$$T_2 - X = 0.1 \times \frac{50}{30} = 0.167 \text{ pu}$$

$$T_3 - X = 0.1 \times \frac{50}{35} = 0.143 \text{ pu}$$

Transmission Lines

$$\text{Base } Z = \frac{(220)^2}{50} = 968 \Omega$$

$$\frac{80}{968} = 0.0826 \quad ; \quad \frac{100}{968} = 0.1033$$



Summary

1

PER UNIT CALCULATIONS

(A Decimal method of using percent)

The per unit system is a method of normalizing system and equipment impedances so that circuit or network problems can be solved in a straightforward manner for voltages, currents and watts and vars without having to convert values whenever there is a change in voltage across a transformer or when a difference in equipment size is encountered. The per unit system can be applied to either single or three phase systems.

S
V
el

① A system KVA or MVA base is chosen first. For quick applications a base of 1, 10 or 100 MVA is most convenient. This will be the 100% or 1 per unit value. ② Next, the line to line voltage (KV) or voltages are selected. These will usually be the nominal ratings on the major transformer at the substation or at the customer location on distribution systems. Again, each voltage value selected becomes the 100% or 1 per unit value. For the KVA and KV selected as base there will be a unique value of per unit base (or 100%) amperes and base ohms to match the base KVA selected.

① ② Selected
To find the base values we first select base KVA and base KV (line to line). Then
③ ④ Derived BOGO

$$\text{Base KVA} = \sqrt{3} \text{ Base KV(line to line)} \times I^*(\text{line})$$

③ Base current $I^*(\text{line}) = \text{Base KVA} / \sqrt{3} \text{ Base KV(line to line)}$

As with any balanced three phase circuit, we can represent it on a line to neutral basis and consider an equivalent wye system. Remember that 100% or 1 per unit line to line voltage is also 100% or 1 per unit line to neutral voltage.

④ Base ohms = Base V(line to neutral) / Base current

$$= \frac{\text{Base KV[line to line]} * \sqrt{3} \text{ Base KV[line to line]}}{K * \sqrt{3}} \quad \text{Base KVA}$$

Speci

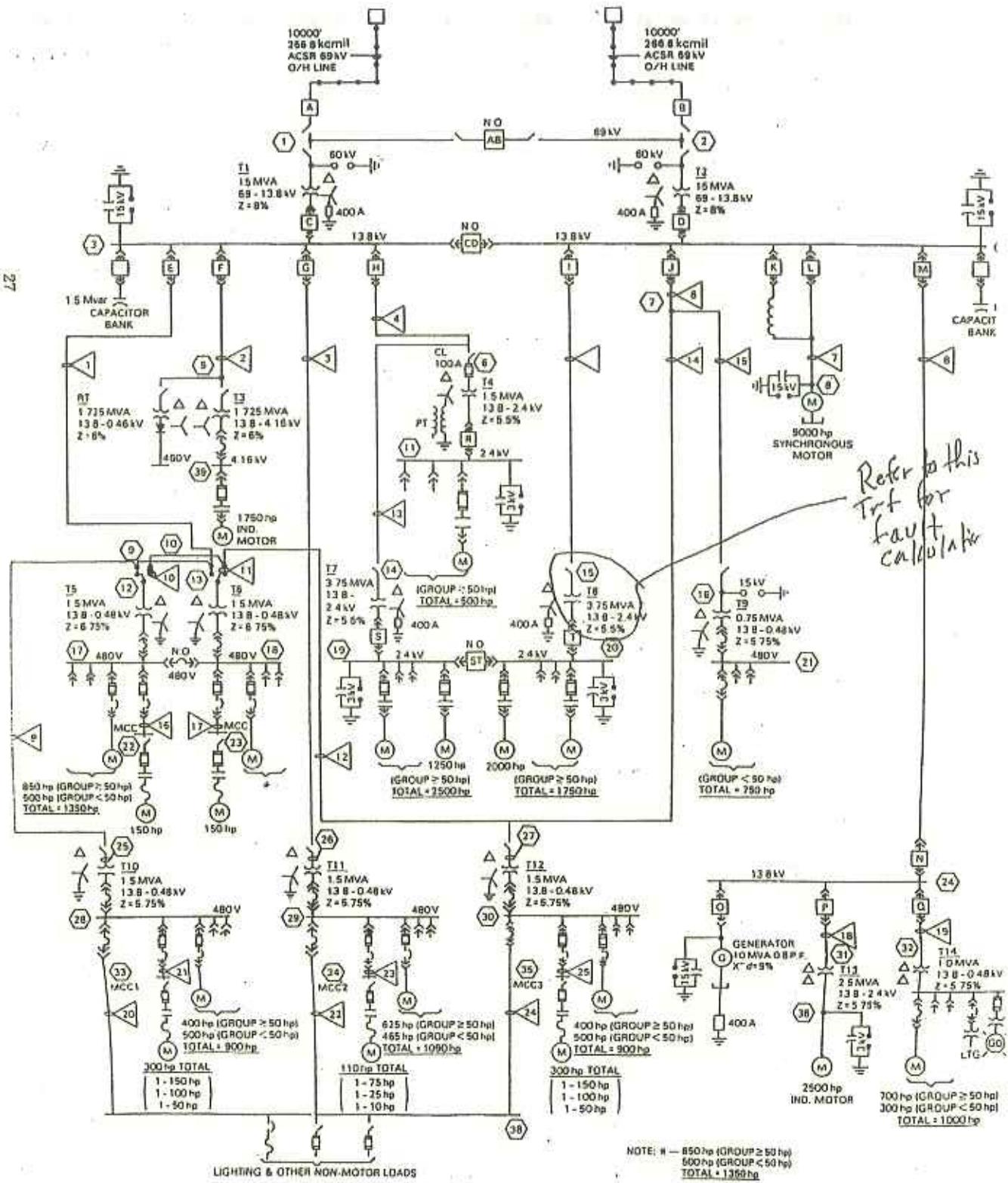
$$= \frac{\text{Base KV}^2[\text{line to line}]}{\text{Base MVA}}$$

system

⑤ To convert a generator or transformer to the base you selected, it is only necessary to remember that it requires a straight ratio.

$$\frac{\text{Nameplate ohms}}{\text{Nameplate KVA}} = \frac{\text{Per unit ohms}}{\text{Base KVA}}$$

Why bother with P.U.?

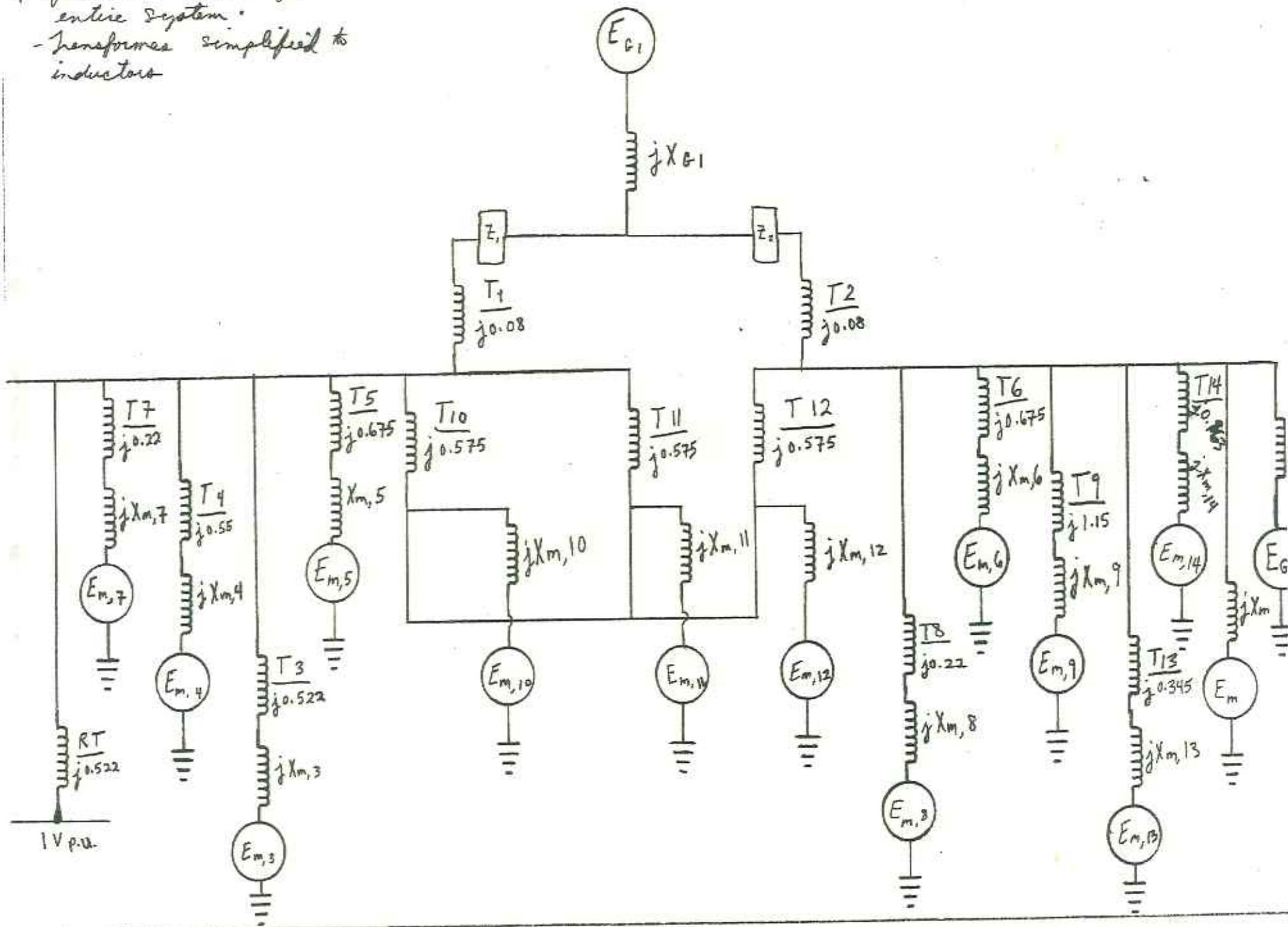


Composite Single-Line Diagram for Typical Large Industrial Power System

- ① Choose S_{base}
- ② From $X_{pu(rated)}$ → $X_{pu(operating)}$
- ③ Simplify

Next time

- * Equivalent circuit for entire system.
- transforms simplified to inductors



You identify each for HW

Symbol Definitions

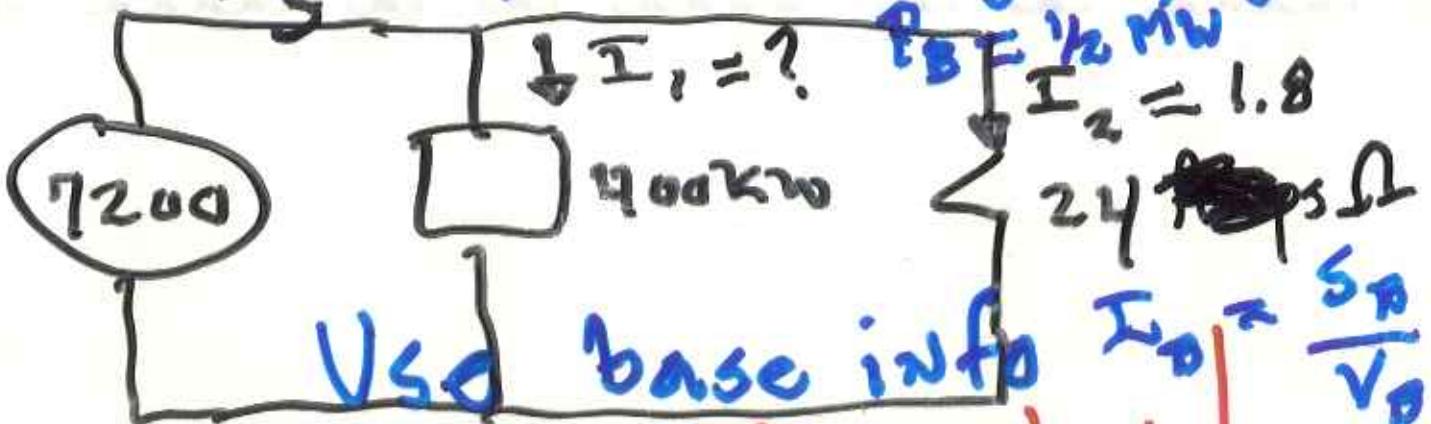
- → || surge arrester (used to contain 3rd Harmonic)
- — — — — surge capacitor
- + + rectifier bank
- m m reactor non-magnetic core
- \ / 3-φ, neutral ungrounded
- \ / 3-φ, neutral grounded
(allowed to test for balance)
- ↔ ↔ — Air circuit Breaker,
removable type DH
- — — circuit Breaker,
non-drawout (oil or vacuum)
- ↔ ↗ Air circuit Breaker,
drawout type
- / — Disconnecting fuse, non-drawout
- — — — — fuse ↔ / — drawout
 fuse
- — — switch
- 3S Potential Transformer
- → || ground
- m m m Two Winding Power
Transformer
- Δ Three phase, three wire delta
connection
- X plus indicator
- △ line flag
- AC machine (generator or motor)
- circuit Breaker
- / — capacitor bank
(used to compensate reactive load)

$$\left(\begin{matrix} X \\ Z \end{matrix}\right)_{\text{(real)}} = X_{\text{PV}} * Z_{\text{base}}$$

$$X_{\text{PV}}^R \text{ (rating)} \quad Z_{\text{base}} \text{ (rating)} = \underbrace{X_{\text{PV}}^{\text{op}}} \underbrace{Z_{\text{base}}}_{\text{(op)}} \quad \frac{V_R^2}{S_R} \text{ (rating)} \quad \frac{V_{\text{op}}^2}{S_{\text{op}}}$$

$$X_{\text{PV}}^R \quad \frac{V_R^2}{S_R} = X_{\text{PV}}^{\text{op}} \quad \frac{V_{\text{op}}^2}{S_{\text{op}}} \\ X_{\text{PV}}^{\text{op}} \equiv X_{\text{PV}}^R * \left[\frac{V_R}{V_{\text{op}}} \right]^2 \frac{S_{\text{op}}}{S_R}$$

$$I_S = I_1 + I_2 \quad \text{Given: } V_B = 4\text{kV}, Z_B = 32\Omega$$



Per Unit Transformation
Same base $E = 4\text{kV}, P = 500\text{kW}$

