Lecture 12 - Analog Communication (I)

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Outline

- Communication Basics
- AM: amplitude modulation
- DSB-SC: double sideband - suppressed carrier
- QAM: quadrature amplitude modulation
- SSB: single sideband

Reference:

- Tretter Chapters 5-8 (being put on e-reserve),
- Haykin, Communication Systems, Third Edition

Basics

The information we wish to send is in the form of a real signal (for example, voice). Some properties of real signals:
Fourier Transform:

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{-j2\pi f t} df. \quad (1)$$

Then for real $g(t)$, we have $g(t) = g^*(t)$, and

$$g^*(t) = \int_{-\infty}^{\infty} G^*(f)\, e^{j2\pi f t} df = \int_{-\infty}^{\infty} G^*(-f')\, e^{-j2\pi f' t} df'. \quad (2)$$

So for real $g(t)$, $G^*(-f) = G(f)$, or

$$|G(f)| = |G(-f)|, \quad \Theta(-f) = \Theta(f). \quad (3)$$

Furthermore, for real, symmetric $g(t) = g(-t)$, the above become

$$G^*(f) = G(f) = G(-f), \quad \Theta(f) = 0, \quad (4)$$

i.e. $G(f)$ is real and symmetric about $f = 0$.

Modulation Theorem

If

$$m(t) \leftrightarrow^{FT} M(f), \tag{5}$$

then

$$m(t)e^{j2\pi f_c t} \leftrightarrow^{FT} M(f - f_c) \tag{6}$$

and

$$m(t)e^{-j2\pi f_c t} \leftrightarrow^{FT} M(f + f_c). \tag{7}$$

This is easily shown from the definition of Fourier Transform. Adding the two, we get

$$m(t)cos(2\pi f_c t) \leftrightarrow^{FT} \frac{1}{2}[M(f - f_c) + M(f + f_c)]. \tag{8}$$

So modulating $m(t)$ by the cosine function produces two frequency-shifted images of the original baseband signal. This is the basis for DSB-SC.

Hilbert Transform

Ref: Haykin.
We use the Fourier Transform to separate out signals according to frequency: G(f). The Hilbert Transform plays a similar role for phase.

$$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} g(t)\frac{1}{t-\tau}d\tau = g(t) * \frac{1}{\pi t}. \quad (9)$$

Convolution in time domain $\equiv$ multiplication in frequency domain. We also know that $\frac{1}{\pi t} \leftrightarrow^{FT} -jsgn(f)$, so can write above in frequency domain:

$$G(\hat{f}) = -jsgn(f)\, G(f). \quad (10)$$

Importance of HT:

1. Used to realize phase selectivity (SSB Modulation/Demodulation);
2. Math basis for realizing bandpass signals.

Hilbert Transform (2)

Example:

$$g(t) = cos(2\pi f_c t) \leftrightarrow^{FT} G(f) = \frac{1}{2}[\delta(f - fc) + \delta(f + f_c)]. \tag{11}$$

Then using eqn(10) above,

$$G(\hat{f}) = -\frac{j}{2}[\delta(f - fc) + \delta(f + f_c)]\, sgn(f) \tag{12}$$

$$G(\hat{f}) = \frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c) \leftrightarrow^{FT} g(\hat{t}) = sin(2\pi f_c t) \tag{13}$$

So the HT changes cosines into sines. Equivalent to 90° phase shift.

Some Properties of Hilbert Transform

1. Signals $g(t)$ and $\hat{g(t)}$ have the same amplitude spectrum. HT operates on phase only

2. $\hat{\hat{g(t)}} = -g(t)$. Easily shown by noting that two HTs applied in series is equivalent to $180°$ degree phase shift, which is equivalent to multiplying by -1.

3. $g(t)$ and $\hat{g(t)}$ are othogonal: $\int_{-\infty}^{\infty} g(t)\,\hat{g(t)}dt = 0$.

Analytic Signals – Baseband

Let g(t) be a real baseband signal. Then it will have an amplitude spectrum symmetric about f=0:

We can form two complex representations of g(t) using Hilbert Transforms:

$$g_+(t) = g(t) + j\hat{g(t)}, \tag{14}$$

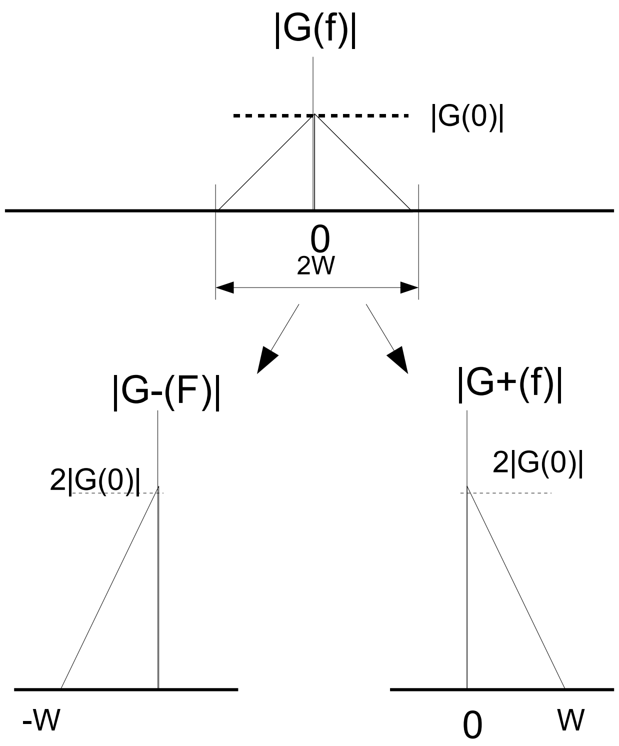
$$g_-(t) = g(t) - j\hat{g(t)}, \tag{15}$$

$g_+(t)$ is called the positive pre-envelope of g(t) and $g_-(t)$ is the negative pre-envelope. Note that $g_-(t) = g_+^*(t)$. We can express g(t) in terms of the $\pm$ envelopes:

$$g(t) = \frac{1}{2}[g_+(t) + g_-(t)]. \tag{16}$$

These split the amplitude spectrum $G(f)$ into two components $G_+(f)$ and $G_-(f)$ which are scaled pieces of the original spectrum having only positive and negative frequencies, respectively.

Analytic Signals – Baseband (2)



Analytic Signals – Passband

We do the same for a real passband signal. This has a symmetric amplitude spectrum with two (typically narrowband) components centered about $\pm f_c$, the carrier frequency.

We can write

$$g(t) = Re\{\tilde{g}(t)e^{j2\pi f_c t}\}. \quad (17)$$

$\tilde{g}(t)$ is called the complex envelope of g(t). In terms of the pre-envelopes, we can write

$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t}, \quad g_-(t) = (\tilde{g}(t))^* e^{-2\pi f_c t}, \quad g(t) = \frac{1}{2}[g_+(t) + g_-(t)]. \quad (18)$$

Analytic Signals – Passband (2)

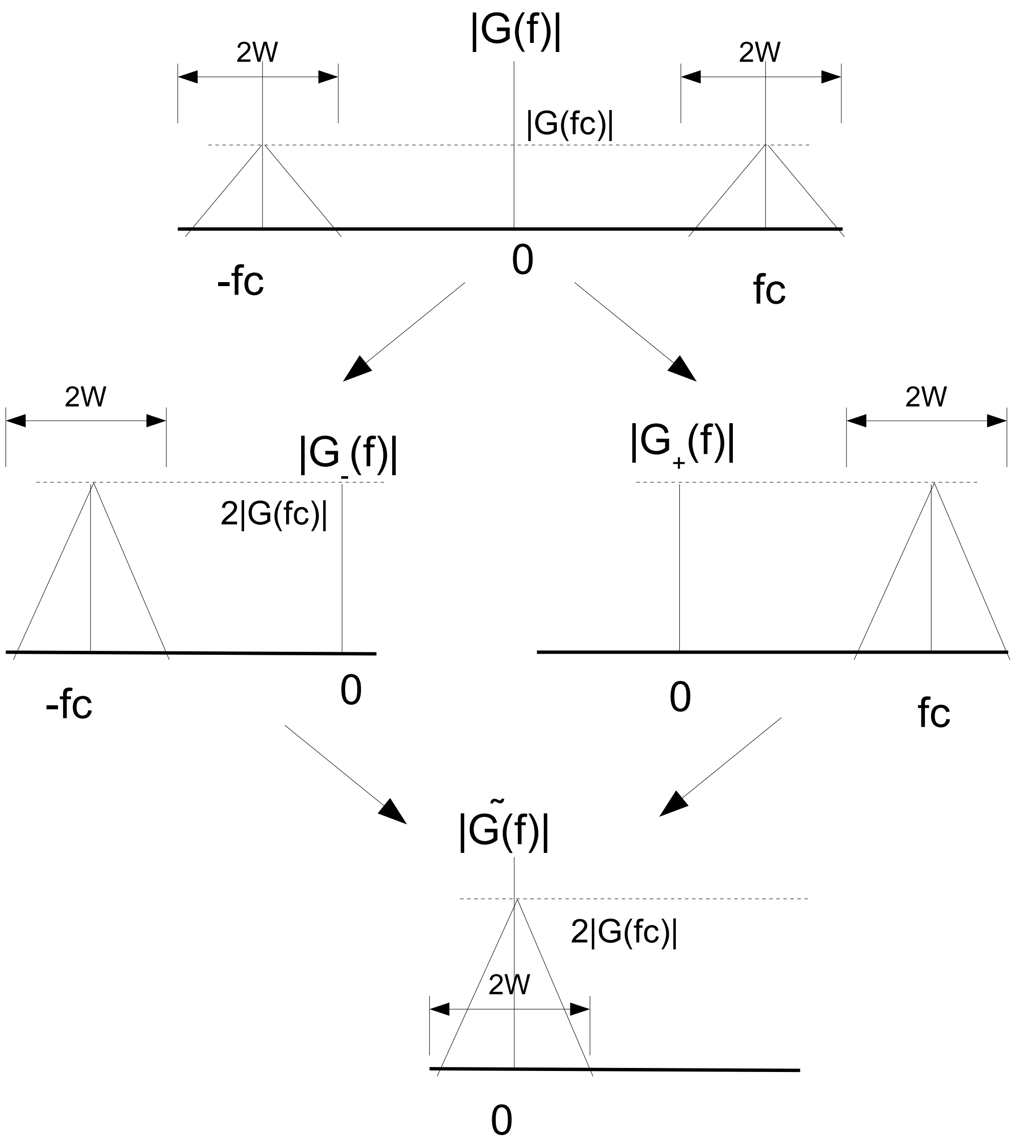
The complex envelope can be split into two components: the in-phase and quadrature components:

$$\tilde{g(t)} = g_I(t) + j g_Q(t). \tag{19}$$

Substituting (19) back into (17), we can write g(t) in terms of in-phase and quadrature components (canonical form):

$$g(t) = g_I(t) cos(2\pi f_c t) - g_Q(t) sin(2\pi f_c t) \tag{20}$$

Analytic Signals – Passband (3)



Amplitude Modulation

First form of modulation (early radio). Simple, especially receiver (crystal set). Least efficient at using radiated power and spectrum.

Start with a carrier signal $c(t) = A_c cos(2\pi f_c t)$. The real radiated signal s(t) given by:

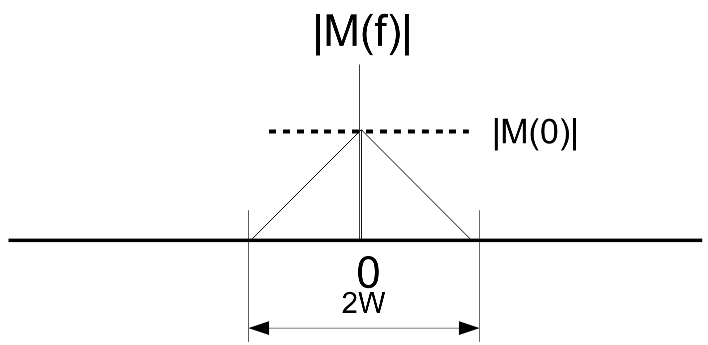
$$s(t) = A_c[1 + k_a m(t)]cos(2\pi f_c t), \quad (21)$$

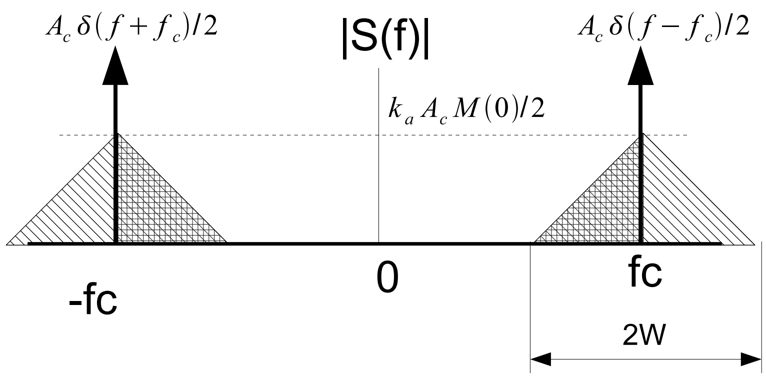
where $m(t)$ is the message (baseband audio signal) and k_a is a constant called the amplitude sensitivity of the modulator. To avoid "over-modulation", a form of distortion, must have $|k_a m(t)| < 1 \; \forall$ t.
Taking the Fourier Transform of (22), we have:

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]. \quad (22)$$

The first bracketed term represents the carrier power (basically wasted) and the second term the two sideband power components, which is where the useful information is.

Amplitude Modulation (2)





Amplitude Modulation (3)

Single-tone example:

Let $m(t) = A_m cos(2\pi f_m t)$. Then s(t) is given by:

$$s(t) = A_c[1 + \mu cos(2\pi f_c t) cos(2\pi f_c t)], \tag{23}$$

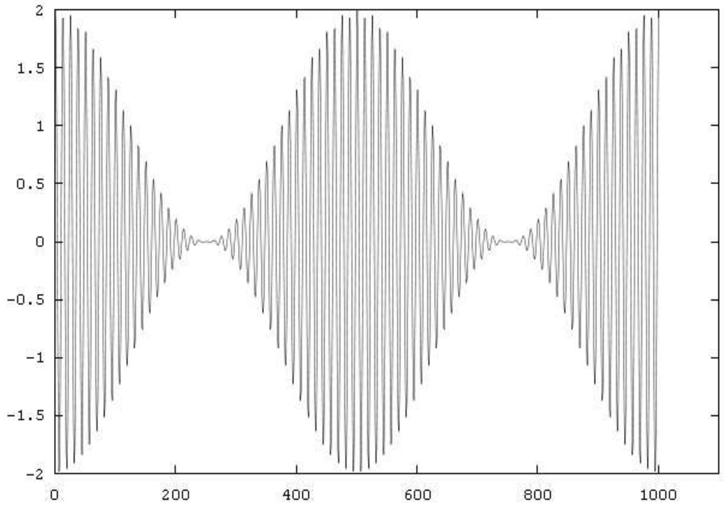
where $\mu = k_a A_m$ is called the modulation factor. Must have $|\mu| < 1$.

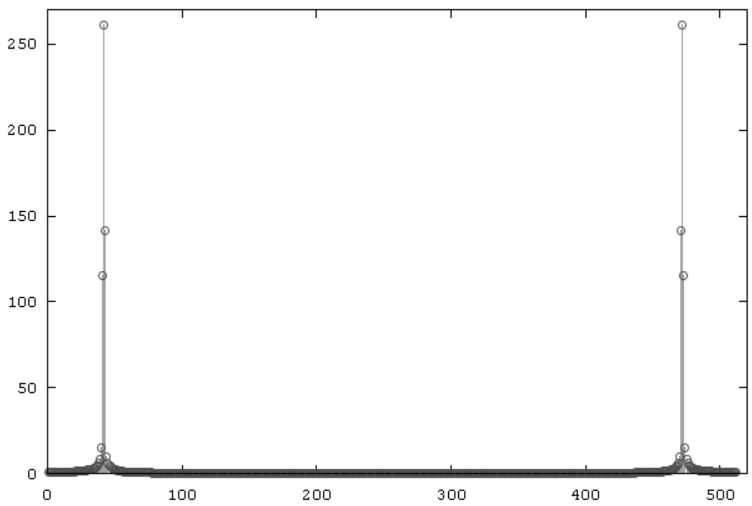
By taking Fourier Transform (see Haykin), the average signal power is

- Carrier power = $\frac{1}{2}A_c^2$
- Upper side-frequency power = lower side-frequency power = $\frac{1}{8}\mu^2 A_c^2$,

$\Rightarrow$the sidebands can never get more that 33% of the total radiated power.

Amplitude Modulation (4)





Amplitude Modulation – Detection

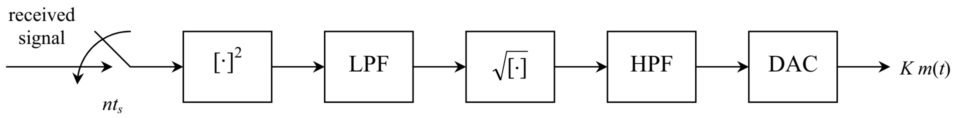
Simple envelope detectors can be used, based on rectifying or squaring the signal to eliminate the carrier. From (21),

$$s^2(t) = A_c^2[1 + k_a m(t)]^2[e^{j2\pi f_c t} + e^{-j2\pi f_c t}]^2/4, \tag{24}$$

or

$$s^2(t) = A_c^2[1 + k_a m(t)]^2[1 + cos(4\pi f_c t)]/2. \tag{25}$$

The cosine term can be filtered out with a low pass filter, and the filter output passed through a square root circuit to recover m(t). Notice that the carrier produces a DC offset which can be eliminated with capacitive coupling (high pass filtering).

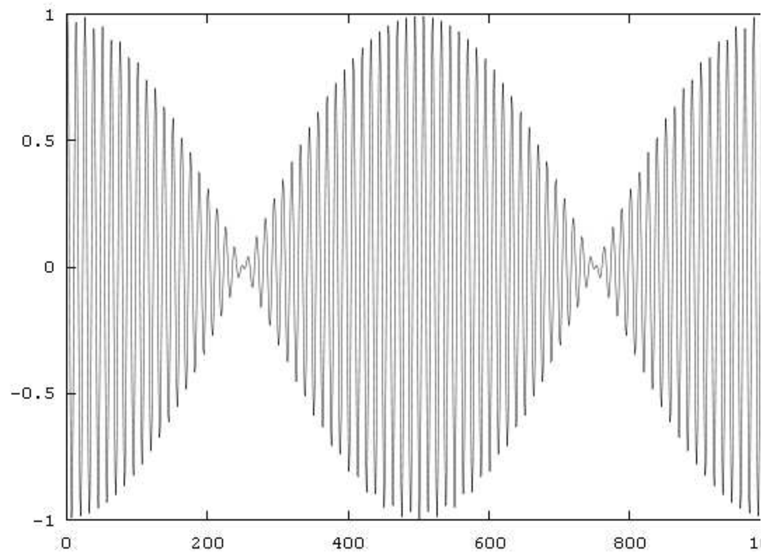


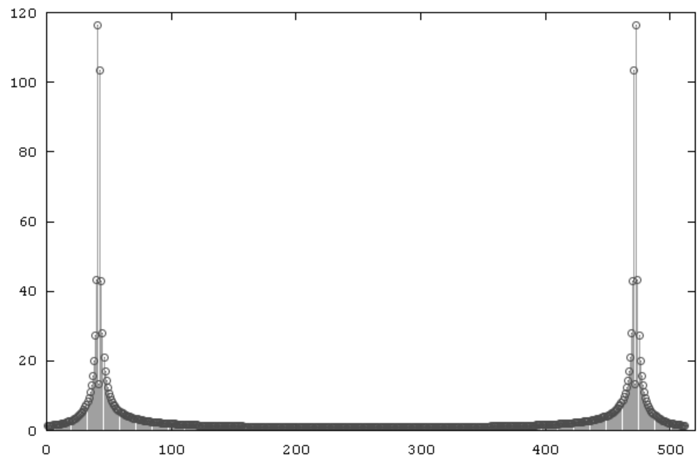
Double Side Band - Suppressed Carrier

A DSB-modulated signal has the following form:

$$s(t) = c(t)m(t) = A_c cos(2\pi f_c t)m(t). \tag{26}$$

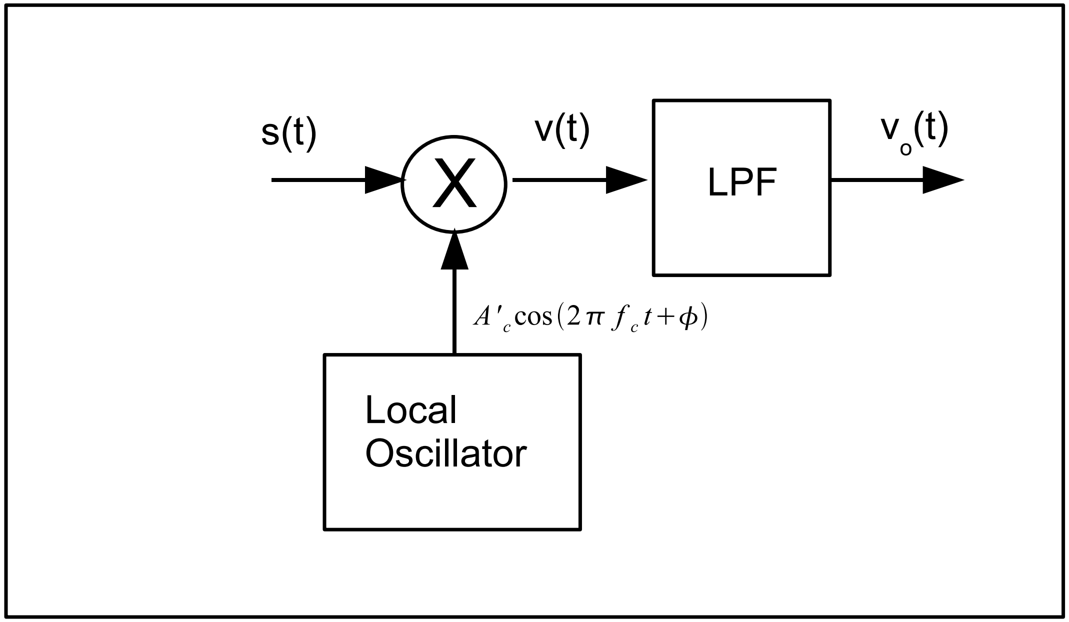
Every time m(t) crosses 0, s(t) undergoes a phase reversal. The spectrum of a DSB-SC signal has NO carrier component.





Demodulating DSB-SC

All suppressed-carrier techniques require more elaboration detection than simple envelope detection – ”coherent” detection. The principle is shown in the figure below:



Demodulating DSB-SC (2)

From the previous figure:

$$v(t) = A'_c cos(2\pi f_c t + \phi)s(t) \tag{27}$$

Substituting for s(t) using eqn(26) and multiplying out, we get

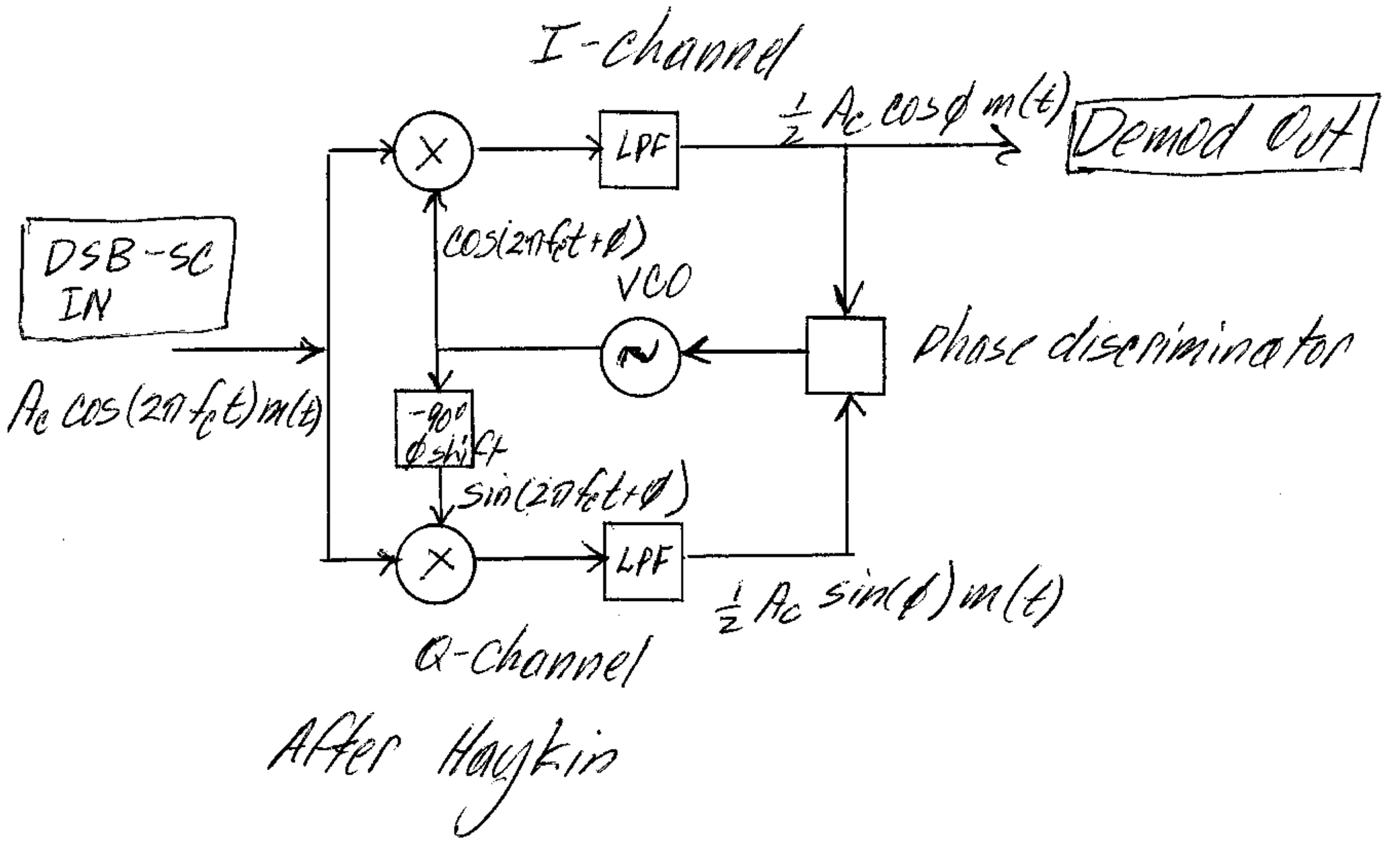
$$v(t) = \frac{1}{2} A_c A'_c [cos(4\pi f_c t + \phi) + cos(\phi)]m(t). \tag{28}$$

The LPF eliminates the first cosine term in brackets and the output is:

$$v_o(t) = \frac{1}{2} A_c A'_c cos(\phi)m(t). \tag{29}$$

We need a control loop to maintain $\phi = 0$: a PLL.

Costas Receiver Block Diagram



Digital Costas Receiver – From Lab7 Notes