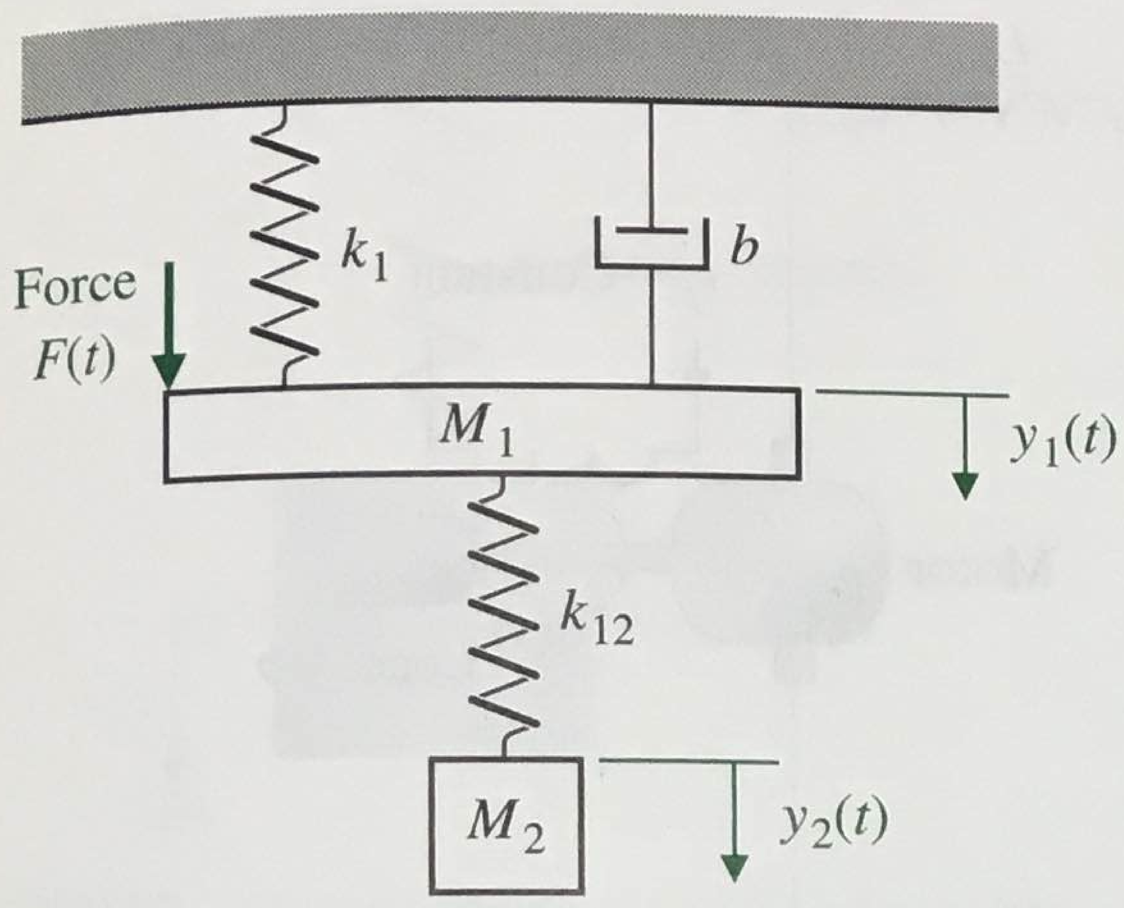

FIGURE P2.2 Vibration absorber.

P2.3 A coupled spring–mass system is shown in Figure P2.3. The masses and springs are assumed to be equal. Obtain the differential equations describing the system.

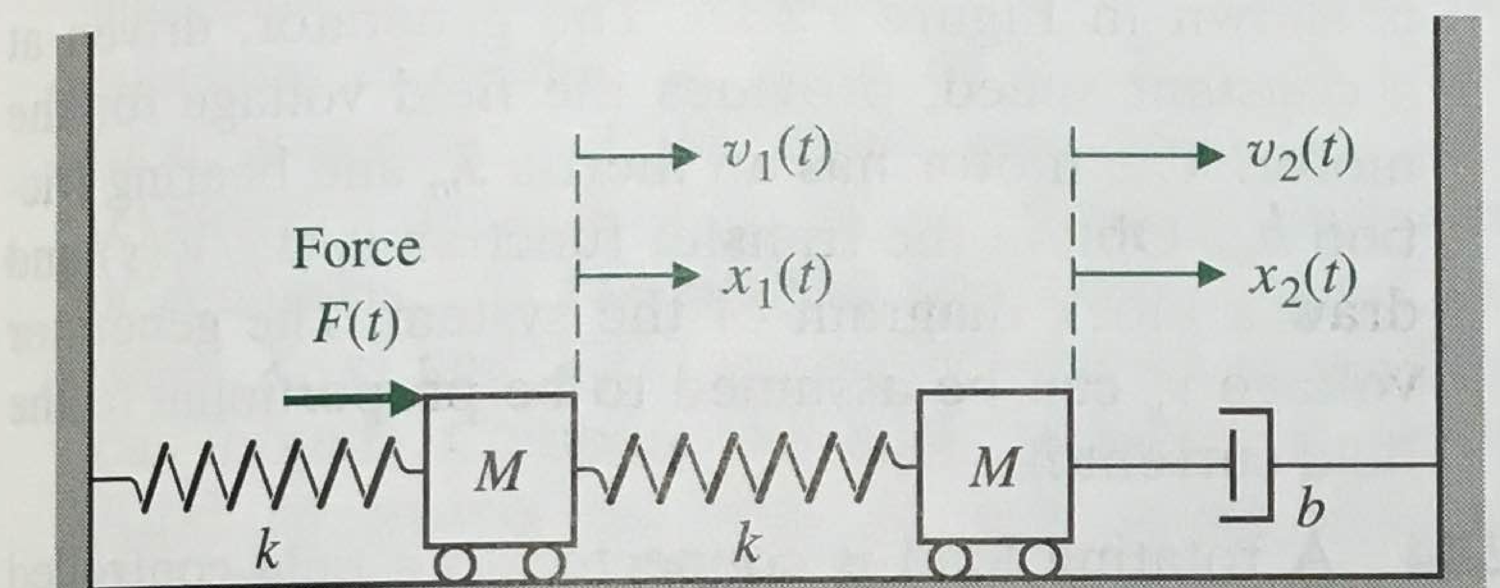

FIGURE P2.3 Two-mass system.

P2.4 A nonlinear amplifier can be described by the following characteristic:

$$v_0(t) = \begin{cases} v_{in}^2 & v_{in} \geq 0 \\ -v_{in}^2 & v_{in} < 0 \end{cases}.$$

The amplifier will be operated over a range of ± 0.5V around the operating point for v_{in}. Describe the amplifier by a linear approximation (a) when the operating point is $v_{in} = 0$ and (b) when the operating point is $v_{in} = 1$V. Obtain a sketch of the nonlinear function and the approximation for each case.

P2.5 Fluid flowing through an orifice can be represented by the nonlinear equation

$$Q = K(P_1 - P_2)^{1/2},$$

where the variables are shown in Figure P2.5 and K is a constant [2]. (a) Determine a linear approximation for the fluid-flow equation. (b) What happens to the approximation obtained in part (a) if the operating point is $P_1 - P_2 = 0$?

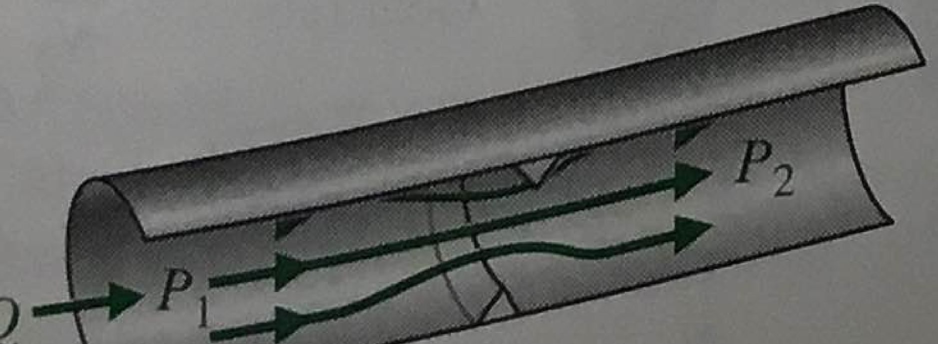

FIGURE P2.5 Flow through an orifice.

P2.6 Using the Laplace transformation, obtain the current $I_2(s)$ of Problem P2.1. Assume that all the initial currents are zero, the initial voltage across capacitor C_1 is zero, $v(t)$ is zero, and the initial voltage across C_2 is 10 volts.

P2.7 Obtain the transfer function of the differentiating circuit shown in Figure P2.7.

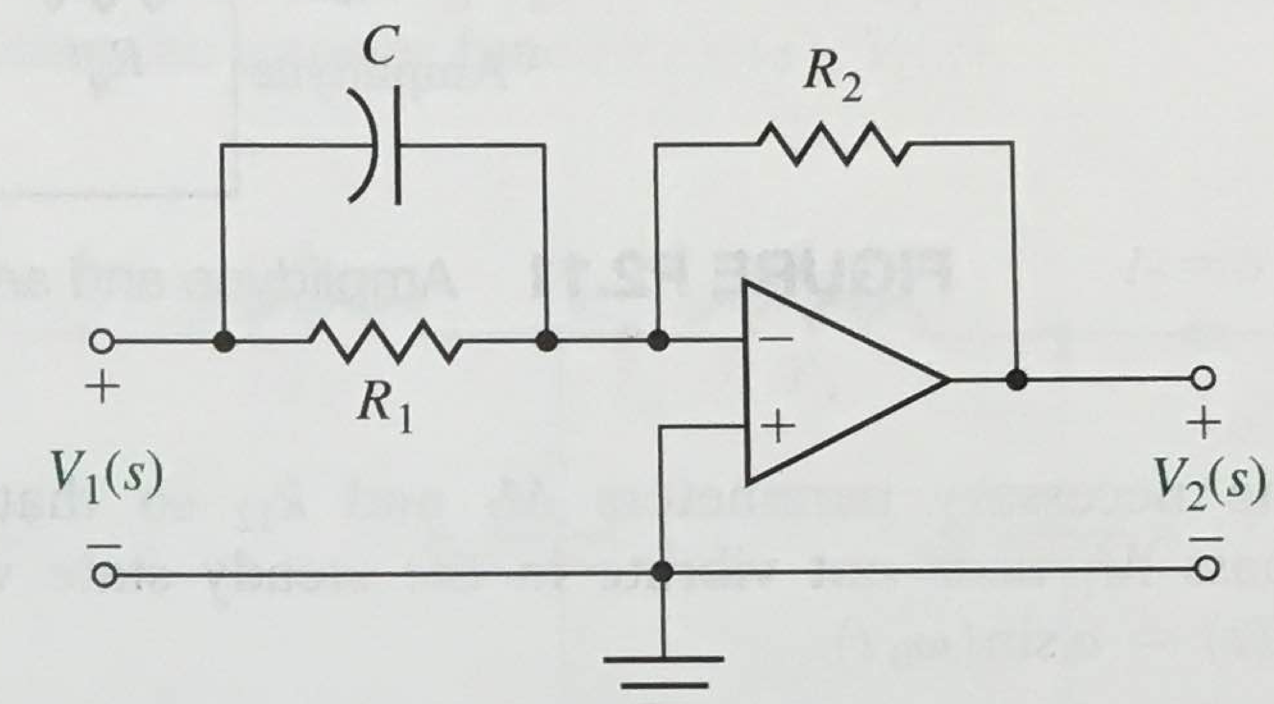

FIGURE P2.7 A differentiating circuit.

P2.8 A bridged-T network is often used in AC control systems as a filter network [8]. The circuit of one bridged-T network is shown in Figure P2.8. Show that the transfer function of the network is

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1 + 2R_1Cs + R_1R_2C^2s^2}{1 + (2R_1 + R_2)Cs + R_1R_2C^2s^2}.$$

Sketch the pole–zero diagram when $R_1 = 1$, $R_2 = 0.5$, and $C = 0.5$.

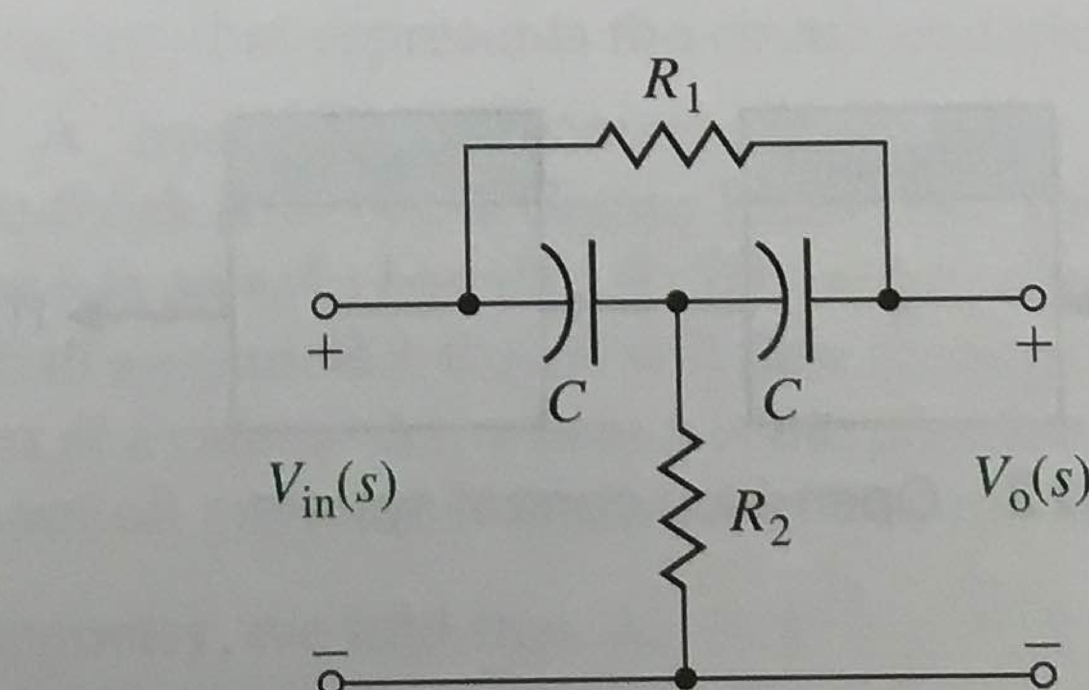

FIGURE P2.8 Bridged-T network.

P2.9 Determine the transfer function $X_1(s)/F(s)$ for the coupled spring–mass system of Problem P2.3. Sketch the s-plane pole–zero diagram for low damping when $M = 1$, $b/k = 1$, and

$$\zeta = \frac{1}{2}\frac{b}{\sqrt{kM}} = 0.1.$$

P2.10 Determine the transfer function $Y_1(s)/F(s)$ for the vibration absorber system of Problem P2.2. Determine

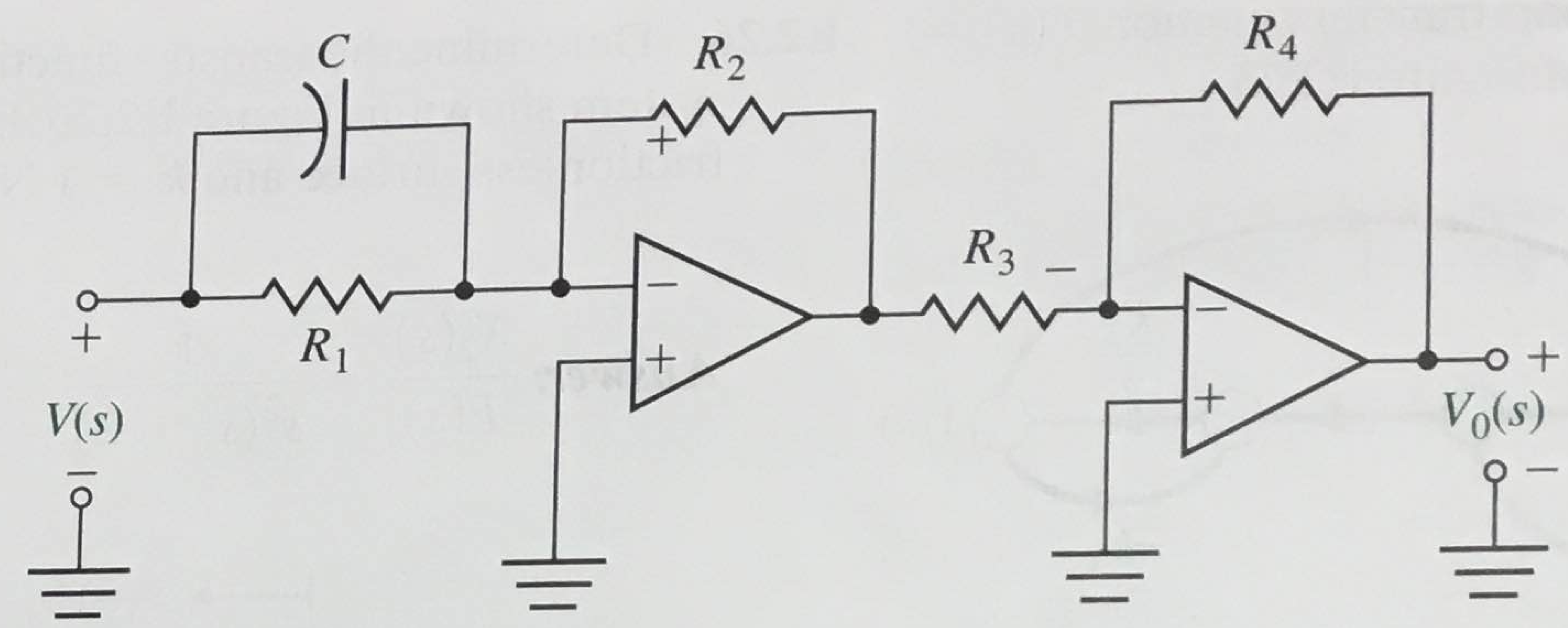

FIGURE E2.28 Op-amp circuit.

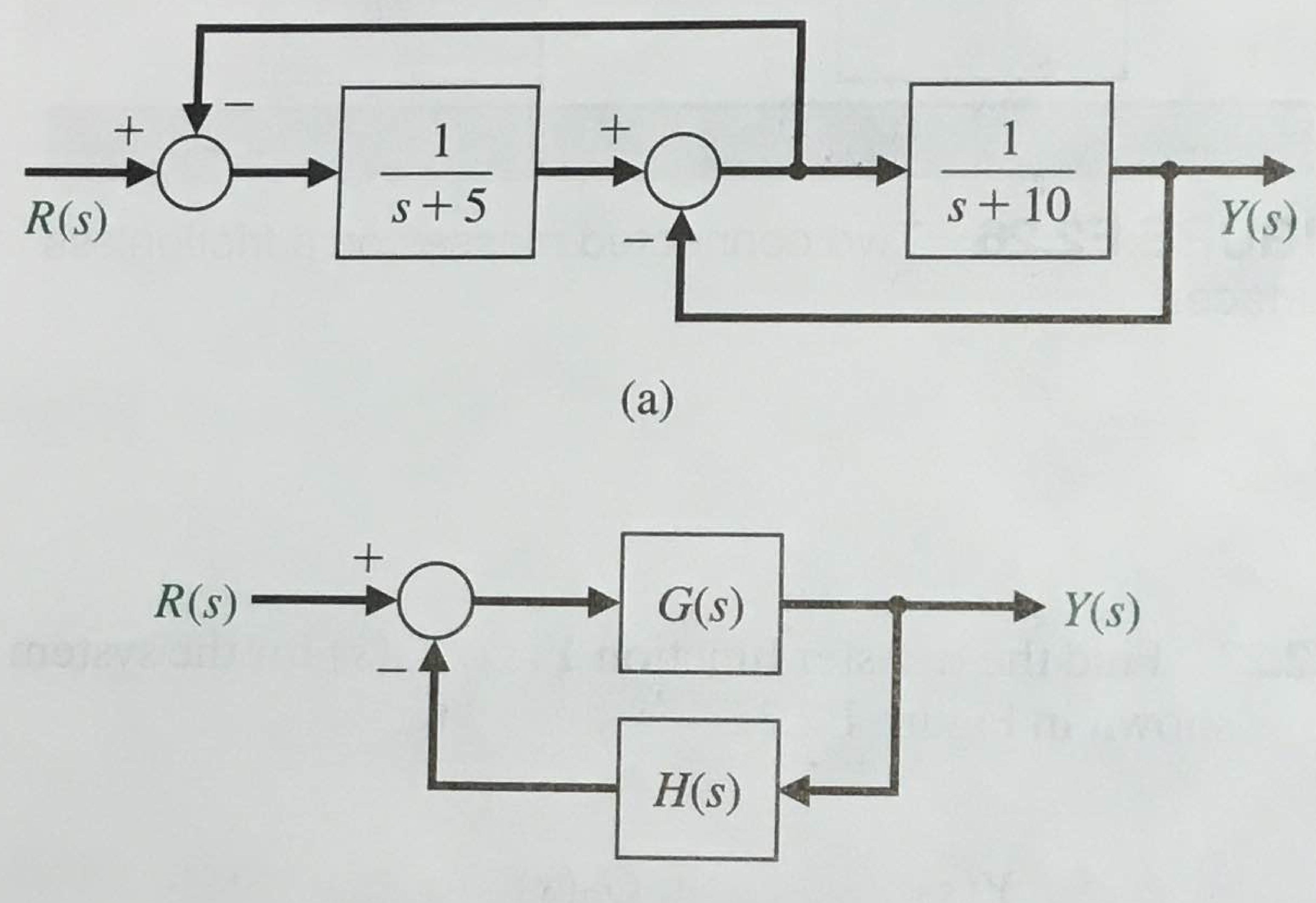

FIGURE E2.29 Block diagram equivalence.

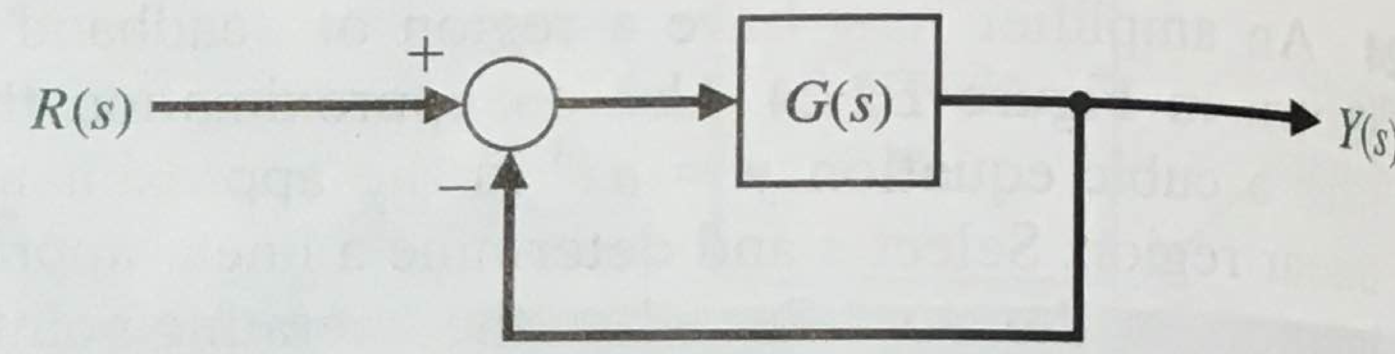

FIGURE E2.30 Unity feedback control system.

E2.31 Determine the partial fraction expansion for $V(s)$ and compute the inverse Laplace transform. The transfer function $V(s)$ is given by

$$V(s) = \frac{500}{s^2 + 8s + 500}.$$

E2.30 A system is shown in Figure E2.30.
(a) Find the closed-loop transfer function $Y(s)/R(s)$ when $G(s) = \dfrac{15}{s^2 + 5s + 15}$.
(b) Determine $Y(s)$ when the input $R(s)$ is a unit step.
(c) Compute $y(t)$.

PROBLEMS

Problems require an extension of the concepts of the chapter to new situations.

P2.1 An electric circuit is shown in Figure P2.1. Obtain a set of simultaneous integrodifferential equations representing the network.

P2.2 A dynamic vibration absorber is shown in Figure P2.2. This system is representative of many situations involving the vibration of machines containing unbalanced components. The parameters M_2 and k_{12} may be chosen so that the main mass M_1 does not vibrate in the steady state when $F(t) = a\sin(\omega_0 t)$. Obtain the differential equations describing the system.

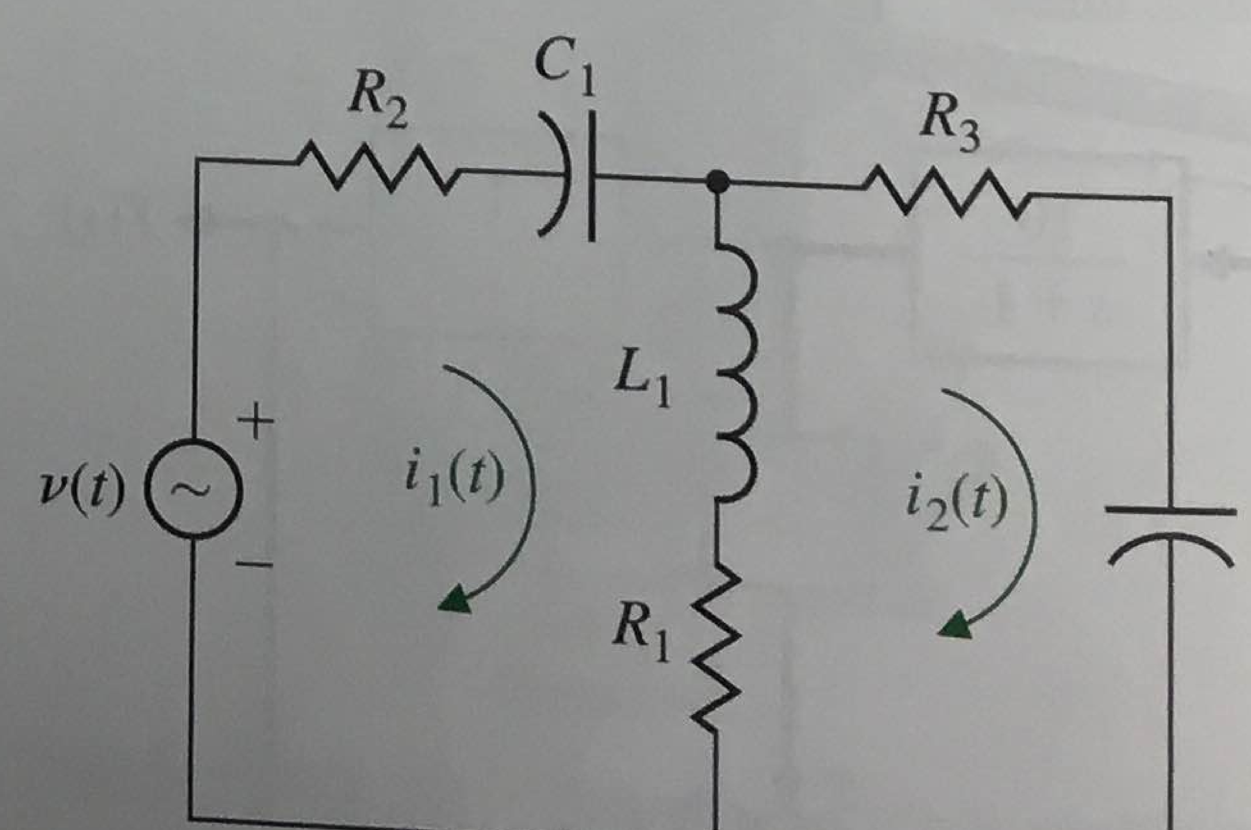

FIGURE P2.1 Electric circuit.

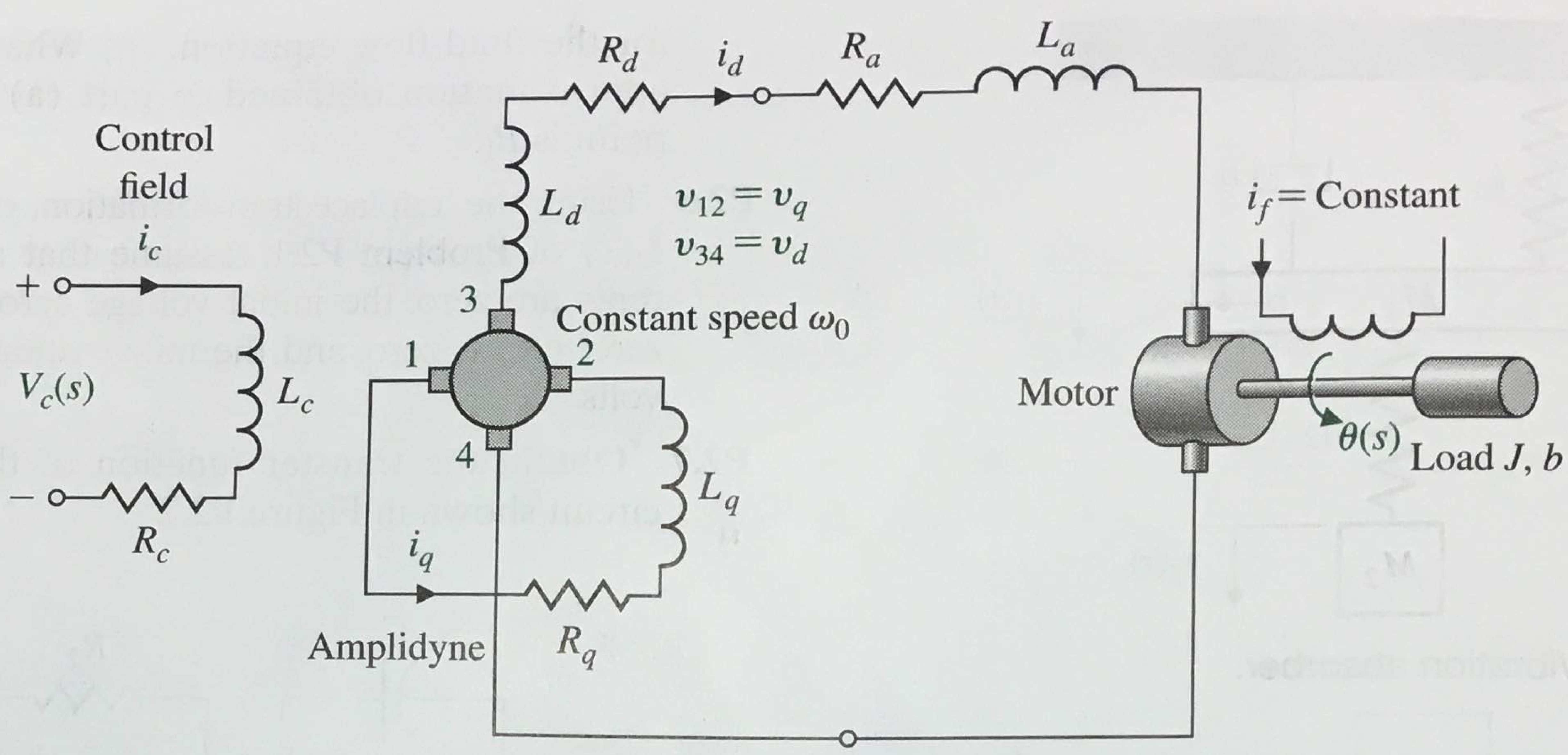

FIGURE P2.11 Amplidyne and armature-controlled motor.

the necessary parameters M_2 and k_{12} so that the mass M_1 does not vibrate in the steady state when $F(t) = a\sin(\omega_0 t)$.

P2.11 For electromechanical systems that require large power amplification, rotary amplifiers are often used [8, 19]. An amplidyne is a power amplifying rotary amplifier. An amplidyne and a servomotor are shown in Figure P2.11. Obtain the transfer function $\theta(s)/V_c(s)$, and draw the block diagram of the system. Assume $v_d = k_2 i_q$ and $v_q = k_1 i_c$.

P2.12 For the open-loop control system described by the block diagram shown in Figure P2.12, determine the value of K such that $y(t) \to 1$ as $t \to \infty$ when $r(t)$ is a unit step input. Assume zero initial conditions.

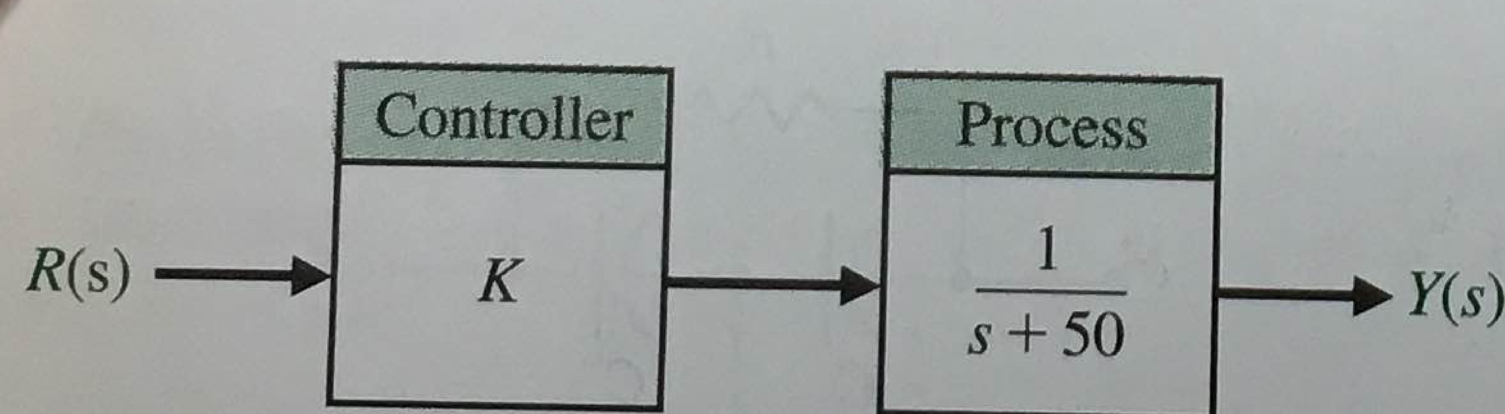

FIGURE P2.12 Open-loop control system.

P2.13 An electromechanical open-loop control system is shown in Figure P2.13. The generator, driven at a constant speed, provides the field voltage for the motor. The motor has an inertia J_m and bearing friction b_m. Obtain the transfer function $\theta_L(s)/V_f(s)$ and draw a block diagram of the system. The generator voltage v_g can be assumed to be proportional to the field current i_f.

P2.14 A rotating load is connected to a field-controlled DC electric motor through a gear system. The motor is assumed to be linear. A test results in the output load reaching a speed of 1 rad/s within 0.5 s when a constant 80 V is applied to the motor terminals. The output steady-state speed is 2.4 rad/s. Determine the transfer function $\theta(s)/V_f(s)$ of the motor, in rad/V. The inductance of the field may be assumed to be negligible. Also, note that the application of 80 V to the motor terminals is a step input of 80 V in magnitude.

P2.15 Consider the spring-mass system depicted in Figure P2.15. Determine a differential equation to describe the motion of the mass m. Obtain the system response $x(t)$ with the initial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$.

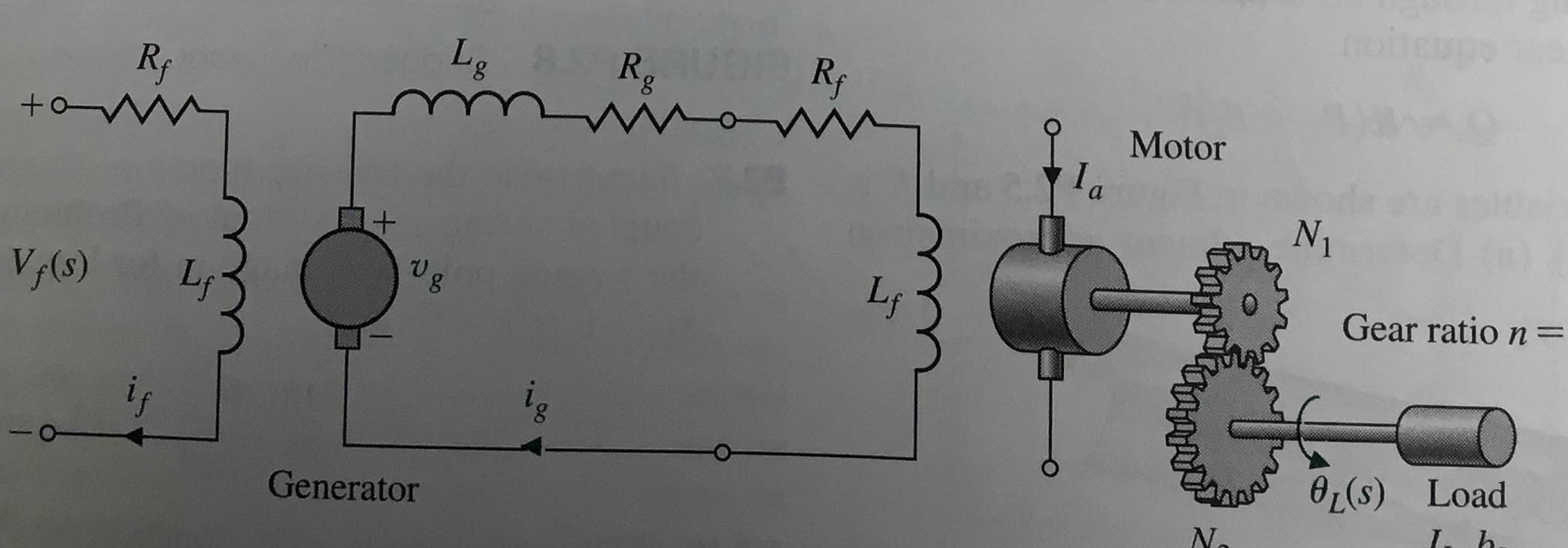

FIGURE P2.13 Motor and generator.

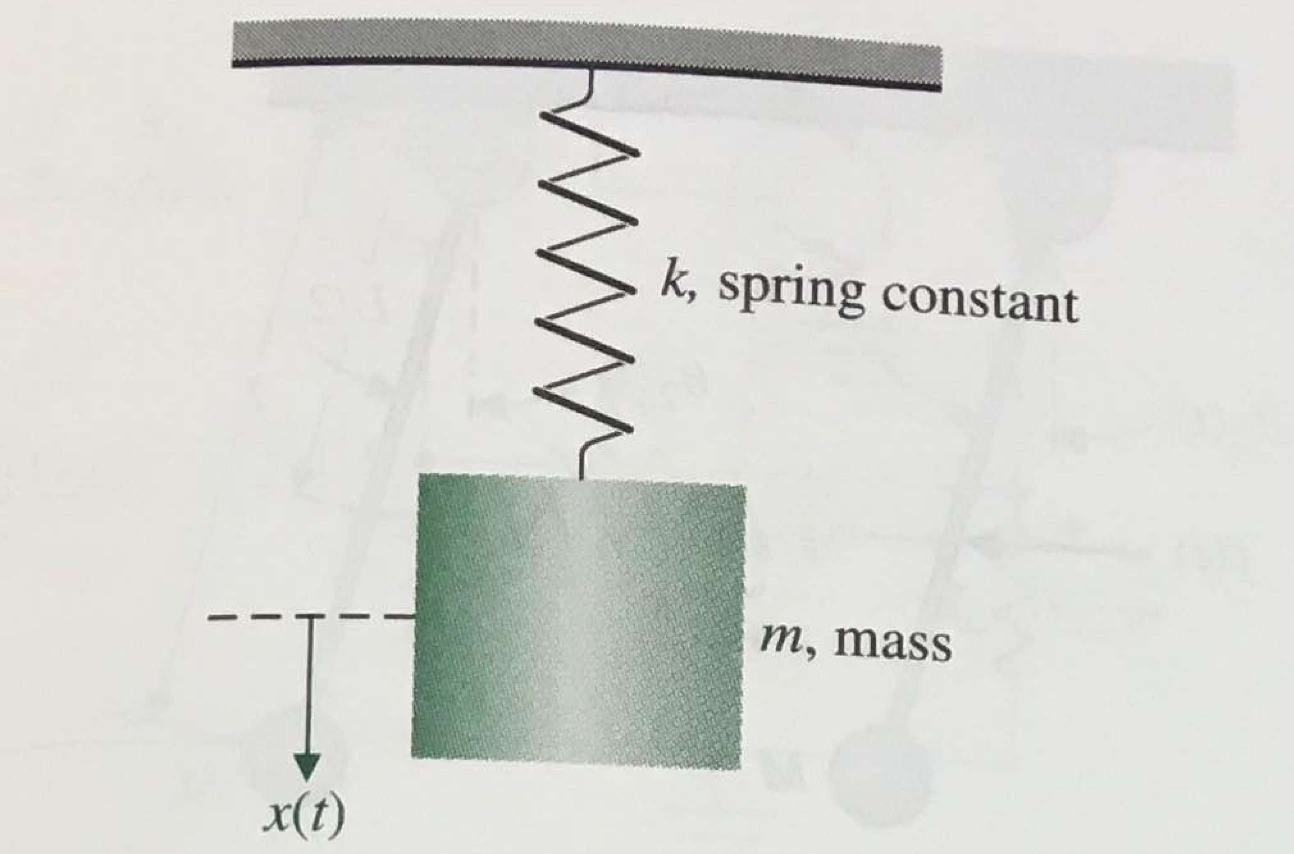

FIGURE P2.15 Suspended spring–mass system.

P2.16 A mechanical system is shown in Figure P2.16, which is subjected to a known displacement $x_3(t)$ with respect to the reference. (a) Determine the two independent equations of motion. (b) Obtain the equations of motion in terms of the Laplace transform, assuming that the initial conditions are zero. (c) Sketch a signal-flow graph representing the system of equations. (d) Obtain the relationship $T_{13}(s)$ between $X_1(s)$ and $X_3(s)$ by using Mason's signal-flow gain formula. Compare the work necessary to obtain $T_{13}(s)$ by matrix methods to that using Mason's signal-flow gain formula.

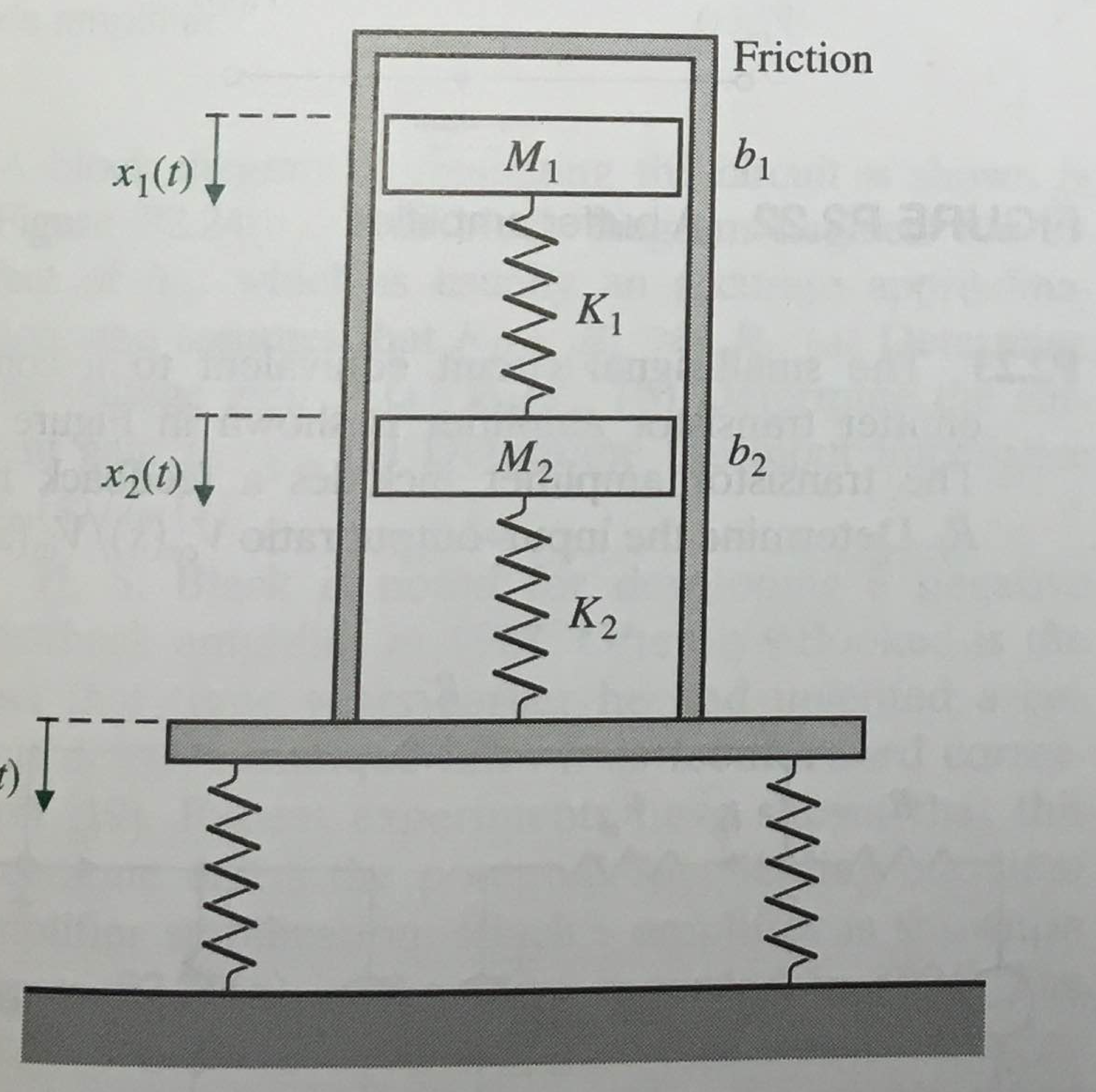

FIGURE P2.16 Mechanical system.

P2.17 Obtain a signal-flow graph to represent the following set of algebraic equations where x_1 and x_2 are to be considered the dependent variables and 6 and 11 are the inputs:

$$x_1 + 1.5x_2 = 6, \quad 2x_1 + 4x_2 = 11.$$

Determine the value of each dependent variable by using the gain formula. After solving for x_1 by Mason's signal-flow gain formula, verify the solution by using Cramer's rule.

P2.18 An LC ladder network is shown in Figure P2.18. One may write the equations describing the network as follows:

$$I_1 = (V_1 - V_a)Y_1, \qquad V_a = (I_1 - I_a)Z_2,$$

$$I_a = (V_a - V_2)Y_3, \qquad V_2 = I_aZ_4.$$

Construct a flow graph from the equations and determine the transfer function $V_2(s)/V_1(s)$.

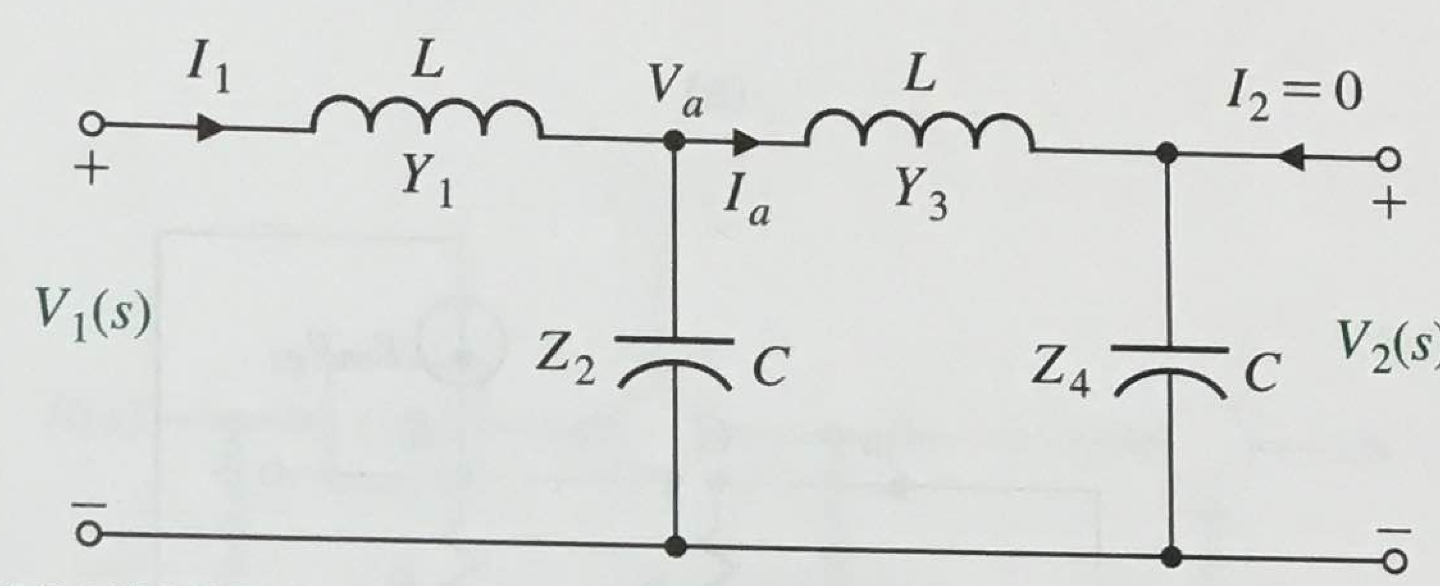

FIGURE P2.18 *LC* ladder network.

P2.19 The source follower amplifier provides lower output impedance and essentially unity gain. The circuit diagram is shown in Figure P2.19(a), and the small-signal model is shown in Figure P2.19(b). This circuit uses an FET and provides a gain of approximately unity. Assume that $R_2 \gg R_1$ for biasing purposes and that $R_g \gg R_2$. (a) Solve for the amplifier gain. (b) Solve for the gain when $g_m = 2000\ \mu\Omega$ and $R_s = 10\ \text{k}\Omega$ where $R_s = R_1 + R_2$. (c) Sketch a block diagram that represents the circuit equations.

P2.20 A hydraulic servomechanism with mechanical feedback is shown in Figure P2.20 [18]. The power piston has an area equal to A. When the valve is moved a small amount Δz, the oil will flow through to the cylinder at a rate $p \cdot \Delta z$, where p is the port coefficient. The input oil pressure is assumed to be constant. From the geometry, we find that $\Delta z = k\dfrac{l_1 - l_2}{l_1}(x - y) - \dfrac{l_2}{l_1}y$.
(a) Determine the closed-loop signal-flow graph or block diagram for this mechanical system. (b) Obtain the closed-loop transfer function $Y(s)/X(s)$.

P2.21 Figure P2.21 shows two pendulums suspended from frictionless pivots and connected at their midpoints by a spring [1]. Assume that each pendulum can be represented by a mass M at the end of a massless bar of length L. Also assume that the displacement is small and linear approximations can be used for $\sin\theta$ and $\cos\theta$. The spring located in the middle of the bars is unstretched when $\theta_1 = \theta_2$. The input force is represented by $f(t)$, which influences the left-hand bar

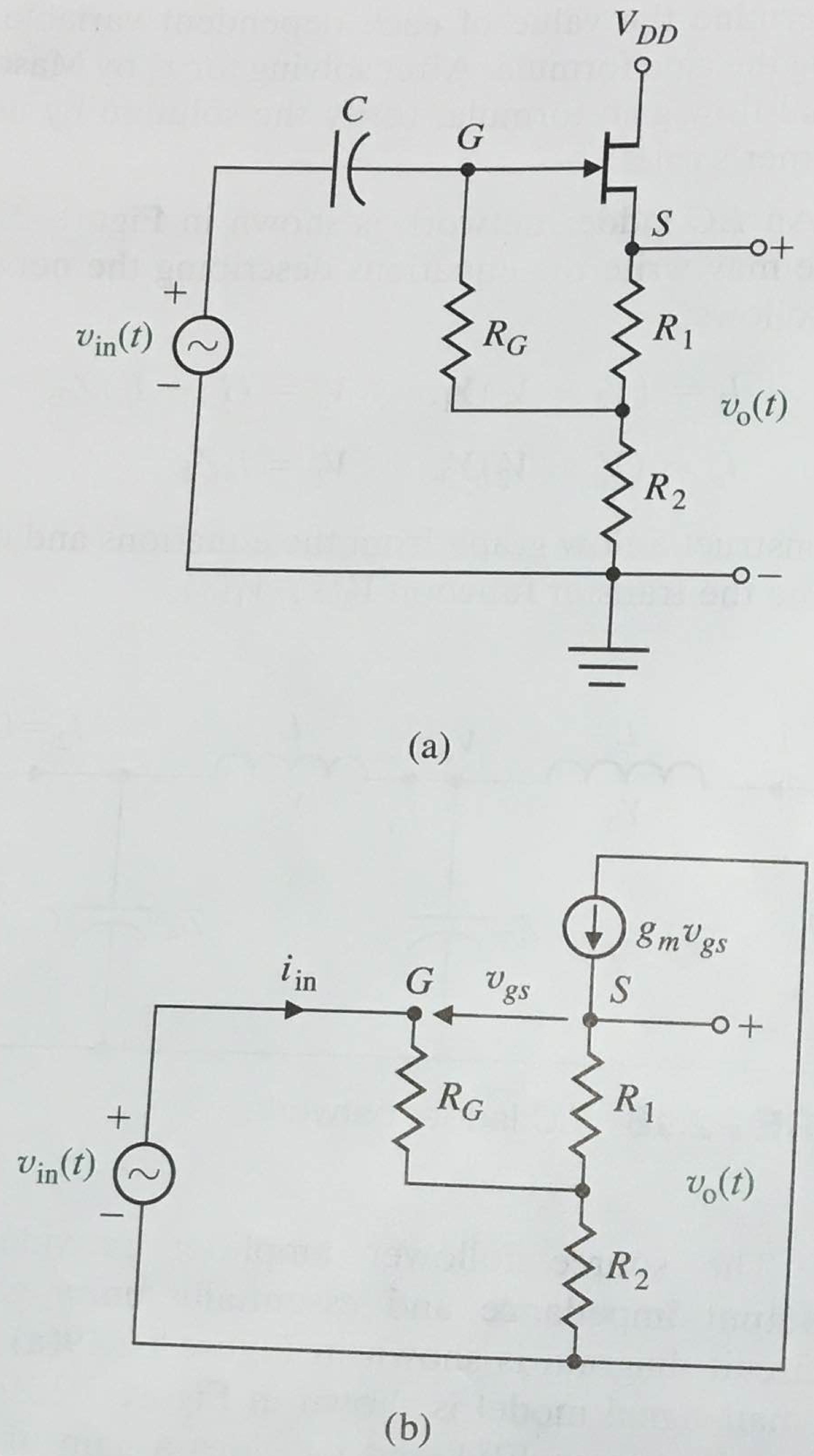

FIGURE P2.19 The source follower or common drain amplifier using an FET.

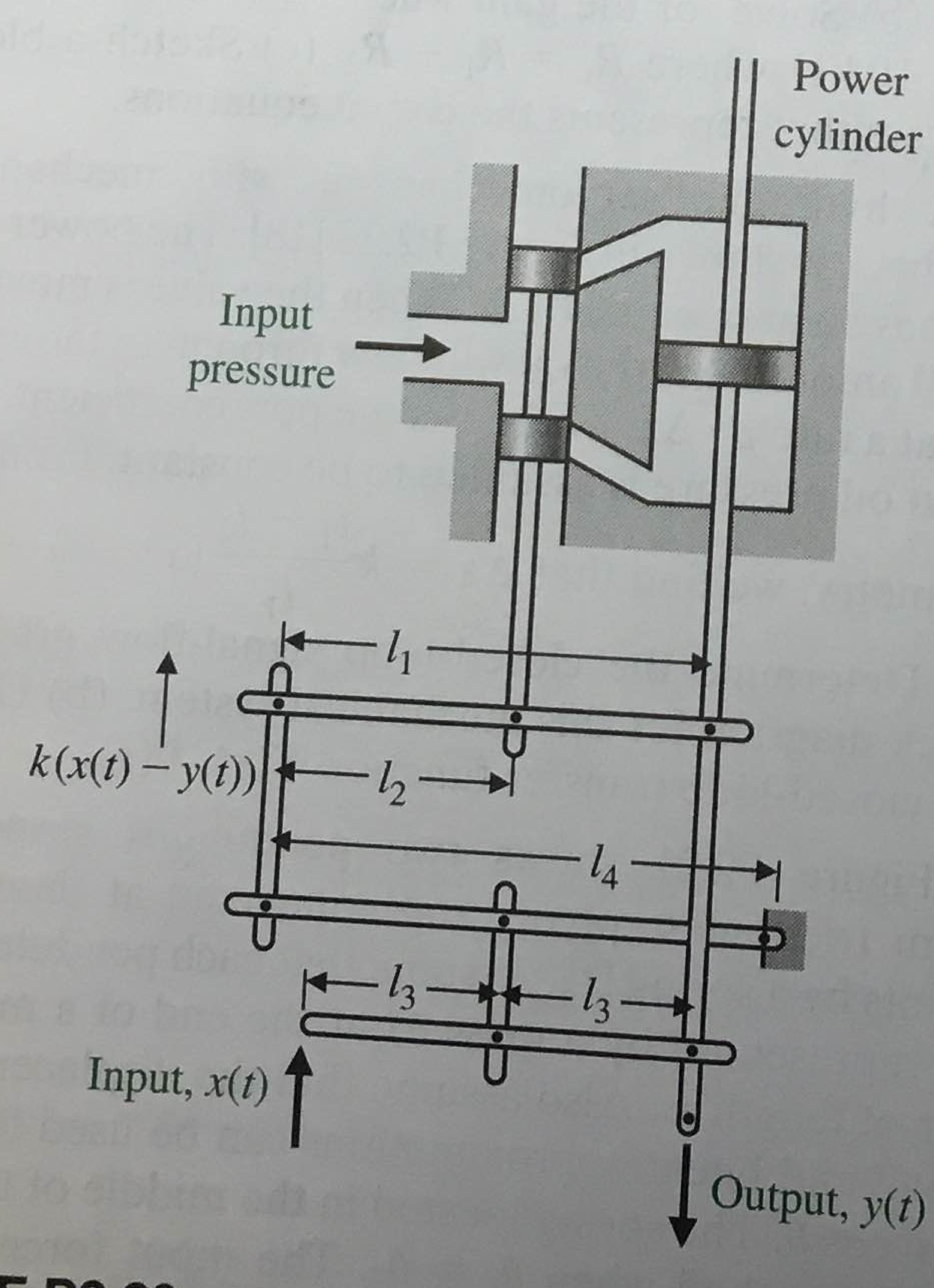

FIGURE P2.20 Hydraulic servomechanism.

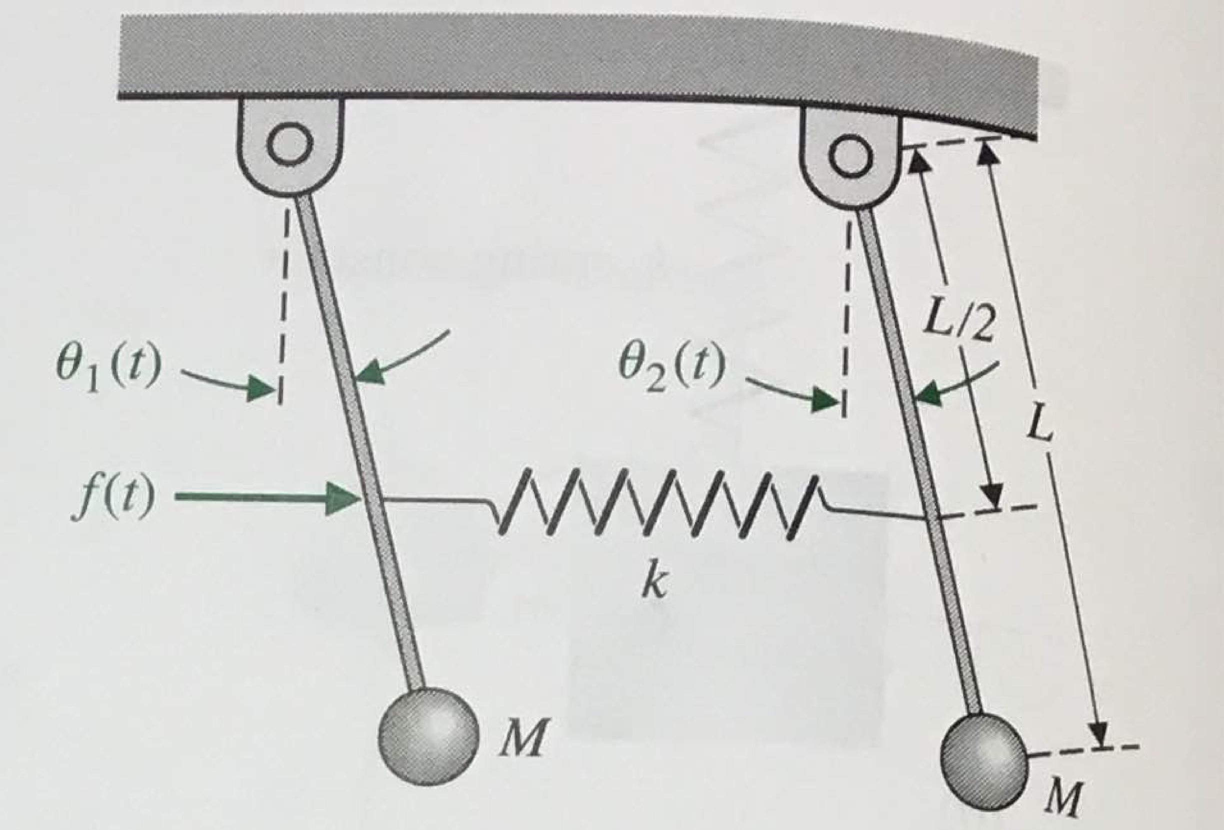

FIGURE P2.21 The bars are each of length L and the spring is located at $L/2$.

only. (a) Obtain the equations of motion, and sketch a block diagram for them. (b) Determine the transfer function $T(s) = \theta_1(s)/F(s)$. (c) Sketch the location of the poles and zeros of $T(s)$ on the s-plane.

P2.22 A voltage follower (buffer amplifier) is shown in Figure P2.22. Show that $T = V_o(s)/V_{in}(s) = 1$. Assume an ideal op-amp.

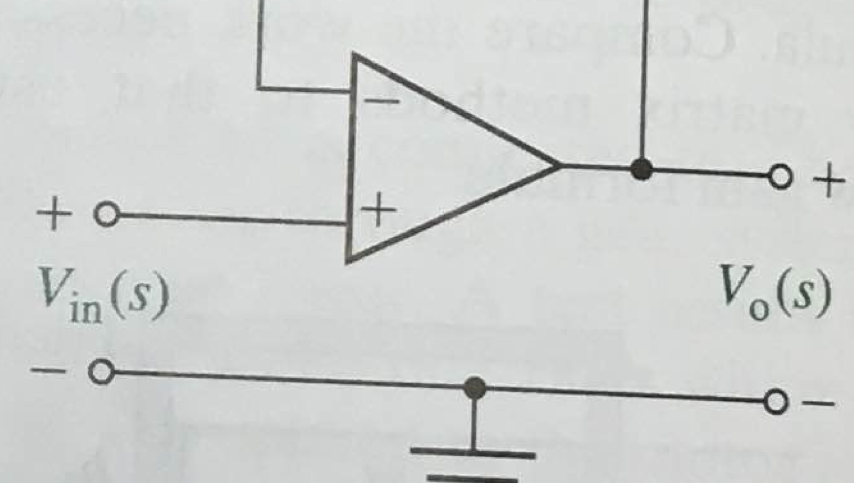

FIGURE P2.22 A buffer amplifier.

P2.23 The small-signal circuit equivalent to a common-emitter transistor amplifier is shown in Figure P2.23. The transistor amplifier includes a feedback resistor R_f. Determine the input–output ratio $V_{ce}(s)/V_{in}(s)$.

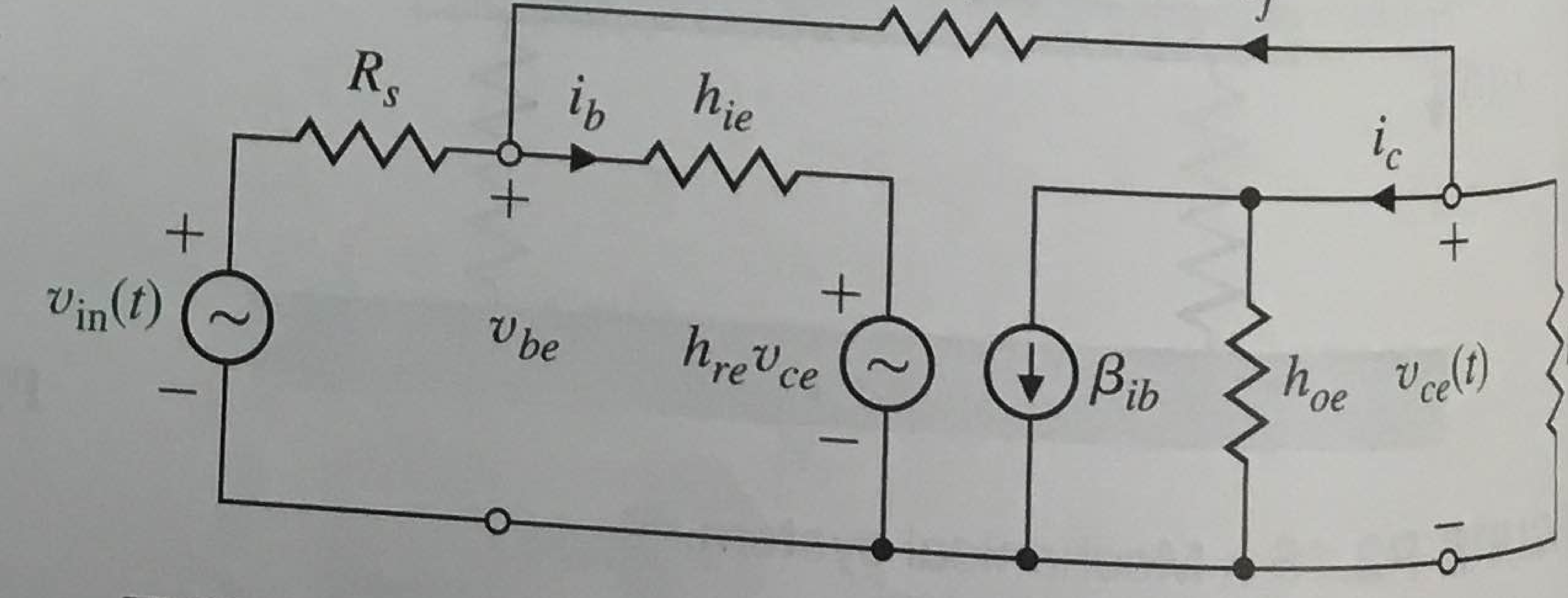

FIGURE P2.23 CE amplifier.

P2.24 A two-transistor series voltage feedback amplifier is shown in Figure P2.24(a). This AC equivalent circuit neglects the bias resistors and the shunt capacitors

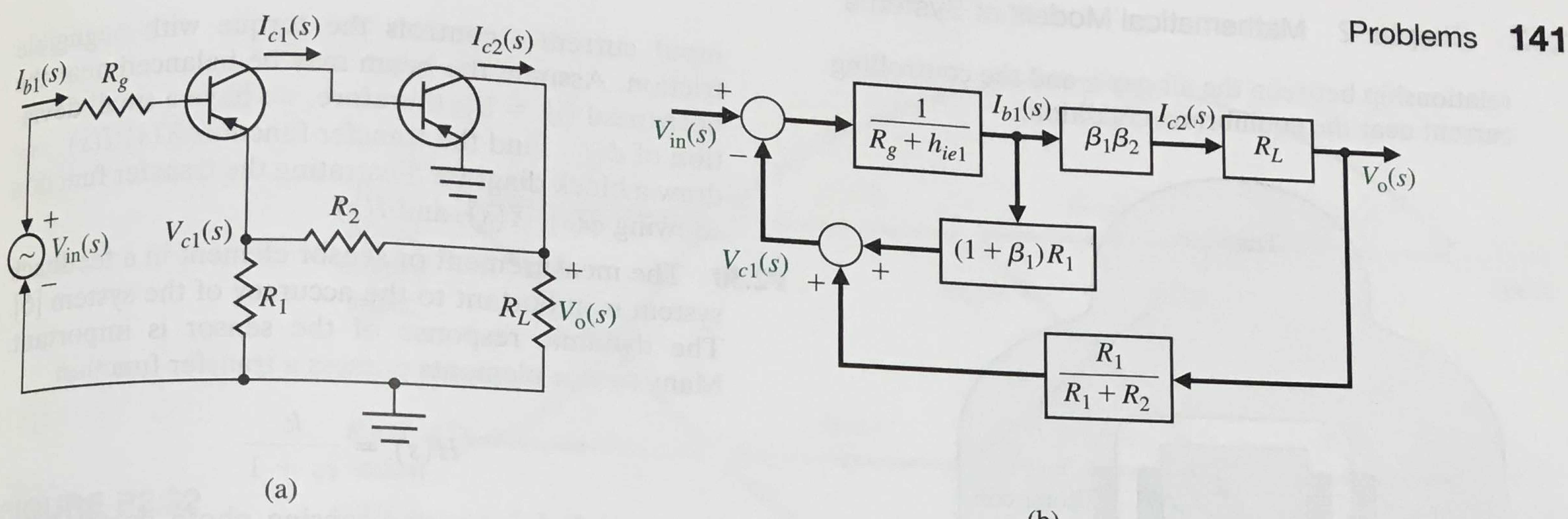

FIGURE P2.24 Feedback amplifier.

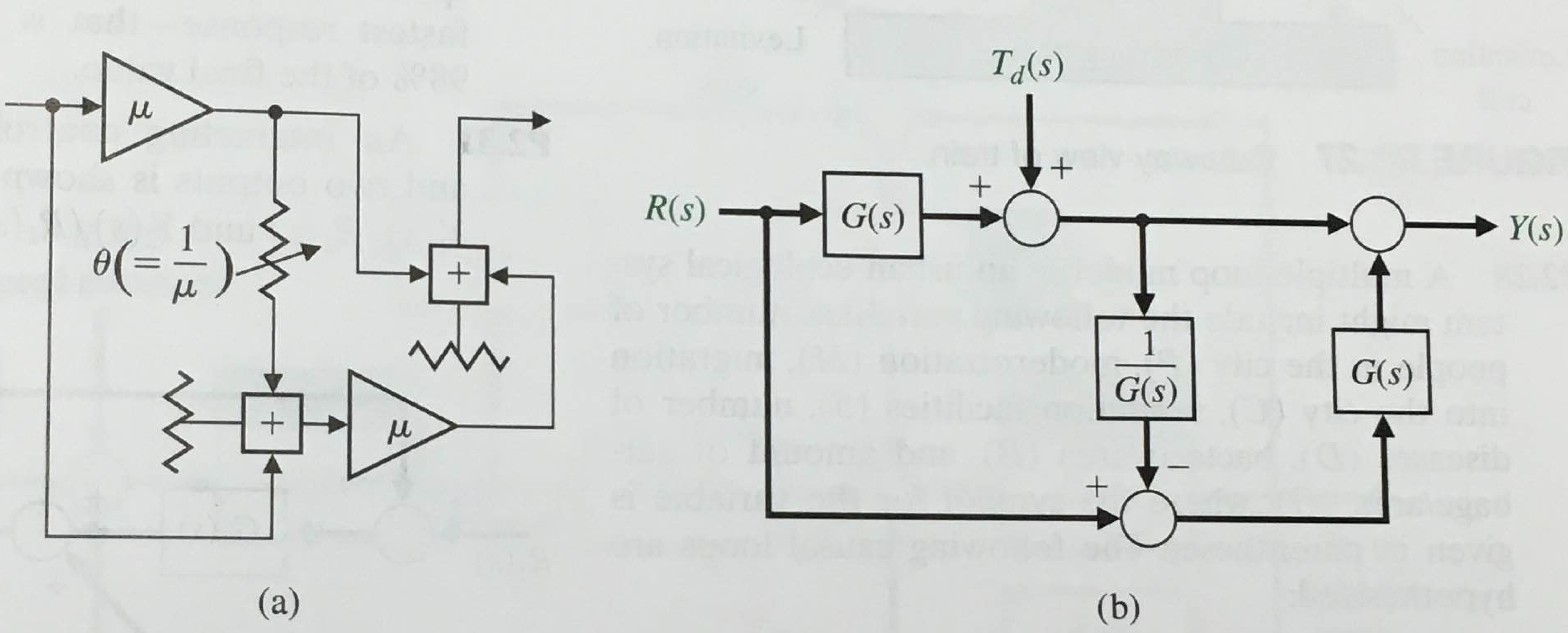

FIGURE P2.25 H. S. Black's amplifier.

A block diagram representing the circuit is shown in Figure P2.24(b). This block diagram neglects the effect of h_{re}, which is usually an accurate approximation, and assumes that $R_2 + R_L \gg R_1$. (a) Determine the voltage gain $V_o(s)/V_{in}(s)$. (b) Determine the current gain i_{c2}/i_{b1}. (c) Determine the input impedance $V_{in}(s)/I_{b1}(s)$.

P2.25 H. S. Black is noted for developing a negative feedback amplifier in 1927. Often overlooked is the fact that three years earlier he had invented a circuit design technique known as feedforward correction [19]. Recent experiments have shown that this technique offers the potential for yielding excellent amplifier stabilization. Black's amplifier is shown in Figure P2.25(a) in the form recorded in 1924. The block diagram is shown in Figure P2.25(b). Determine the transfer function between the output $Y(s)$ and the input $R(s)$ and between the output and the disturbance $T_d(s)$. $G(s)$ is used to denote the amplifier represented by μ in Figure P2.25(a).

P2.26 A robot includes significant flexibility in the arm members with a heavy load in the gripper [6, 20]. A two-mass model of the robot is shown in Figure P2.26. Find the transfer function $Y(s)/F(s)$.

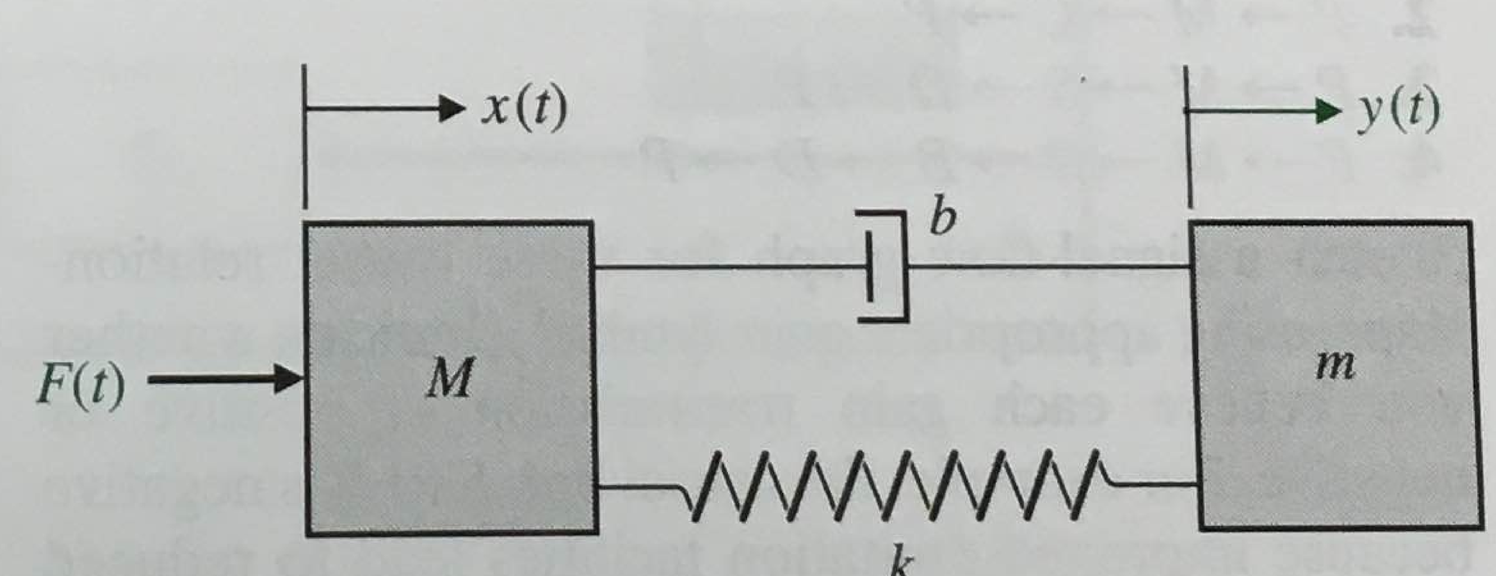

FIGURE P2.26 The spring-mass-damper model of a robot arm.

P2.27 Magnetic levitation trains provide a high-speed, very low friction alternative to steel wheels on steel rails. The train floats on an air gap as shown in Figure P2.27 [25]. The levitation force F_L is controlled by the coil current i in the levitation coils and may be approximated by

$$F_L = k\frac{i^2}{z^2},$$

where z is the air gap. This force is opposed by the downward force $F = mg$. Determine the linearized relationship between the air gap z and the controlling current near the equilibrium condition.

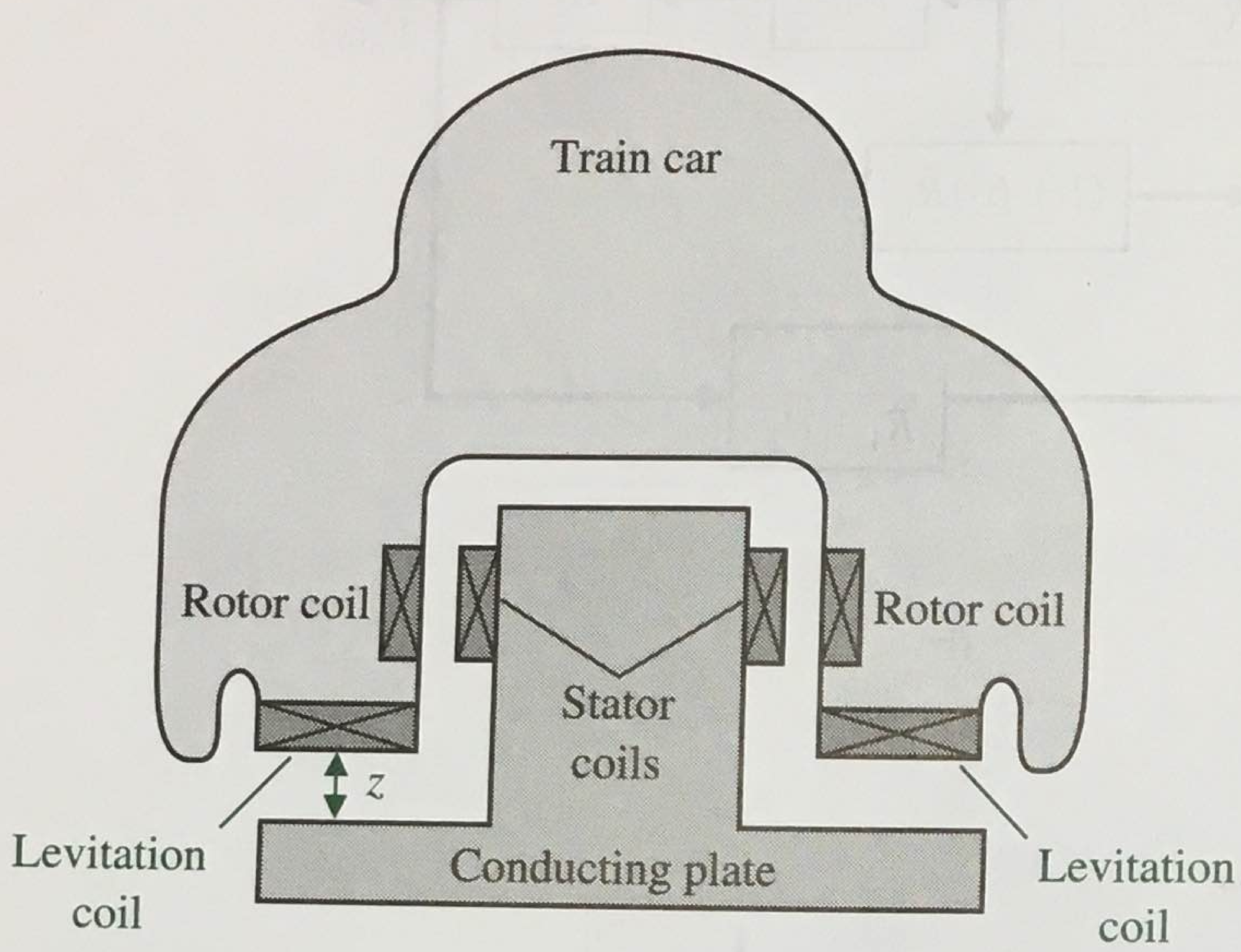

FIGURE P2.27 Cutaway view of train.

P2.28 A multiple-loop model of an urban ecological system might include the following variables: number of people in the city (P), modernization (M), migration into the city (C), sanitation facilities (S), number of diseases (D), bacteria/area (B), and amount of garbage/area (G), where the symbol for the variable is given in parentheses. The following causal loops are hypothesized:

1. $P \rightarrow G \rightarrow B \rightarrow D \rightarrow P$
2. $P \rightarrow M \rightarrow C \rightarrow P$
3. $P \rightarrow M \rightarrow S \rightarrow D \rightarrow P$
4. $P \rightarrow M \rightarrow S \rightarrow B \rightarrow D \rightarrow P$

Sketch a signal-flow graph for these causal relationships, using appropriate gain symbols. Indicate whether you believe each gain transmission is positive or negative. For example, the causal link S to B is negative because improved sanitation facilities lead to reduced bacteria/area. Which of the four loops are positive feedback loops and which are negative feedback loops?

P2.29 We desire to balance a rolling ball on a tilting beam as shown in Figure P2.29. We will assume the motor input current i controls the torque with negligible friction. Assume the beam may be balanced near the horizontal ($\phi = 0$); therefore, we have a small deviation of $\phi(t)$. Find the transfer function $X(s)/I(s)$, and draw a block diagram illustrating the transfer function showing $\phi(s)$, $X(s)$, and $I(s)$.

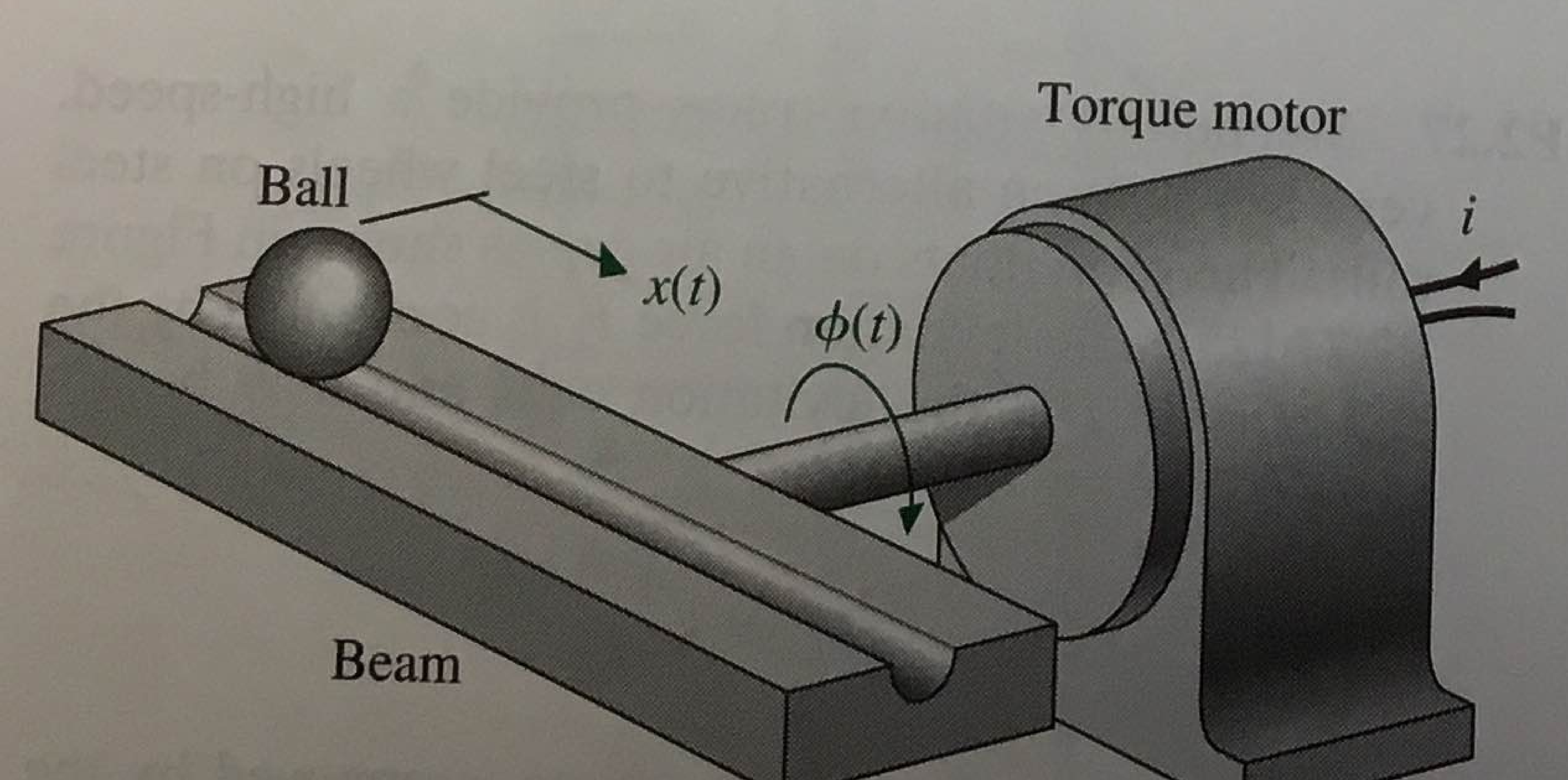

FIGURE P2.29 Tilting beam and ball.

P2.30 The measurement or sensor element in a feedback system is important to the accuracy of the system [6]. The dynamic response of the sensor is important. Many sensor elements possess a transfer function

$$H(s) = \frac{k}{\tau s + 1}$$

Suppose that a position-sensing photo detector has $\tau = 5\ \mu s$ and $0.999 < k < 1.001$. Obtain the step response of the system, and find the k resulting in the fastest response—that is, the fastest time to reach 98% of the final value.

P2.31 An interacting control system with two inputs and two outputs is shown in Figure P2.31. Solve for $Y_1(s)/R_1(s)$ and $Y_2(s)/R_1(s)$ when $R_2 = 0$.

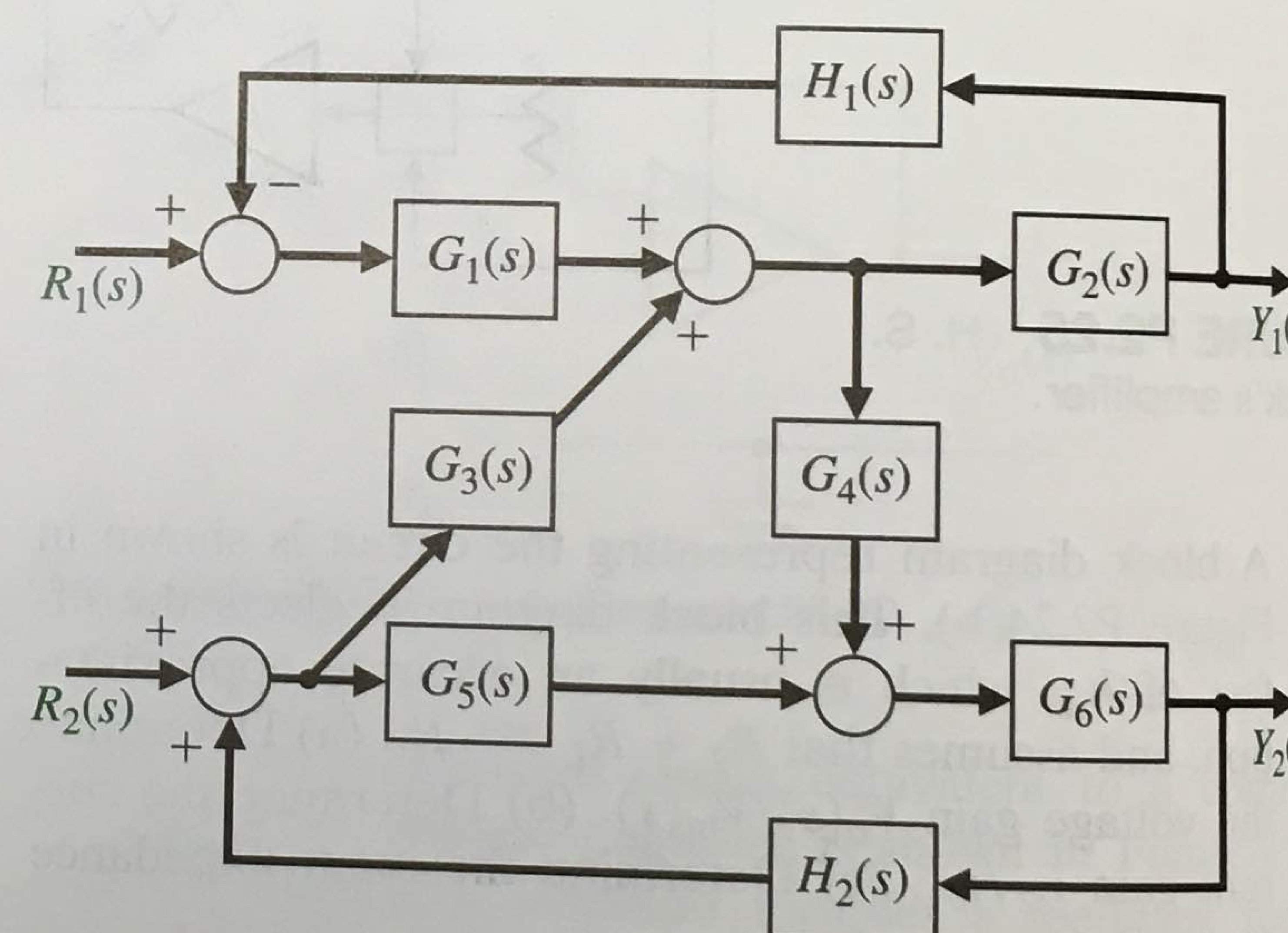

FIGURE P2.31 Interacting system.

P2.32 A system consists of two electric motors that are coupled by a continuous flexible belt. The belt also passes over a swinging arm that is instrumented to allow measurement of the belt speed and tension. The basic control problem is to regulate the belt speed and tension by varying the motor torques.

An example of a practical system similar to that shown occurs in textile fiber manufacturing processes when yarn is wound from one spool to another at high speed. Between the two spools, the yarn is processed in a way that may require the yarn speed and tension to be controlled within defined limits. A model of the system is shown in Figure P2.32. Find $Y_2(s)/R_1(s)$. Determine a relationship for the system that will make Y_2 independent of R_1.

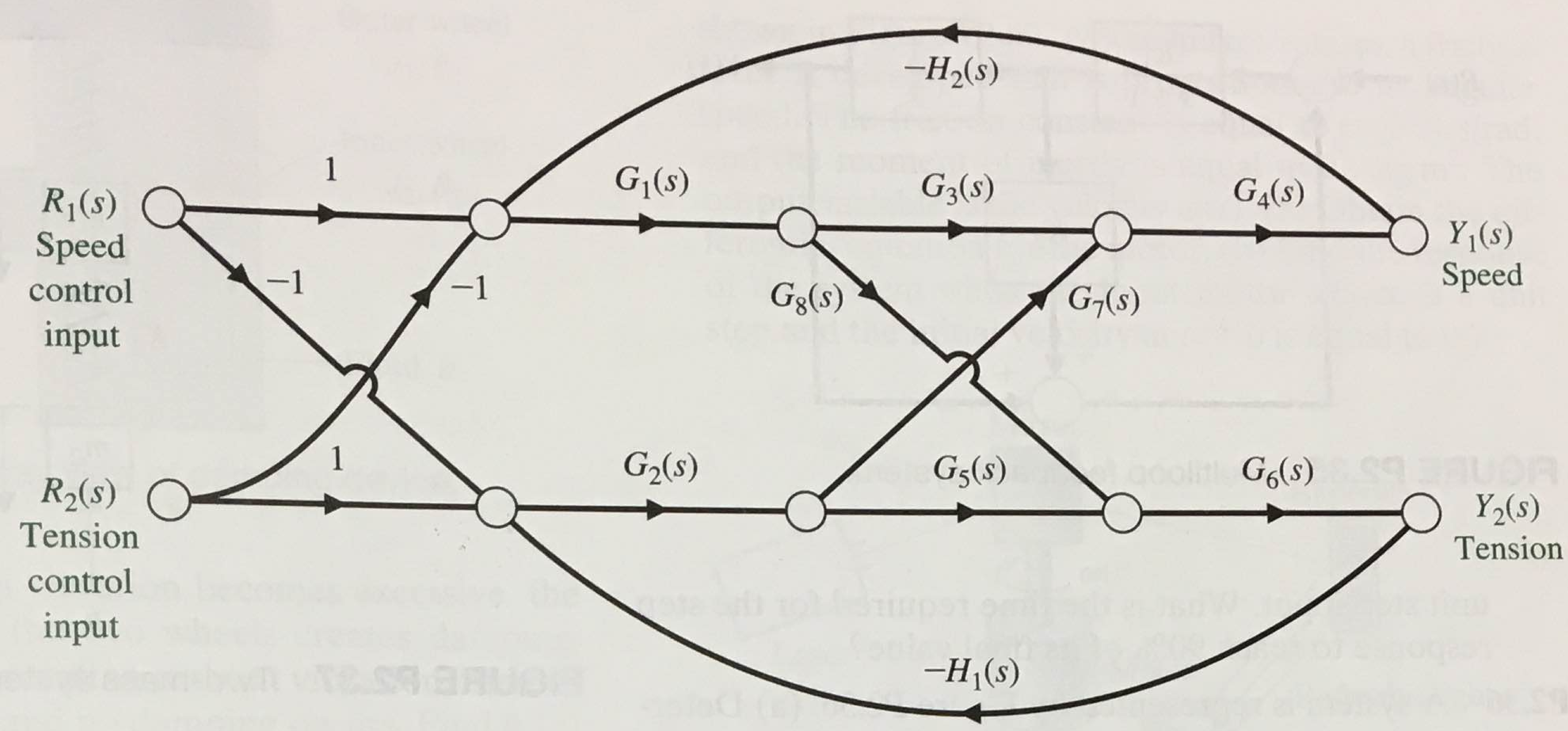

FIGURE P2.32 A model of the coupled motor drives.

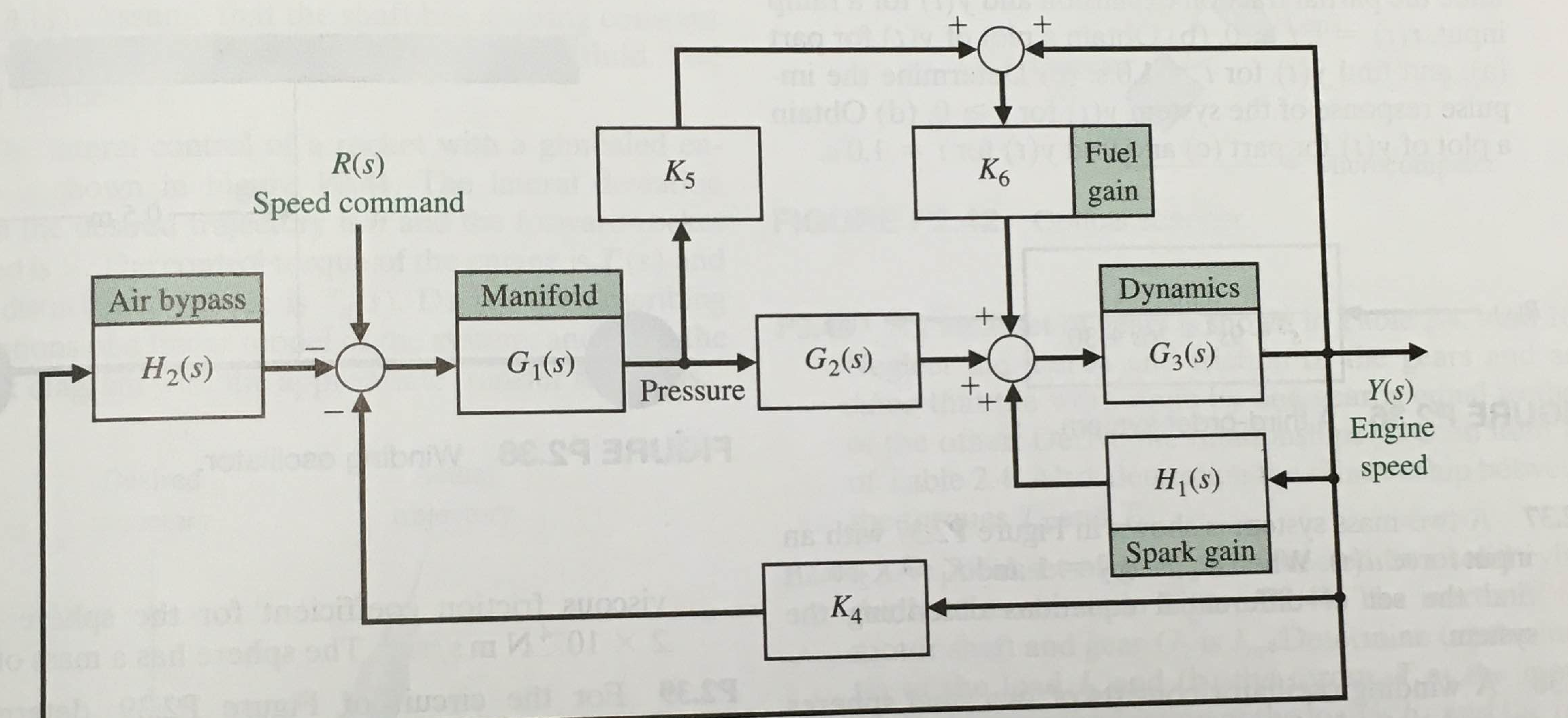

FIGURE P2.33 Idle speed control system.

P2.33 Find the transfer function for $Y(s)/R(s)$ for the idle-speed control system for a fuel-injected engine as shown in Figure P2.33.

P2.34 The suspension system for one wheel of an old-fashioned pickup truck is illustrated in Figure P2.34. The mass of the vehicle is m_1 and the mass of the wheel is m_2. The suspension spring has a spring constant k_1 and the tire has a spring constant k_2. The damping constant of the shock absorber is b. Obtain the transfer function $Y_1(s)/X(s)$, which represents the vehicle response to bumps in the road.

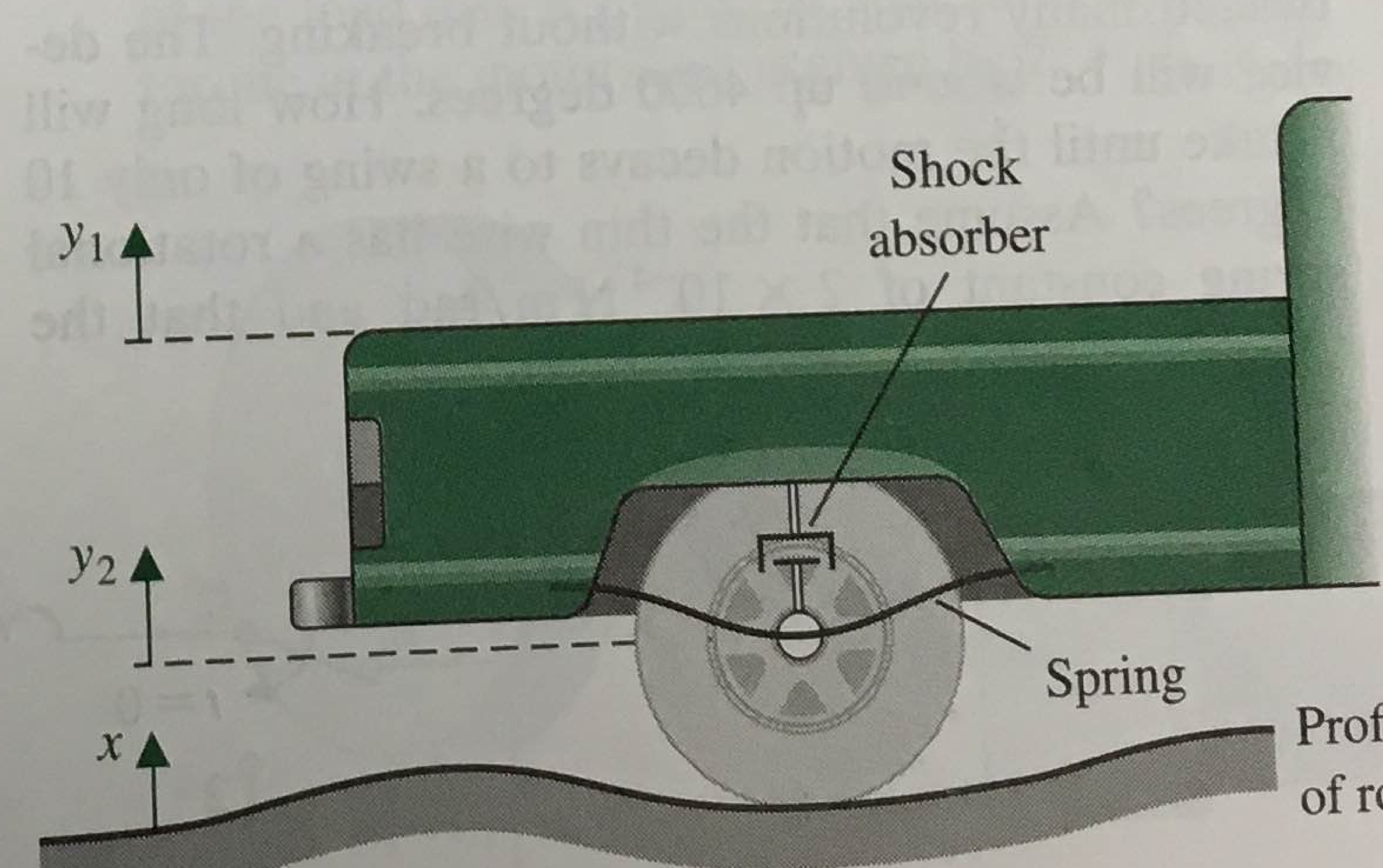

FIGURE P2.34 Pickup truck suspension.

P2.35 A feedback control system has the structure shown in Figure P2.35. Determine the closed-loop transfer function $Y(s)/R(s)$ (a) by block diagram manipulation and (b) by using a signal-flow graph and Mason's signal-flow gain formula. (c) Select the gains K_1 and K_2 so that the closed-loop response to a step input is critically damped with two equal roots at $s = -10$. (d) Plot the critically damped response for a

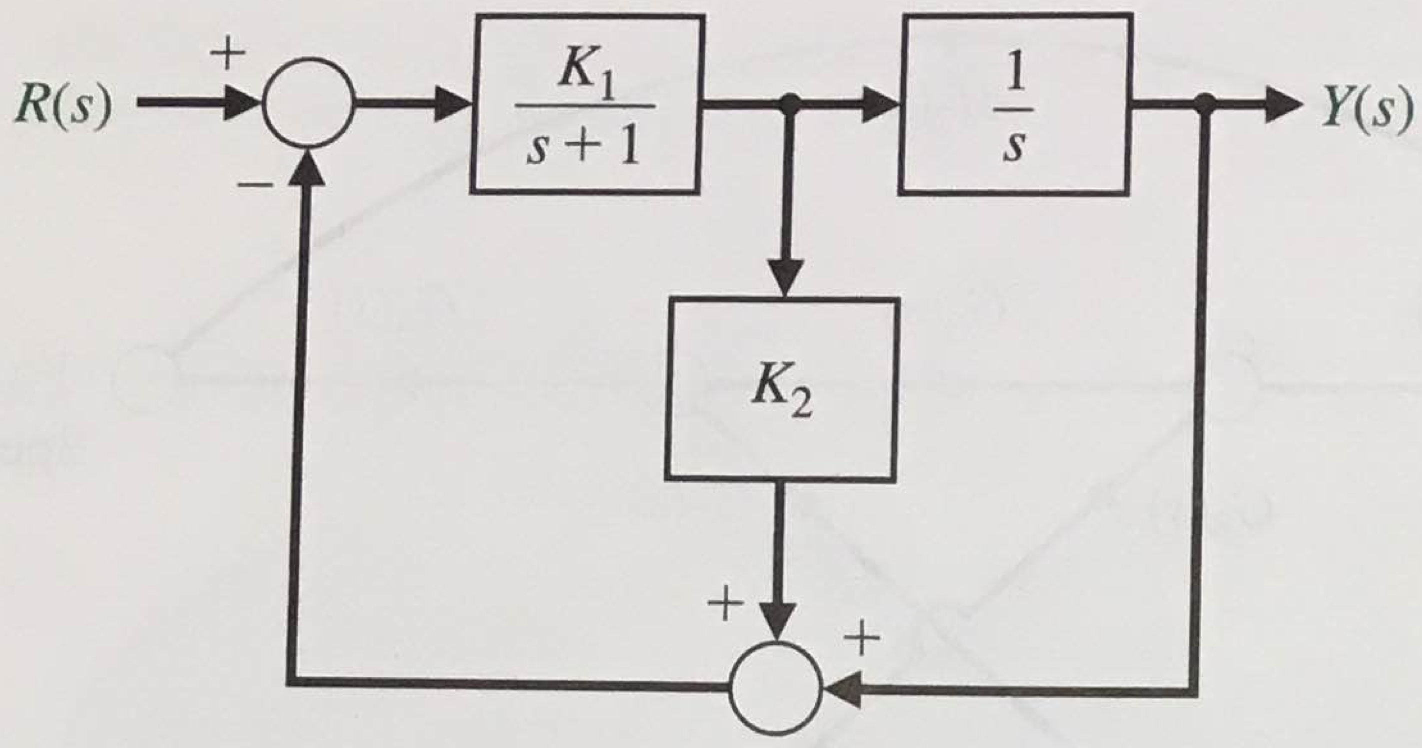

FIGURE P2.35 Multiloop feedback system.

unit step input. What is the time required for the step response to reach 90% of its final value?

P2.36 A system is represented by Figure P2.36. (a) Determine the partial fraction expansion and $y(t)$ for a ramp input, $r(t) = t, t \geq 0$. (b) Obtain a plot of $y(t)$ for part (a), and find $y(t)$ for $t = 1.0$ s. (c) Determine the impulse response of the system $y(t)$ for $t \geq 0$. (d) Obtain a plot of $y(t)$ for part (c) and find $y(t)$ for $t = 1.0$ s.

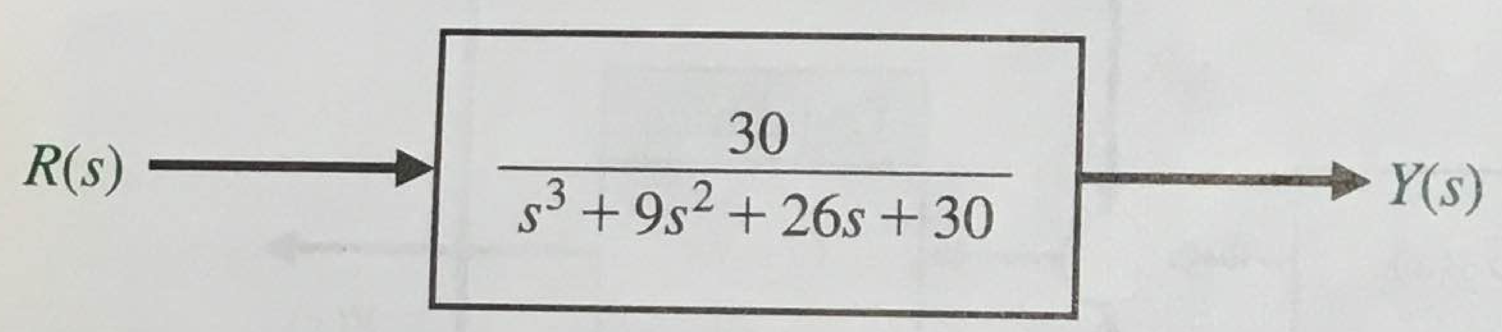

FIGURE P2.36 A third-order system.

P2.37 A two-mass system is shown in Figure P2.37 with an input force $u(t)$. When $m_1 = m_2 = 1$ and $K_1 = K_2 = 1$, find the set of differential equations describing the system.

P2.38 A winding oscillator consists of two steel spheres on each end of a long slender rod, as shown in Figure P2.38. The rod is hung on a thin wire that can be twisted many revolutions without breaking. The device will be wound up 4000 degrees. How long will it take until the motion decays to a swing of only 10 degrees? Assume that the thin wire has a rotational spring constant of 2×10^{-4} N m/rad and that the viscous friction coefficient for the sphere in air is 2×10^{-4} N m s/rad. The sphere has a mass of 1 kg.

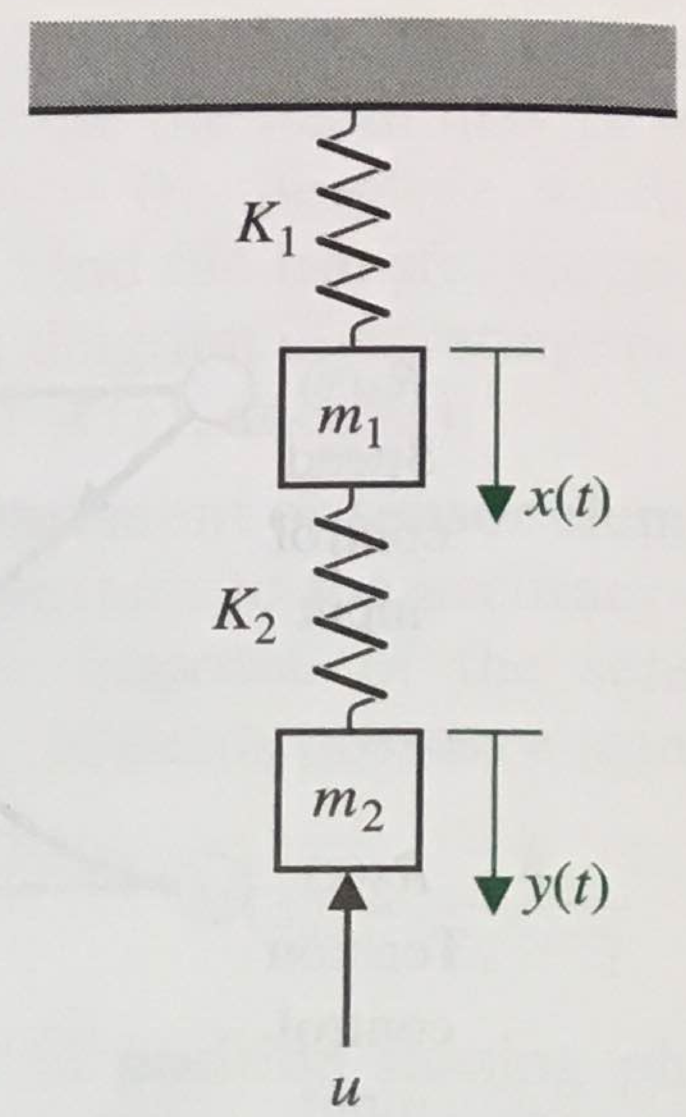

FIGURE P2.37 Two-mass system.

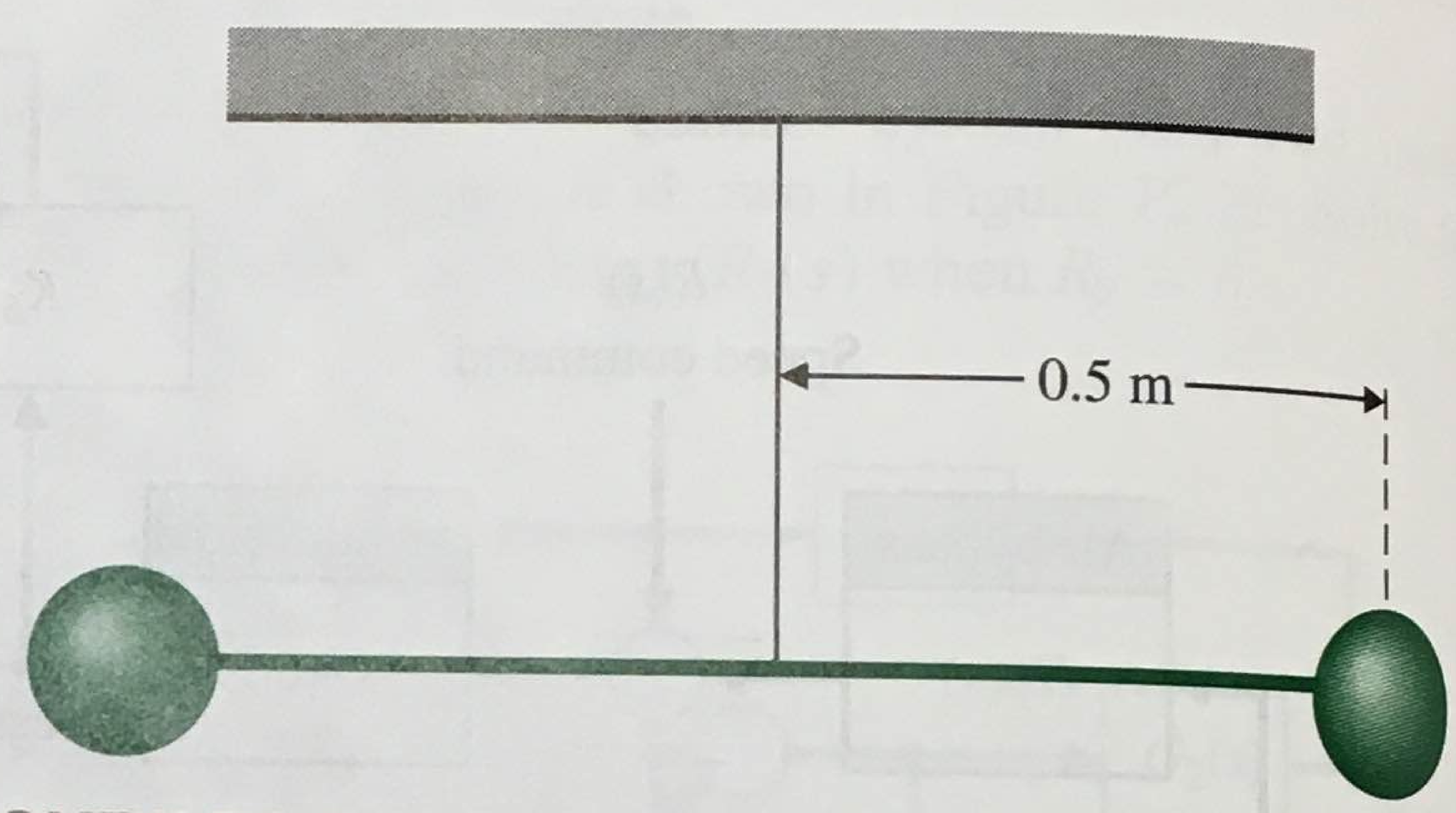

FIGURE P2.38 Winding oscillator.

P2.39 For the circuit of Figure P2.39, determine the transform of the output voltage $V_0(s)$. Assume that the circuit is in steady state when $t < 0$. Assume that the switch moves instantaneously from contact 1 to contact 2 at $t = 0$.

P2.40 A damping device is used to reduce the undesired vibrations of machines. A viscous fluid, such as a heavy oil, is placed between the wheels, as shown in

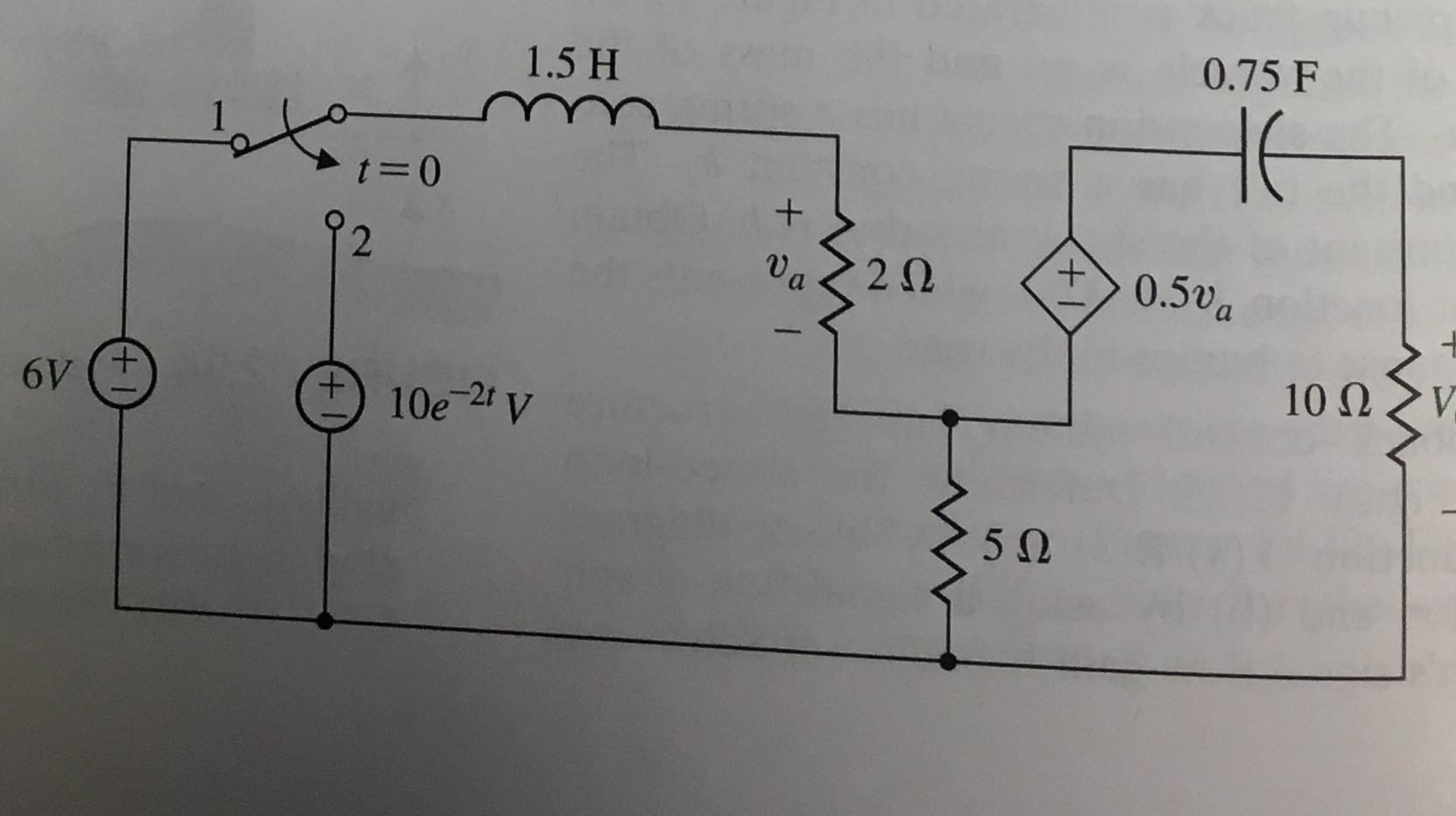

FIGURE P2.39 Model of an electronic circuit.

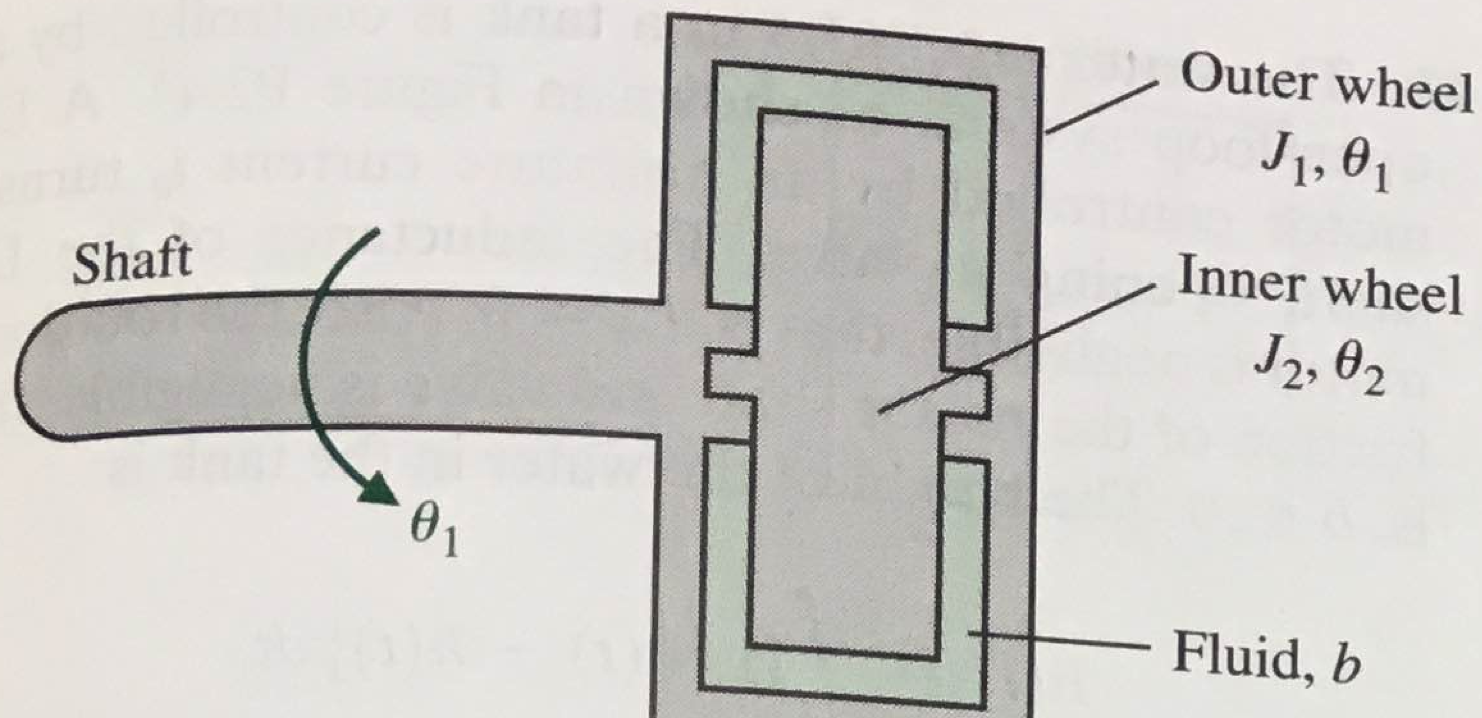

FIGURE P2.40 Cutaway view of damping device.

Figure P2.40. When vibration becomes excessive, the relative motion of the two wheels creates damping. When the device is rotating without vibration, there is no relative motion and no damping occurs. Find $\theta_1(s)$ and $\theta_2(s)$. Assume that the shaft has a spring constant K and that b is the damping constant of the fluid. The load torque is T.

P2.41 The lateral control of a rocket with a gimbaled engine is shown in Figure P2.41. The lateral deviation from the desired trajectory is h and the forward rocket speed is V. The control torque of the engine is $T_c(s)$ and the disturbance torque is $T_d(s)$. Derive the describing equations of a linear model of the system, and draw the block diagram with the appropriate transfer functions.

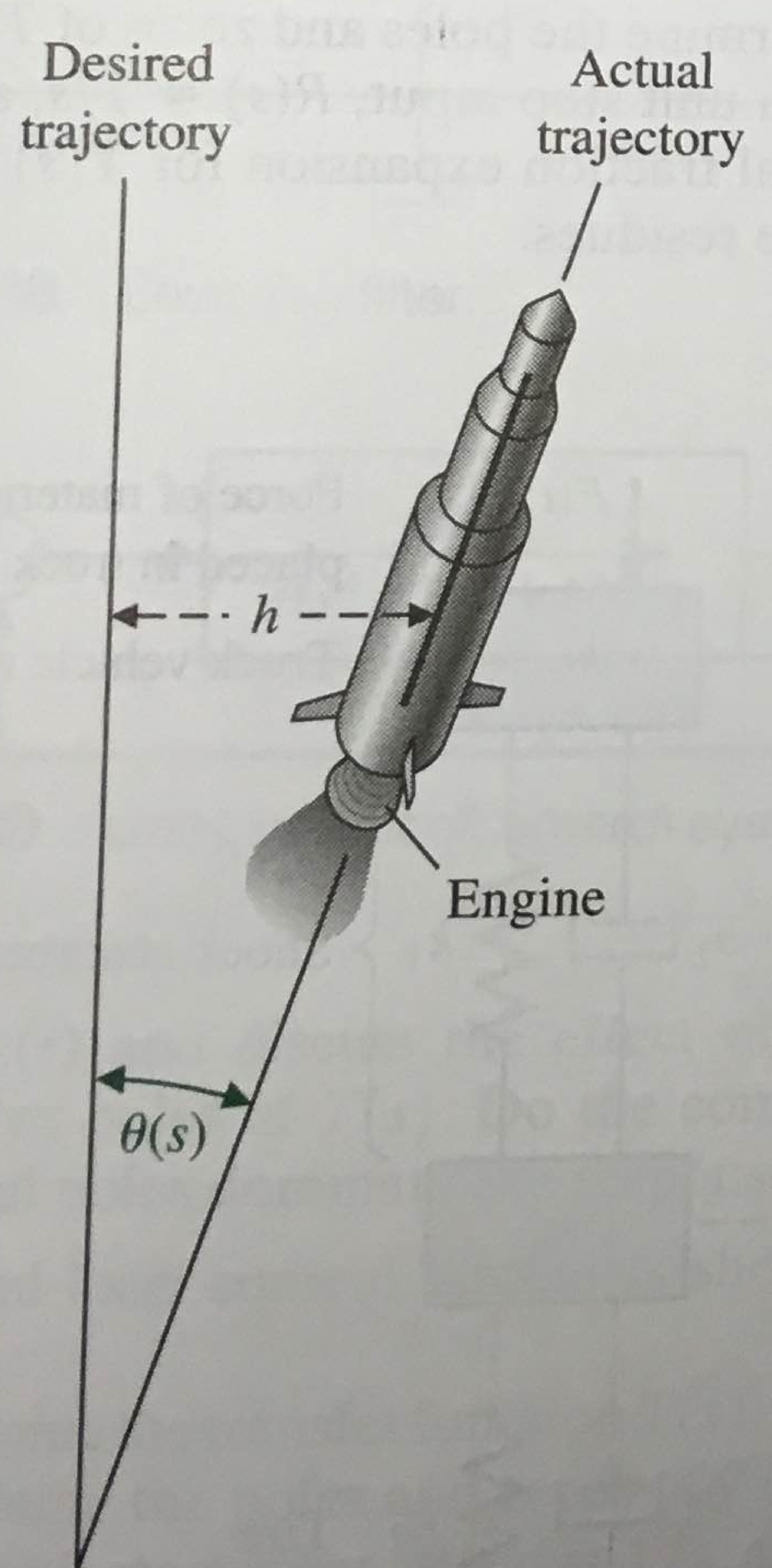

FIGURE P2.41 Rocket with gimbaled engine.

P2.42 In many applications, such as reading product codes in supermarkets and in printing and manufacturing, an optical scanner is utilized to read codes, as shown in Figure P2.42. As the mirror rotates, a friction force is developed that is proportional to its angular speed. The friction constant is equal to 0.06 N s/rad, and the moment of inertia is equal to 0.1 kg m^2. The output variable is the velocity $\omega(t)$. (a) Obtain the differential equation for the motor. (b) Find the response of the system when the input motor torque is a unit step and the initial velocity at $t = 0$ is equal to 0.7.

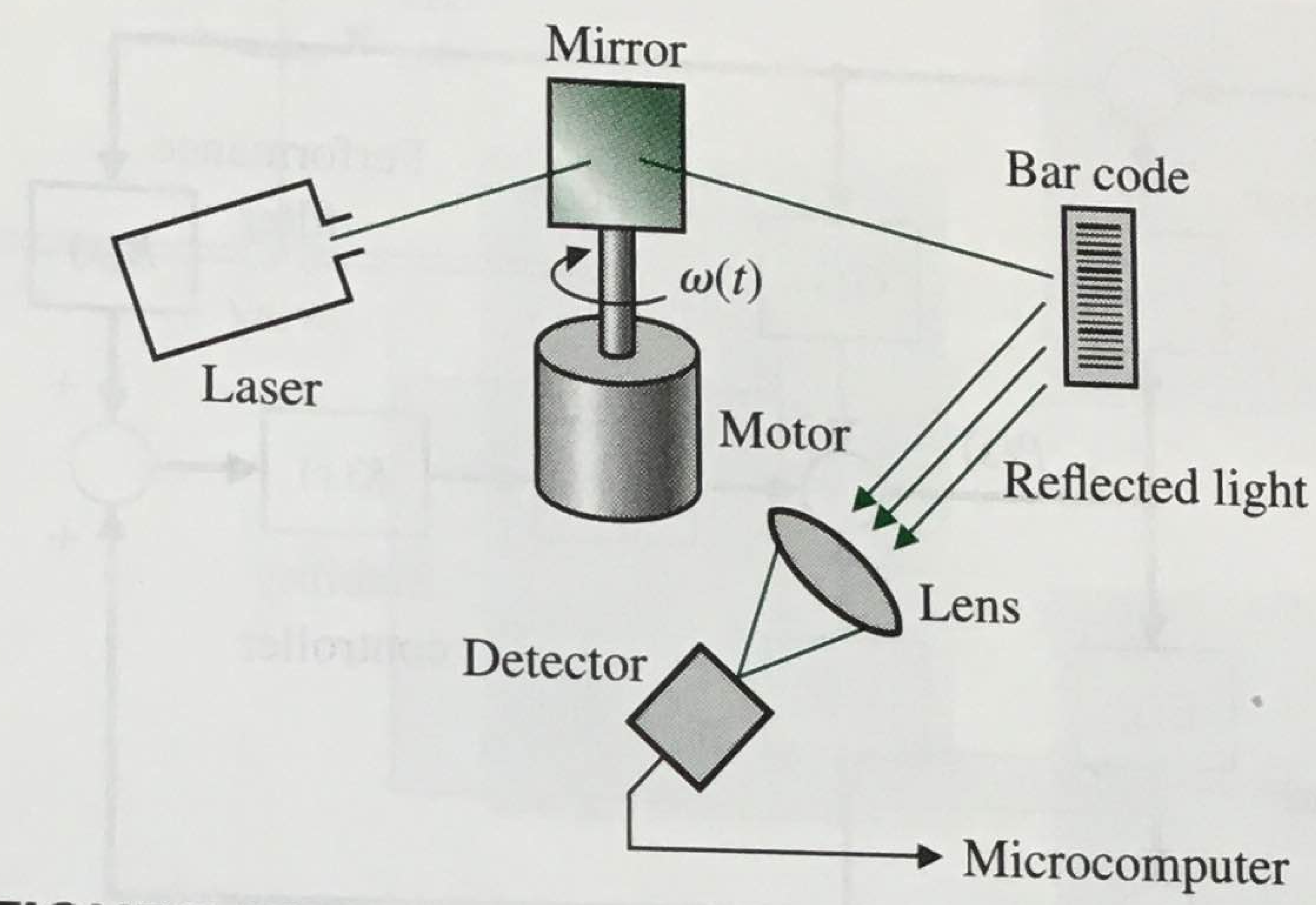

FIGURE P2.42 Optical scanner.

P2.43 An ideal set of gears is shown in Table 2.4, item 10. Neglect the inertia and friction of the gears and assume that the work done by one gear is equal to that of the other. Derive the relationships given in item 10 of Table 2.4. Also, determine the relationship between the torques T_m and T_L.

P2.44 An ideal set of gears is connected to a solid cylinder load as shown in Figure P2.44. The inertia of the motor shaft and gear G_2 is J_m. Determine (a) the inertia of the load J_L and (b) the torque T at the motor shaft. Assume the friction at the load is b_L and the friction at the motor shaft is b_m. Also assume the density of the load disk is ρ and the gear ratio is n. Hint: The torque at the motorshaft is given by $T = T_1 + T_m$.

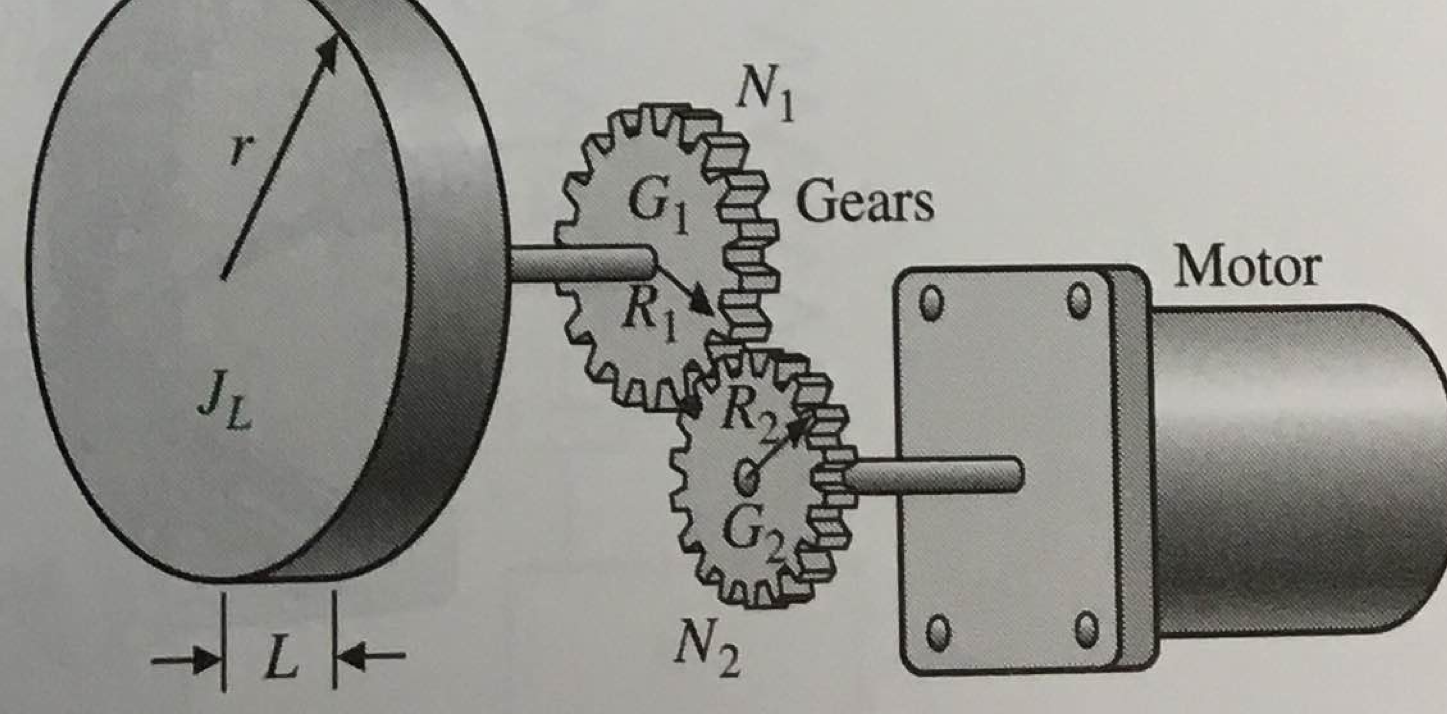

FIGURE P2.44 Motor, gears, and load.

P2.45 To exploit the strength advantage of robot manipulators and the intellectual advantage of humans, a class of manipulators called **extenders** has been examined [22].

The extender is defined as an active manipulator worn by a human to augment the human's strength. The human provides an input $U(s)$, as shown in Figure P2.45. The endpoint of the extender is $P(s)$. Determine the output $P(s)$ for both $U(s)$ and $F(s)$ in the form

$$P(s) = T_1(s)U(s) + T_2(s)F(s).$$

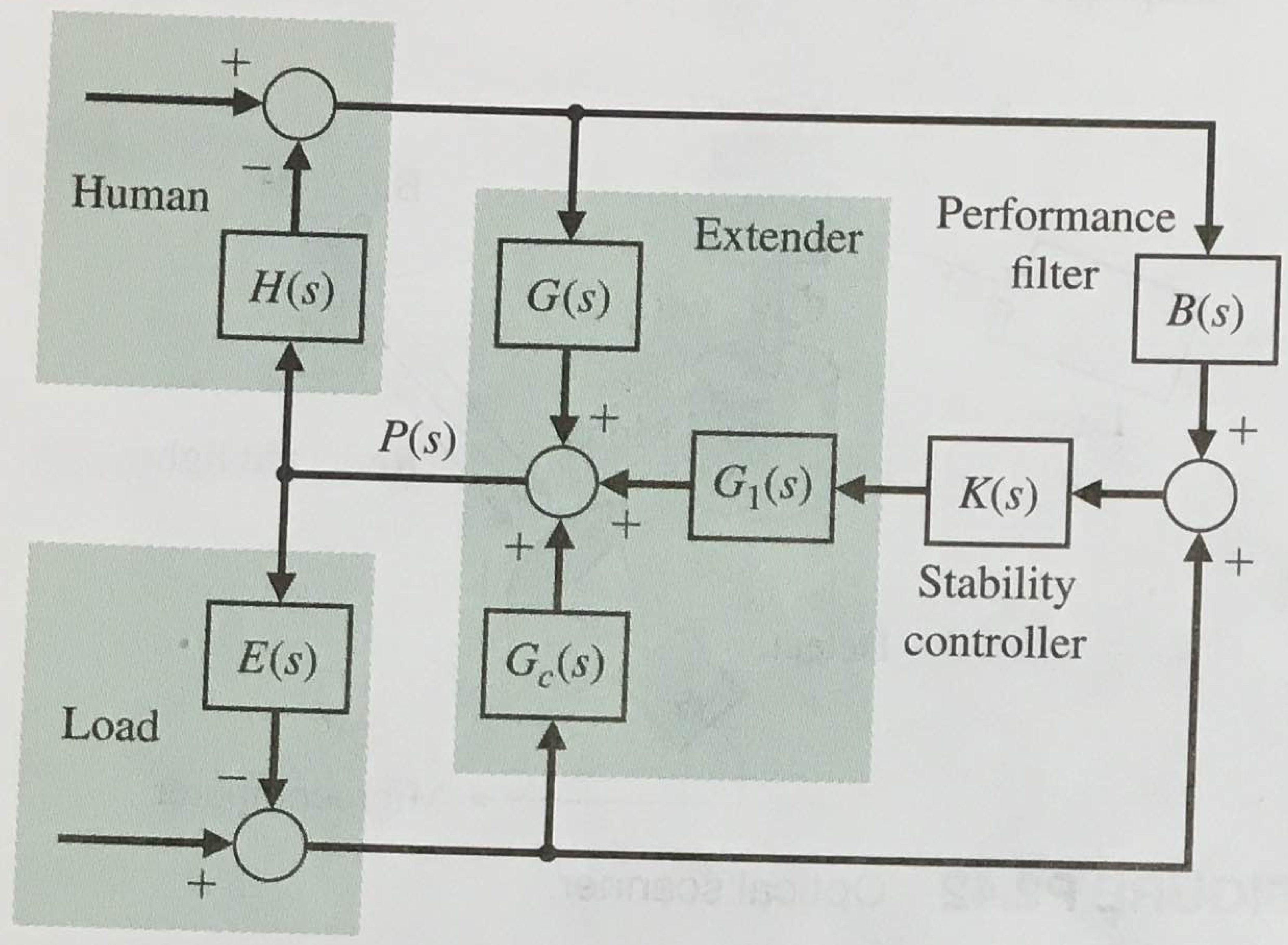

FIGURE P2.45 Model of extender.

P2.46 A load added to a truck results in a force $F(s)$ on the support spring, and the tire flexes as shown in Figure P2.46(a). The model for the tire movement is shown in Figure P2.46(b). Determine the transfer function $X_1(s)/F(s)$.

P2.47 The water level $h(t)$ in a tank is controlled by an open-loop system, as shown in Figure P2.47. A DC motor controlled by an armature current i_a turns a shaft, opening a valve. The inductance of the DC motor is negligible, that is, $L_a = 0$. Also, the rotational friction of the motor shaft and valve is negligible, that is, $b = 0$. The height of the water in the tank is

$$h(t) = \int [1.6\theta(t) - h(t)]\,dt,$$

the motor constant is $K_m = 10$, and the inertia of the motor shaft and valve is $J = 6 \times 10^{-3}$ kg m^2. Determine (a) the differential equation for $h(t)$ and $v(t)$ and (b) the transfer function $H(s)/V(t)$.

P2.48 The circuit shown in Figure P2.48 is called a lead-lag filter.

(a) Find the transfer function $V_2(s)/V_1(s)$. Assume an ideal op-amp.

(b) Determine $V_2(s)/V_1(s)$ when $R_1 = 250$ kΩ, $R_2 = 200$ kΩ, $C_1 = 2\ \mu F$, and $C_2 = 0.1\ \mu F$.

(c) Determine the partial fraction expansion for $V_2(s)/V_1(s)$.

P2.49 A closed-loop control system is shown in Figure P2.49.

(a) Determine the transfer function

$$T(s) = Y(s)/R(s).$$

(b) Determine the poles and zeros of $T(s)$.

(c) Use a unit step input, $R(s) = 1/s$, and obtain the partial fraction expansion for $Y(s)$ and the value of the residues.

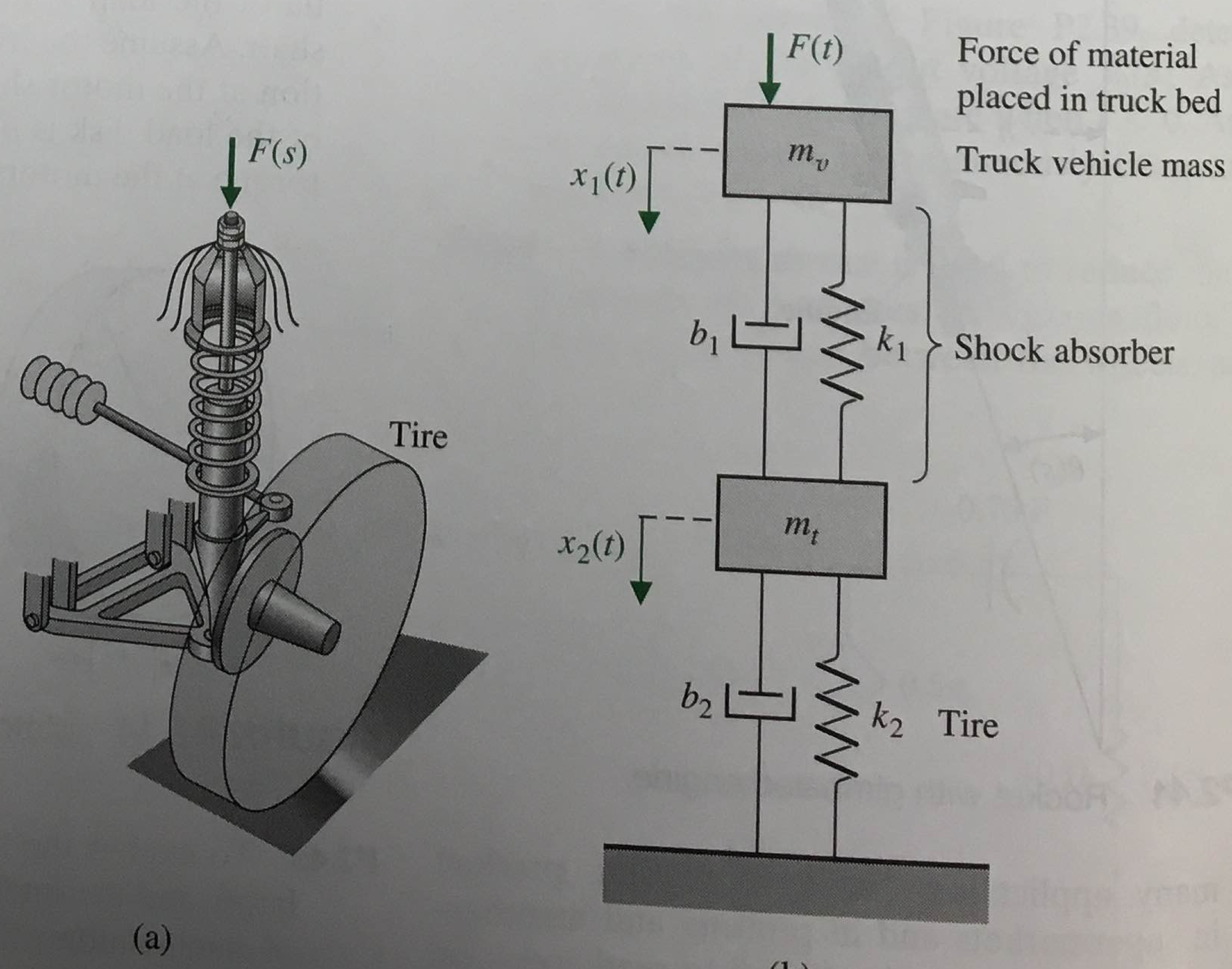

FIGURE P2.46 Truck support model.

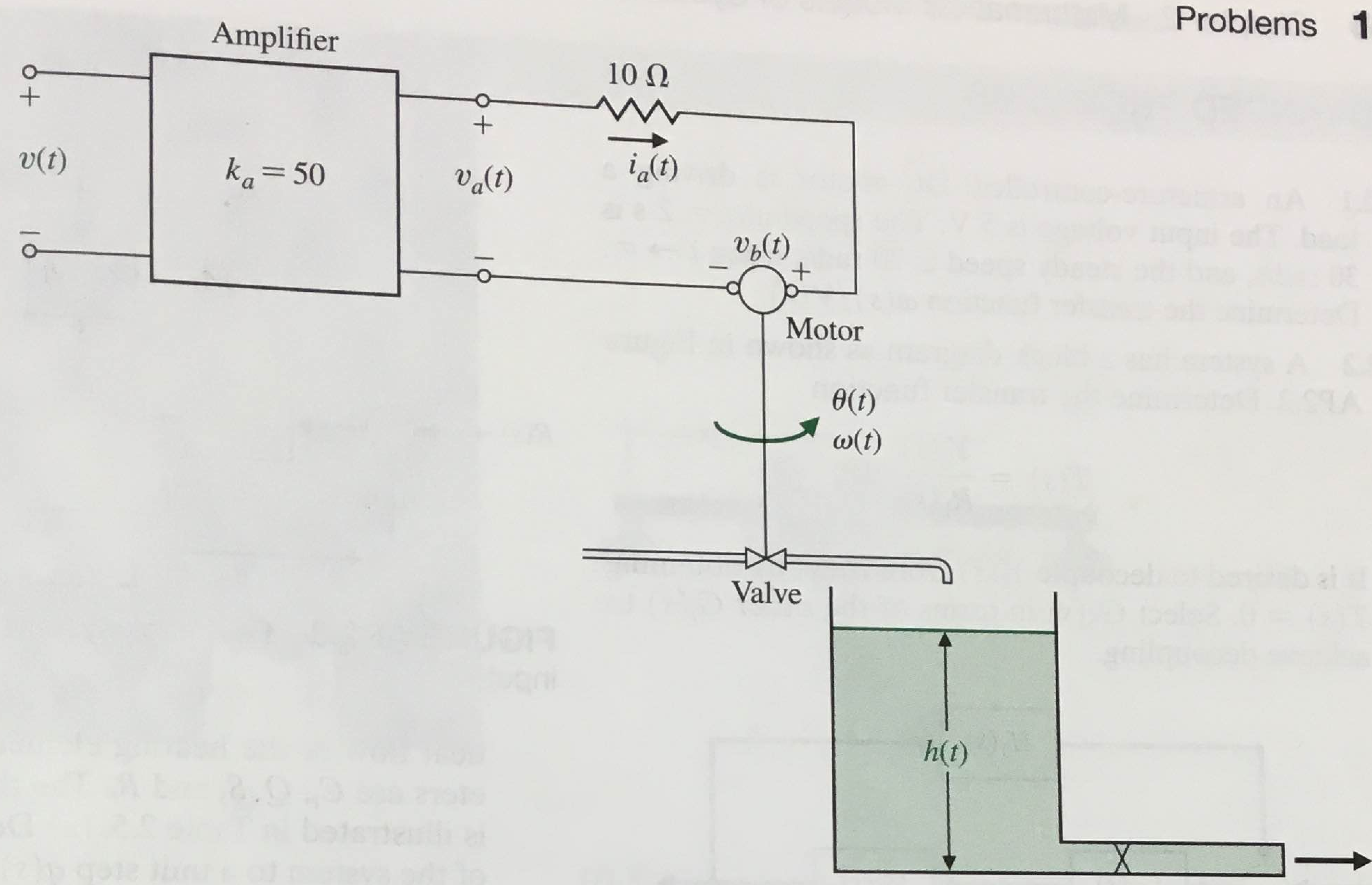

FIGURE P2.47 Open-loop control system for the water level of a tank.

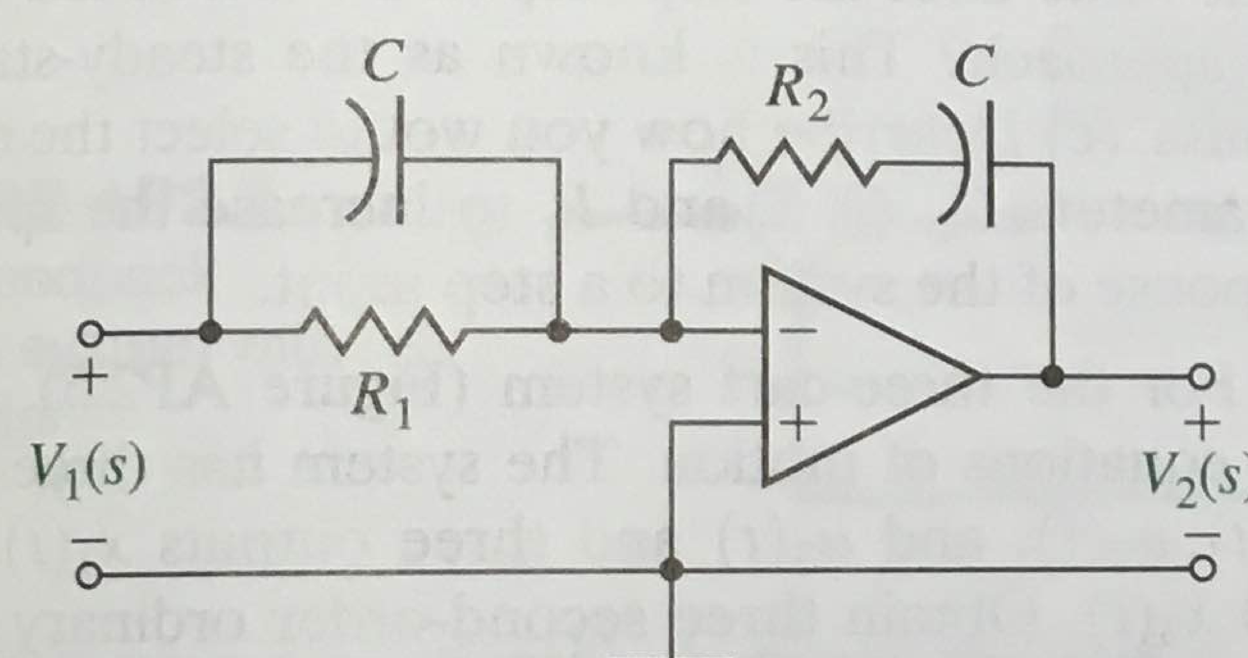

FIGURE P2.48 Lead-lag filter.

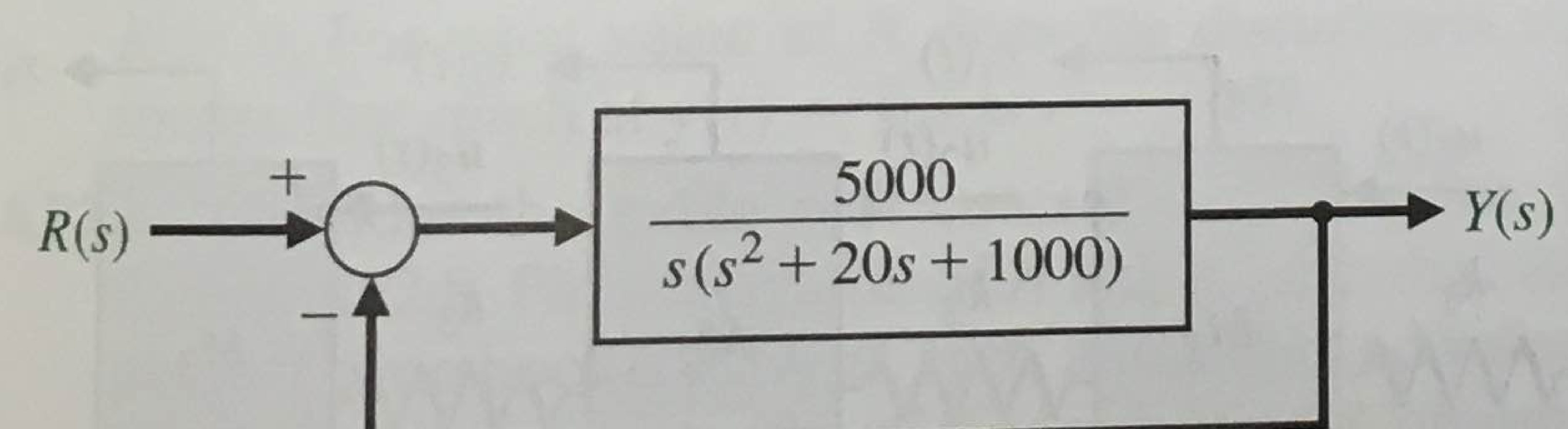

FIGURE P2.49 Unity feedback control system.

(d) Plot $y(t)$ and discuss the effect of the real and complex poles of $T(s)$. Do the complex poles or the real poles dominate the response?

P2.50 A closed-loop control system is shown in Figure P2.50.

(a) Determine the transfer function $T(s) = Y(s)/R(s)$.

(b) Determine the poles and zeros of $T(s)$.

(c) Use a unit step input, $R(s) = 1/s$, and obtain the partial fraction expansion for $Y(s)$ and the value of the residues.

(d) Plot $y(t)$ and discuss the effect of the real and complex poles of $T(s)$. Do the complex poles or the real poles dominate the response?

(e) Predict the final value of $y(t)$ for the unit step input.

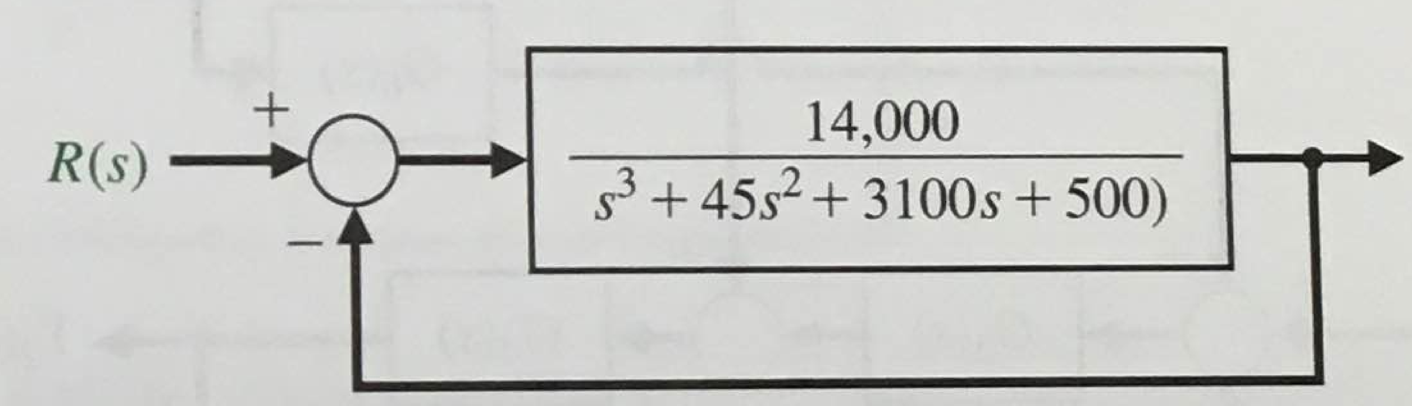

FIGURE P2.50 Third-order feedback system.

P2.51 Consider the two-mass system in Figure P2.51. Find the set of differential equations describing the system.

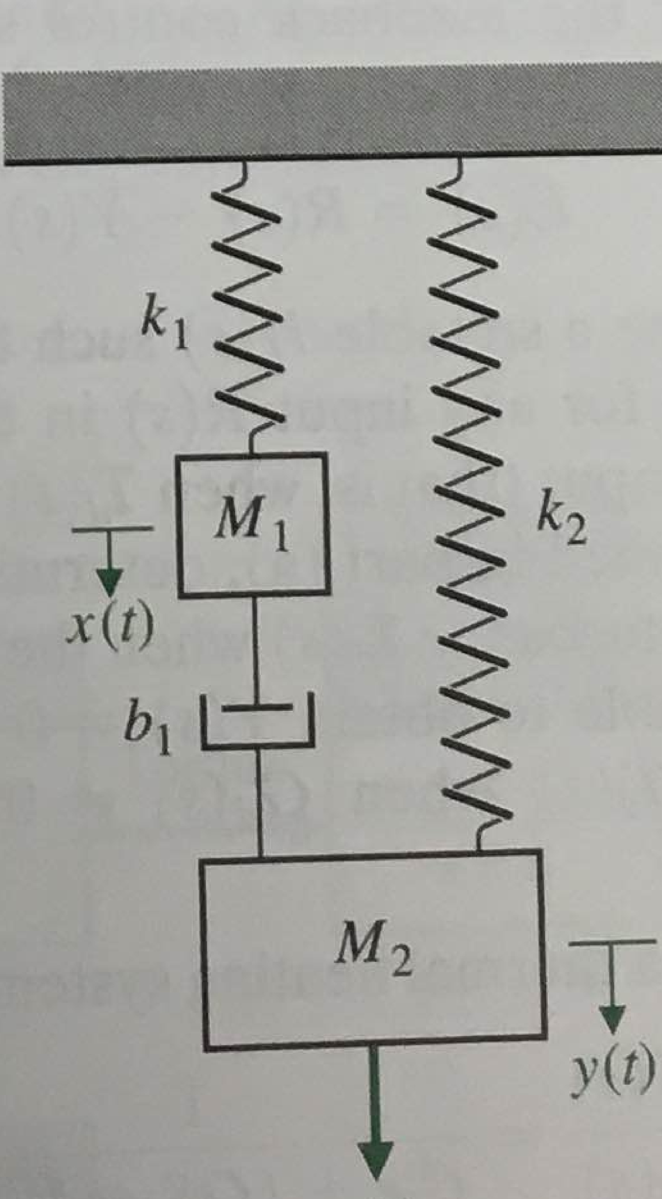

FIGURE P2.51 Two-mass system with two springs and one damper.