United States Army Corps of Engineers Engineering Manual EM 1110-2-1601

CECW-EH-D Engineer Manual 1110-2-1601	Department of the Army U.S. Army Corps of Engineers Washington, DC 20314-1000	EM 1110-2-1601 1 July 1991/ 30 June 1994
	Engineering and Design HYDRAULIC DESIGN OF FLOOD CONTROL CHANNELS	
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	Engineering and Design HYDRAULIC DESIGN OF FLOOD CONTROL CHANNELS	

Chapter 2

Open Channel Hydraulic Theory

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Physical Hydraulic Elements

- Channel slope
- Cross-sectional area
- Wetted perimeter
- "Most efficient section"
- Boundary surface roughness

Invert Slope

- Controlled by surrounding topgraphy
- Can be altered to achieve desired project goals, but…
  - Cut/fill can be expensive
  - Effect on hydraulics and costs
  - Need to examine out of reach influences

Channel Cross-Section

- Balance capacity with costs
- Typically:
  - Trapezoidal for rural areas
  - Rectangular for urban reaches
  - V-bottom for minimizing effects of sediment

Roughness

- Absolute roughness
  - Theoretical interest only
- Effective roughness
  - Dimension of length
  - "Effective roughness height"
- Composite roughness
  - Most common approach

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Hydraulic Design Elements

- Assumptions
- Friction Losses
- Friction Coefficients
- Flow Classification

Assumptions

- Familiar with concepts of:
  - Uniform flow
  - Gradually varied flow
  - Conservation of Energy
  - Conservation of Momentum
  - Conversion of friction loss terms
- Have or have access to Chow (1959)

Hydraulic Design Elements

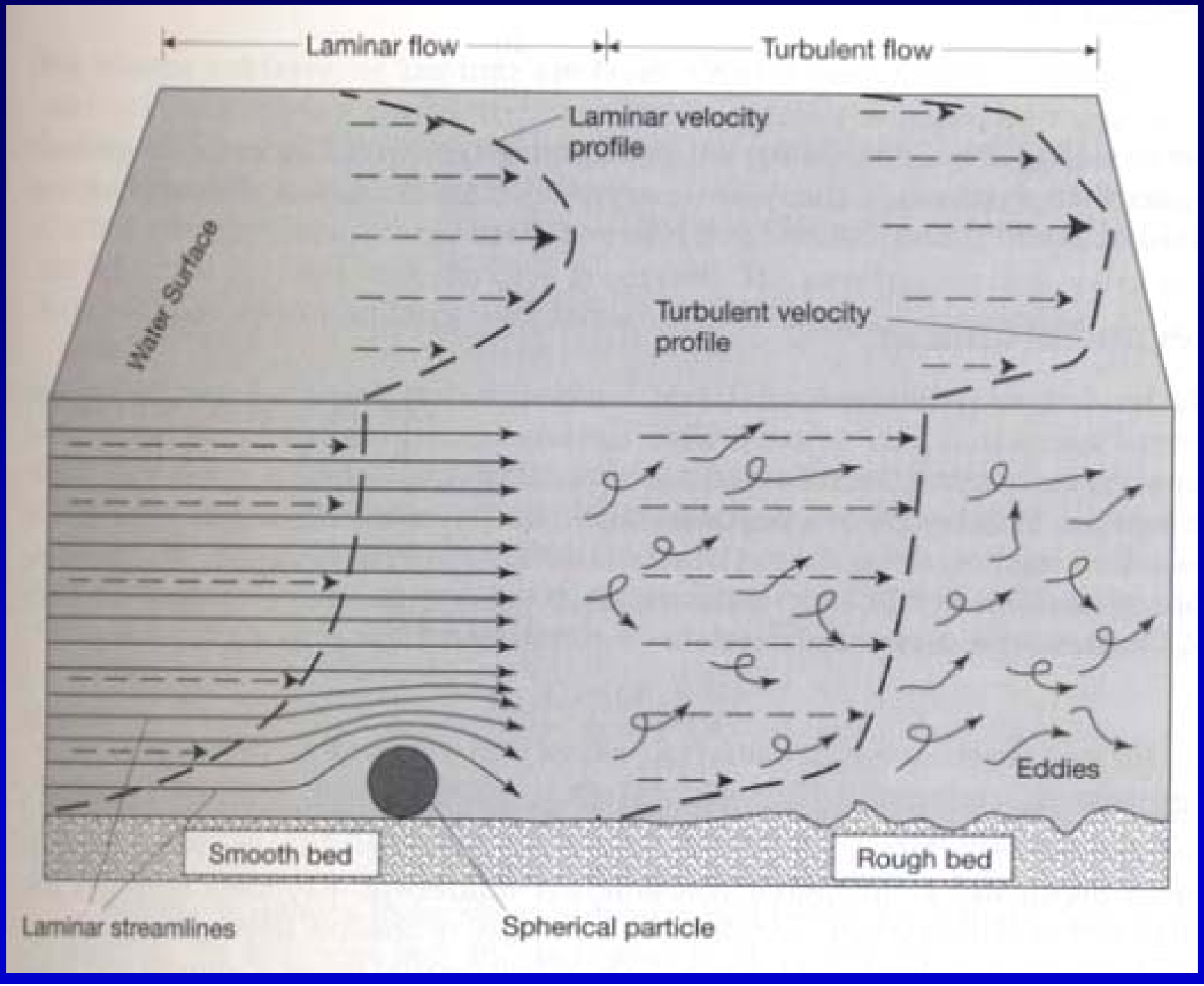
- Assumptions
- Friction Losses
- Friction Coefficients
- Flow Classification

Friction Losses

- Three main equations/concepts
  - Chezy
  - Manning
  - Darcy-Weisbach
- Included
  - Losses due to friction
  - Uniform turbulence and eddy losses
- Not Included
  - Local turbulence and eddy losses

Types of Fluid Flow

- Laminar Flow – flow persists as unidirectional movement
  - Molecules flow parallel
  - Movement up and down by diffusion
- Turbulent Flow – highly distorted flow
  - Large scale flow perpendicular to direction of flow
  - Transfer of movement up and down by macroscale processes
- Turbulence = irregular and random component of fluid motion
- Eddies = highly turbulent water masses


Laminar flow
Turbulent flow
Laminar velocity profile
Water Surface
Turbulent velocity profile
Eddies
Smooth bed
Rough bed
Laminar streamlines
Spherical particle

Laminar vs Turbulent Flow

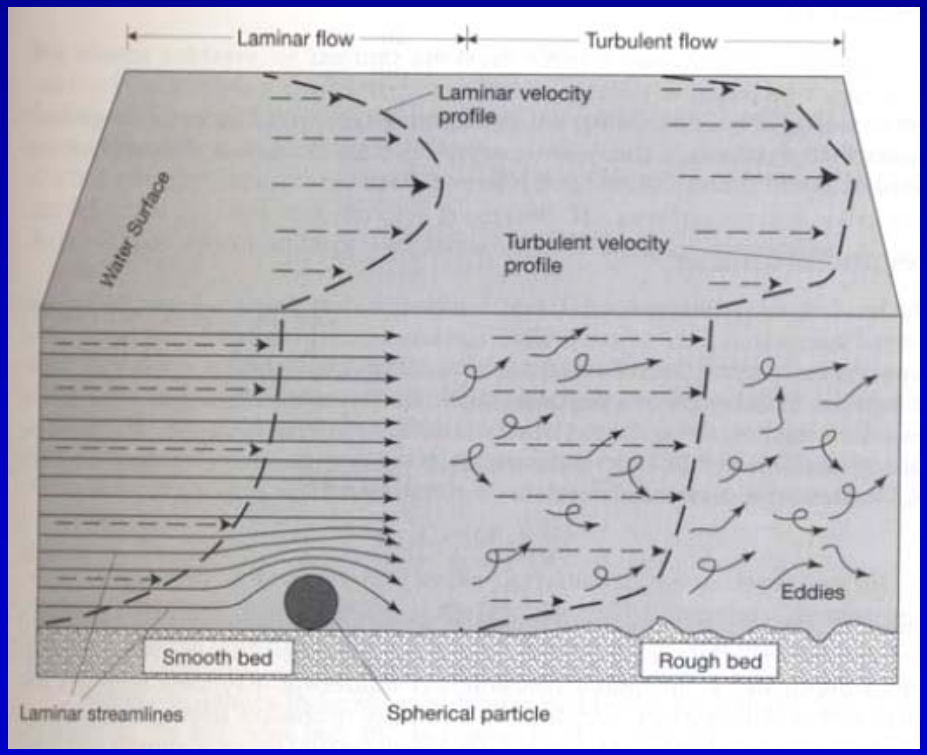
- Laminar flow – velocity constant at a point over time
- Turbulence
  - Most flows = turbulent
  - Slow settling velocity – upward motion of water particles
  - Increases effectiveness of fluid in eroding and entraining particles from the bed; but less efficient transport agent
  - Velocity measured at a point over time – tends towards an average value; but varies from instant to instant

Friction Losses

Type of Energy Losses

In consideration of the energy losses in fluid flow, they are generally broken down into 2 types:

- Major losses
- Minor losses



Friction Losses

Major Head Losses

Major head losses are a form of energy considered to take place continuously along the path of flow.

These losses are generally called "Friction Head Losses".

Friction Losses

Minor Head Losses

Minor head losses are those losses generally created by increased turbulence and resistance to flow at points in the stream where the direction of flow is changed or where other obstruction tale place.

The most commonly encountered forms of minor head losses are as follows:

- Hf ➡ head losses due to a sudden of gradual enlargement of the cross section of flow.
- Hc ➡ head losses due to a sudden of gradual contraction of the cross section of flow.
- Hg ➡ head losses due to obstruction in the path of flow (gates, valves, metering devices, and so on)
- Hb ➡ head losses occurring at bends and changes in direction of the flow path.

Friction Losses

Total Head Losses

The total head loss in a stream of flowing fluid is equal to the sum of all the head losses along its path

Total Head Losses = Major Head Losses + Minor Head Losses

$$H_l = h_f + h_e + h_c + h_g + h_b$$

Hydraulic Design Elements

- Assumptions
- Friction Losses
- Friction Coefficients
- Flow Classification

Recap

- If depth & velocity remain constant over a length of channel which has a constant cross-section and slope, then the flow is **uniform** and water surface will be parallel to streambed. The depth is called the normal depth.
- If depth and/or velocity change over distance then it is **varied** flow, which can be divided into gradually varied or rapidly varied

Steady Flow

- Law of continuity
  - $Q = V_1A_1 = V_2A_2 = \ldots$
  - Q = discharge, V = Average velocity, A = cross sectional area
  - Must have continuous, **steady-varied** flow
- Flow may also be:
  - Unsteady-varied
  - Unsteady-uniform
  - Steady-uniform

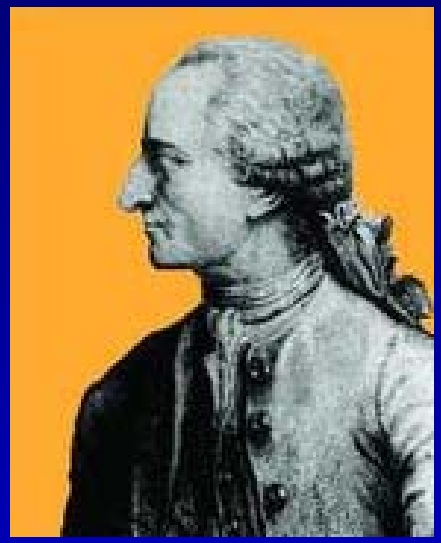
Uniform flow

- Gravity (force causing motion)
- Friction (force opposing motion)
- For uniform flow the gravity force will equal the friction force and the energy line, the water surface and the streambed will be parallel (have the same slope)

Analysis of Major Energy Losses

Chezy Analysis and Formula

Antoine Chezy, in 1775



A French civil engineer developed the Chézy equation, which relates the uniform flow velocity to channel roughness, hydraulic radius, and bed slope.

$$v = C\sqrt{Rs}$$

v = average velocity of flow
R = hydraulic radius
S = slope of the channel
C = coefficient depending upon the various characteristics of the channel and their comparison with those of another similar channel

Chezy's Equation

- The gravitation force is approximately proportional to the velocity squared
- $V = \sqrt{(\rho g/\alpha)RS}$
  - V is the mean velocity (ft/s)
  - α is a coefficient mainly dependent on channel roughness

Chezy's "C"

- The term $\sqrt{(\rho g/\alpha)}$ was later replaced with a single coefficient C
- C has units of $(ft^{1/2}/s)$
- And varies from:
  - 30 (small rough channels)
  - 90 (large smooth ones)

TABLE XI. VALUES OF C FOR USE IN THE CHEZY FORMULA

$$V = C\sqrt{rs}$$

(Art. 286, p. 315.)

Slope	r	n .020	n .025	n .030	n .035	n .040	n .045	n .050	n .055	n .060
.0001	3.28	91	73	60	52	46	40	36	33	30
	10	111	92	78	69	62	55	50	46	42
	20	122	102	89	79	71	65	60	55	51
	50	134	114	100	91	83	76	71	67	63
	100	140	121	108	98	91	84	79	74	70
.0002	10	108	89	76	67	60	53	49	45	41
	20	117	98	85	76	68	61	57	53	49
	50	126	108	94	85	78	71	66	62	58
	100	131	113	99	90	83	77	72	68	64
.0004	10	107	88	75	66	59	53	48	44	41
	20	115	96	83	73	66	60	55	51	48
	50	123	104	91	82	75	68	63	59	56
	100	127	108	96	87	80	73	68	64	61
.0010	10	105	87	74	65	58	52	47	44	40
	20	113	94	81	72	65	59	54	50	47
	50	120	101	89	79	72	66	61	57	54
	100	124	105	94	85	77	71	66	62	59
.010	10	105	86	74	65	58	51	47	43	40
	20	112	93	80	71	64	58	53	49	46
	50	119	100	87	78	71	65	60	56	53
	100	122	104	91	82	75	69	65	61	58

NOTE. — For $r = 3.28$ feet, n constant, C is constant for all values of slope. For slopes greater than 0.01, or fall of 52.8 feet per mile, C remains nearly constant.

From "River Discharge," by Hoyt and Grover.

Chezy

$$S_f = \frac{V^2}{C^2 R}$$

Where:

S_f = EGL slope

V = velocity

C = Chezy coefficient

R = Hydraulic radius

Chezy

Hydraulically smooth channels

$$C = 32.6 \log_{10}\left(\frac{5.2R_n}{C}\right)$$

Where:

C = Chezy coefficient

R_n = Reynolds number

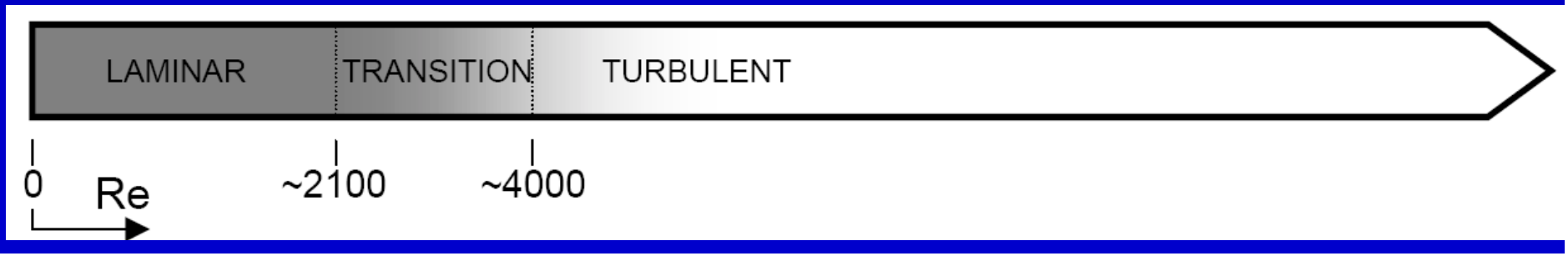
Reynolds Number

$$\mathrm{Re} = \frac{VL}{\upsilon}$$

or $\dfrac{\text{Internal Forces}}{\text{Viscous Forces}}$

Where:

R_e = Reynolds number
L = characteristic length
v = kinematic viscosity
V = velocity



Chezy

Hydraulically rough channels

$$C = 32.6 \log_{10} \left(\frac{12.2R}{k} \right)$$

Where:

C = Chezy coefficient

k = effective roughness

R = hydraulic radius

Hydraulically Rough or Smooth?

- Hydraulically rough
  - $k_s > k_c$
- Hydraulically smooth
  - $K_s < k_c$

Hydraulically Rough or Smooth?

$$k_c = \left(\frac{5C}{\sqrt{g}}\right)\left(\frac{\nu}{V}\right)$$

Where:

k_c = Equivalent roughness

C = Chezy C

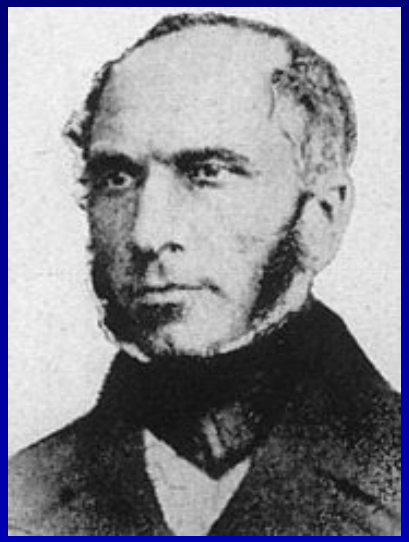
ν = kinematic viscosity

g = gravity

V = Velocity

Darcy-Weibach Formula

Henry Darcy, 1857



Showed that f is not dimensionless but dependent upon numerous parameters

- -the roughness of conduit wall
- -the velocity of flow
- -the viscosity and density of fluid
- -the diameter of pipe

Darcy-Weibach Formula

Julius Weisbach,

Developed a formula for pipe flow, based to some extent on the Chezy formula.

$$h_f = f x \frac{l}{d} x \frac{v^2}{2g}$$

f = friction factor
d = pipe diameter
l = pipe length
h_f = head losses over pipe length
v = velocity

Analysis of Major Energy Losses

Darcy-Weibach Formula

Values for f for water flowing in straight smooth pipe are shown in table.

Diameter, mm	Velocity, m/s						
	0.10	0.25	0.50	1.00	1.50	3.00	5.00
150	0.032	0.029	0.027	0.026	0.024	0.024	0.023
200	0.030	0.027	0.025	0.023	0.022	0.022	0.022
300	0.028	0.024	0.022	0.021	0.020	0.020	0.020
450	0.025	0.021	0.020	0.019	0.018	0.018	0.018
600	0.024	0.020	0.018	0.018	0.017	0.017	0.017
900	0.020	0.018	0.017	0.016	0.016	0.015	0.015

Darcy – Weisbach *f*

$$S_f = \frac{fV^2}{8Rg}$$

Where:

S_f = EGL slope

V = velocity

f = Darcy-Weisbach coefficient

R = Hydraulic radius

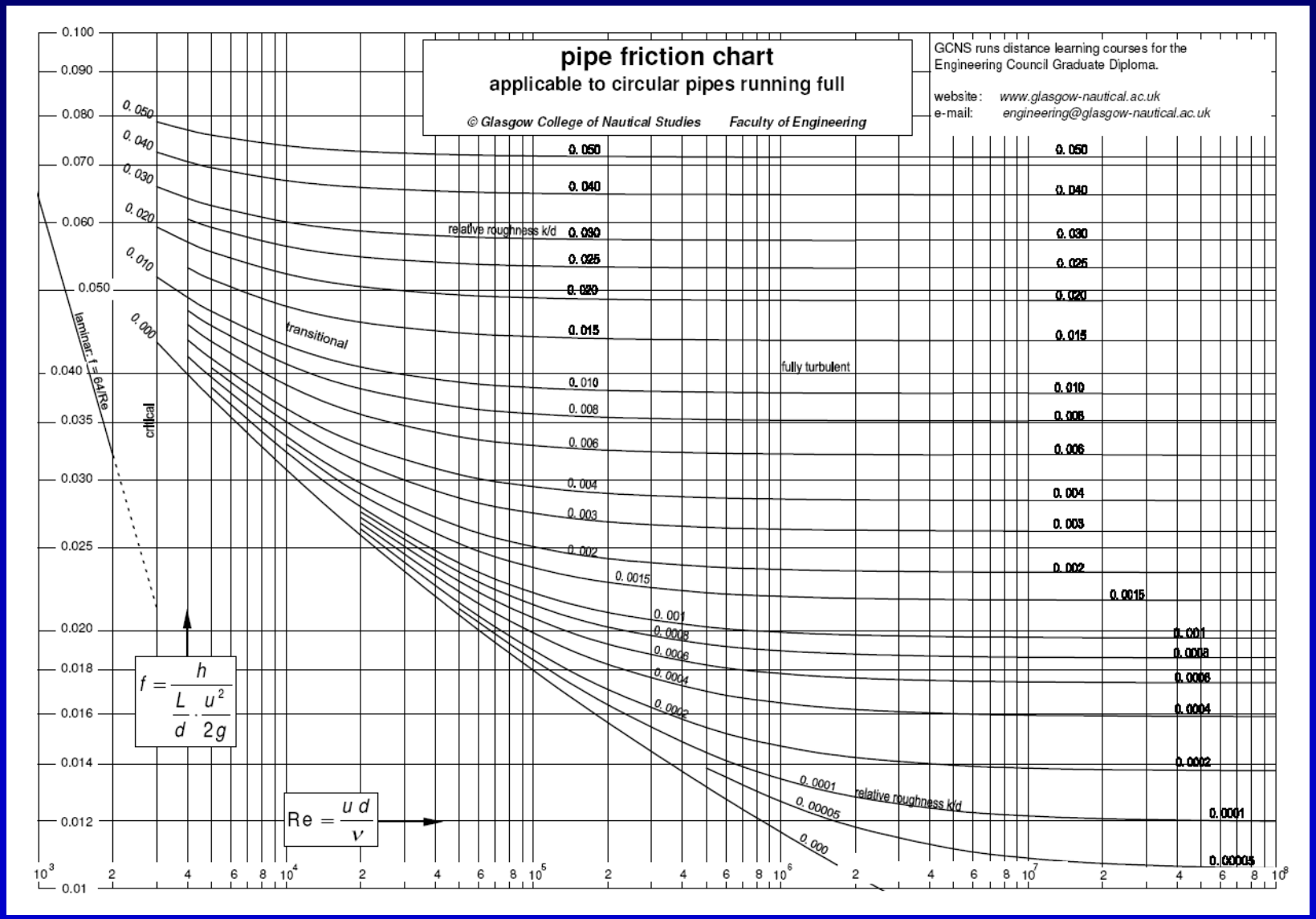
Reynolds Number

Moody Diagram

- L.F. Moody developed the diagram which expresses the relationships between
  - -Reynolds number
  - -f for various ranges of values of k/d
- Moody diagram
  - -all curves are concave upward and tend to flatten out as value of R_e increase.

$$R_e = 400\frac{d}{k}\log(3.7\frac{d}{k})$$

Moody Diagram



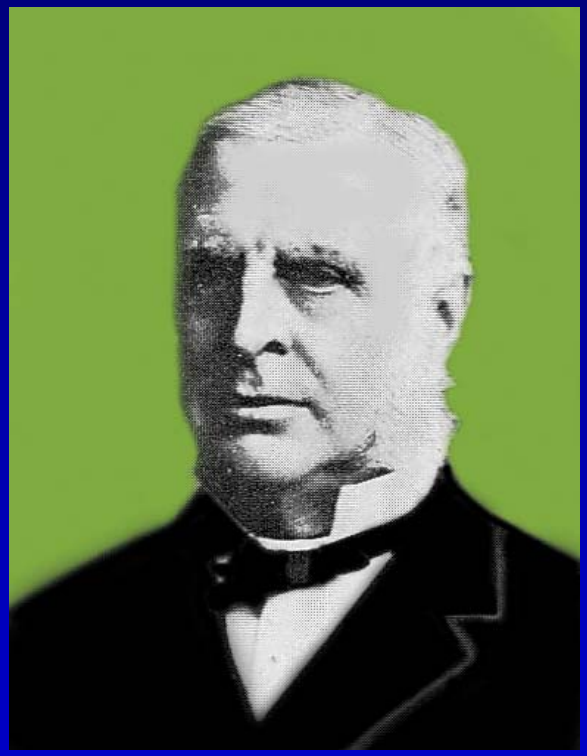
Friction Factor

Reynolds Number

Open-Channel Flow Formulas

Manning Formula

Robert Manning, in 1885



Developed Manning formula used for open channel flow conditions.

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$

v = velocity of flow, m/s
R = hydraulic radius, m
S = slope of the energy gradient
n = a roughness coefficient

Manning's Equation

- $V = 1/n\ R^{2/3} S^{1/2}$
- $Q = 1/n\ AR^{2/3} S^{1/2}$
  - Q is discharge (ft3/s)
  - n is a coefficient known as "Manning's n"
  - SI units (imperial are multiplied by 1.486)
- $n = sd^{1/6}$
  - d is the median diameter in (mm) d_{50} for which 50% of streambed particles are smaller
- $n = (0.1129R^{1/6})/(1.16+2\log(R/d_{84}))$
  - d_{84} is the diameter (m) for which 84% of streambed particles are smaller

Manning

$$S_f = \frac{V^2 n^2}{2.21 R^{4/3}}$$

Where:

V = velocity

S_f = EGL slope

n = Manning's n

R = Hydraulic radius

Manning's *n* Examples

TABLE 6.1 Manning roughness coefficients (n) for different boundary types.

Boundary	Manning n ($ft^{1/6}$)
Very smooth surfaces such as glass, plastic, or brass	0.010
Very smooth concrete and planed timber	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Good wood	0.014
Vitrified clay	0.015
Shot concrete, untroweled, and earth channels in best condition	0.017
Straight unlined earth canals in good condition	0.020
Rivers and earth canals in fair condition; some growth	0.025
Winding natural streams and canals in poor condition; considerable moss growth	0.035
Mountain streams with rocky beds and rivers with variable sections and some vegetation along banks	0.041–0.050

Source: *Handbook of Applied Hydrology,* ed. by Ven T. Chow, copyright 1964 McGraw-Hill Publishing Co., Inc.

Manning's *n* Examples

TABLE 6.2 Variation of Manning *n* values with changes in bed form occurring under different flow conditions.

Bed Form	Manning *n* ($ft^{1/6}$)
Lower regime	
Ripples	0.017–0.028
Dunes	0.018–0.035
Washed-out dunes or transition	0.014–0.024
Upper regime	
Plane bed	0.011–0.015
Standing waves	0.012–0.016
Antidunes	0.012–0.020

Source: *Handbook of Applied Hydrology*, ed. by Ven T. Chow, copyright 1964 McGraw-Hill Publishing Co., Inc.

Coefficient Relation

$$\frac{C}{1.486} = \frac{R^{1/6}}{n} = \frac{10.8}{f^{1/2}}$$

Plate 3 - *k* to *C* and *f*

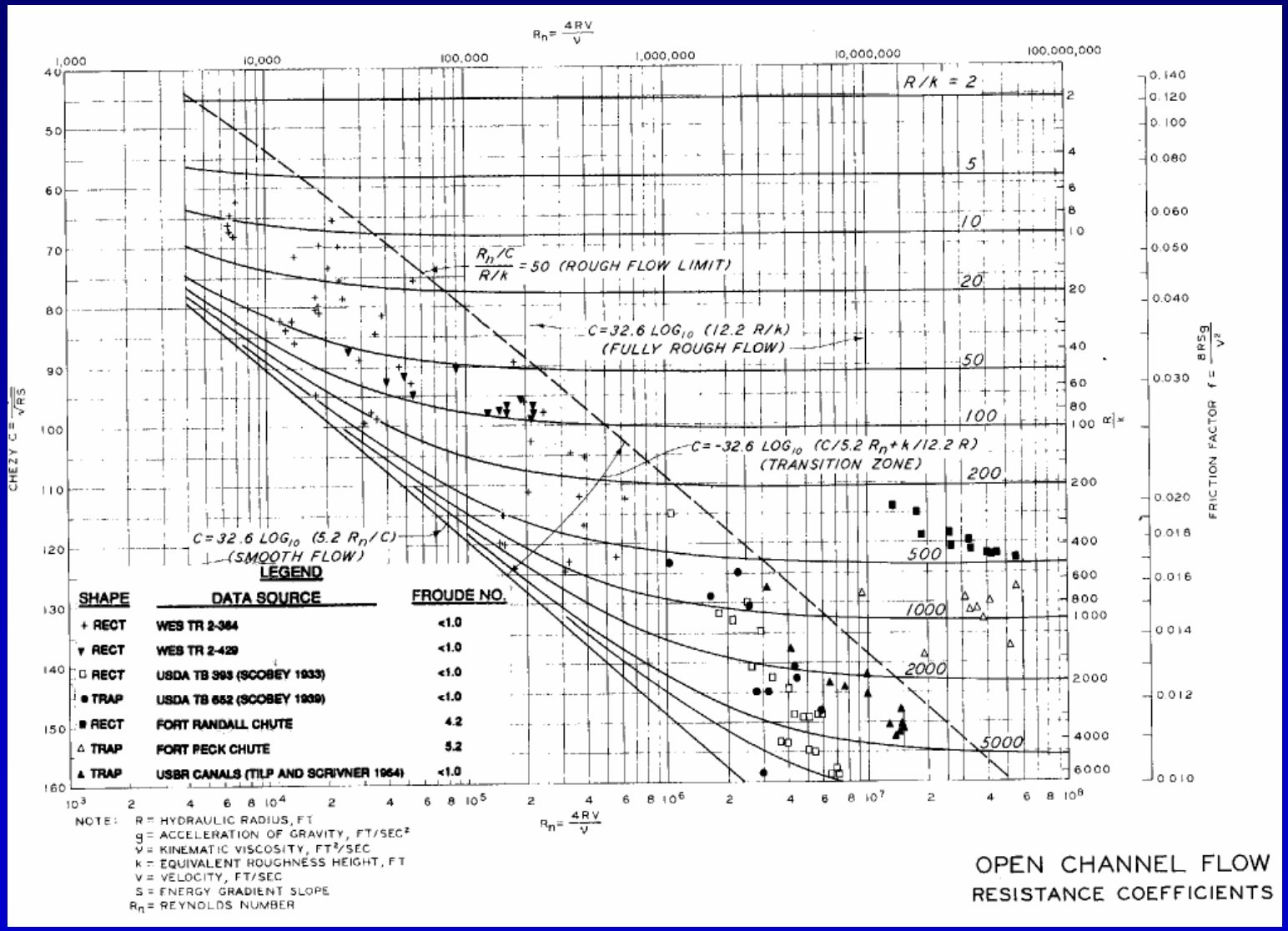
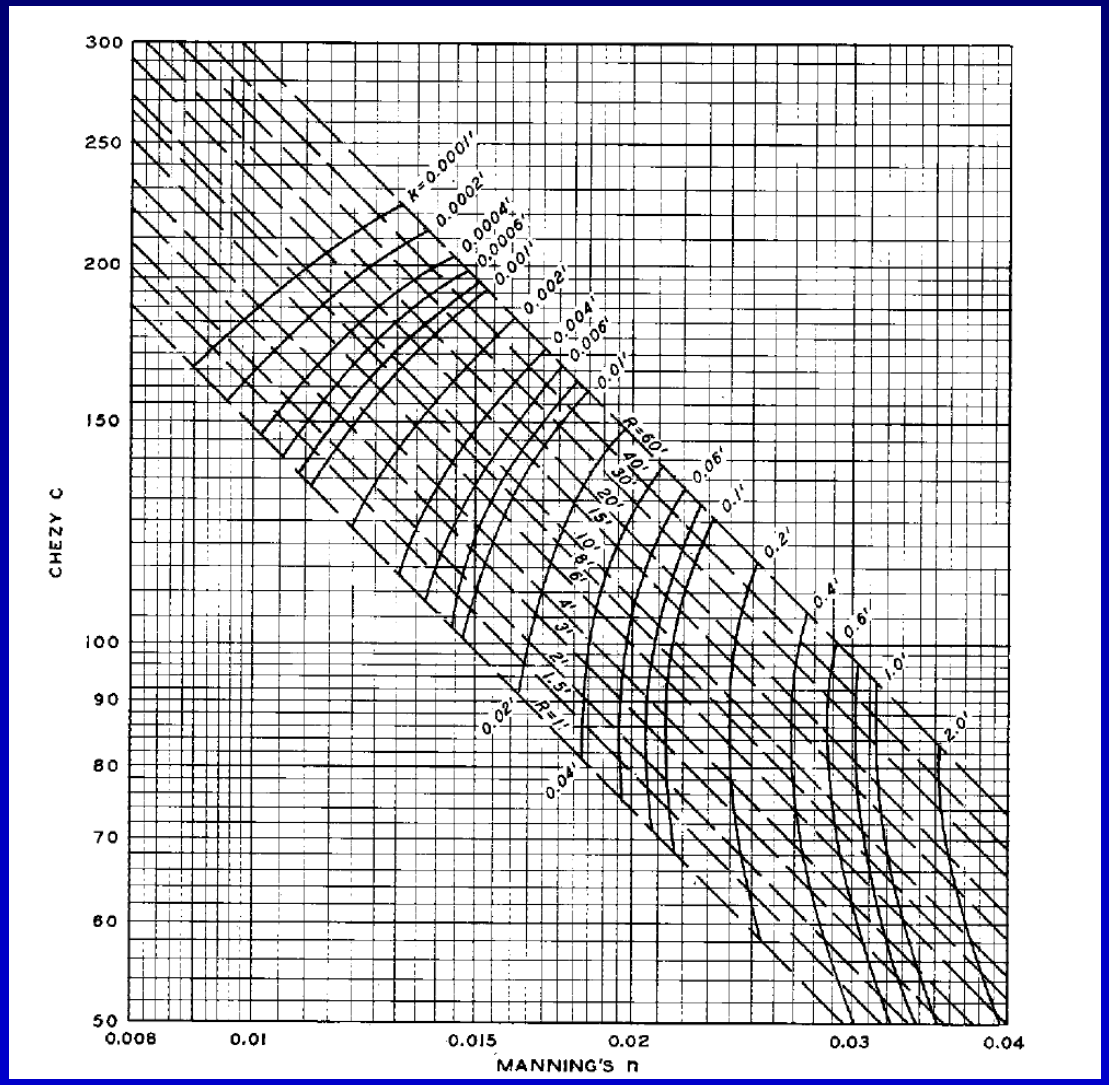


Plate 3

Hydraulically Rough Channels



OPEN CHANNELS

C-n-R-k RELATION

$0.008 < n < 0.04$

BASIC EQUATIONS

$$C = 32.6 \log_{10} 12.2R/k$$

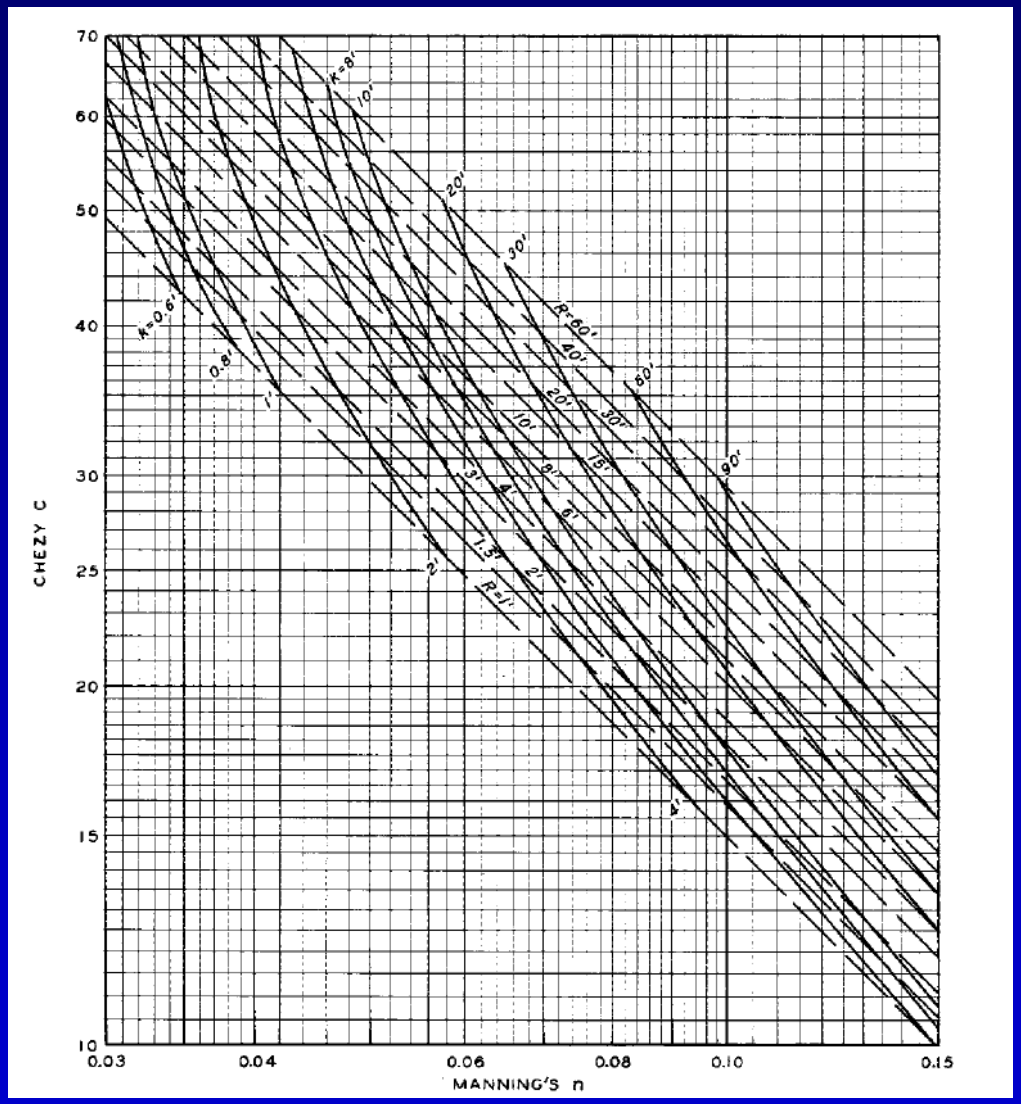
$$n = \frac{R^{1/6}}{23.85 + 21.95 \log_{10} R/k}$$

WHERE:

- C = CHEZY COEFFICIENT
- n = MANNING'S RESISTANCE COEFFICIENT
- R = HYDRAULIC RADIUS, FT
- k = EQUIVALENT ROUGHNESS HEIGHT, FT

Plate 5

Hydraulically Rough Channels



OPEN CHANNELS

C-n-R-K RELATION

$0.03 < n < 0.15$

BASIC EQUATIONS

$C = 32.6 \, LOG_{10} \, 12.2R/K$

$$n = \frac{R^{1/6}}{23.85 + 21.95 \, LOG_{10} \, R/K}$$

WHERE:

- C = CHEZY COEFFICIENT
- n = MANNING'S RESISTANCE COEFFICIENT
- R = HYDRAULIC RADIUS, FT
- K = EQUIVALENT ROUGHNESS HEIGHT, FT

Stop 15 SEP 08

Plate 4

Rules of Thumb

- Most channels are hydraulically rough
- *k* values for natural rivers:
  - 0.1 to 3.0 (ft)
- Typically larger than spherical diameters of bed materials – why??

Rules of Thumb

- If field data is available
  - Determine k from:

$$C = 32.6 \log_{10}\left(\frac{12.2R}{k}\right)$$

EQ 2-6

$$S_f = \frac{V^2}{C^2 R}$$

Rules of Thumb

- If field data is available
  - Determine k from Equation 2-6
- Value of k can be assumed for flow levels at or lower than observed condition
- Appendix C outlines method for determining an effective roughness

Why use *k*?

- Can be assumed to be spherical diameter of average bed material
  - What if losses other than friction?
- Consider relations:

$$\frac{C}{1.486} = \frac{R^{1/6}}{n}$$

$$C = 32.6 \log_{10}\left(\frac{12.2R}{k}\right)$$

k does not vary with R
n varies with $R^{1/6}$

However…

- *k* must be evaluated for each subsection
- Use subsections:
  - Differing bed materials
  - Bed forms present
  - Expansions and contractions
  - Form roughness significant

Why do we need to know all this stuff?

- Environmental flows
  - Depth of flow
  - Min particle size to prevent erosion
- The depth of flow is obviously important in calculating possible flood depths….
- But its not that simple, we actually use this for back water calculations.

Hydraulic Design Elements

- Assumptions
- Friction Losses
- Friction Coefficients
- Flow Classification

Flow Classification

- Tranquil and rapid flow
- Pulsating rapid flow
- Varied Flow

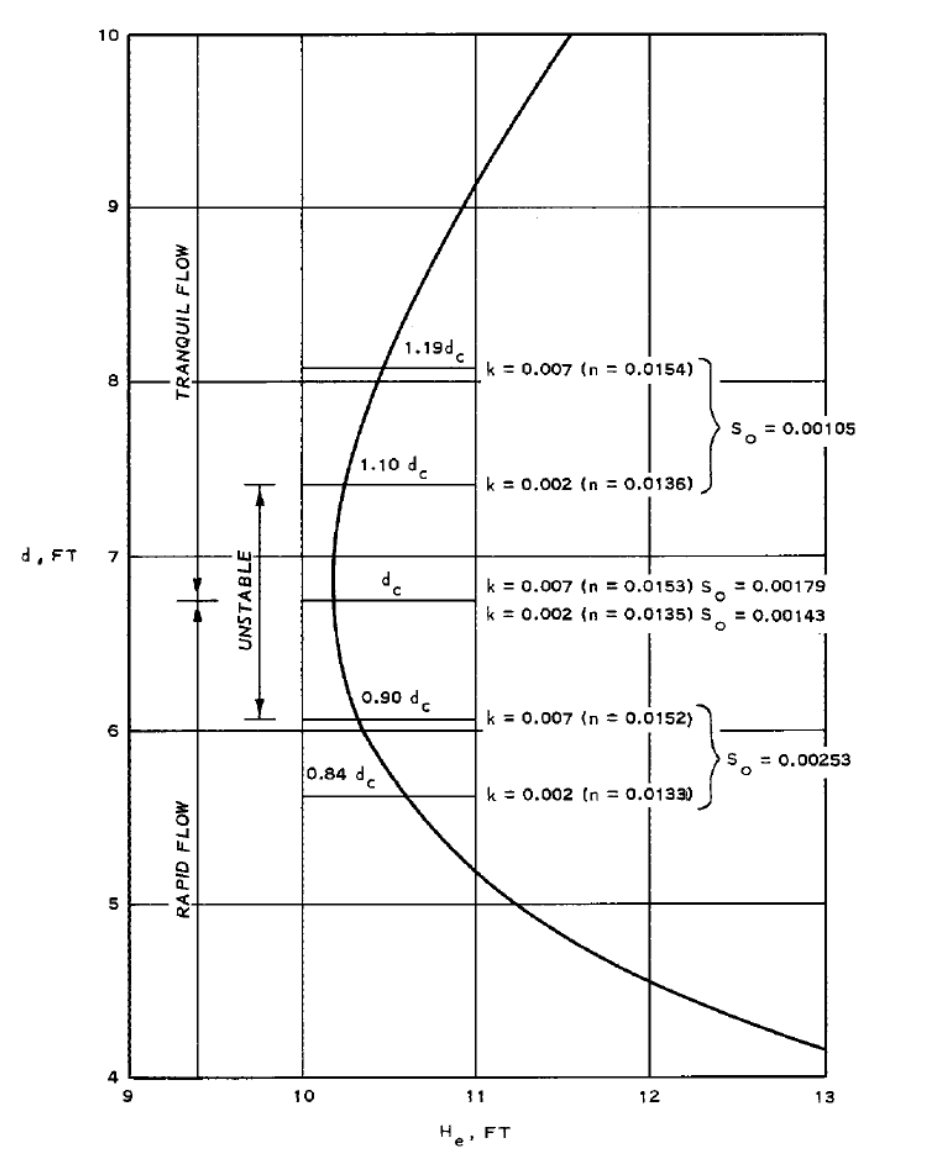
Tranquil vs. Rapid Flow

- Distinction centered around specific energy and critical depth

$$H_e = d + \alpha \frac{V^2}{2g}$$

$$d_c = \left(\frac{q^2}{g} \right)^{1/3}$$

Depth vs. Specific Energy



NOTE: q = 100 CFS/FT

$H_e = d + \alpha V^2/2g$

k = EQUIVALENT ROUGHNESS HEIGHT, FT

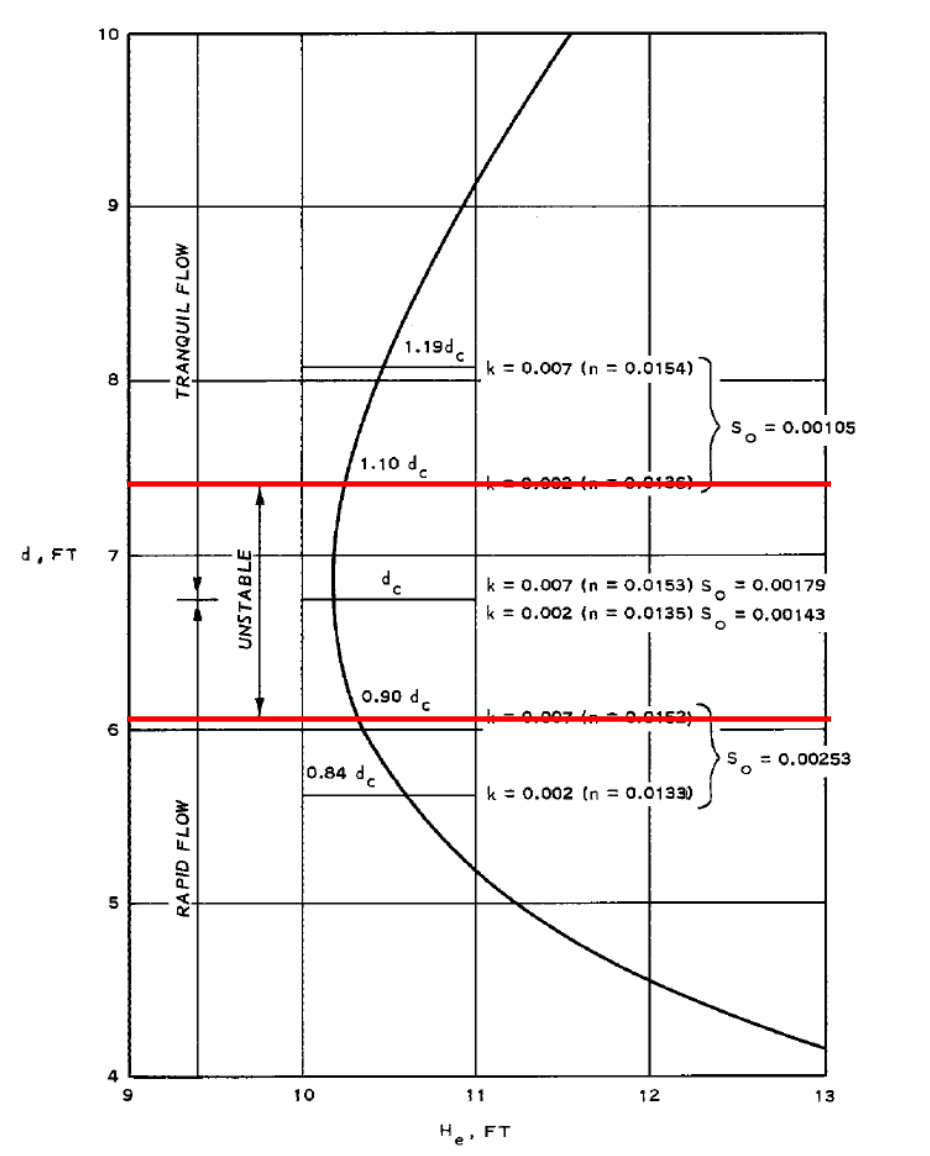
n = MANNING'S n

S_o = INVERT SLOPE

d = DEPTH

d_c = CRITICAL DEPTHS

Depth vs. Specific Energy

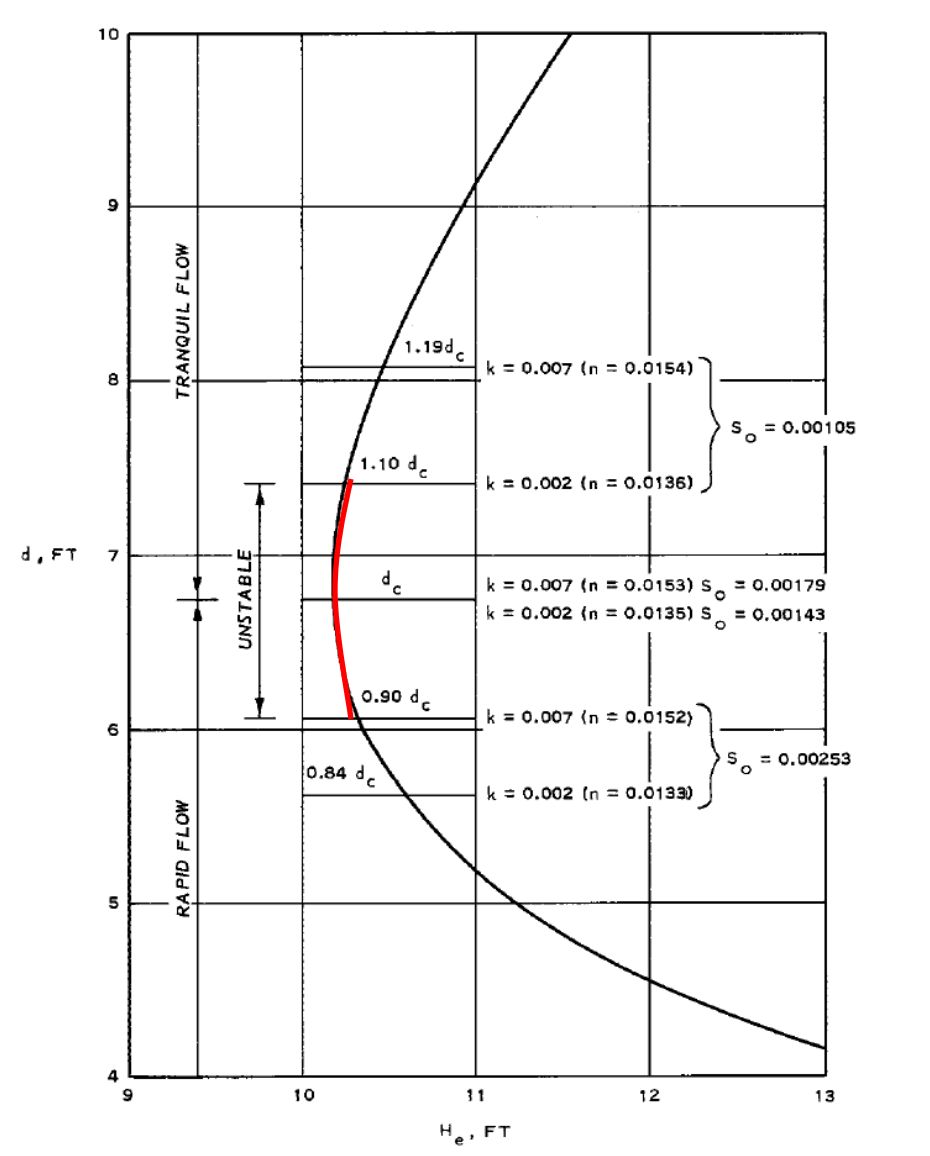


Tranquil Flow d > 1.1dc
or Fr < 0.86

Rapid Flow d < 0.9dc
or Fr > 1.13

NOTE: $q = 100$ CFS/FT
$H_e = d + \alpha V^2/2g$
k = EQUIVALENT ROUGHNESS HEIGHT, FT
n = MANNING'S n
S_o = INVERT SLOPE
d = DEPTH
d_c = CRITICAL DEPTHS

Depth vs. Specific Energy



If in instability region:

Evaluate both high and low resistance values

Adjust slope (or k) to design in proper zone

NOTE: q = 100 CFS/FT

$H_e = d + \alpha V^2/2g$

k = EQUIVALENT ROUGHNESS HEIGHT, FT

n = MANNING'S n

S_o = INVERT SLOPE

d = DEPTH

d_c = CRITICAL DEPTHS

Pulsating Rapid Flow

- Occurs at Froude numbers much greater than 1
- Formation of slugs
- Typically on steep slopes with shallow depths
- What is effect?

Pulsating Rapid Flow

- Need to determine roughness
- Concept of limiting Froude number
- Rather difficult to quantify

Roughness

$$\frac{0.0463R^{1/6}}{n} = 4.04 - \log_{10}\left(\frac{F}{F_s}\right)^{2/3}$$

EQ. 2-10

Where:

R = Hydraulic radius

n = Manning's n

F = Froude number of flow

Fs = Limiting Froude number

Limiting Froude Number

$$F_s = \frac{\xi}{\sqrt{g}\ \zeta^{3/2}\ (1 + Z\zeta)}$$

EQ. 2-11

Where:

Fs = Limiting Froude number

g = Gravity

ξ = Flow function

ζ = depth/width ratio

Z = Channel side slope

Limiting Froude Number

$$\xi = \frac{Q}{b^{5/2}}$$

$$\zeta = \frac{d}{b}$$

Where:

ξ = flow function

ζ = depth/width ratio

d = flow depth

b = bottom width

Pulsating Rapid Flow

Q = 1000
b = 10
y = 3

z = 2
Fs = 2.12
??

z = 0
Fs = 3.4
??

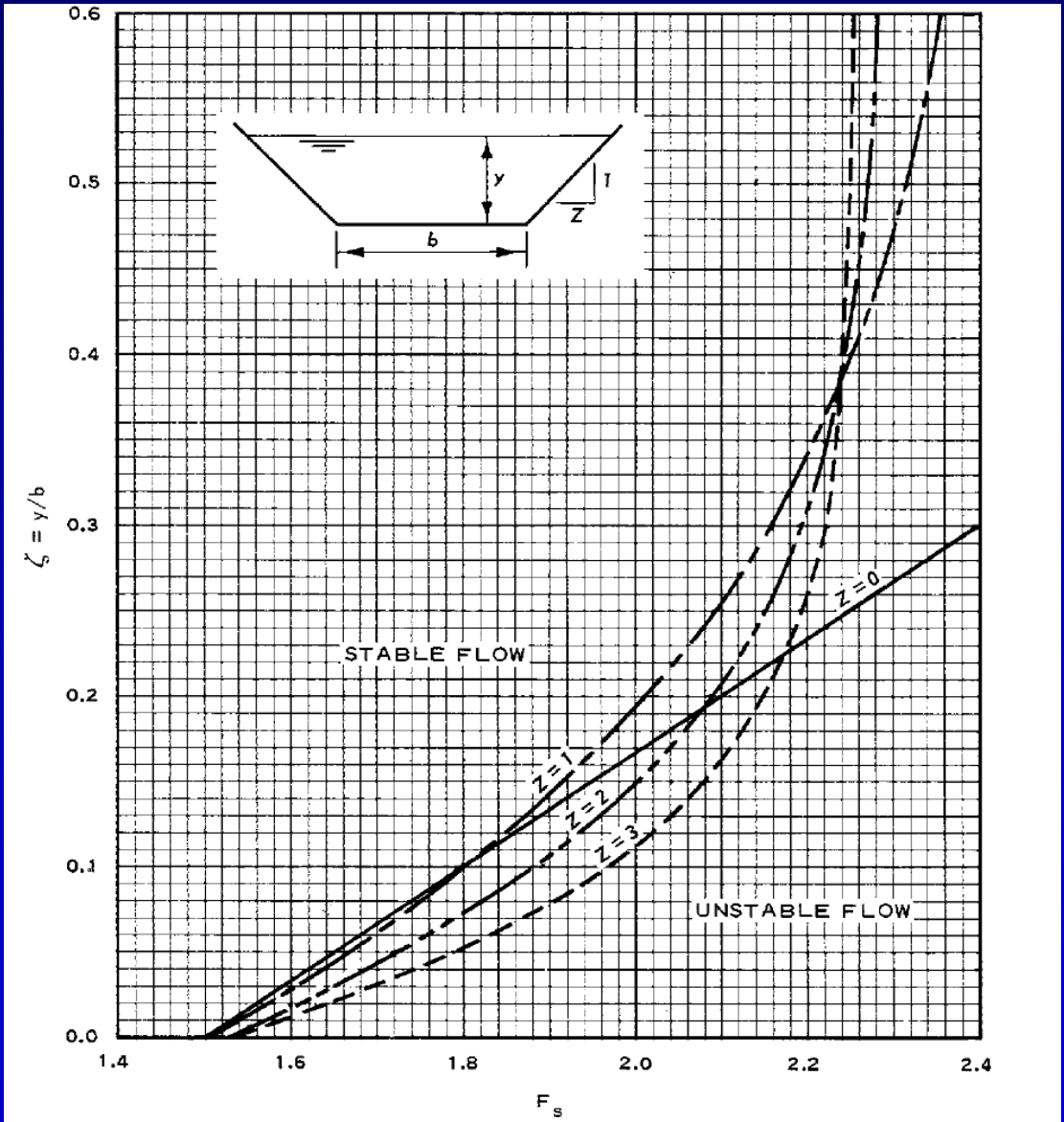


Plate 7

Varied Flow Profiles

- Prismatic Channels
  - Unvarying cross sections
  - Constant invert slope
  - Straight alignment
- How to determine profile
  - Direct integration
  - Direct step
  - Standard step

Start 11 SEP

Direct Step Method

$$\Delta x = \frac{E_2 - E_1}{S_o - S_f}$$

- Calculation steps
  - Determine starting depth
  - Assume second depth and compute parameters associated with that depth
  - Solve for step length

Direct Step Method

- Compute
  - Upstream direction
    - Subcritical flow
  - Downstream direction
    - Supercritical flow
- Utilize small changes of y so that:

$$S_f = \frac{n^2 V^2}{2.22 R^{4/3}}$$

$$S_f = \frac{f V^2}{8 g R}$$

Standard Step Method

$$WS_2 + \frac{\alpha_2 V_2^2}{2g} = WS_1 + \frac{\alpha_1 V_1^2}{2g} + h_e$$

$$h_e = L\bar{S}_f + C\left|\frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g}\right|$$

- Calculation steps
  - Assume WS_2 at upstream location
  - Compute associated velocity head
  - Solve for Sf
  - Solve for WS_2
  - Compare with assumed WS_2
  - Iterate as needed

Comparison

Comparison between direct step and standard step methods.			
No.	Characteristic	Direct step method	Standard step method
1	Ease of computation	Easy (hours)	Difficult (months)
2	Type of cross section	prismatic	any (prismatic or nonprismatic)
3	Independent variable	flow depth	length of channel
4	Dependent variable	length of channel	flow depth
5	Calculation advances ->	directly	by iteration (trial and error)
6	Accuracy increases with ->	a smaller flow depth increment	lesser cross-sectional variability
7	Type of cross-section input	one typical cross section (prismatic)	many cross sections (nonprismatic)
8	Data needs	minimal	extensive
9	Tools	spread sheet (or programming)	HEC-RAS (or programing)
10	Reliability	Answer always possible	Answer sometimes not possible (depends on the type of cross-sectional data)

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Flow Through Bridges

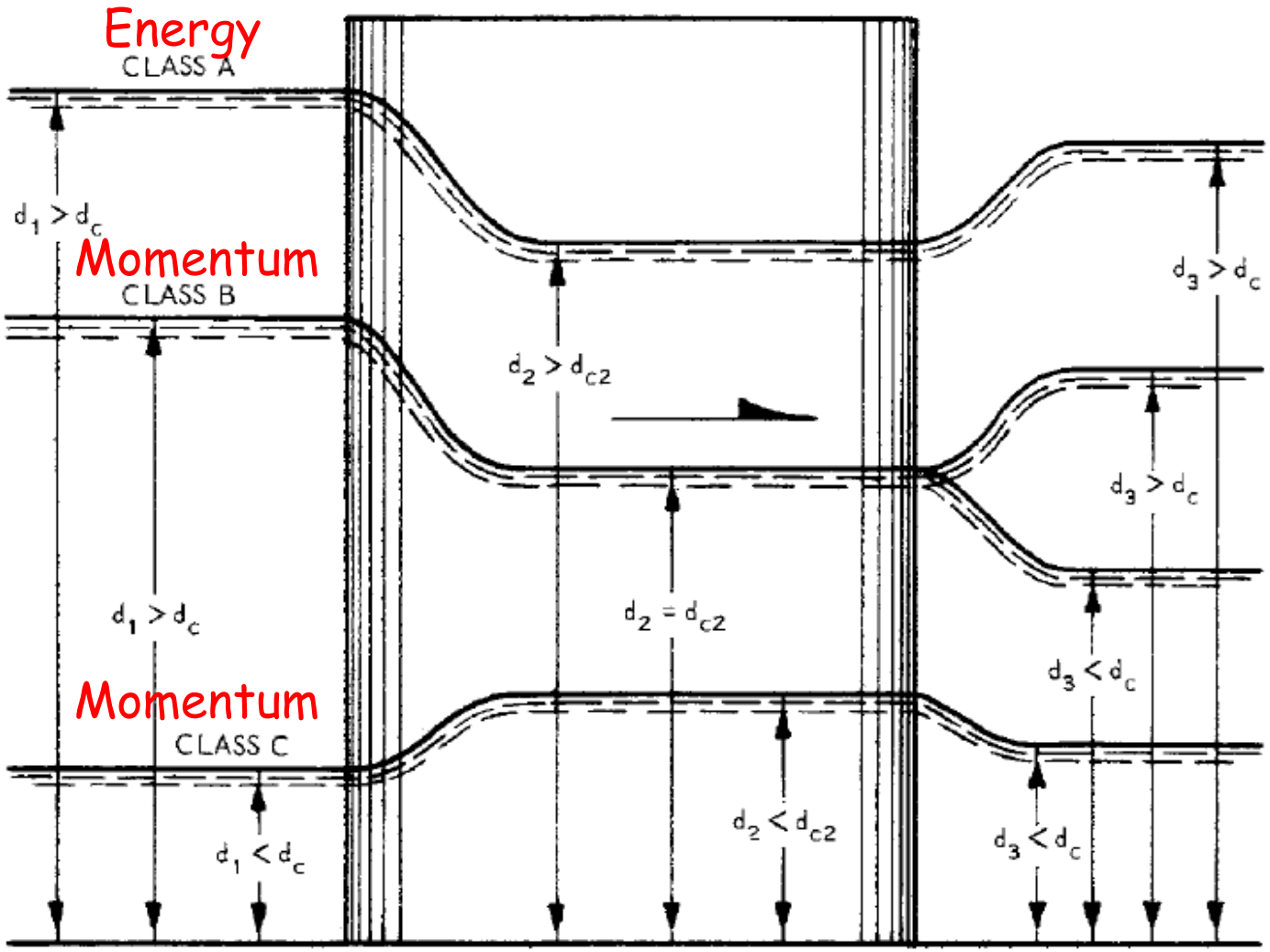
- Energy losses
- Turbulence
- Water surface elevations

Abutment Losses

- Rapid flow
  - Stay out of channel !
- Tranquil flow
  - Flow depth between abutments is > yc
  - Preliminary layout
    - 1.0 for expansion loss
    - 0.5 for contraction loss
- Bradley (1978) – Design charts

Pier Losses

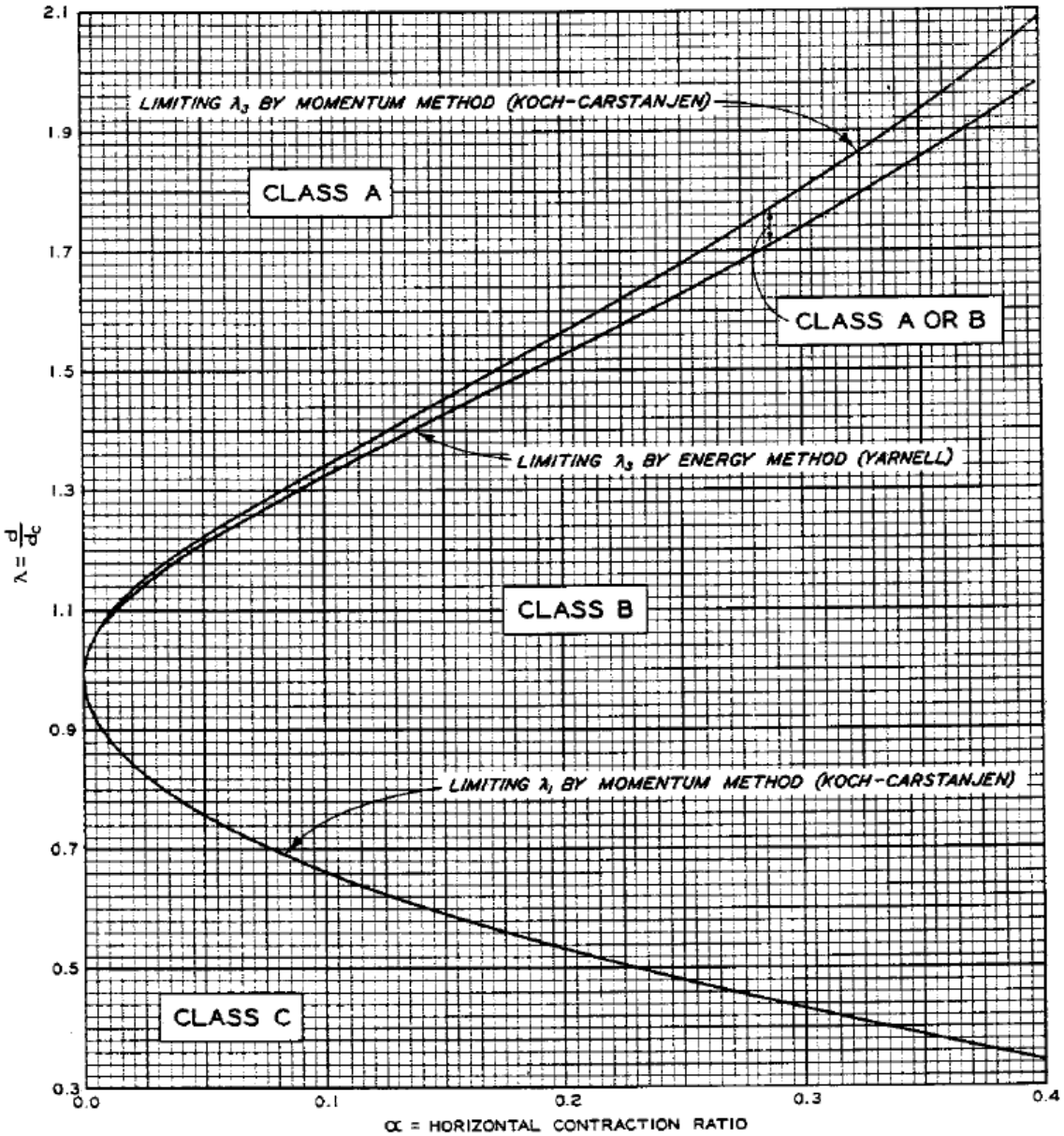
- Three conditions (Chow 1959)
  - Class A
  - Class B
  - Class C



ELEVATION

Plate 10

Pier Losses



EQUATIONS FOR LIMITING λ

λ_3 – ENERGY METHOD (YARNELL) (CHOW 1959)

$$\alpha = 1 - \left[\frac{3\lambda_3^2}{2\lambda_3^3+1}\right]^{3/2}$$

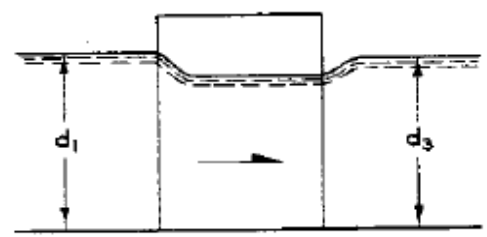
λ_3 – MOMENTUM METHOD (KOCH 1926)

$$\alpha = 1 - \left[\frac{3\lambda_3}{(1-\alpha)\lambda_3^3+2}\right]^{3}$$

λ_1 – MOMENTUM METHOD (KOCH 1926)

$$\alpha = 1 - \left[\frac{3\lambda_1}{\lambda_1^3+2}\right]^{3/4}$$

NOTE: $\lambda_1 = d_1/d_c$
$\lambda_3 = d_3/d_c$
d_1 = UPSTREAM WATER DEPTH
d_3 = DOWNSTREAM WATER DEPTH
d_c = CRITICAL DEPTH WITHIN THE UNOBSTRUCTED CHANNEL SECTION
α = HORIZONTAL CONTRACTION RATIO (Σ PIER WIDTHS ÷ CHANNEL WIDTH)
d = DEPTH WITHOUT BRIDGE PIERS



DEFINITION SKETCH

Plate 11

Energy Method

$$E1 = E2 + hl_{1-2}$$

$$E2 = E3 = hl_{2-3}$$

Where:

E1 = Energy at Section 1

E2 = Energy at Section 2

E3 = Energy at Section 3

hl_{1-2} = Losses between sections 1 and 2

hl_{2-3} = losses between sections 2 and 3

Momentum Method

$$M = \beta \left(\frac{\gamma Q V}{g} \right)$$

Where:

M = momentum per unit time (lbs)

β = momentum correction factor (1.0)

Q = discharge (cfs)

V = average channel velocity (fps)

Momentum Method

$$m_1 - m_p + \frac{\gamma Q^2}{gA_1^2}(A_1 - A_p) = m_2 + \frac{\gamma Q^2}{gA_2}$$

$$= m_3 - m_p + \frac{\gamma Q^2}{gA_3}$$

Where:

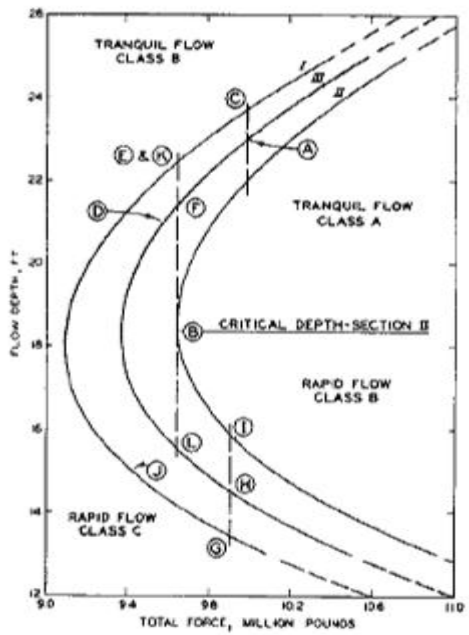
m_1 = hydrostatic force in section 1

m_p = hydrostatic force on pier ends

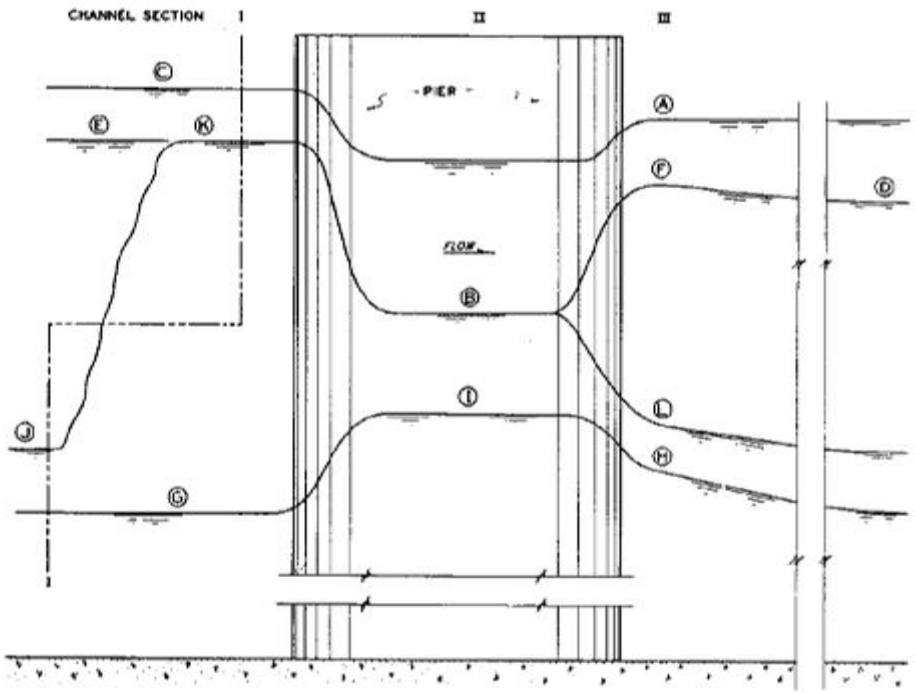
m_2 = hydrostatic force in section 2

m_3 = hydrostatic force in section 3

Pier Losses



d. FORCE CURVES FOR CHANNEL SECTIONS I, II, AND III



b. FLOW PROFILES

$$I = m_1 - m_{P_1} + \frac{vQ^2}{gA_1^2}(A_1 - A_{P_1})$$

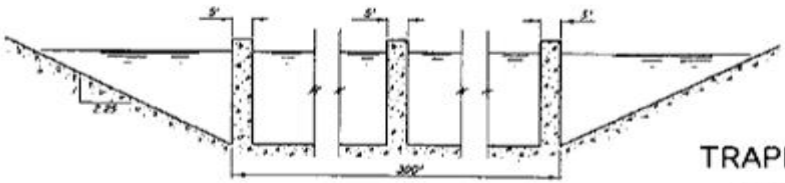
$$II = m_2 + \frac{vQ^2}{gA_2}$$

$$III = m_3 - m_{P_3} + \frac{vQ^2}{gA_3}$$

$$\frac{vQ^2}{gA} = \text{MOMENTUM FORCE}$$

$$m = v\bar{y}A$$

c. FORCE EQUATIONS FOR CHANNEL SECTIONS I, II, AND III



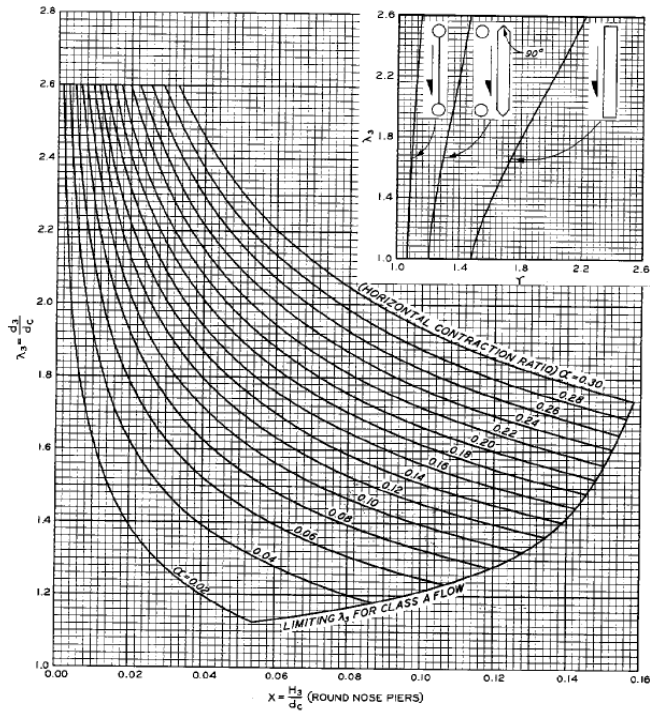
d. CHANNEL SECTION II
Q = 140,000 CFS

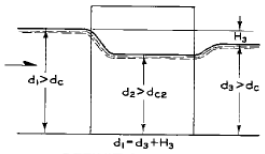
TRAPEZOIDAL SECTION
MOMENTUM METHOD
EXAMPLE CURVES

Plate 12

Analysis Tools

Plate 14





DEFINITION SKETCH

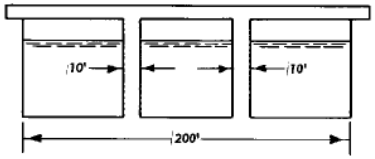
NOTE: d_c = CRITICAL DEPTH WITHIN THE UNOBSTRUCTED CHANNEL SECTION
d_{c2} = CRITICAL DEPTH WITHIN THE PIER SECTION
$H_3 = Xd_c$ (ROUND NOSE PIERS)
$H_3 = Xd_cY$ (INDICATED SHAPES)

RECTANGULAR SECTION
ENERGY METHOD
CLASS A FLOW

Plate 15

GIVEN:

Rectangular channel section
Round nose piers
Channel discharge (Q) = 40,000 cfs
Channel width (W_c) = 200 ft
Total pier width (W_p) = 20 ft
Depth without bridge piers (d) = 14.3 ft



COMPUTE:

1. Horizontal contraction ratio (α)

$$\alpha = \frac{W_p}{W_c} = \frac{20}{200} = 0.10$$

2. Discharge (q) per ft of channel width

$$q = \frac{Q}{W_c} = \frac{40{,}000}{200} = 200 \text{ cfs}$$

3. Critical depth (d_c) in unobstructed channel
   From Chart 610-8, d_c = 10.8 ft
   for q = 200 cfs.

4. $\lambda = d/d_c = 14.3/10.8$
   $= 1.324$

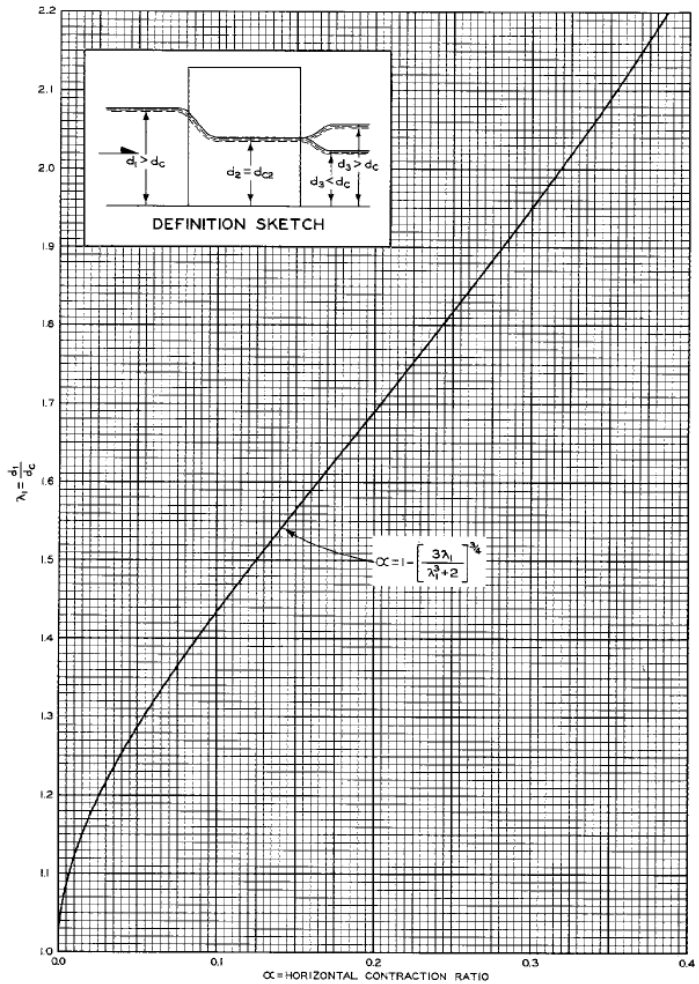
5. Flow classification
   On Plate 11, intersection of α = 0.10 and λ = 1.324 is in zone marked Class A or B.

6. Upstream depth (d_1)
   a. Class A flow - Energy Method
      $d_1 = d_3 + H_3$ (Plate 14)
      $H_3 = Xd_c$
      X = 0.127 for α = 0.10 and $\lambda_3 = \lambda = 1.324$
      $H_3 = 0.127 \times 10.8 = 1.37$
      $d_1 = 14.3 + 1.37 = 15.67$ ft
   b. Class B flow - Momentum Method
      $d_1 = \lambda_1 d_c$ (Plate 16)
      $\lambda_1 = 1.435$ for α = 0.10
      $d_1 = 1.435 \times 10.8 = 15.50$ ft
   c. Class B flow - Energy Method
      $d_1 = \lambda_1 d_c$ (Plate 17)
      $\lambda_1 = 1.460$ for α = 0.10
      $d_1 = 1.460 \times 10.8 = 15.77$ ft

RECTANGULAR SECTION
ENERGY METHOD
SAMPLE COMPUTATION

Analysis Tools

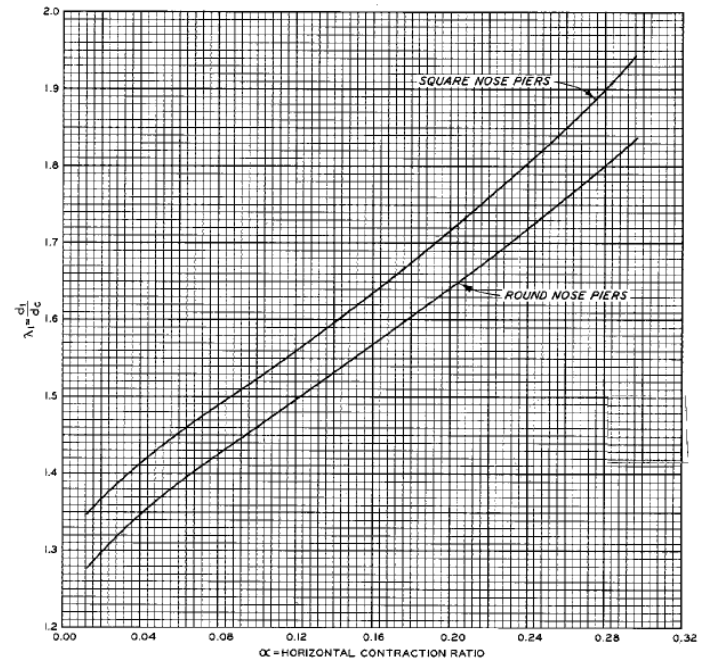
Plate 16



NOTE: $\lambda_1 = d_1/d_c$
d_1 = UPSTREAM WATER DEPTH
d_c = CRITICAL DEPTH WITHIN THE UNOBSTRUCTED CHANNEL SECTION
d_{c2} = CRITICAL DEPTH WITHIN THE PIER SECTION
α = HORIZONTAL CONTRACTION RATIO

RECTANGULAR SECTION
MOMENTUM METHOD
CLASS B FLOW

Plate 17

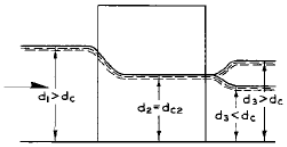


EQUATIONS

$$\frac{1}{(1-\alpha)^{2/3}} = \frac{1}{3\lambda_L^2} + \frac{2\lambda_L}{3}$$

$$\lambda_1 = \lambda_L + \frac{0.5 + K_B(5.5\,\alpha^3 + 0.08)}{2\lambda_1^2}$$

NOTE: $\lambda_1 = d_1/d_c$
$\lambda_3 = d_3/d_c$
λ_L = LIMITING λ_3 BY ENERGY METHOD
d_1 = UPSTREAM WATER DEPTH
d_3 = DOWNSTREAM WATER DEPTH
d_c = CRITICAL DEPTH WITHIN THE UNOBSTRUCTED CHANNEL SECTION
d_{c2} = CRITICAL DEPTH WITHIN THE PIER SECTION
α = HORIZONTAL CONTRACTION RATIO
K_B = YARNELL PIER-SHAPE COEFFICIENT (1.0 FOR ROUND NOSE) (5.0 FOR SQUARE NOSE)



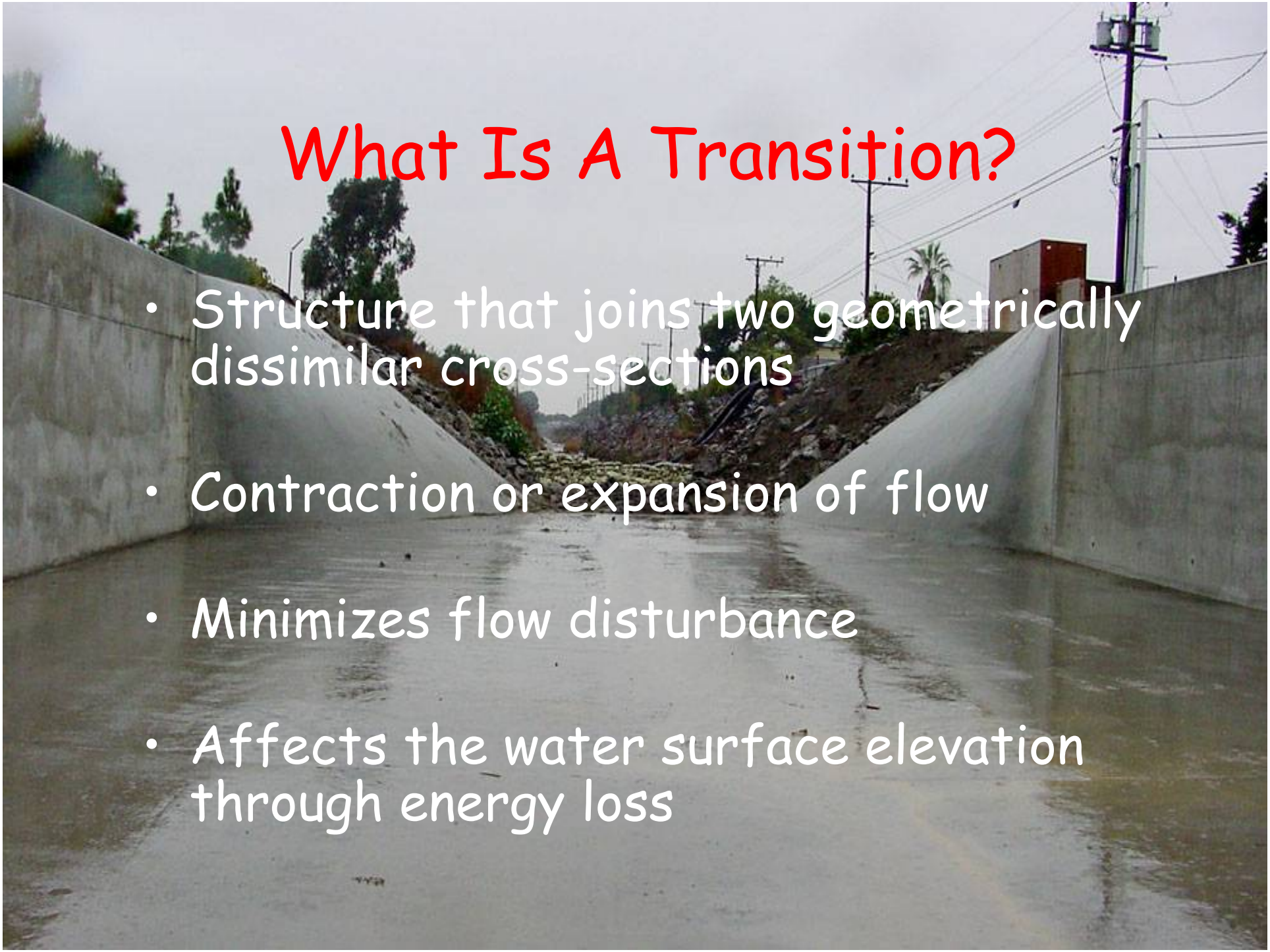
DEFINITION SKETCH

RECTANGULAR SECTION
ENERGY METHOD
CLASS B FLOW

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Start 22 SEP

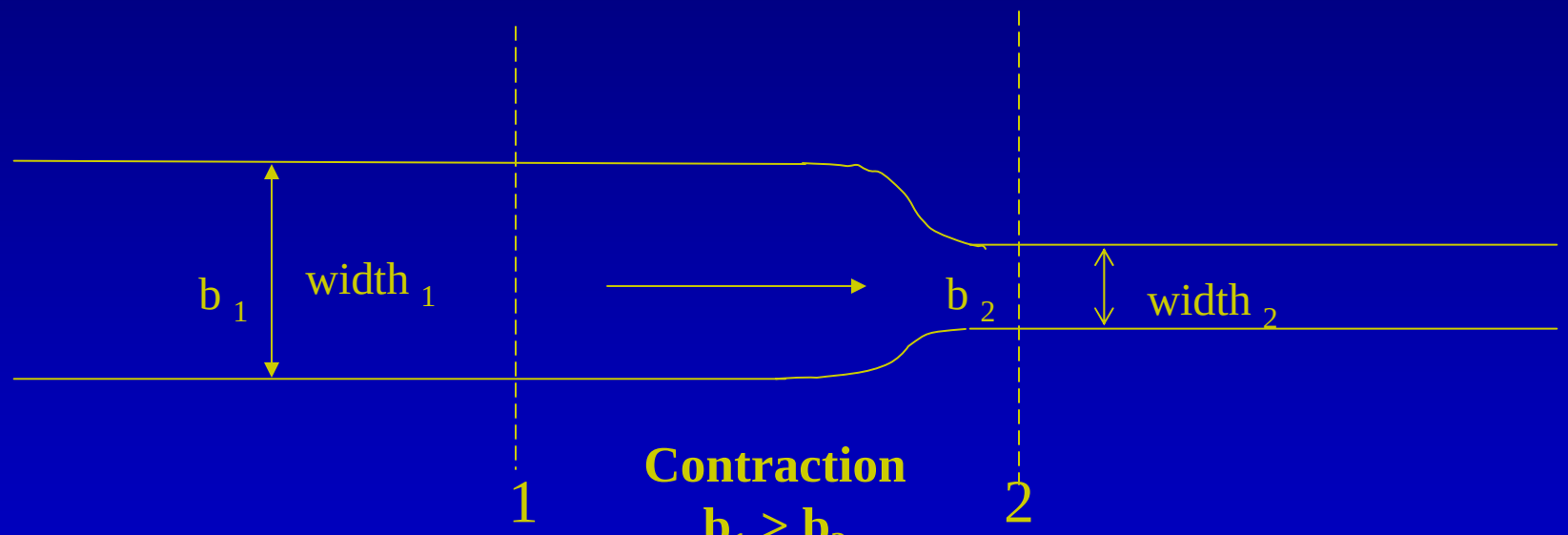


What Is A Transition?

- Structure that joins two geometrically dissimilar cross-sections
- Contraction or expansion of flow
- Minimizes flow disturbance
- Affects the water surface elevation through energy loss

Transitions
Contraction & Expansion

Plan View



Contraction
$\mathbf{b_1 > b_2}$

Expansion
$\mathbf{b_1 < b_2}$

Application of Transition Structures

- Approach to bridge and culvert crossings
- A placeholder between existing and future improvements
- To create a choke in the channel

Analysis of Transition Structures

Principles of Open Channel Hydraulics

Three Governing Principles

- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy

$$E_1 = E_2 + Losses$$

Energy Equation (Bernoulli's)

elevation head

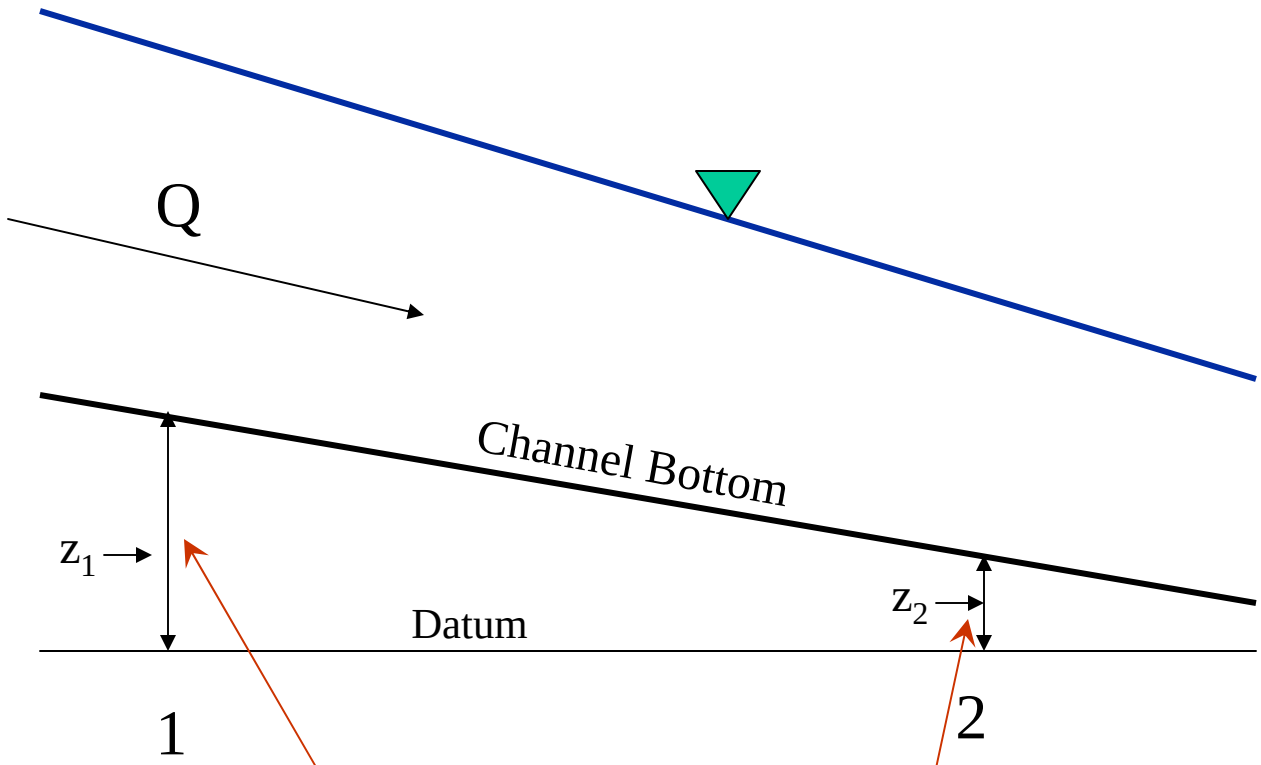
$$\frac{v_1^2}{2g} + y_1 + z_1 + \frac{p_1}{\gamma} = \frac{v_2^2}{2g} + y_2 + z_2 + \frac{p_2}{\gamma} + h_{L,1-2}$$

velocity head

pressure head

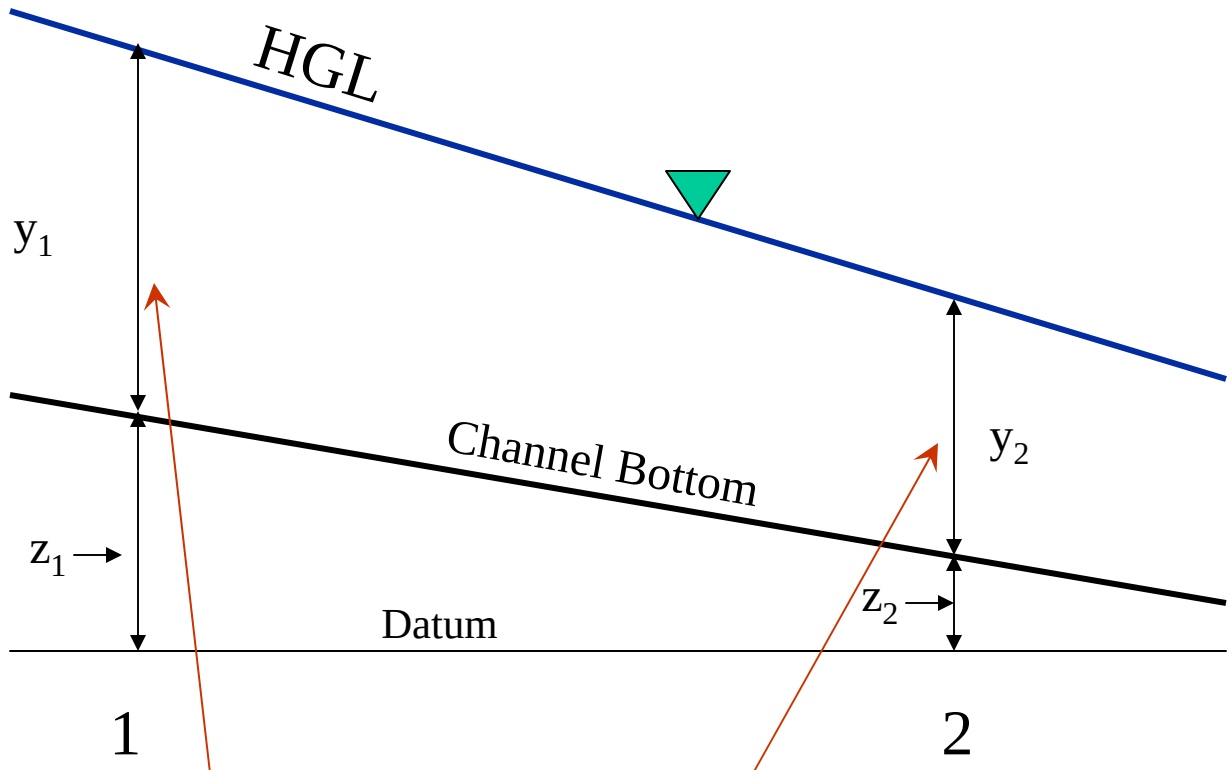
Energy loss between sections 1 and 2

Open Channel Energy Equation



$$\frac{v_1^2}{2g} + y_1 + z_1 = \frac{v_2^2}{2g} + y_2 + z_2 + h_{L,1-2}$$

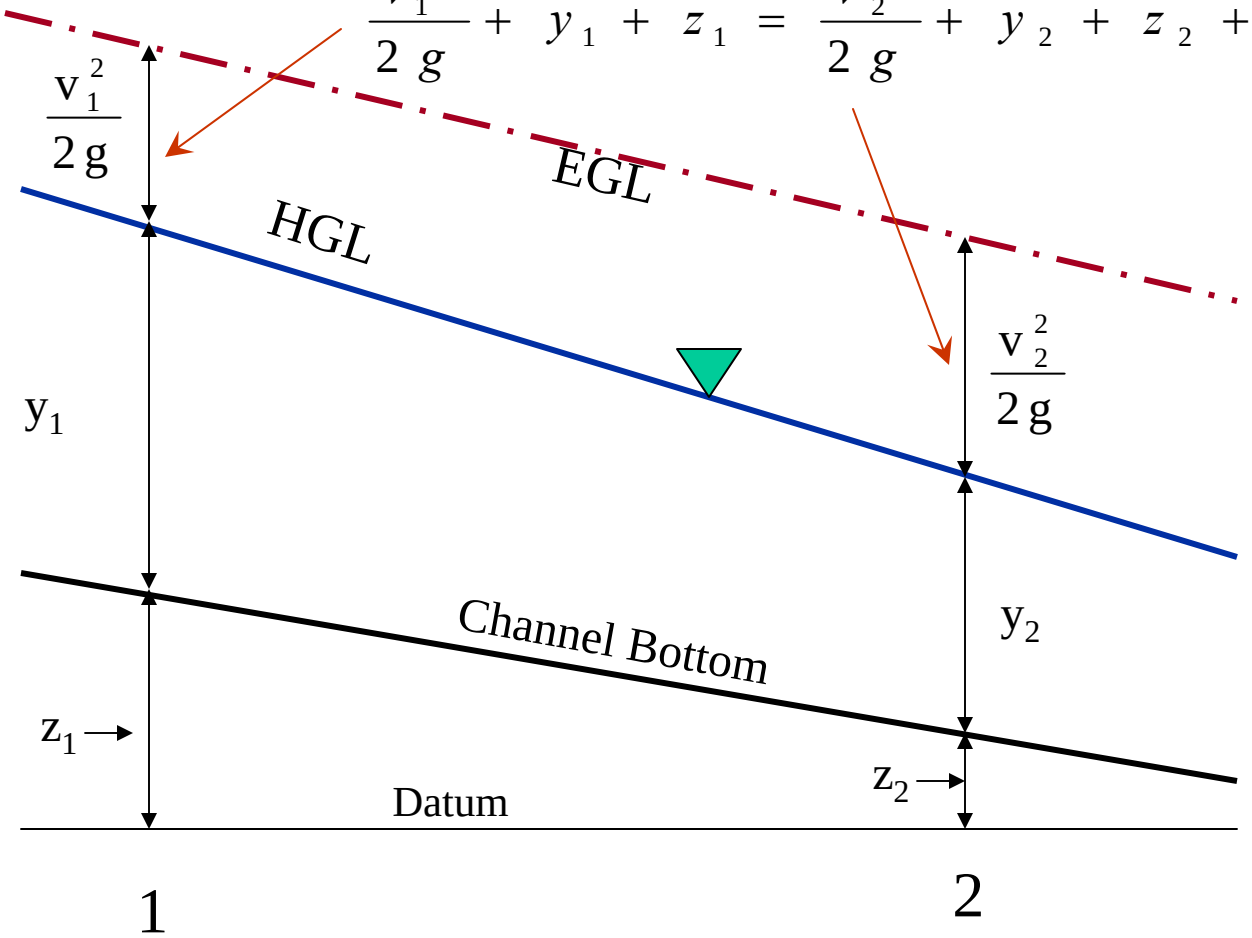
Open Channel Energy Equation



$$\frac{v_1^2}{2g} + y_1 + z_1 = \frac{v_2^2}{2g} + y_2 + z_2 + h_{L,1-2}$$

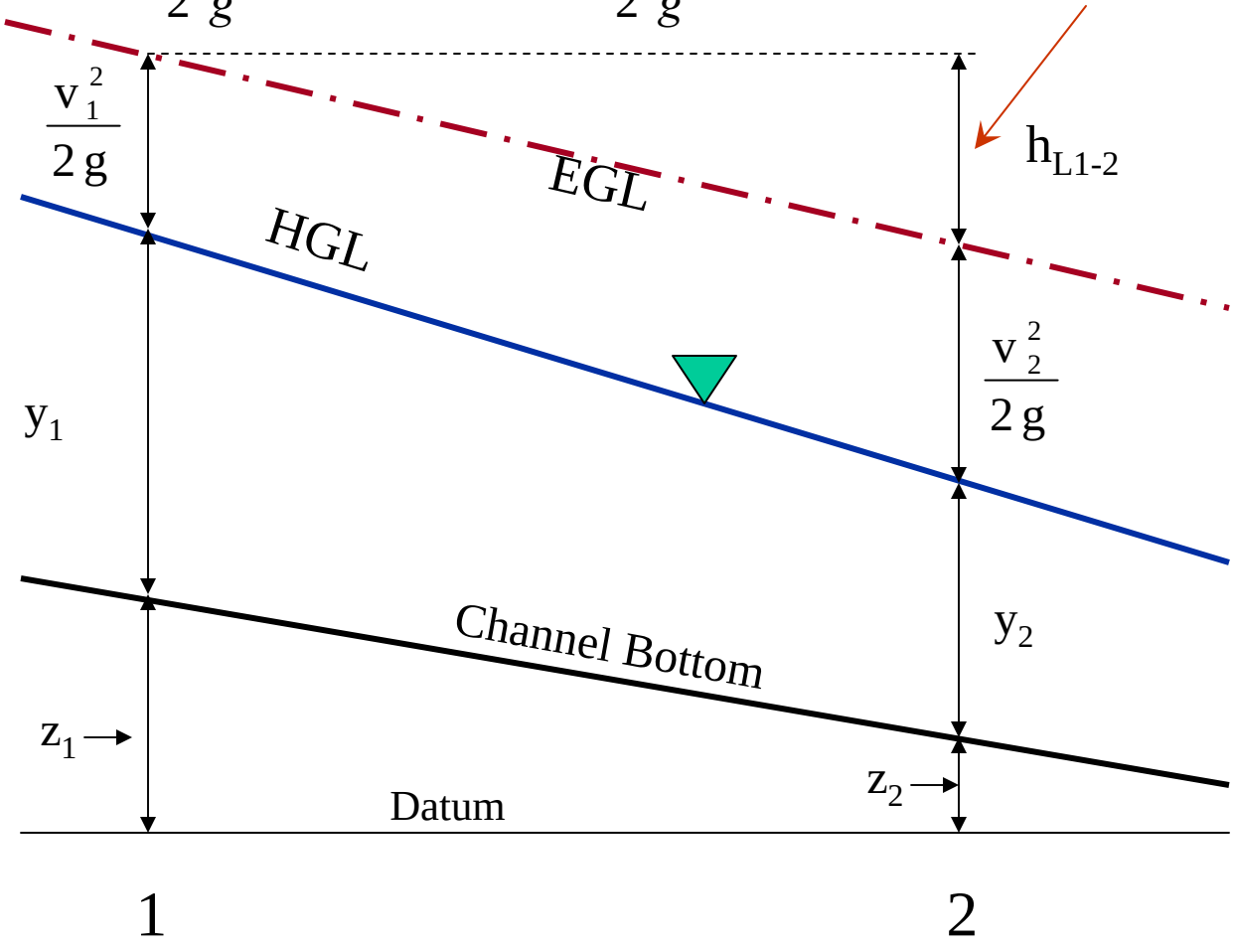
Open Channel Energy Equation

$$\frac{v_1^2}{2g} + y_1 + z_1 = \frac{v_2^2}{2g} + y_2 + z_2 + h_{L,1-2}$$



Open Channel Energy Equation

$$\frac{v_1^2}{2g} + y_1 + z_1 = \frac{v_2^2}{2g} + y_2 + z_2 + h_{L,1-2}$$



Open Channel Energy Equation

- Head Losses

$$h_{L1-2} = Major\ Losses + Minor\ Losses$$

Friction between fluid and it's flow boundary

Change in velocity or change in flow

Open Channel Energy Equation

- Losses due to Transitions are considered Minor Losses

$$h_{L1-2} = Major\ Losses + Minor\ Losses$$

Change in velocity or change in flow

Open Channel Energy Equation

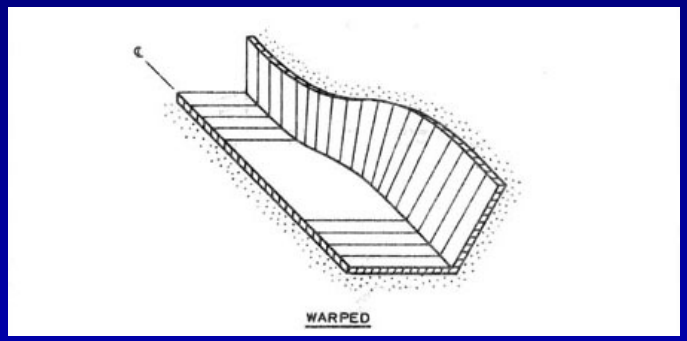
- General Equation for Minor Losses

$$h_{Lm} = \frac{K_m V^2}{2g}$$

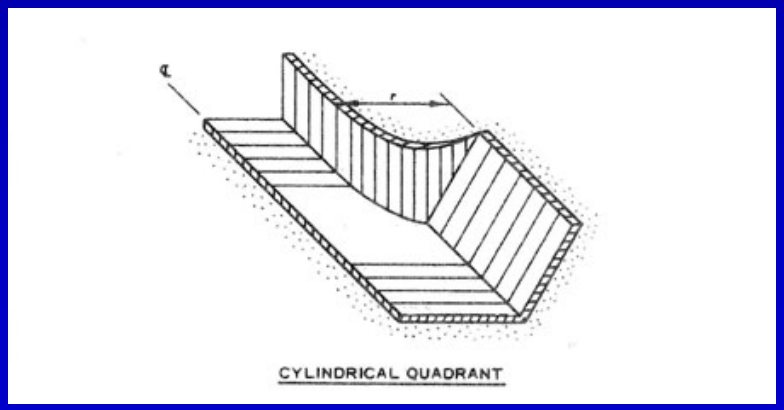
Transition Types

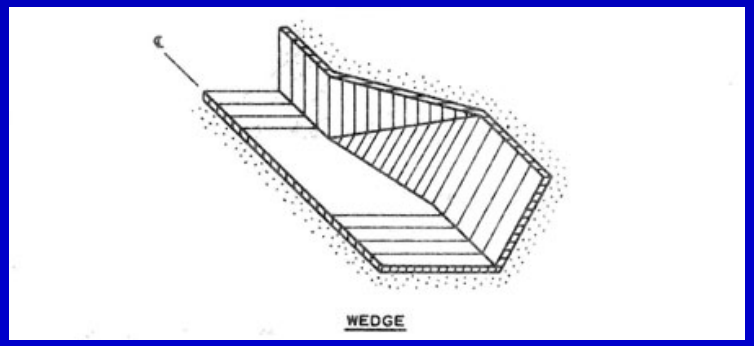
- For connecting trapezoidal and rectangular sections
  - Cylindrical quadrant
  - Warped
  - Wedge
- Rectangular sections
  - Straight-line
- Abrupt/square

Types of Transitions



- Warped
- Straight Line
- Cylindrical Quadrant
- Wedge (longer or shorter than warped?)
- Abrupt (square)





Abrupt Transition

Channel Transitions

- Subcritical Flow
  - Each type can be used in either direction
  - Cylindrical quadrant used for:
    - Expansions from rectangular to trapezoidal
    - Contractions from trapezoidal to rectangular
  - Straight-line transition or quadrant for rectangular channels

Channel Transitions

- Supercritical Flow
  - Cylindrical quadrant
    - Subcritical in trap section to supercritical in rectangular
  - Straight-line for contractions in rectangular sections
  - Special shape for expansions in rectangular channel

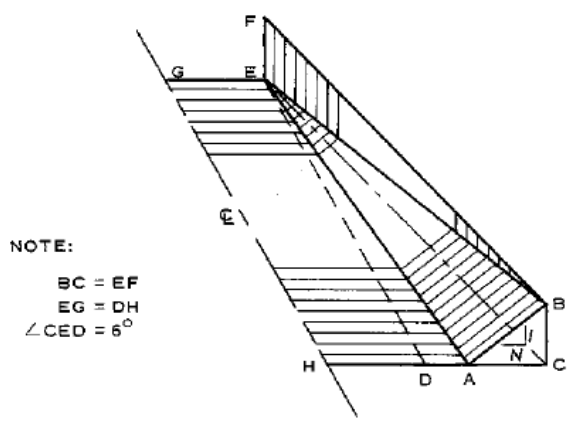
Transition Design

- Subcritical Flow
  - Rectangular to trapezoidal
    - Wedge Type
    - Plate 20

TABLE OF GEOMETRIC VALUES

N	AD/BC	DE/BC	AE/BC	CE/BC	BE/BC	ANG AEC[a]	ANG BEC[a]
2.0	0.4000	22.8344	22.8379	22.9602	22.9820	4.9964	2.4939
2.0	0.6000	24.7373	24.7446	24.8735	24.8936	4.6106	2.3022
2.0	0.8000	26.6402	26.6522	26.7869	26.8056	4.2799	2.1380
2.0	1.0000	28.5438	28.5605	28.7002	28.7177	3.9935	1.9955
2.0	1.2000	30.4459	30.4695	30.6136	30.6299	3.7429	1.8709
2.0	1.4000	32.3488	32.3790	32.5269	32.5423	3.5219	1.7609
2.0	1.6000	34.2516	34.2890	34.4403	34.4548	3.3255	1.6632
2.5	0.4000	27.5916	27.5945	27.7436	27.7616	5.1694	2.0643
2.5	0.6000	29.4945	29.5006	29.6569	29.6738	4.8346	1.9312
2.5	0.8000	31.3973	31.4075	31.5703	31.5861	4.5404	1.8143
2.5	1.0000	33.3002	33.3152	33.4836	33.4986	4.2799	1.7106
2.5	1.2000	35.2031	35.2235	35.3970	35.4111	4.0477	1.6182
2.5	1.4000	37.1059	37.1323	37.3103	37.3237	3.8393	1.5353
2.5	1.6000	39.0088	39.0416	39.2237	39.2364	3.6513	1.4604
3.0	0.4000	32.3488	32.3512	32.5269	32.5423	5.2916	1.7609
3.0	0.6000	34.2516	34.2569	34.4403	34.4548	4.9964	1.6632
3.0	0.8000	36.1545	36.1633	36.3536	36.3674	4.7324	1.5757
3.0	1.0000	38.0574	38.0705	38.2670	38.2801	4.4948	1.4969
3.0	1.2000	39.9602	39.9783	40.1803	40.1928	4.2799	1.4257
3.0	1.4000	41.8631	41.8865	42.0937	42.1056	4.0846	1.3609
3.0	1.6000	43.7660	43.7952	44.0070	44.0184	3.9063	1.3017

[a] DEGREES



NOTE:

BC = EF
EG = DH
∠CED = 6°

DEFINITION SKETCH

Plate 20

Transition Design

- Subcritical Flow
  - Rectangular to trapezoidal
    - Plate 20
  - 6° maximum change in flow line
  - Water surface profiles determined by step computations
  - < 20 percent change in velocity between steps
  - Adjust to make water surface as straight as practicable

Transition Design

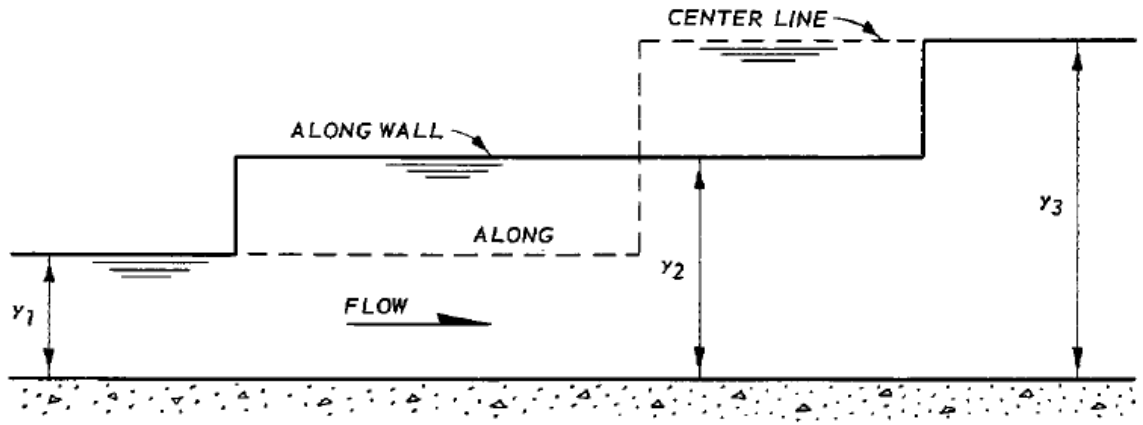
- Supercritical Flow
  - Between trapezoidal and rectangular
    - Typically incorporate wedge or straight-line transition
    - Need to minimize/contain standing waves
    - Rule of thumb:

Recommended Convergence and Divergence Rates

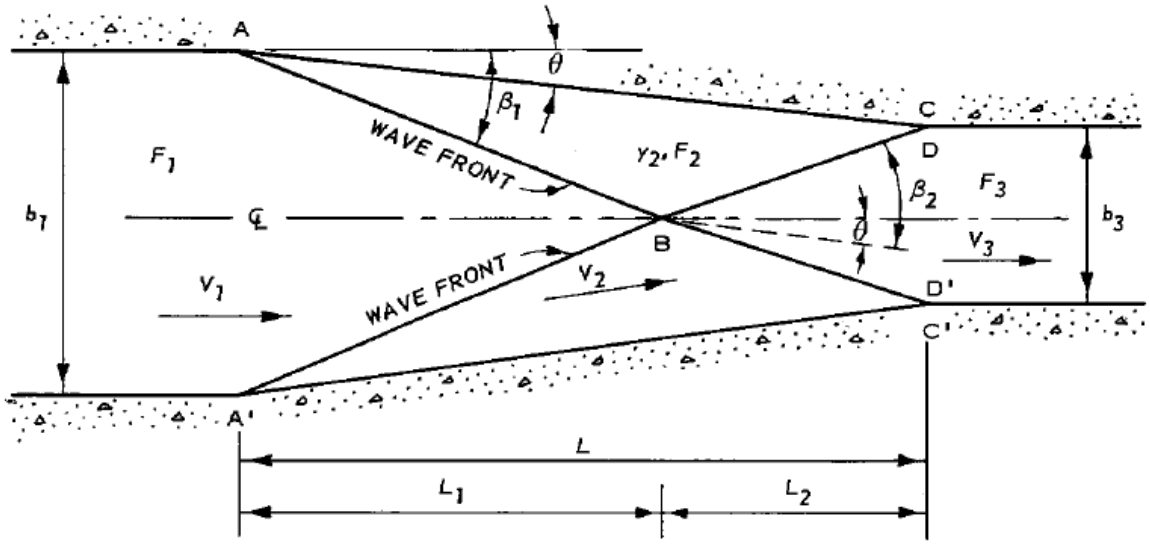
Mean channel velocity (fps)	Wall flare (horizontal to longitudinal)
1-10	1:10
15-30	1:15
30-40	1:20

Transition Design

- Supercritical Flow
  - Rectangular contractions:



a. SCHEMATIC PROFILE

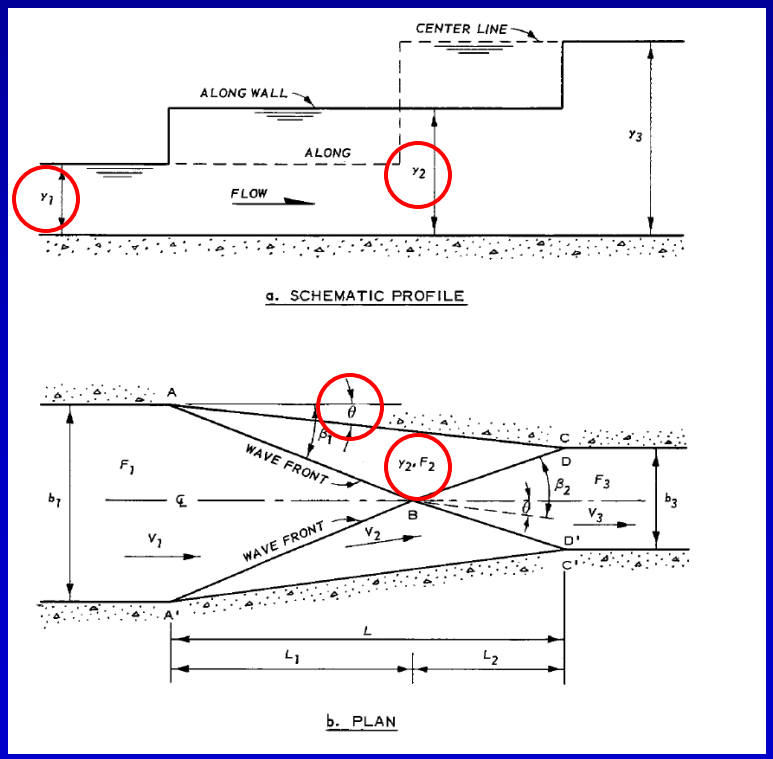


b. PLAN

Plate 21

Transition Design

- Supercritical Flow
  - Rectangular contractions:



$$\tan\theta = \frac{\tan\beta_1\left(\sqrt{1 + 8F_1^2\sin^2\beta_1} - 3\right)}{2\tan^2\beta_1 + \sqrt{1 + 8F_1^2\sin^2\beta_1} - 1}$$

EQ 2-22

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_1^2\sin^2\beta_1} - 1\right)$$

EQ 2-23

$$F_2^2 = \frac{y_1}{y_2}\left[F_1^2 - \frac{1}{2}\frac{y_1}{y_2}\left(\frac{y_2}{y_1} - 1\right)\left(\frac{y_2}{y_1} + 1\right)^2\right]$$

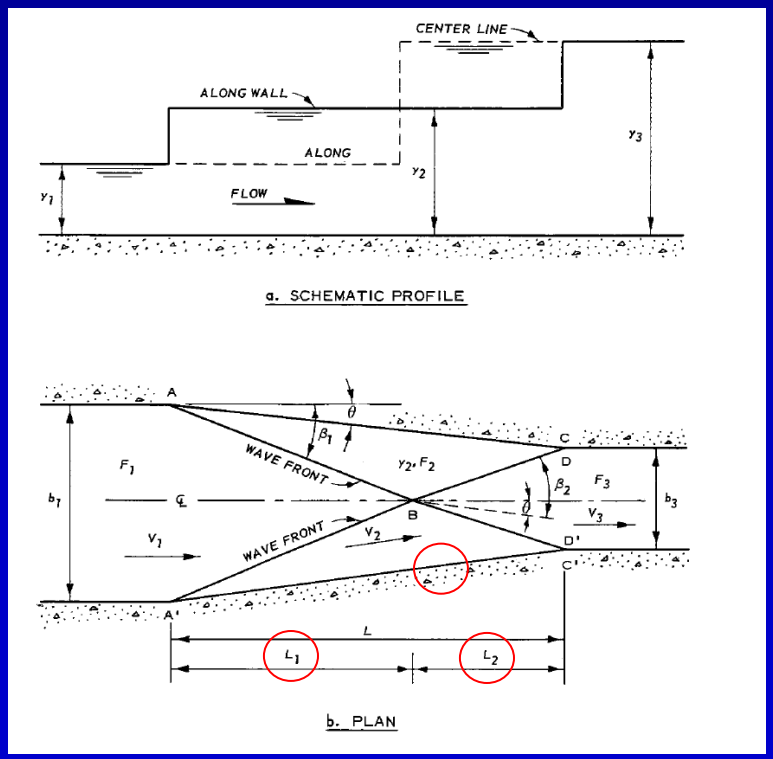
EQ 2-24

Transition Design

- Supercritical Flow
  - Rectangular contractions:

$$L = \frac{b_1 - b_3}{2 \tan \theta}$$

EQ 2-25



However, disturbances are minimized when $L = L_1 + L_2$ so..

$$L_1 = \frac{b_1}{2 \tan \beta_1}$$

EQ 2-26

$$L_2 = \frac{b_3}{2 \tan (\beta_2 - \theta)}$$

EQ 2-27

Transition Design

- Supercritical Flow
  - Rectangular contractions:
  - Correct design requires choosing a value of Θ so that $L = L_1 + L_2$
  - Either solve equations simultaneously or use Plate 22
  - Plate 23 is a "go-by"

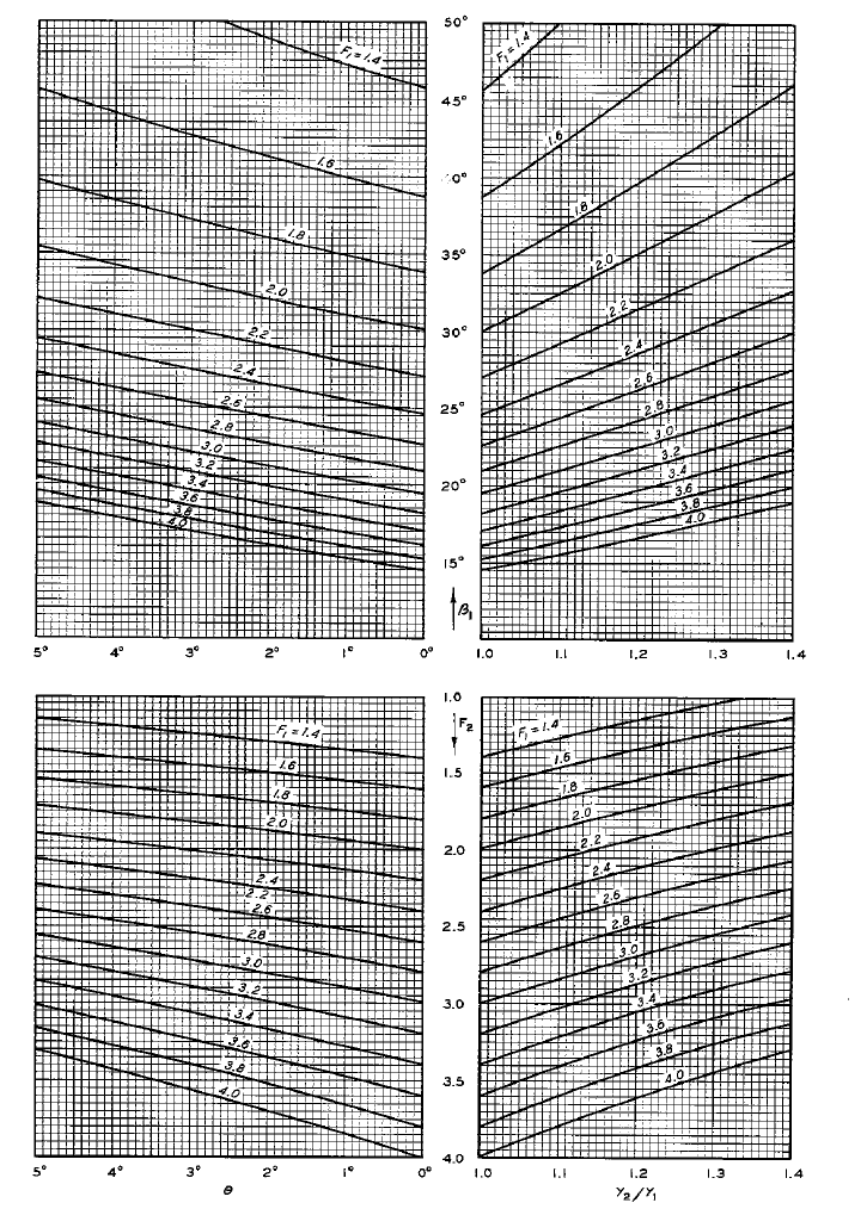


Plate 22

GIVEN

FROUDE NO. $F_1 = 3.5$

FLOW DEPTH $y_1 = 5$ FT

CHANNEL WIDTH $b_1 = 160$ FT

CHANNEL WIDTH $b_3 = 140$ FT

REQUIRED

CONVERGENCE ANGLE θ

FROUDE NO. F_3

FLOW DEPTH y_3

TRANSITION LENGTH $L = L_1 + L_2$

PROCEDURE

ASSUME VALUES OF θ AND BY REPETITIVE USE OF PLATE 22 SOLVE EQUATIONS 2-25, 2-26, AND 2-27 UNTIL $L = L_1 + L_2$. IF COL 6 IS GREATER THAN COL 9, CONTINUE COMPUTATION USING VALUE OF θ WITH F_2 IN SAME MANNER AS WAS DONE WITH F_1. TO COMPUTE COL 11 THROUGH 18, EACH SUBSCRIPT IN PLATE 22 IS ASSUMED TO BE INCREASED BY ONE UNIT.

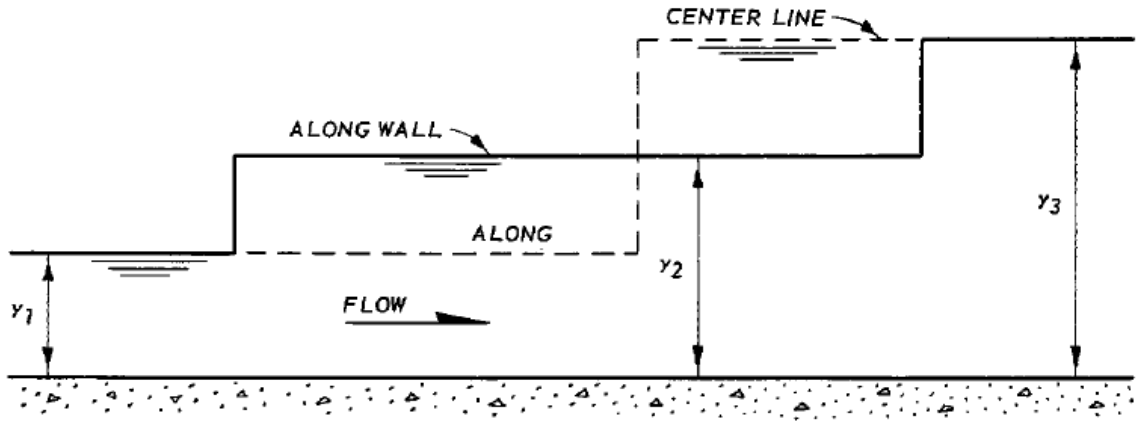
COMPUTATION

(1) θ	(2) $\frac{y_2}{y_1}$	(3) y_2, FT	(4) β_1, DEG	(5) TAN θ	(6) L, FT $= (b_1 - b_3) \div 2$ (TAN θ)
3.0	1.20	6.00	19.2	0.0524	191
2.0	1.12	5.60	18.3	0.0349	287
1.3	1.07	5.35	17.6	0.0227	441
1.2	1.06	5.30	17.5	0.0209	478

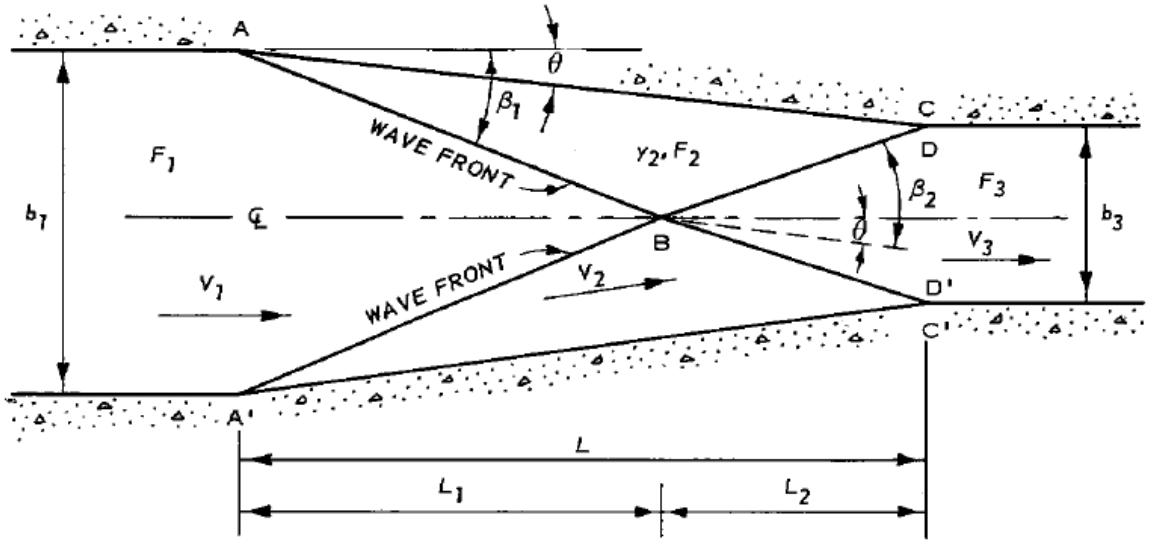
(7) TAN β	(8) $\frac{b_1}{2}$, FT	(9) L_1, FT = (8) ÷ (7)	(10) F_2	(11) $\frac{y_3}{y_2}$	(12) y_3, FT
0.348	80	230	($L_1 > L$; ASSUME SMALLER θ)		
0.331	80	242	3.25	1.12	6.27
0.319	80	251	3.34	1.08	5.78
0.315	80	254	3.35	1.08	5.72

(13) β_2	(14) F_3	(15) $\frac{b_3}{2}$	(16) TAN $(\beta_2 - \theta)$	(17) L_2, FT = (15) ÷ (16)	(18) $L_1 + L_2$, FT
---	---	---	---	---	---
19.7	3.05	70	0.319	219	461 > L
18.5	3.20	70	0.310	226	477 > L
18.2	3.22	70	0.306	229	483 ≈ L

Plate 23



a. SCHEMATIC PROFILE



b. PLAN

$$L = \frac{b_1 - b_3}{2 \tan \theta}$$

$$L_1 = \frac{b_1}{2 \tan \beta_1}$$

$$L_2 = \frac{b_3}{2 \tan (\beta_2 - \theta)}$$

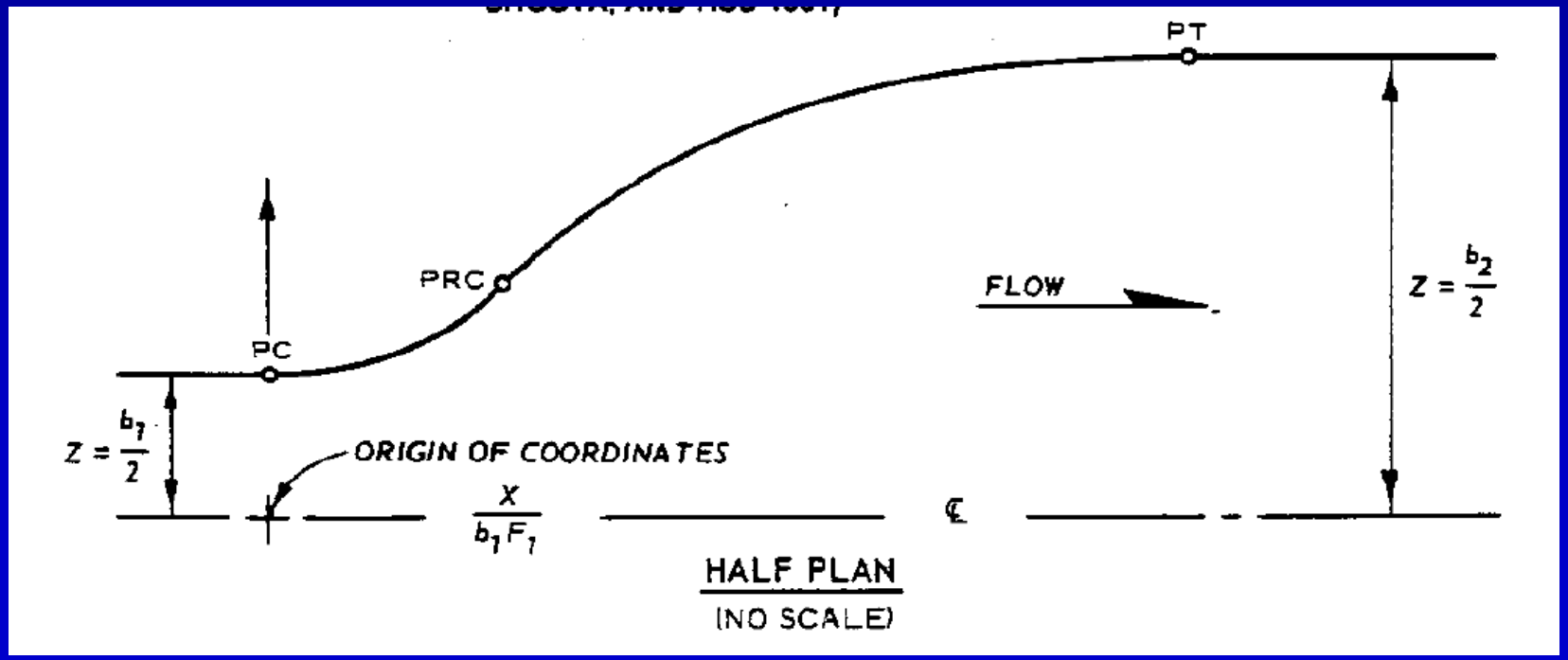
Plate 21

Transition Design

- Supercritical Flow
  - Rectangular expansions
    - Model studies have shown that changes in flow direction are much more gradual in expansions than in contractions
    - Theory and empirical data show that a curved transition helps regulate wave propagation
    - Model testing has indicated that downstream depths can be significantly greater than what theory predicts

Transition Design

- Supercritical Flow
  - Rectangular expansions



Transition Design

- Supercritical Flow
  - Rectangular expansions

$$\frac{Z}{b_1} = \frac{1}{2}\left(\frac{X}{b_1 F_1}\right)^{3/2} + \frac{1}{2}$$

EQ 2-25

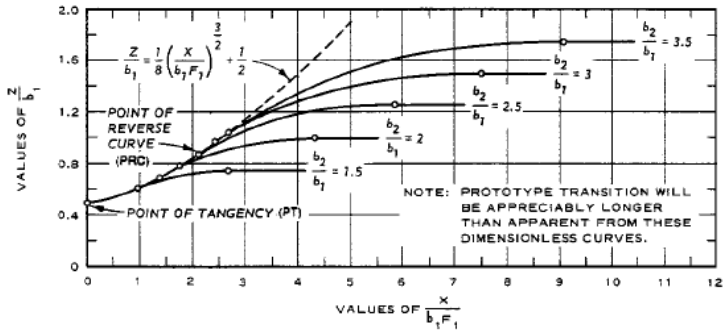
Where:

Z = Transverse distance from channel centerline

b_1 = approach channel width

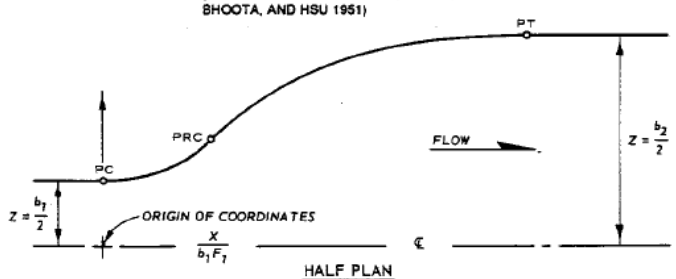
X = longitudinal distance from beginning of expansion

F_1 = approach Froude number



a. GENERALIZED DESIGN CURVES

(REPRODUCED FROM FIG. 59, ROUSE, BHOOTA, AND HSU 1951)



HALF PLAN

(NO SCALE)

POINTS	$\frac{Z}{b_1}$	$\frac{X}{b_1F_1}$
PC	$\frac{1}{2}$	0
PRC	$\frac{11}{60}\left(\frac{b_2}{b_1}\right) + \frac{19}{60}$	$\left[\frac{22}{15}\left(\frac{b_2}{b_1} - 1\right)\right]^{2/3}$
PT	$\frac{1}{2}\frac{b_2}{b_1}$	$\frac{13}{4}\left(\frac{b_2}{b_1}\right) - \frac{9}{4}$
PC TO PRC	$\frac{1}{8}\left(\frac{X}{b_1F_1}\right)^{3/2} + \frac{1}{2}$	0 TO $\left(\frac{X}{b_1F_1}\right)_{PRC}$
PRC TO PT	$\frac{b_2}{2b_1} - q\left[\left(\frac{X}{b_1F_1}\right)_{PT} - \frac{X}{b_1F_1}\right]^r$	$\left(\frac{X}{b_1F_1}\right)_{PRC}$ TO $\left(\frac{X}{b_1F_1}\right)_{PT}$

WHERE

$$q = \frac{\frac{b_2}{2b_1} - \left(\frac{Z}{b_1}\right)_{PRC}}{\left[\left(\frac{X}{b_1F_1}\right)_{PT} - \left(\frac{X}{b_1F_1}\right)_{PRC}\right]^r} \quad \text{AND} \quad r = \frac{\left(\frac{X}{b_1F_1}\right)_{PT} - \left(\frac{X}{b_1F_1}\right)_{PRC}}{\frac{b_2}{2b_1} - \left(\frac{Z}{b_1}\right)_{PRC}}\left(\frac{3}{16}\right)\left(\frac{X}{b_1F_1}\right)^{1/2}_{PRC}$$

b. EQUATIONS APPROXIMATING CURVES

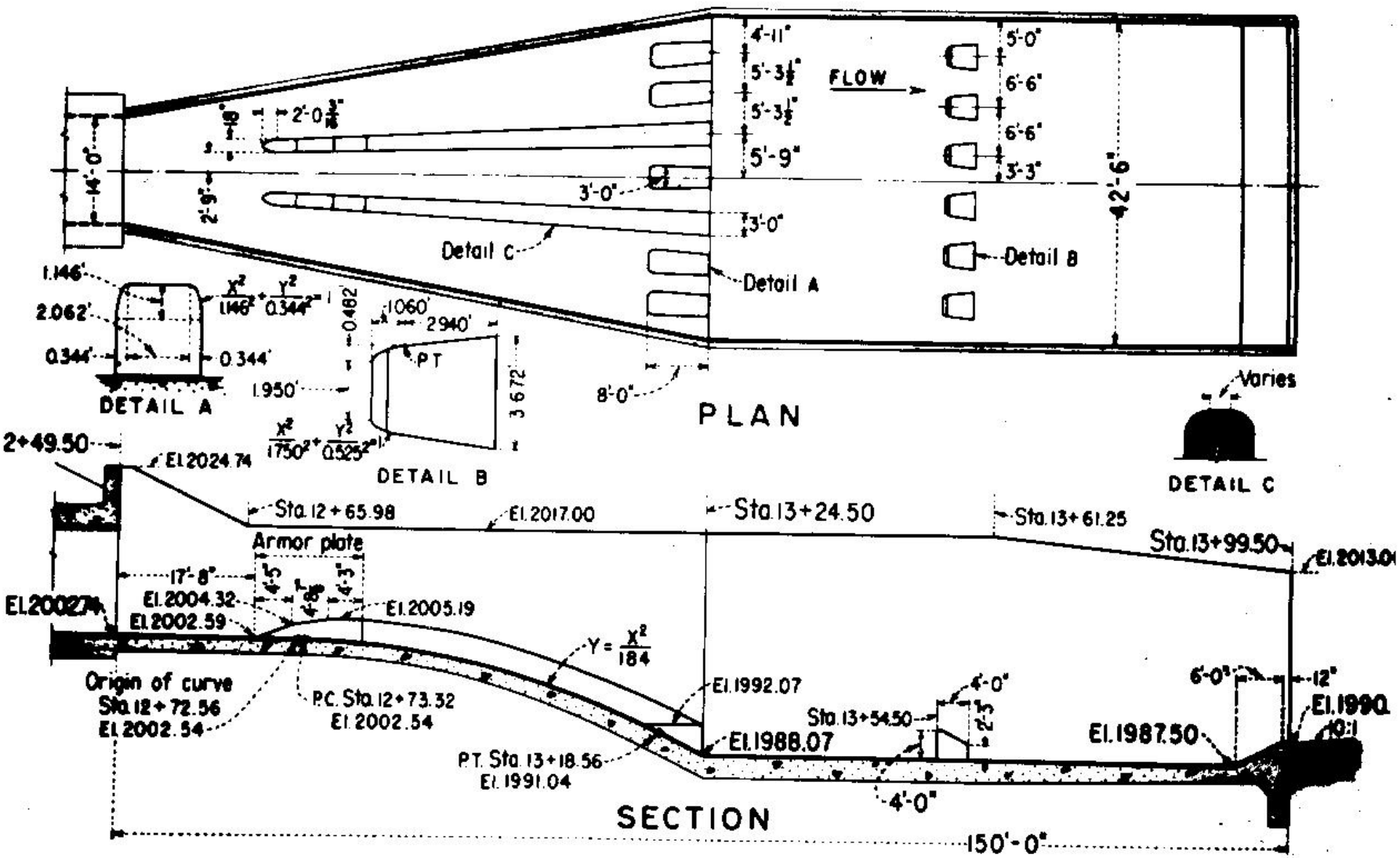
Plate 24

Transition Design

- Supercritical Flow
  - Non-rectangular transitions:
    - No real design guidance, typically need to model

Transition Design

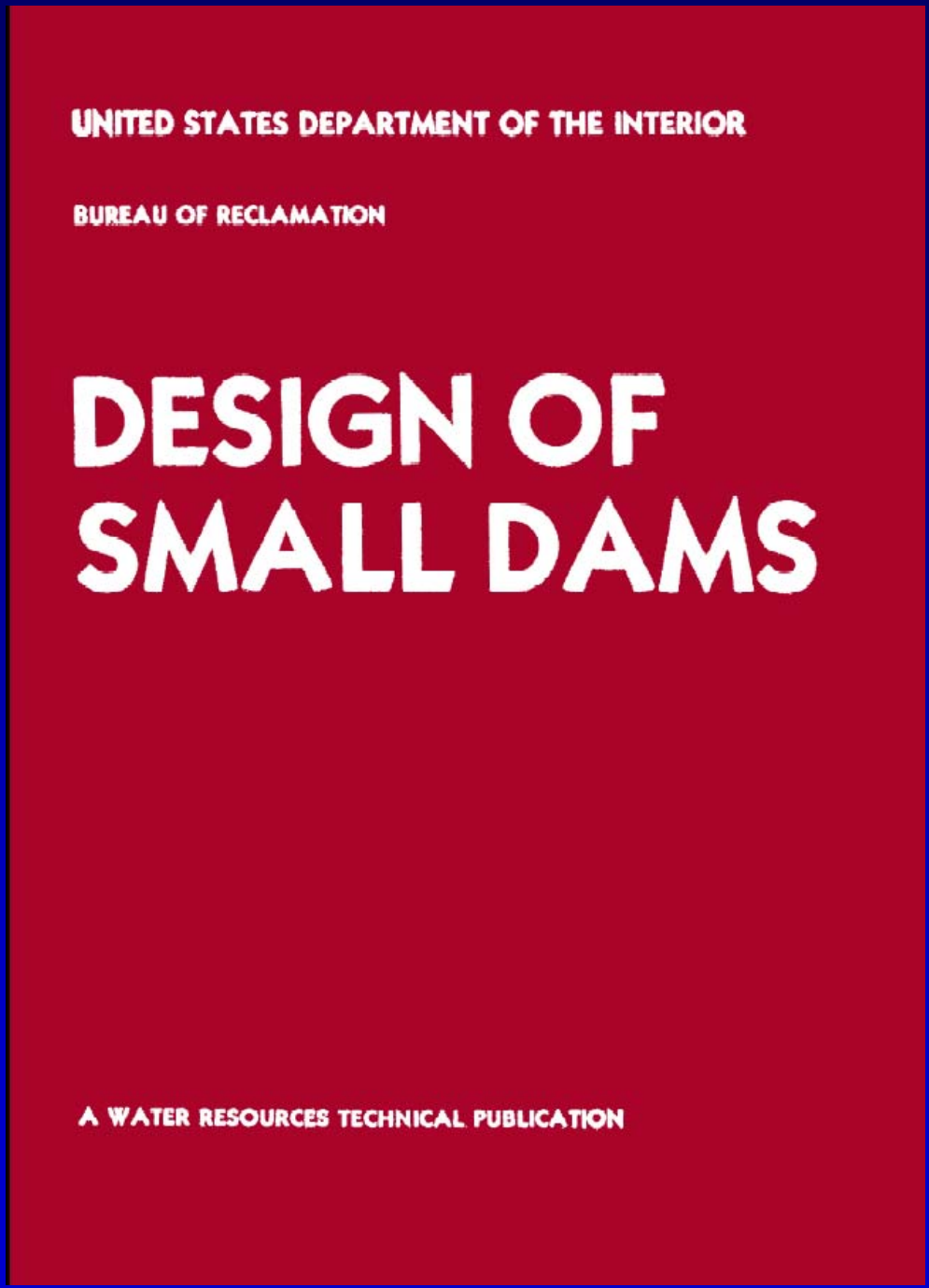
- Supercritical to Subcritical Flow
  - Typically expansions from rectangular to trapezoidal
  - Wedge type transition
  - Can be rapid or gradual
  - Jump needs to be contained within transition
  - Scenario??



HEART BUTTE DAM

SPILLWAY AND OUTLET WORKS

STILLING BASIN DETAILS


UNITED STATES DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
DESIGN OF SMALL DAMS
A WATER RESOURCES TECHNICAL PUBLICATION

Transition Losses

- Subcritical flow
  - Design should minimize energy losses and costs
- Supercritical Flow
  - Can be substantial and drive the design

Transition Loss Equations

Contraction:

$$h = K_c \frac{V_2^{\,2} - V_1^{\,2}}{2g}$$

Expansion:

$$h = K_e \frac{V_1^{\,2} - V_2^{\,2}}{2g}$$

Transition Loss Coefficients

SHAPE	K_c (Contraction)	K_e (Expansion)
Abrupt (Square)	0.30	0.80
Straight Line		
10°	0.10	0.20
15°	0.10	0.30
20°	0.10	0.40
30°	0.10	0.70
Warped Design	0.10	0.20

Design Considerations

- Define purpose of structure
- Define project constraints
  - Right-of-Way
  - Site conditions
  - Economic feasibility
  - Location of transition structure
- Design will be case by case

General Guidelines for Design

1. Define design parameters
   - Design discharge, Q (cfs)
   - Geometry of existing and proposed channel section
2. Determine location of transition structure
3. Length of transition structure
4. Determine transition head loss, H_t
5. Determine existing WSE at beginning and end of preliminary location of transition
6. Based on initial water surface elevation calculation (output), refine design of transition

Key Points to Remember

- Transitions in Subcritical Flow are analyzed using the Energy Equation
- Transition losses are energy losses associated with a change in velocity
- If possible, avoid design of transitions in supercritical and unstable range of flows
- Design of transitions is case by case and comes with experience

References

1. Chow, V. T. , **Open Channel Hydraulics**, McGraw Hill Book Co., Inc. New York (1959)
2. Ippen, A. T., "Mechanics of Supercritical Flow", **Transactions, ASCE**, Vol. 116 (1951)
3. Ippen, A. T. and Dawson, J. H., "Design of Channel Contractions", **Transactions, ASCE**, Vol. 116 (1951)
4. Rouse, H., Bhoota, B. V., and Hsu, En-Yun, "Design of Channel Expansions", **Transactions, ASCE** Vol. 116 (1951)
5. Knapp, R. T., "Design of Channel Curves for Supercritical Flow", **Transactions, ASCE**, Vol. 116 (1951)
6. Morris, Henry M. and Wiggert, James M., **Applied Hydraulics in Engineering**, Second Edition, John Wiley & Sons
7. U. S. Army, Office, Chief of Engineer, **Hydraulic Design of Flood Control Channels**, Engineer Manual EM 1110-2-1601 (1970)
8. Mostafa, M. Gamal, **Open Channel Transitions in Subcritical Flow**, Final Report No. ERC-78-396FR, (1978)
9. **Orange County Flood Control District Design Manual**, O.C.F.C. D., Orange County, California
10. Majaj, Nadeem H., "Appendix A – Basics of Hydraulics", **Basin Analysis Software User's Manual**, Hydraulic Solutions, Inc. (2003)

Note: Graphics and text depicting the Energy Equation were obtained from another slide presentation on the web. Author was not named.

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Start 6 OCT 08

Flow in Curved Channels

- Superelevation
- Limiting curvature
- Bend losses
- Shear stress

Flow in Curved Channels

- Superelevation
- Secondary currents
- Shift in maximum velocity

Flow in Curved Channels

- Superelevation

$$\Delta y = C\frac{V^2 W}{gr}$$ EQ. 2-31

Where:

- ✦y = rise in water surface
- C = coefficient
- V = average velocity
- W = channel width
- g = gravity
- r = radius of curvature

Table 2-4
Superelevation Formula Coefficients

Flow Type	Channel Cross Section	Type of Curve	Value of C
Tranquil	Rectangular	Simple Circular	0.5
Tranquil	Trapezoidal	Simple Circular	0.5
Rapid	Rectangular	Simple Circular	1.0
Rapid	Trapezoidal	Simple Circular	1.0
Rapid	Rectangular	Spiral Transitions	0.5
Rapid	Tapezoidal	Spiral Transitions	1.0
Rapid	Rectangular	Spiral Banked	0.5

Flow in Curved Channels

- Superelevation
  - Subcritical
    - Increase wall height along outside of curve to account for rise in water surface elevation
    - Can have waves on inside of channel bend

Flow in Curved Channels

- Superelevation
  - Supercritical
    - Effect propagated downstream requiring increased wall height
    - Mitigation
      - Spiral transition curves
      - Spiral banked curves
      - Limiting curvature

Flow in Curved Channels

- Spiral transition curves
  - Induce curvature and superelevation at a gradual rate
  - Gradually increases from infinity to a maximum value at plan view center point

$$L_s = 1.82 \frac{VW}{\sqrt{gy}}$$

EQ. 2-32

Flow in Curved Channels

- Spiral banked curves
  - Used in rectangular channels
  - Rotate channel bottom about centerline
  - Maximum banking twice ✦y from EQ 2-31
    - Inside bend depressed by ✦y
    - Outside bend elevated by ✦y
  - Permits wall heights to be equal on both sides of channel

Flow in Curved Channels

- Limiting curvature
  - Subcritical flow
    - Rc >3 times channel width
  - Supercritical flow
    - Use Equation 2.34

Flow in Curved Channels

$$r_{\min} \geq \frac{4V^2W}{gy}$$ EQ. 2-34

Where:

r_{min} = minimum radius of curvature

V = average velocity

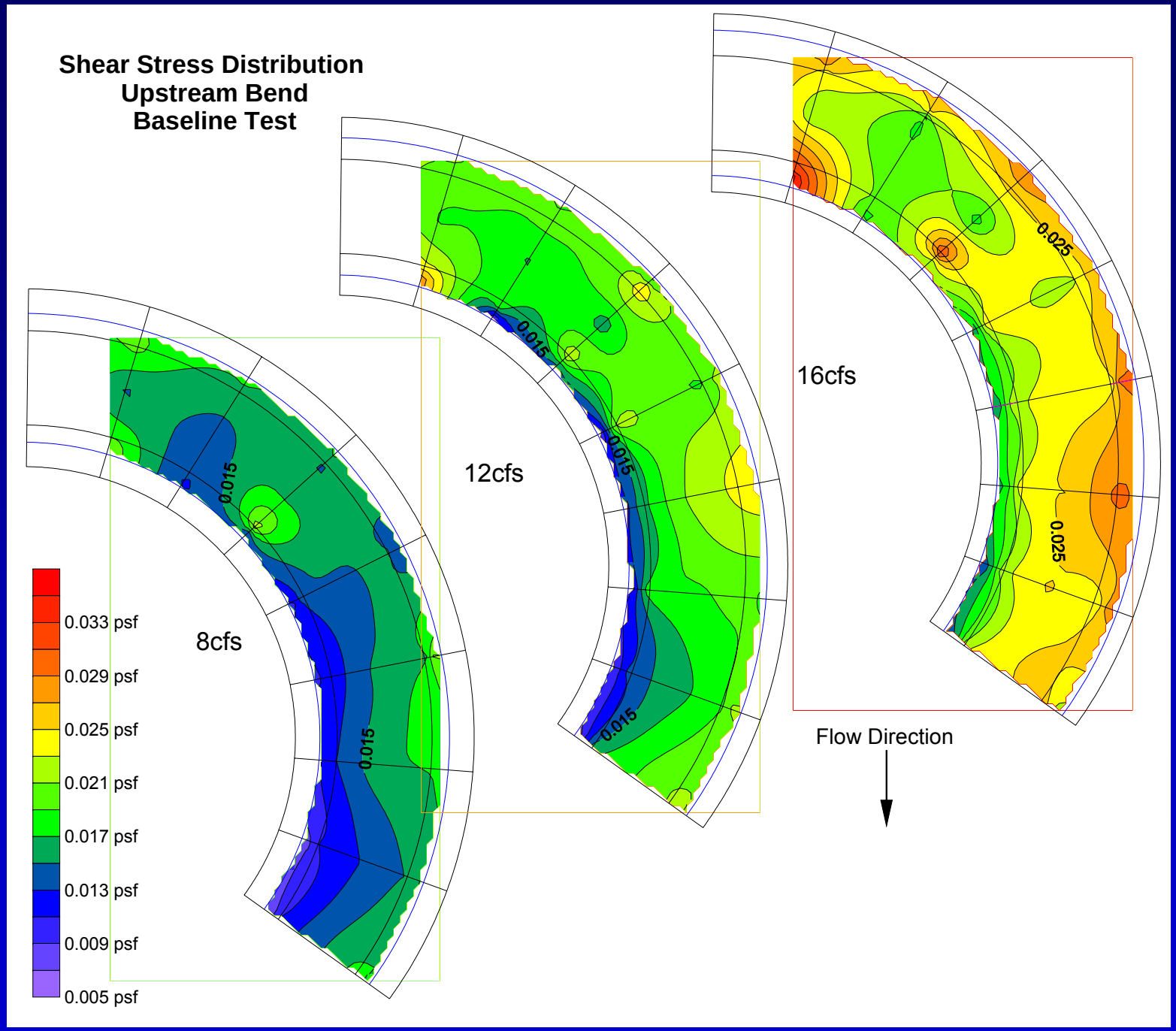
W = channel width at design water surface

y = flow depth

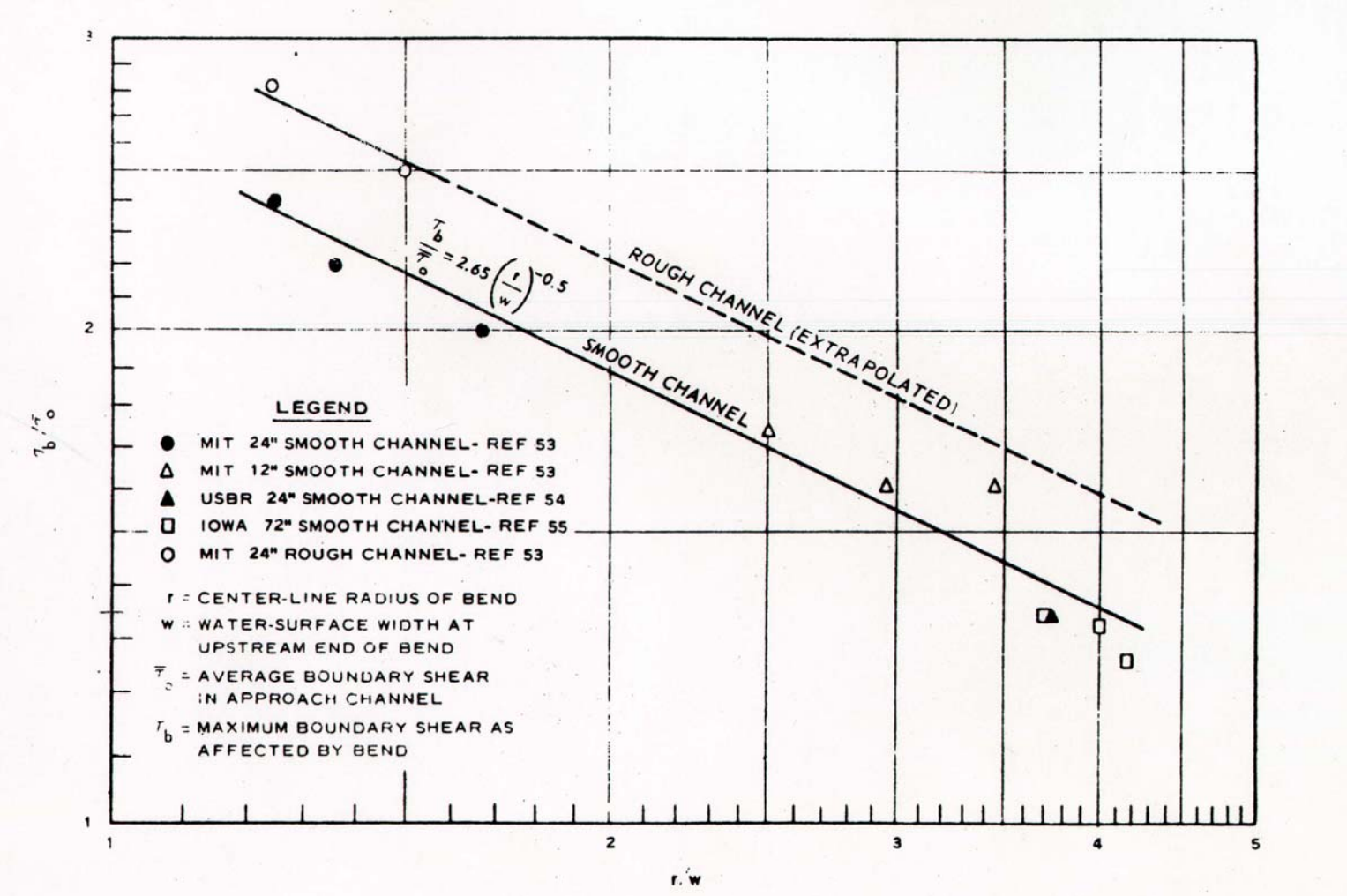
Flow in Curved Channels

- Bend losses
  - Scobey (1933) recommended increasing n by 0.001 for each 20° of curvature per 100 feet of channel
  - Maximum increase of 0.003
  - Minor losses in HEC-RAS, assumed to be negligent
  - Effective roughness due to secondary currents
  - Recent experiments indicate that losses can be significant for values of $r_c/W < \sim 4$

Curved Channels and Shear Stress

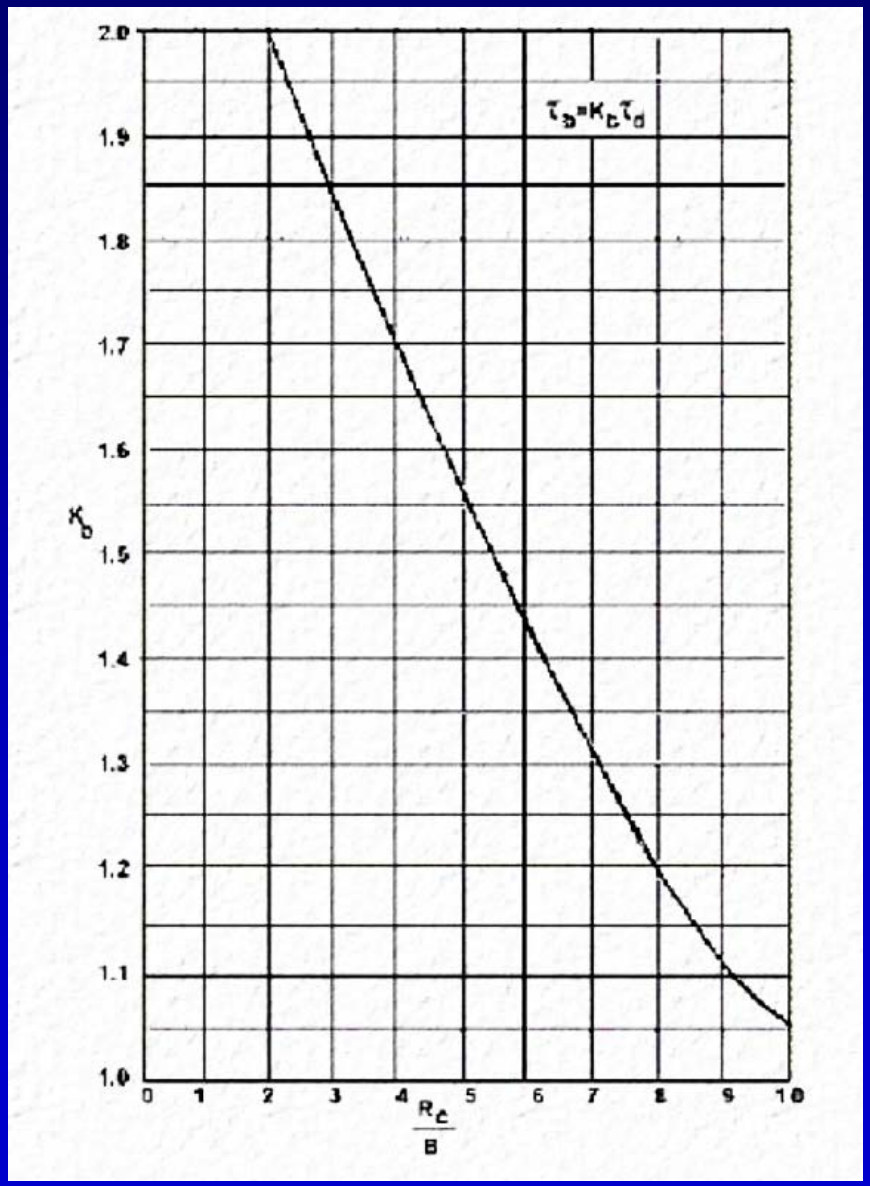

Shear Stress Distribution
Upstream Bend
Baseline Test
8cfs
12cfs
16cfs
0.015
0.015
0.015
0.015
0.015
0.025
0.025
0.033 psf
0.029 psf
0.025 psf
0.021 psf
0.017 psf
0.013 psf
0.009 psf
0.005 psf
Flow Direction

Design Guidance: EM-1601 (1970)



MAXIMUM SHEAR AT CHANNEL BENDS

Design Guidance: HEC-15 (1988)



- R_c = Centerline Radius of Curvature
- B = Channel bottom width
- $K_b = \tau_b / \tau_o$
- τ_b = Maximum bend shear stress
- τ_o = Average approach shear stress

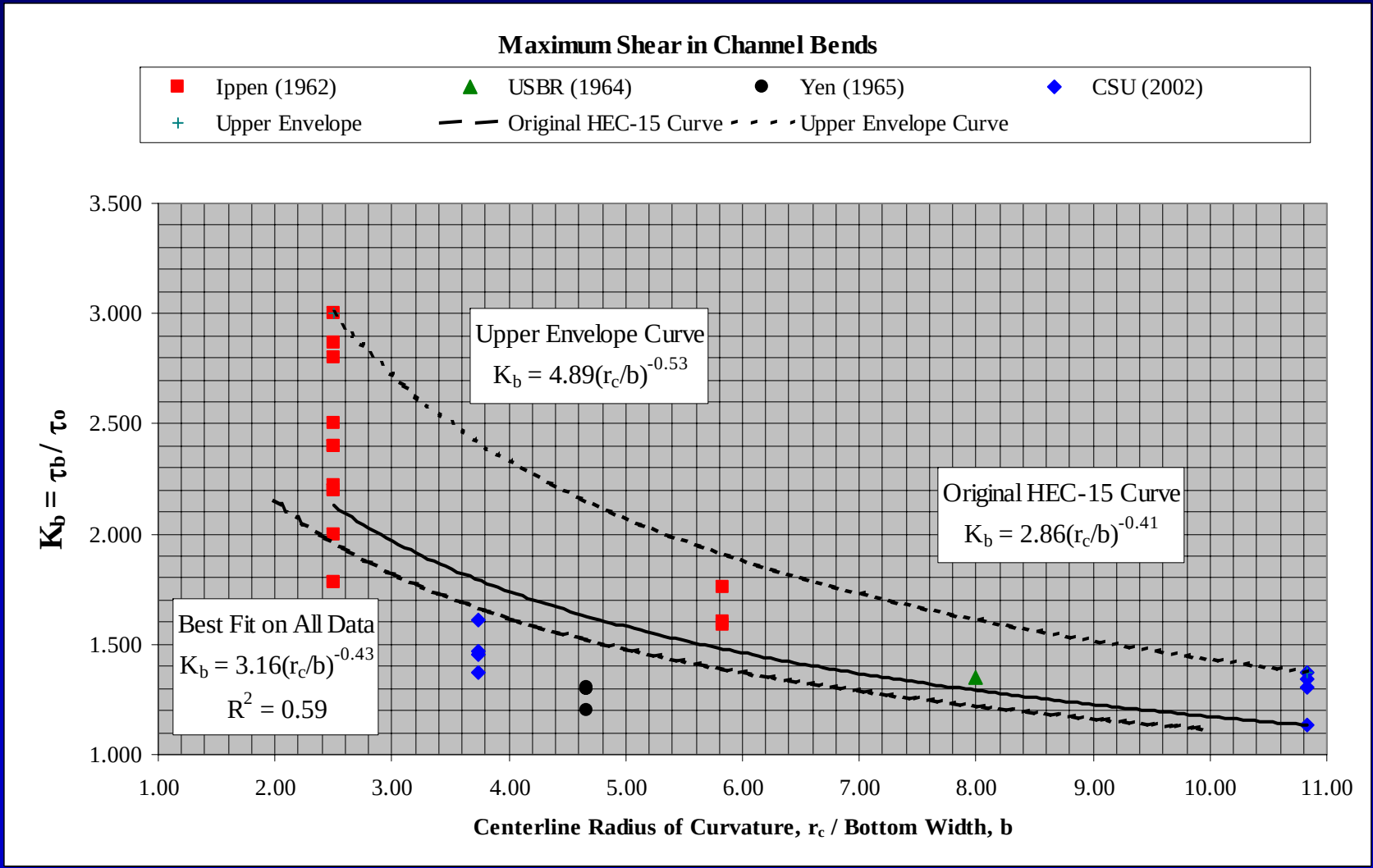
Data Information

- Ippen, A. T., et. al. (1962) ; 16 data points
- USBR (1964) ; 1 data point
- Yen, B.C. (1965) ; 5 data points
- CSU / USBR (2002) ; 8 data points
- Characteristics (for all sets)
  - Trapezoidal channels
  - Preston Tube utilized for shear measurements
  - Rigid boundary

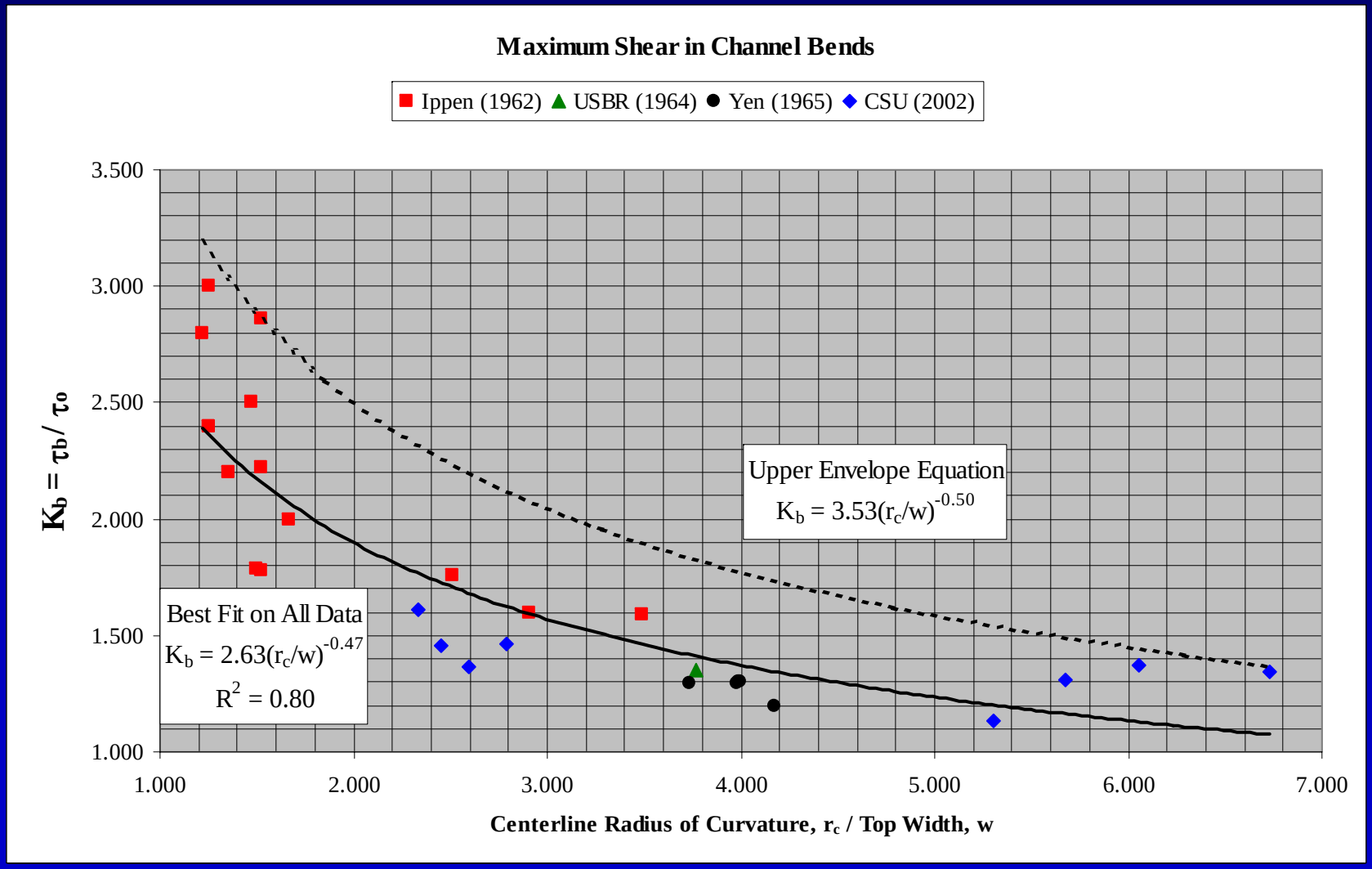
Bend Geometry Ranges

Variable	Range
Side Slope	1 to 2
Bottom Width	1 to 10.2 ft
Top Width	1.67 to 16.27 ft
Centerline Radius of Curvature	5 to 65 ft
Centerline Radius of Curvature / Top Width	1.22 to 6.73
Centerline Radius of Curvature / Bottom Width	2.5 to 10.83

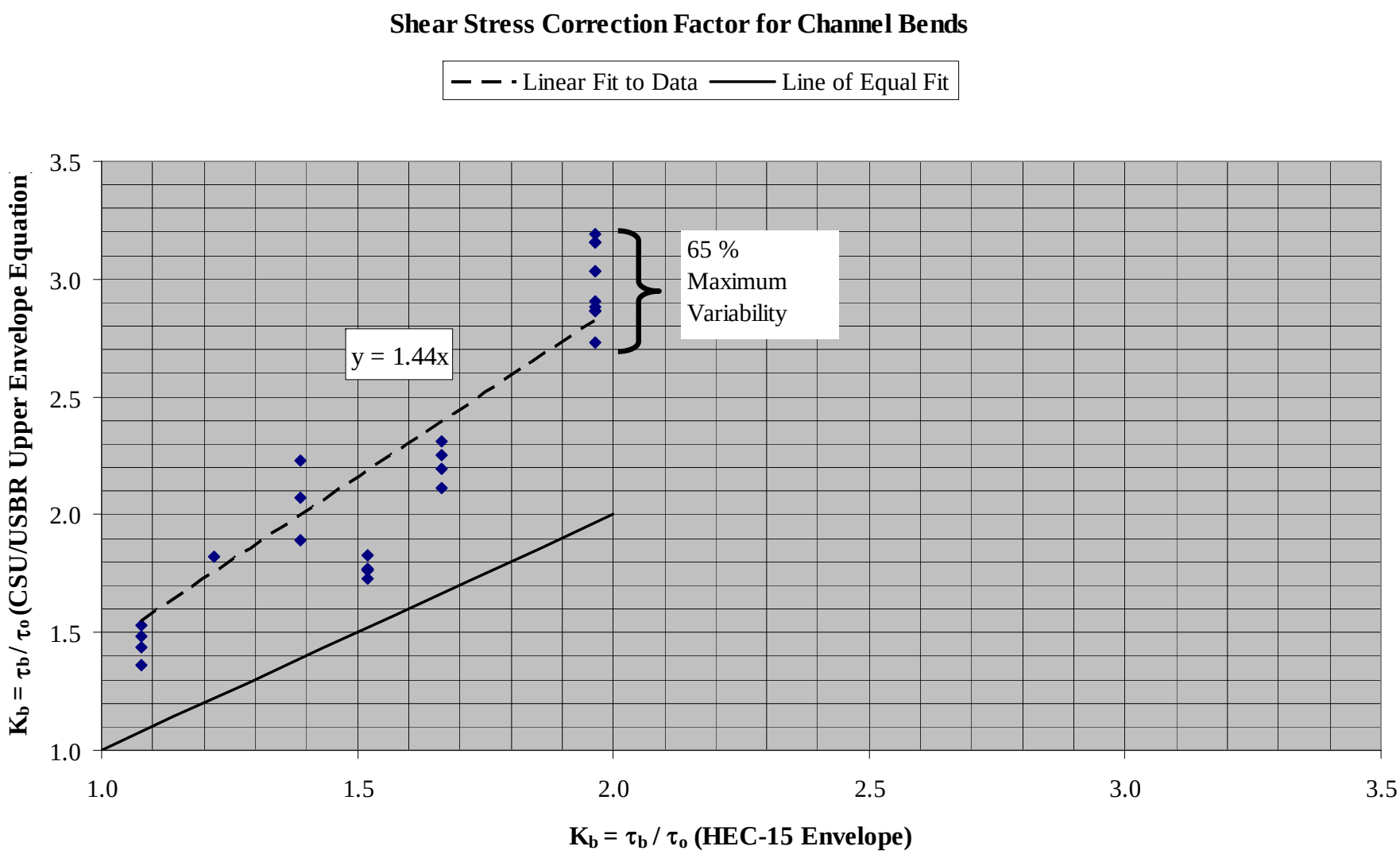
HEC-15 Method



EM-1601 Method

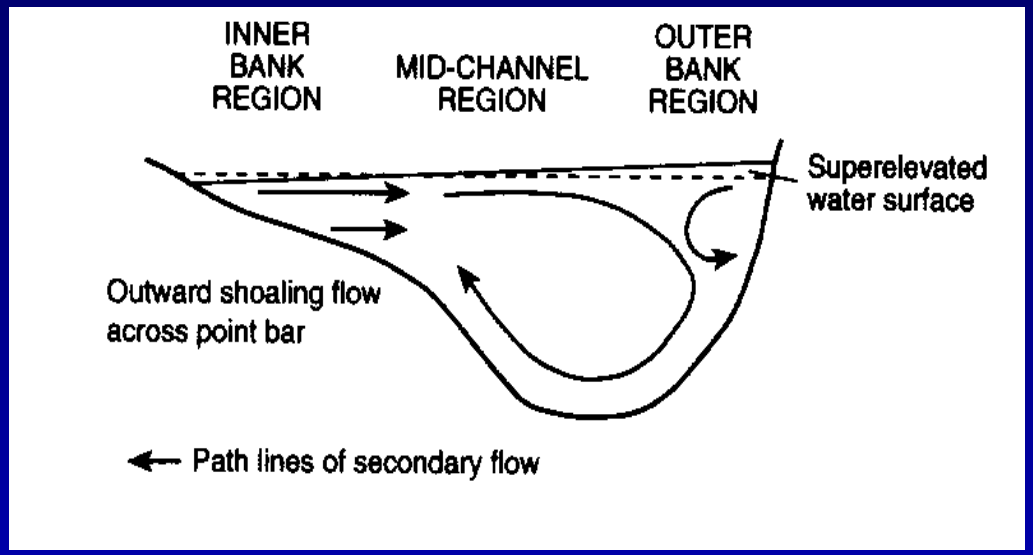


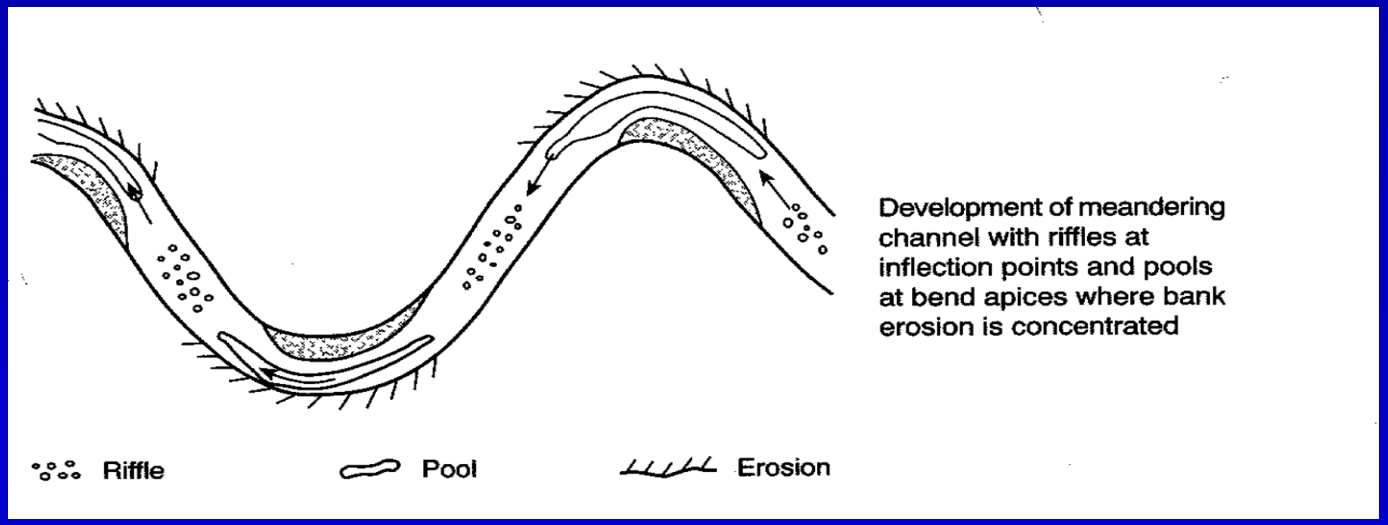
Comparison of Methods



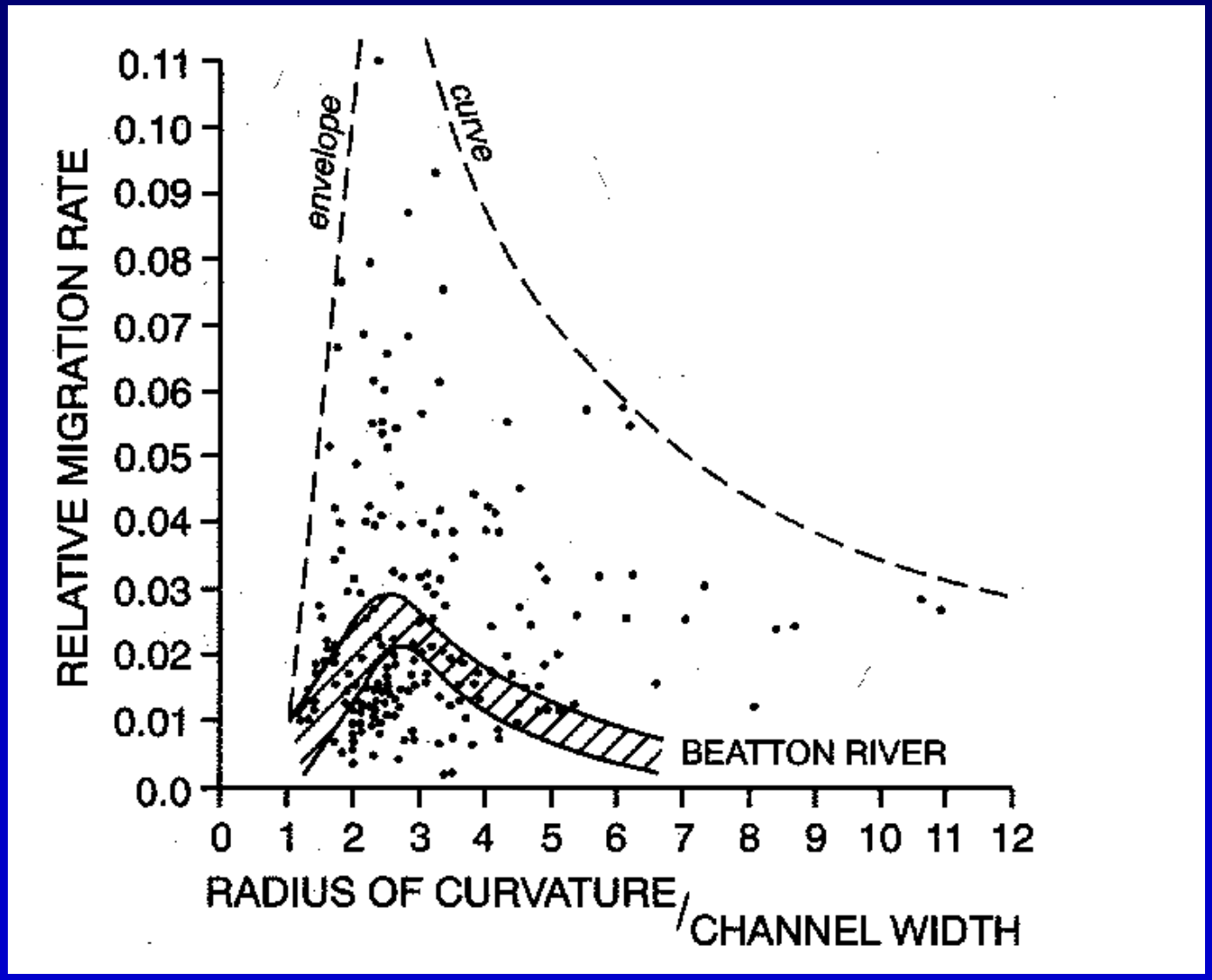
Reality

3-D problem


INNER BANK REGION
MID-CHANNEL REGION
OUTER BANK REGION
Superelevated water surface
Outward shoaling flow across point bar
Path lines of secondary flow


Development of meandering channel with riffles at inflection points and pools at bend apices where bank erosion is concentrated
Riffle
Pool
Erosion

Application



Boundary shear stress has been determined to be a fundamental part of channel migration (Brown, 1988)

Theory - Overview

- Boundary Shear Stress in a stream bend can be expressed as

$$\tau_b = \tau_{viscous} + \tau_{turbulent}$$

$\tau_{viscous}$ = viscous boundary shear stress in a stream bend

$\tau_{turbulent}$ = turbulent boundary shear stress in a stream bend

Start 8 OCT 08

Theory - Overview

- And in further detail as

$$\tau_b = \overbrace{\tau_o + \tau_{lat} + \tau_g}^{\text{Viscous}} + \overbrace{\tau_{zx} + \tau_{yx} + \tau_{zy}}^{\text{Turbulent}}$$

τ_b = total boundary shear stress in a stream bend

τ_o = average longitudinal viscous shear stress

τ_{lat} = average lateral viscous shear stress

τ_g = average additional viscous shear stress due to channel geometry

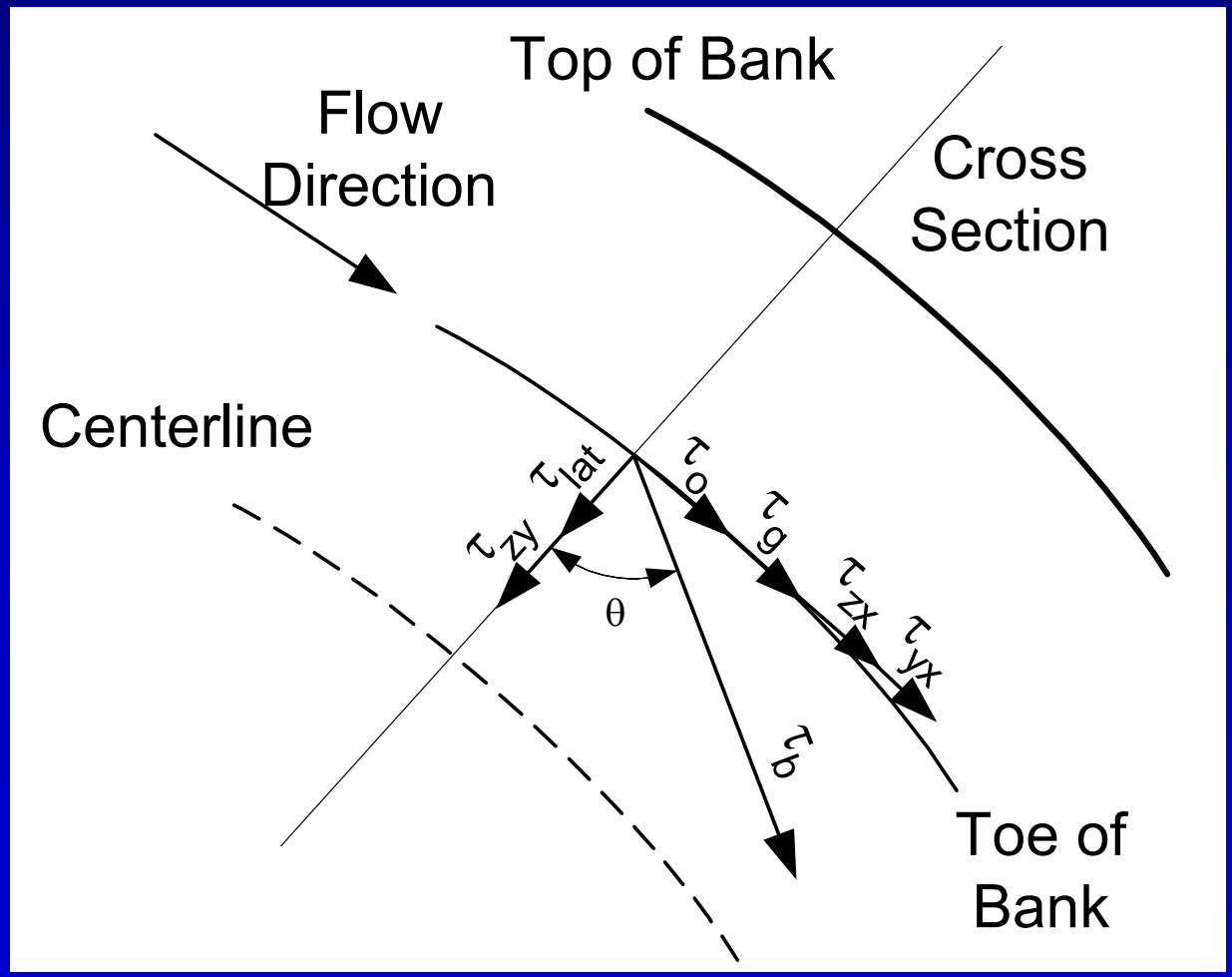
τ_{zx} = average vertical turbulent shear stress component in longitudinal direction

τ_{yx} = average lateral turbulent shear stress component in longitudinal direction

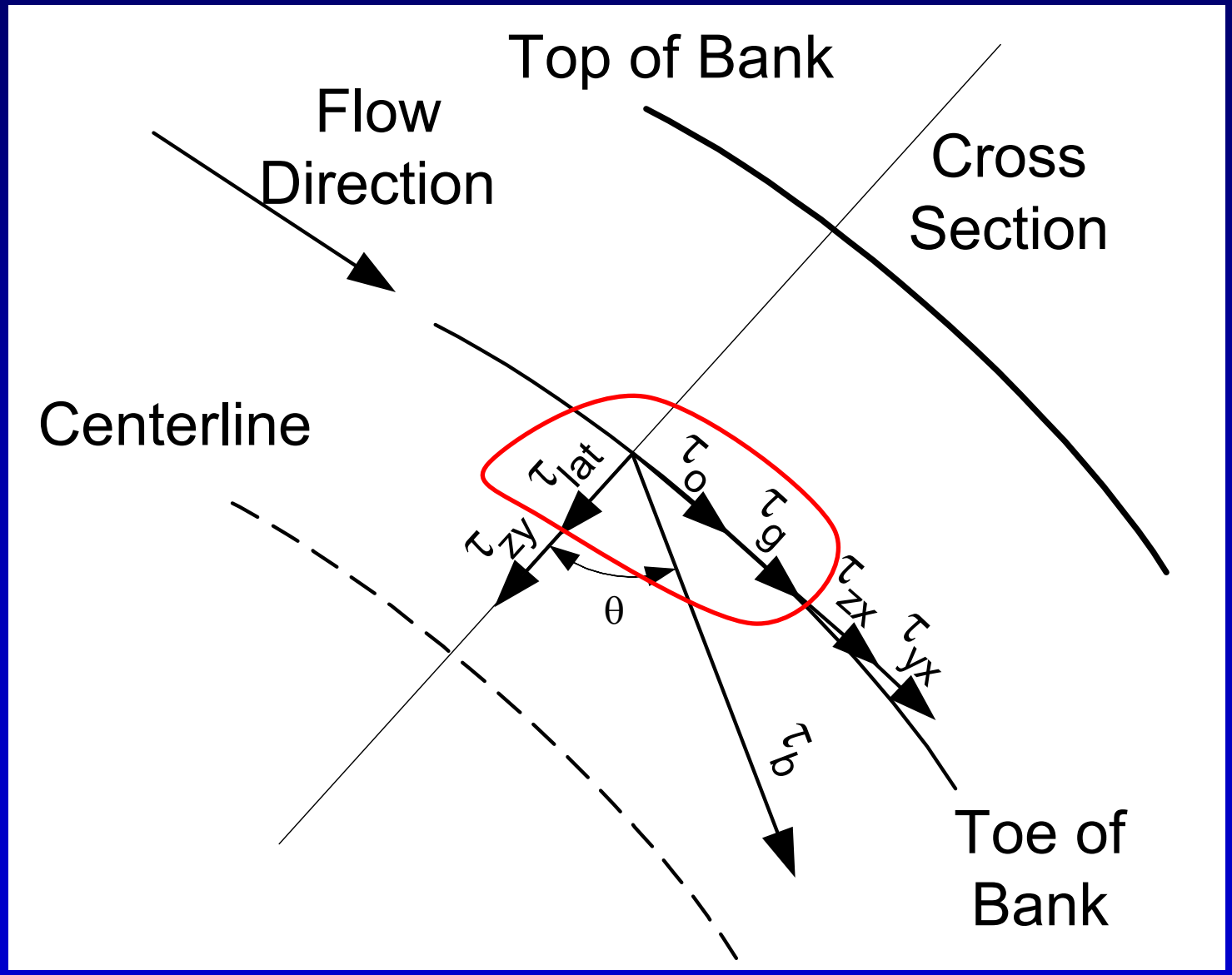
τ_{zy} = average vertical turbulent shear stress component in lateral direction

Theory - Illustration

$$\tau_b = \overbrace{\tau_o + \tau_{lat} + \tau_g}^{\text{Viscous}} + \overbrace{\tau_{zx} + \tau_{yx} + \tau_{zy}}^{\text{Turbulent}}$$



Theory - Illustration



Theory – Viscous Shear Stress

$$\tau_o = \gamma R S_f$$

Where: τ_o = average longitudinal viscous shear stress
γ = unit weight of water
R = hydraulic radius
S_f = friction slope

Theory – Viscous Shear Stress

$$\partial\tau_{lat} = \rho\overline{V_z}\partial V_y - \frac{\rho\overline{V_x^2}\partial z}{r} + \gamma S_y \partial z$$

Where: τ_{lat} = average lateral viscous shear stress
γ = unit weight of water
r = centerline radius of curvature
ρ = density of water
S_y = lateral water surface slope
V_x = average velocity in the x-direction
V_y = average velocity in the y-direction

Theory – Viscous Shear Stress

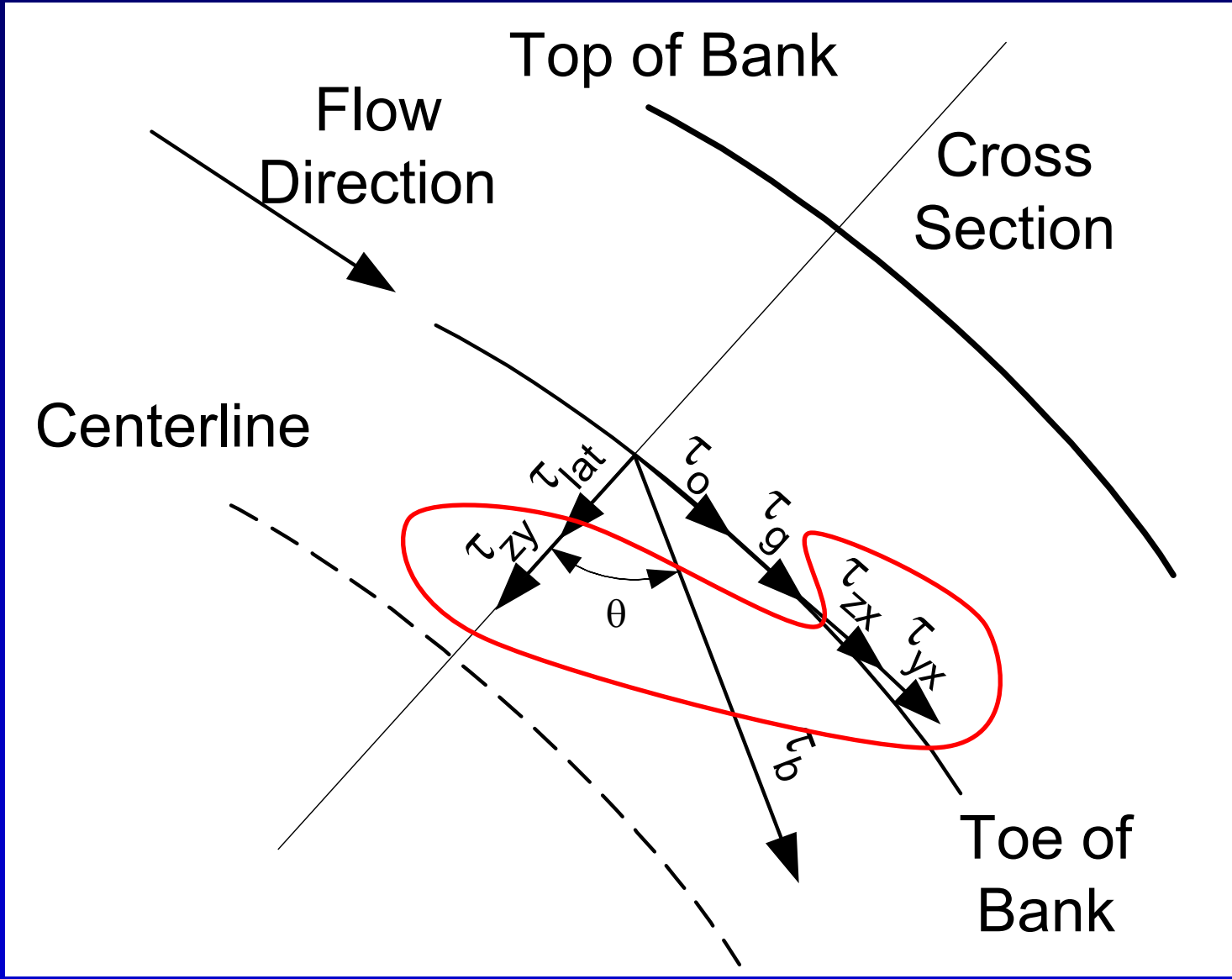
$$\tau_g = \tau_s - \tau_o$$

Where: τ_g = average additional viscous shear stress due to channel geometry

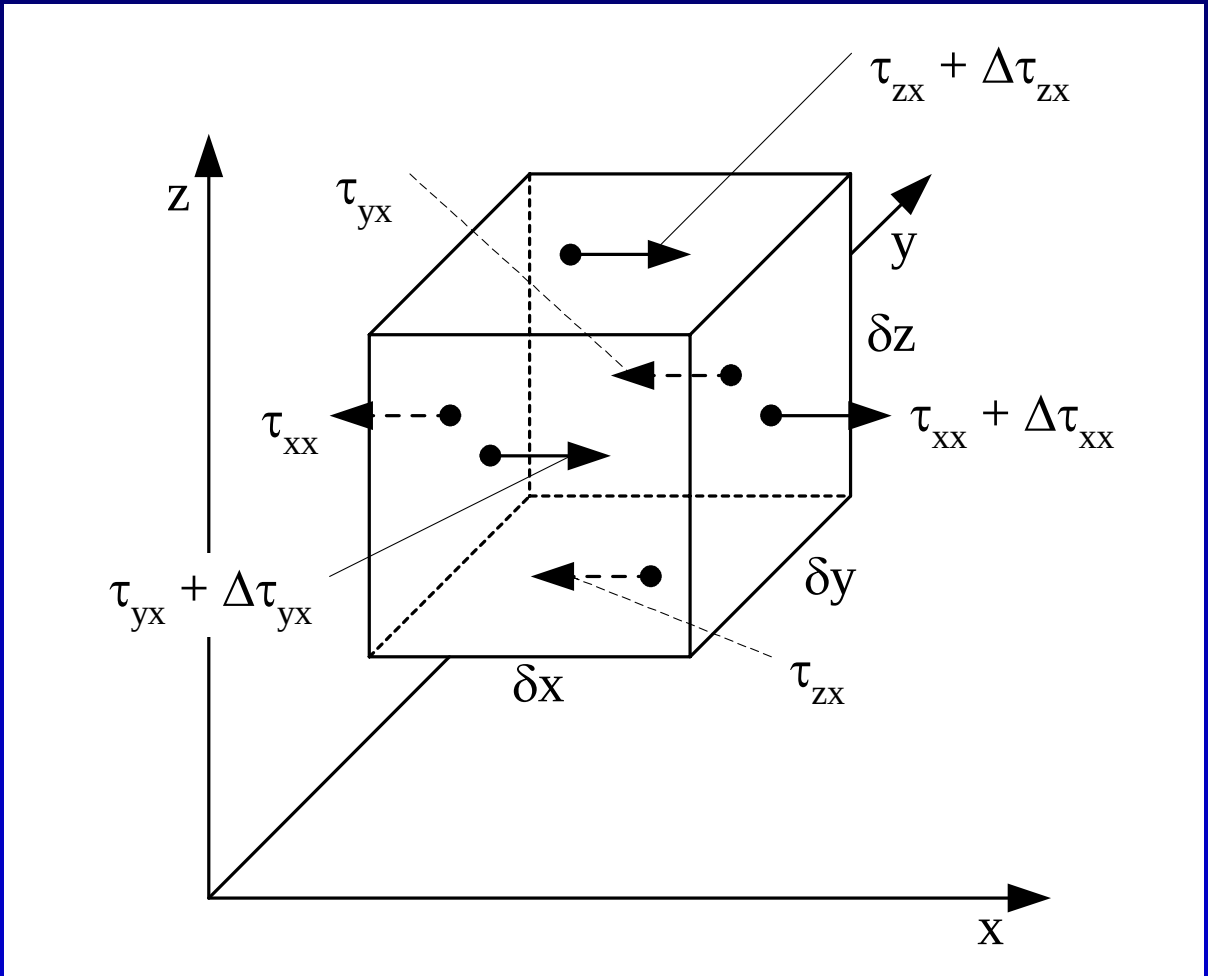
τ_o = average longitudinal viscous shear stress

τ_s = average longitudinal boundary shear stress due to super elevation

Theory - Illustration



Theory - Illustration



Theory – Turbulent Shear Stresses

$$\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}_{app} = \begin{bmatrix} -\rho\overline{(u')^2} & -\rho\overline{(u'v')} & -\rho\overline{(u'w')} \\ -\rho\overline{(v'u')} & -\rho\overline{(v')^2} & -\rho\overline{(v'w')} \\ -\rho\overline{(w'u')} & -\rho\overline{(w'v')} & -\rho\overline{(w')^2} \end{bmatrix} =$$

normal to the x-plane applied in the x-direction	normal to the x-plane applied in the y-direction	normal to the x-plane applied in the z-direction
normal to the y-plane applied in the x-direction	normal to the y-plance applied in the y-direction	normal to the y-plane applied in the z-direction
normal to the z-plane applied in the x-direction	normal to the z-plane applied in the y-direction	normal to the z-plane applied in the z-direction

Theory – Turbulent Shear Stresses

$$\begin{bmatrix} \tau_{xx} & & \\ \tau_{yx} & \tau_{yy} & \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}_{app} = \begin{bmatrix} -\rho\overline{(u')^2} & & \\ -\rho\overline{(v'u')} & -\rho\overline{(v')^2} & \\ -\rho\overline{(w'u')} & -\rho\overline{(w'v')} & -\rho\overline{(w')^2} \end{bmatrix} =$$

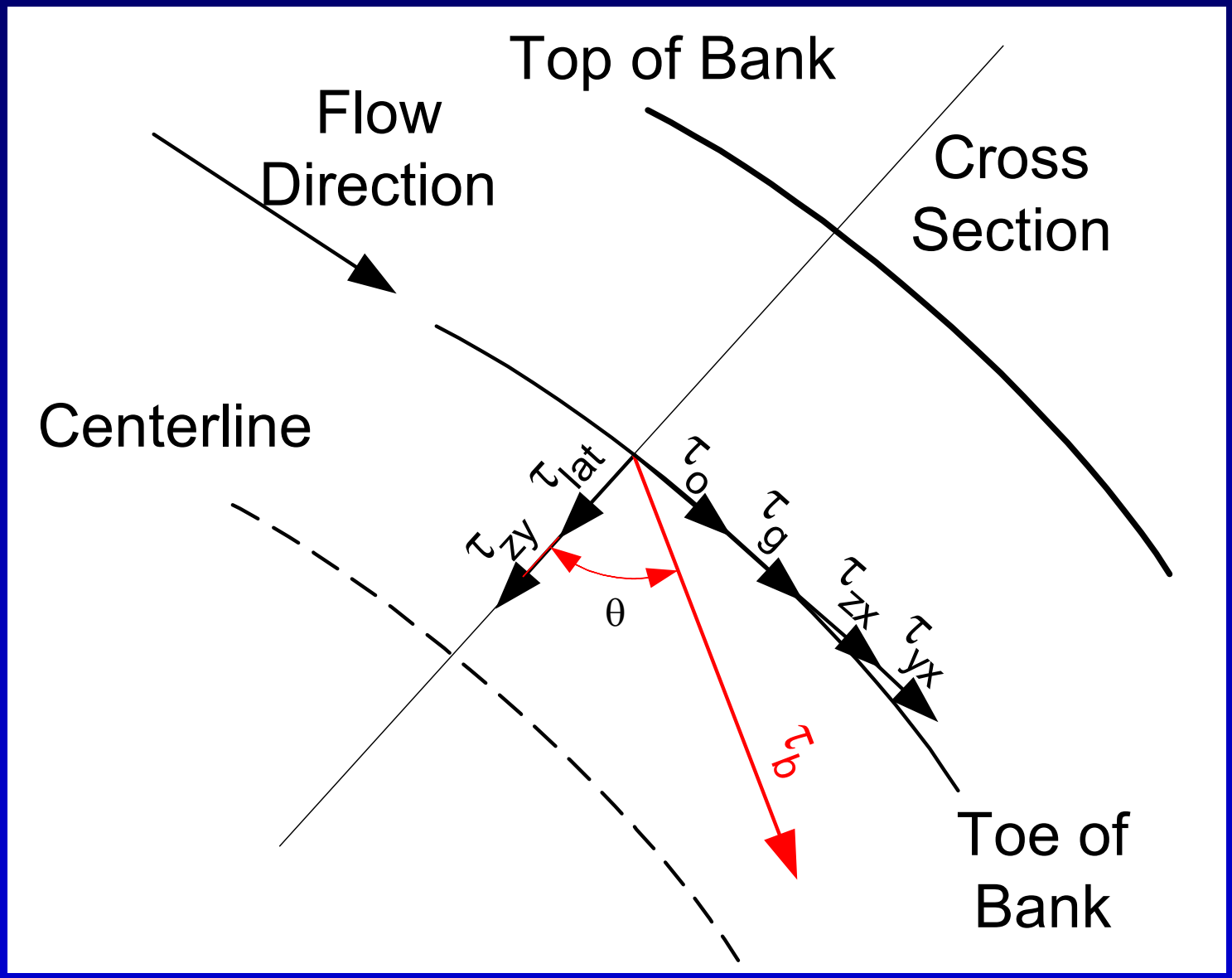
normal to the x-plane applied in the x-direction		
normal to the y-plane applied in the x-direction	normal to the y-plance applied in the y-direction	
normal to the z-plane applied in the x-direction	normal to the z-plane applied in the y-direction	normal to the z-plane applied in the z-direction

Theory – Turbulent Shear Stresses

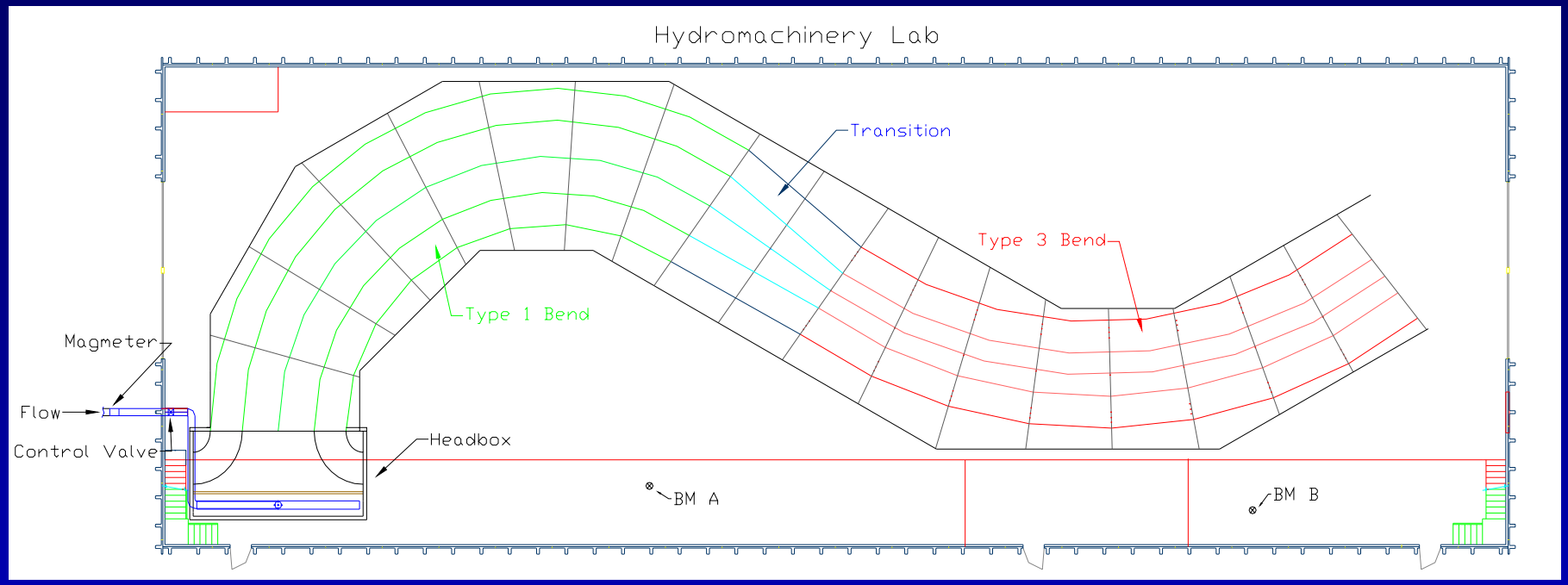
$$\begin{bmatrix} & & \\ \tau_{yx} & & \\ \tau_{zx} & \tau_{zy} & \end{bmatrix}_{app} = \begin{bmatrix} & & \\ -\rho\left(\overline{v'u'}\right) & & \\ -\rho\left(\overline{w'u'}\right) & -\rho\left(\overline{w'v'}\right) & \end{bmatrix} =$$

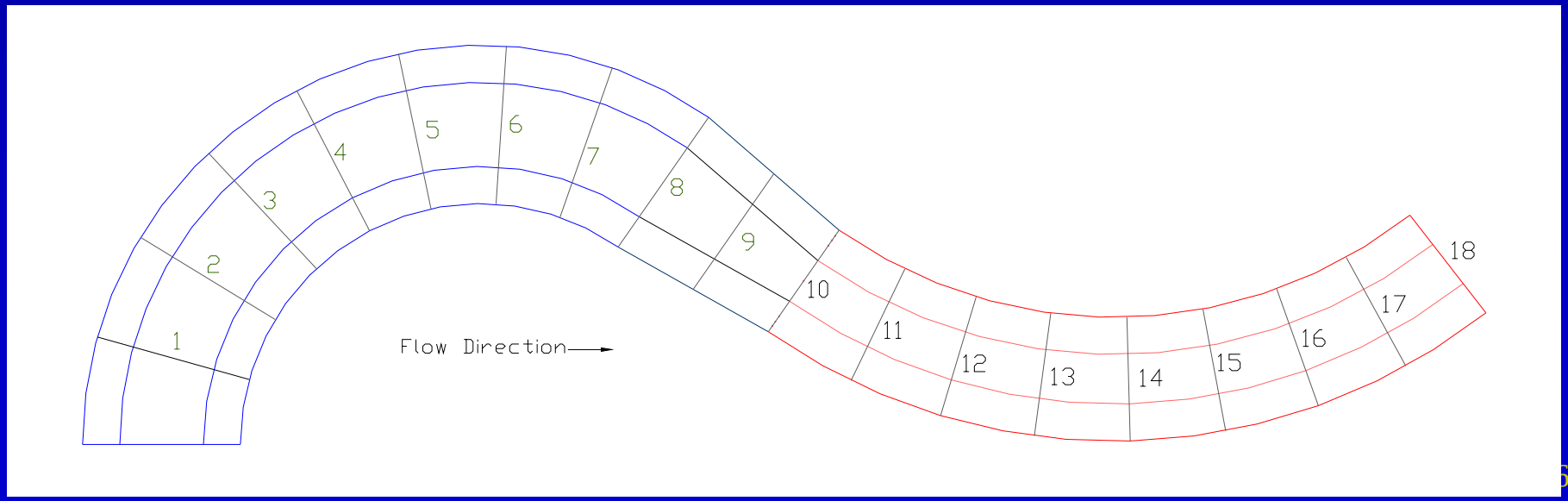
normal to the y-plane applied in the x-direction		
normal to the z-plane applied in the x-direction	normal to the z-plane applied in the y-direction	

Theory - Illustration

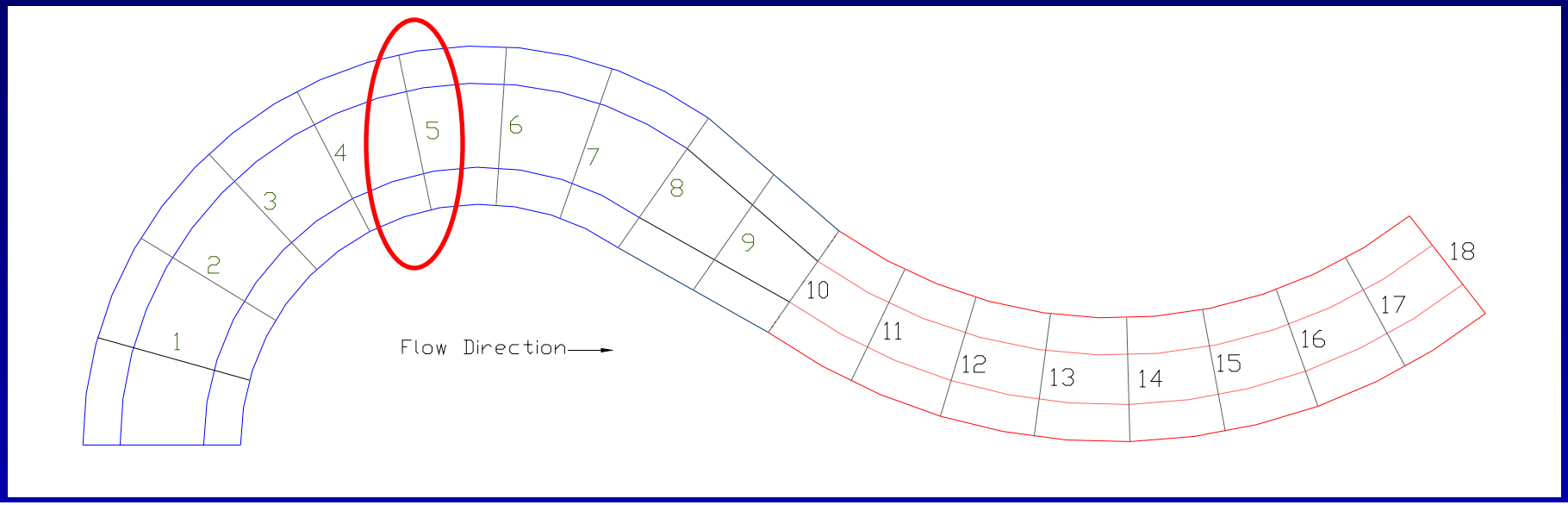


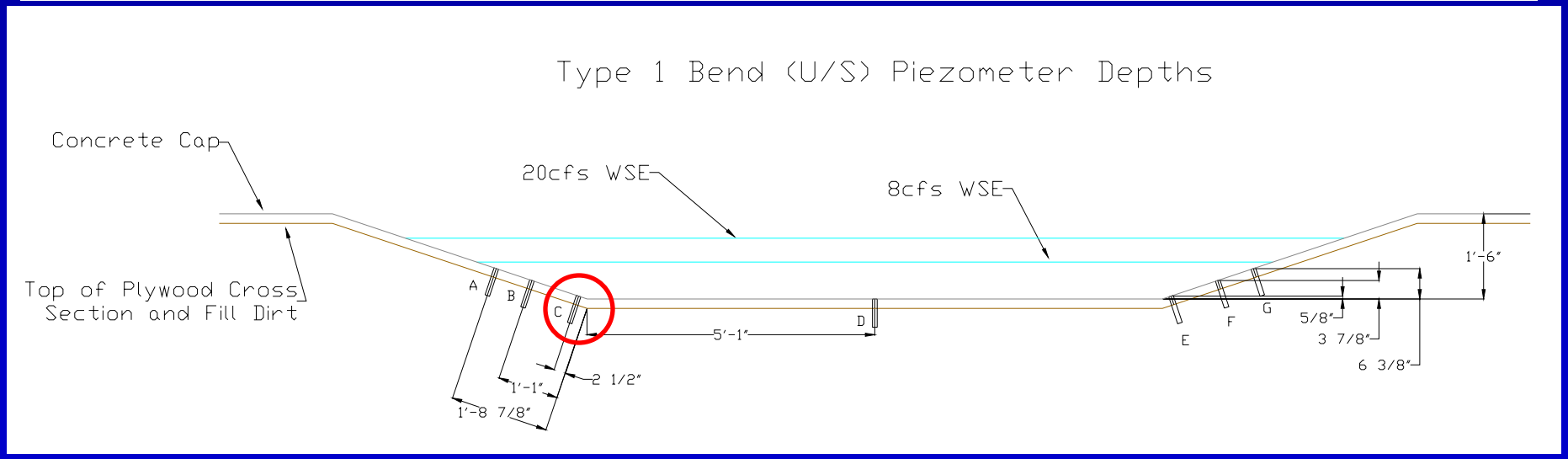
Illustration





Illustration





Example Data

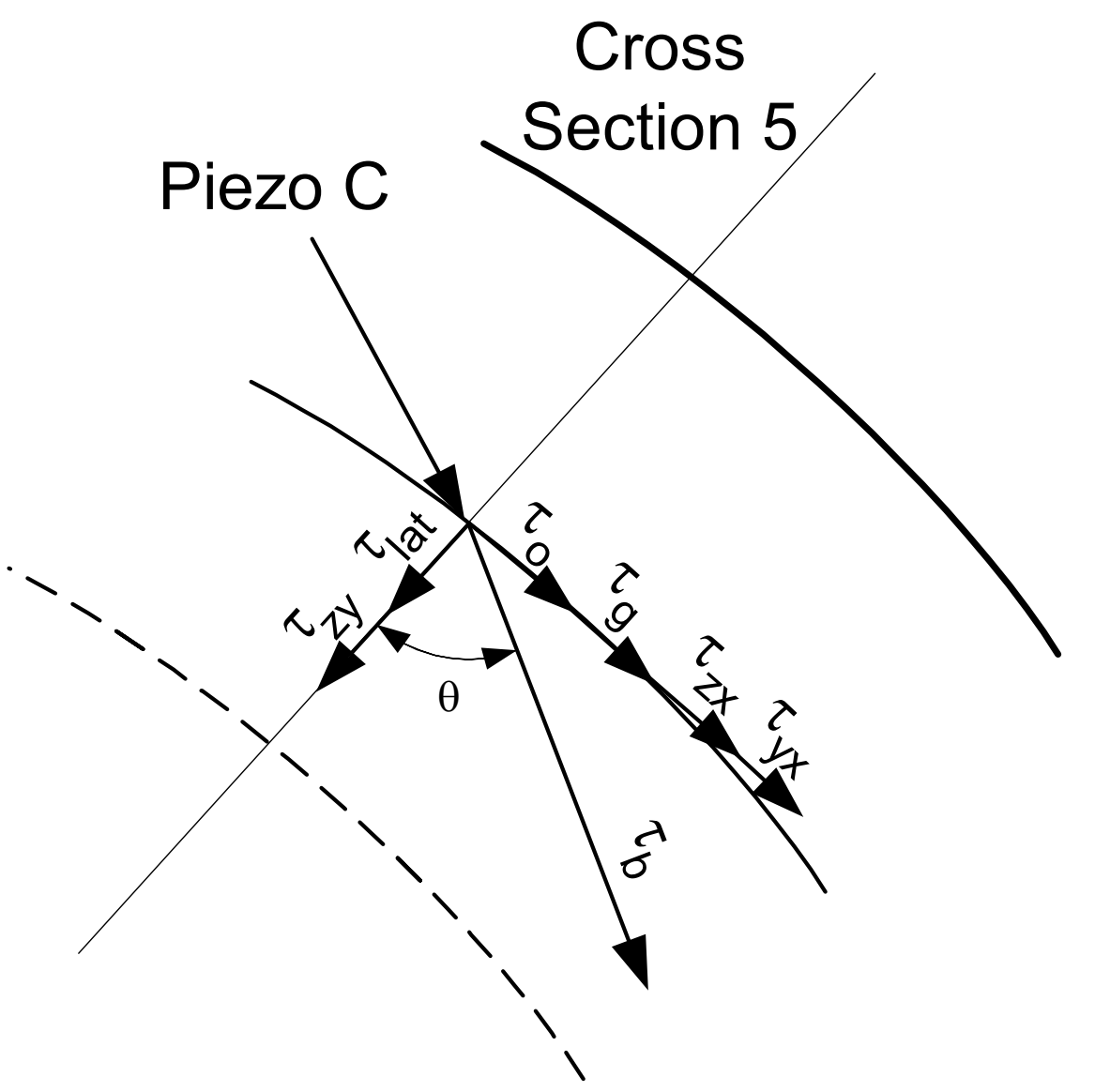
Model discharge was 20 cfs.

τ_o was calculated to be 0.0225 psf

Cross Section	1	2	3	4	5	6	7	8
Piezometer	C	C	C	C	C	C	C	C
τ_p (lb/ft^2)	0.022	0.029	0.038	0.028	0.026	0.024	0.028	0.033
τ_g (lb/ft^2)	0.000	0.006	0.015	0.005	0.003	0.001	0.005	0.011
% of τ_o	0%	28%	68%	23%	14%	5%	23%	49%
τ_{lat} (lb/ft^2)*	0.000	0.003	0.007	0.003	0.002	0.001	0.002	0.005
% of τ_o	0%	13%	31%	13%	10%	4%	9%	22%
* These values were estimated.								

XS #	Piezo	% Depth	τ_{yx}	% of τ_o	τ_{zx}	% of τ_o	τ_{zy}	% of τ_o
			(psf)		(psf)		(psf)	
1	C	100	0.009	42%	0.009	41%	0.001	4%
2	C	100	0.009	40%	0.008	36%	0.001	5%
3	C	100	0.006	26%	0.007	29%	0.001	4%
4	C	100	0.001	3%	0.007	32%	0.002	8%
5	C	100	0.005	21%	0.007	32%	0.003	12%
6	C	100	0.002	9%	0.006	26%	0.000	0%
7	C	100	0.005	24%	0.006	28%	0.002	9%
8	C	100	0.004	19%	0.008	36%	0.002	9%

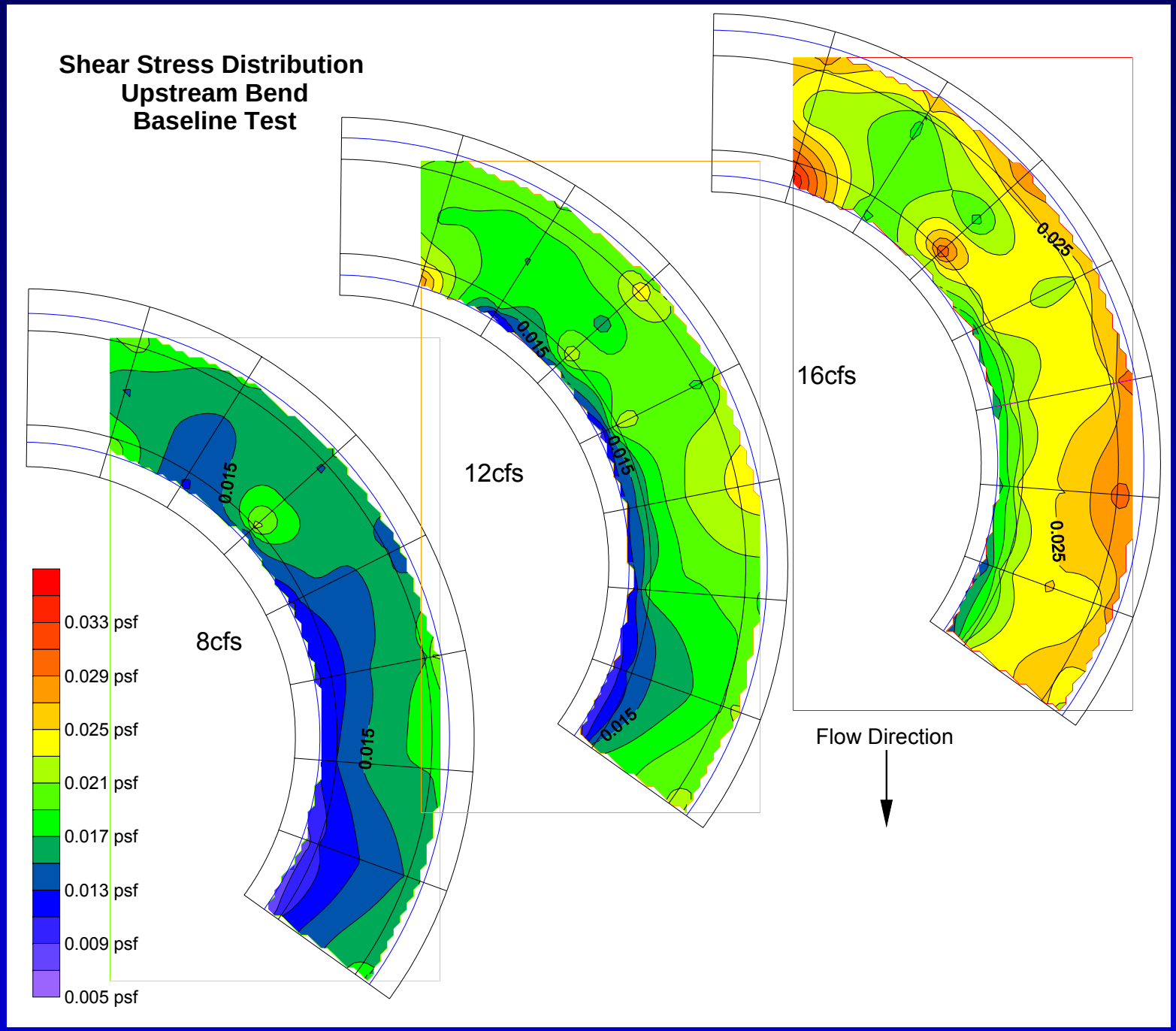
Illustration of Example



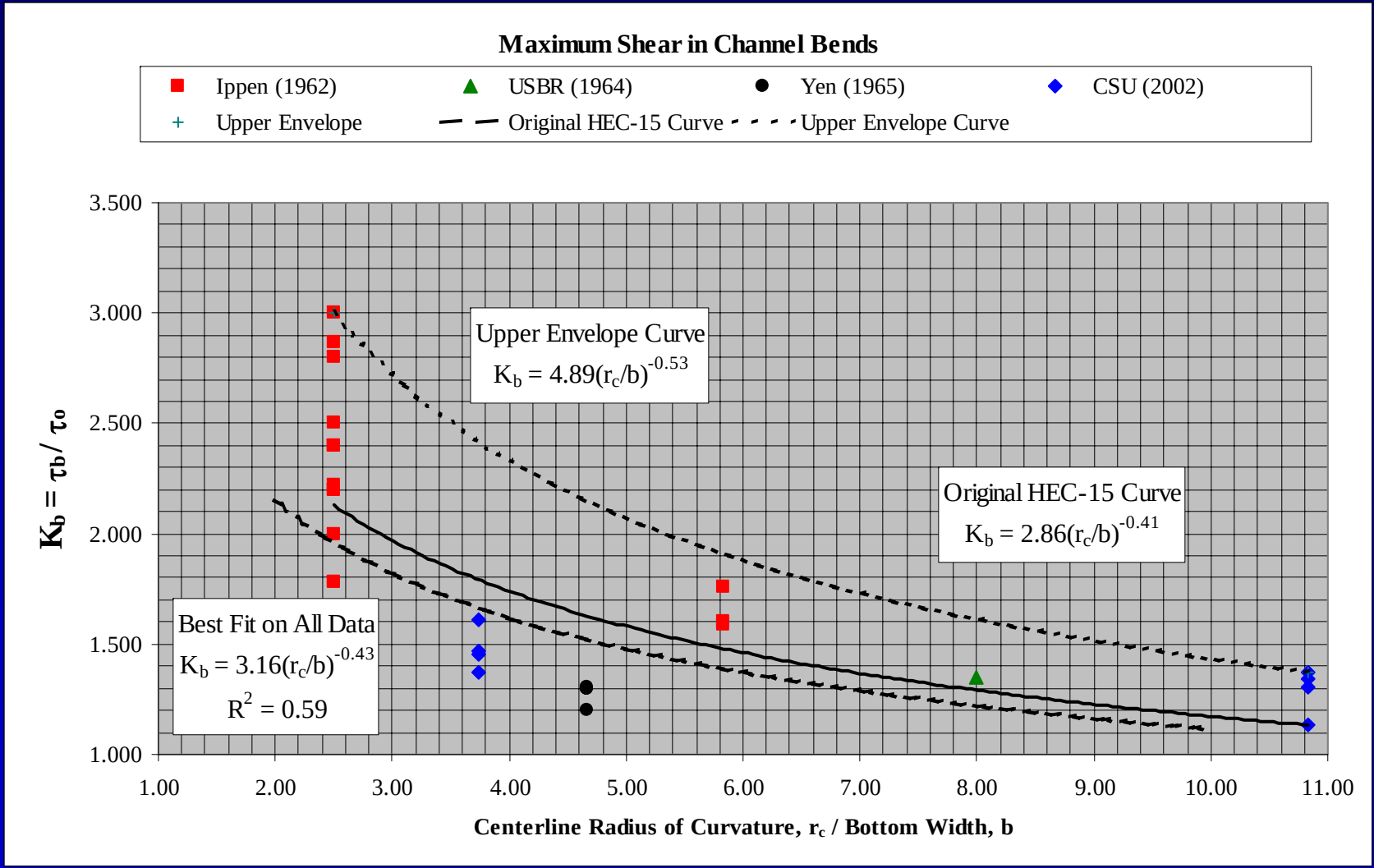
In this particular example, τ_b, would have a magnitude of 0.038 psf with an angle θ of 82.4 degrees, which is a total of 68% larger than τ_o.

Summary of Theory

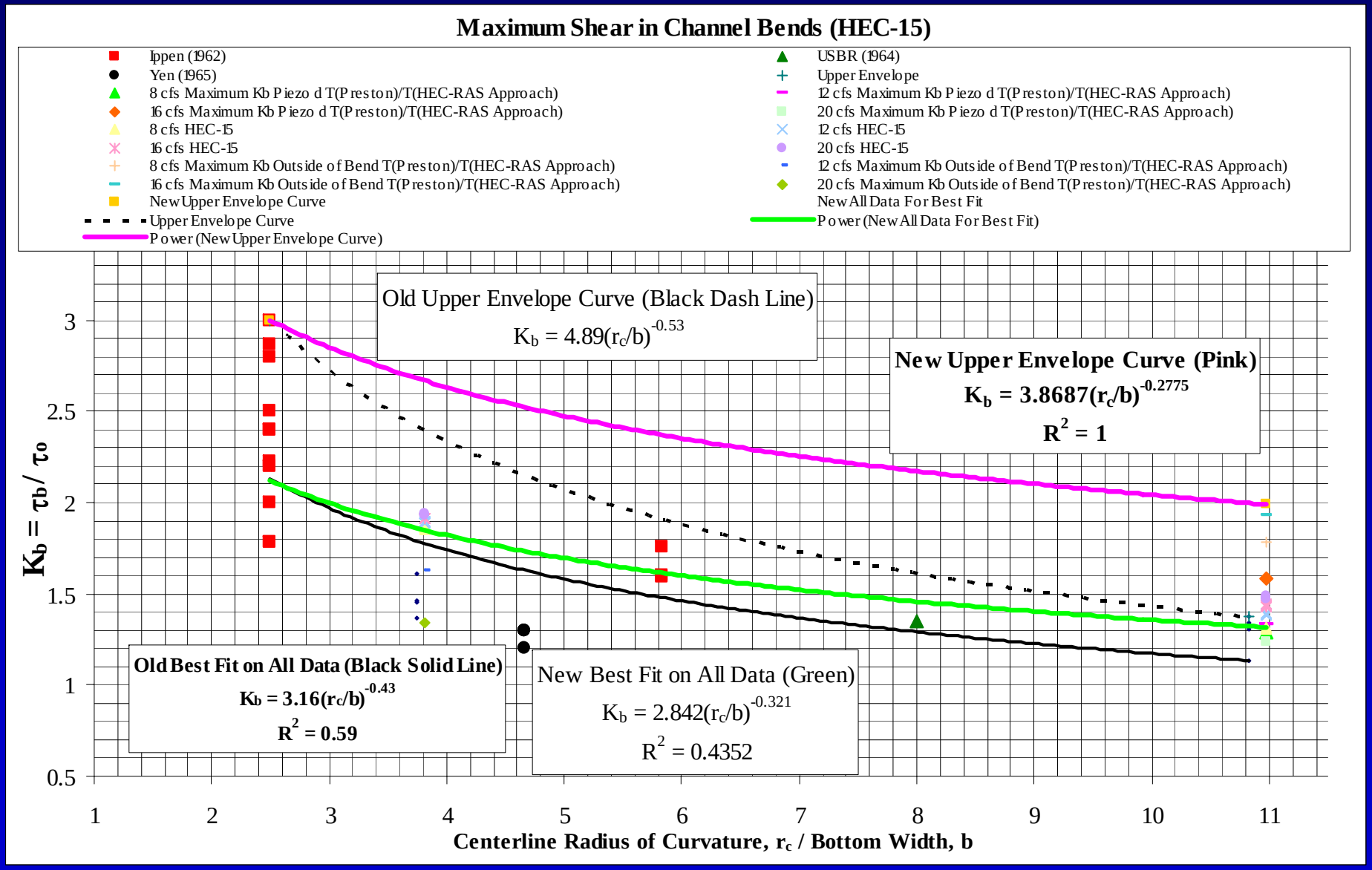
- Total boundary shear stress in a stream bend can be theoretically calculated, however:
  - All terms except, τ_o, require a detailed set of data
  - The dataset would require:
    - Direct shear measurements
    - Water surface elevations throughout and across the bend
    - Turbulent three-dimensional velocities
    - Detailed channel geometry
  - All of which are not generally available


Shear Stress Distribution
Upstream Bend
Baseline Test
8cfs
12cfs
16cfs
0.015
0.015
0.015
0.015
0.015
0.025
0.025
0.033 psf
0.029 psf
0.025 psf
0.021 psf
0.017 psf
0.013 psf
0.009 psf
0.005 psf
Flow Direction

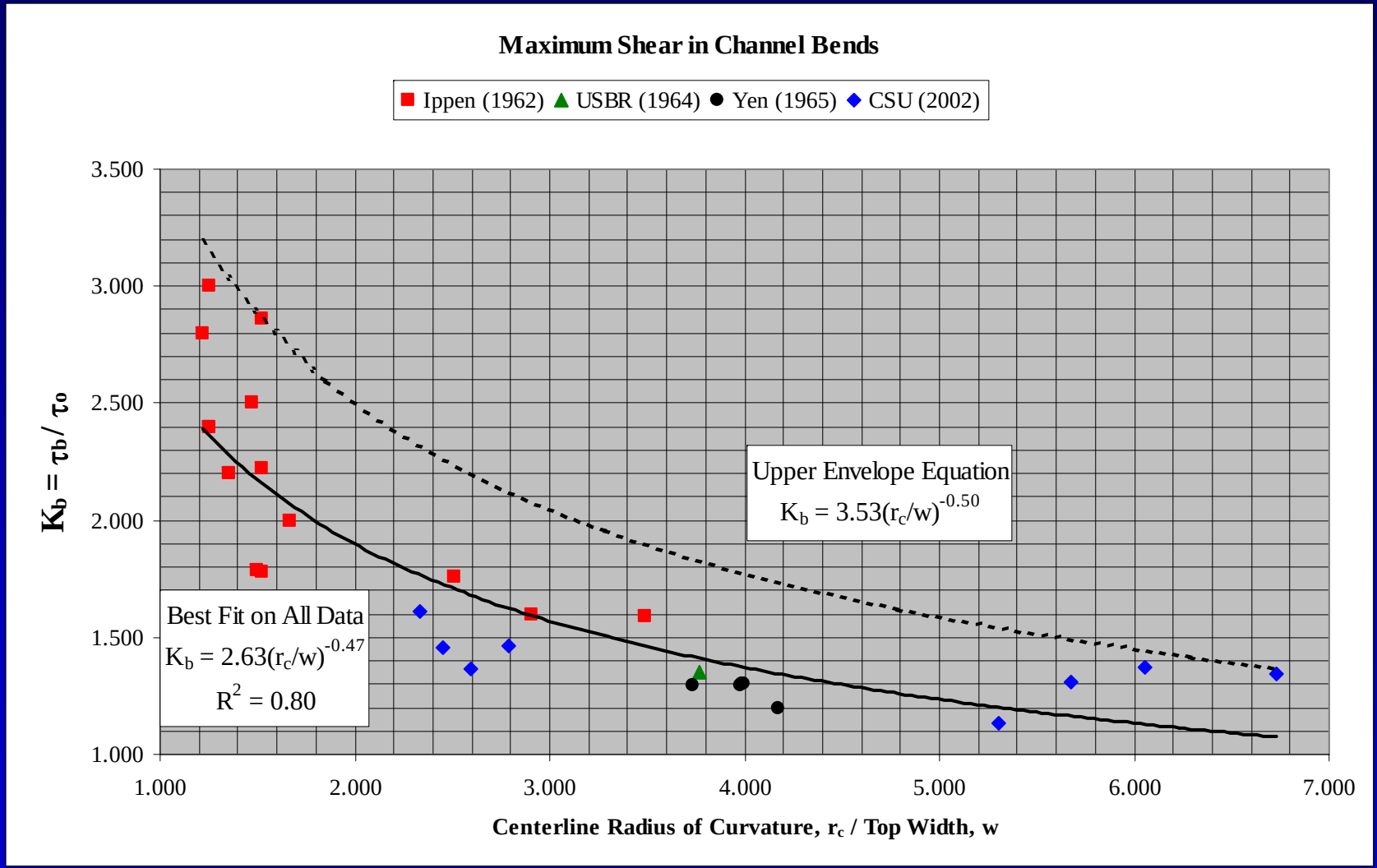
HEC-15 Method



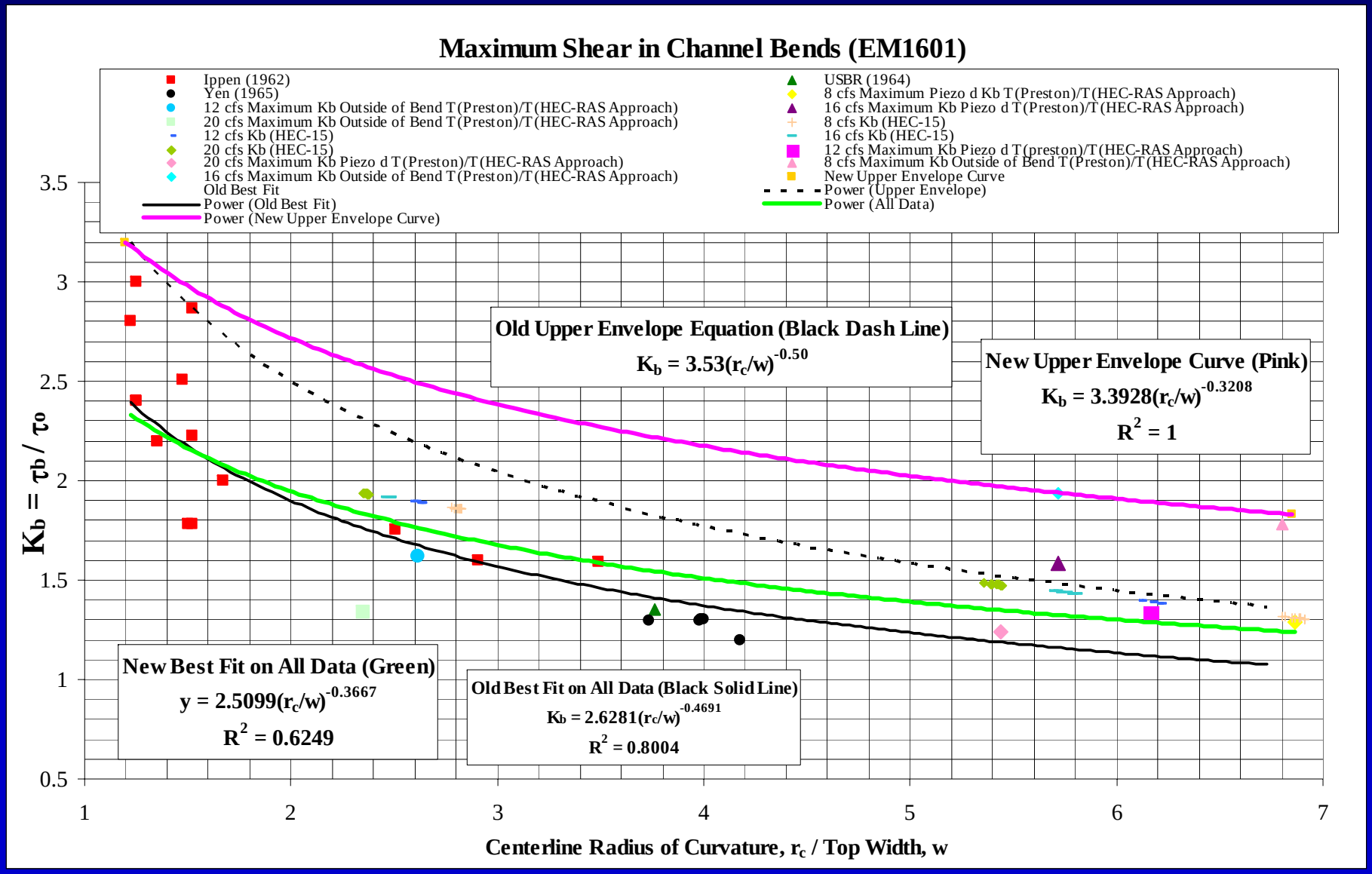
HEC-15 Method



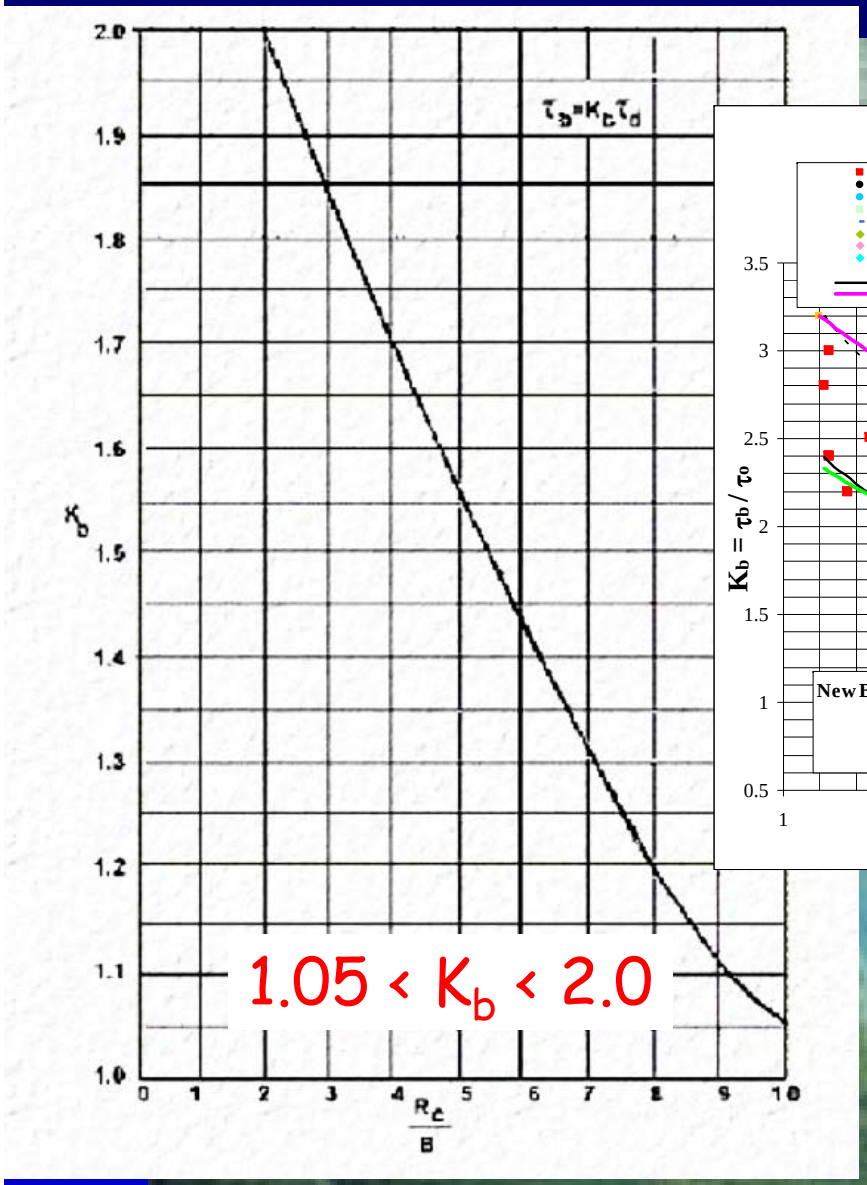
EM-1601 Method

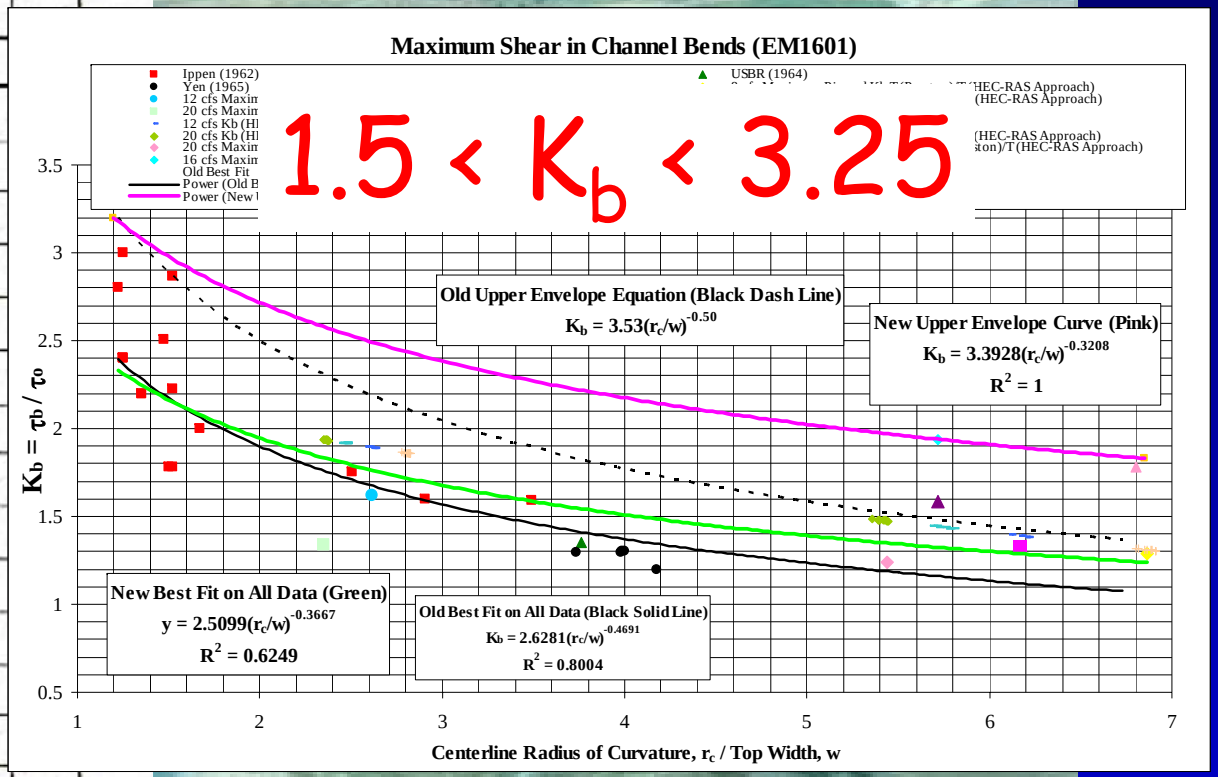


EM-1601 Method



Reality


$\tau_b = K_b \tau_d$
K_b
$\frac{R_c}{B}$
1.05 < K_b < 2.0


Maximum Shear in Channel Bends (EM1601)
Ippen (1962)
Yen (1965)
USBR (1964)
1.5 < K_b < 3.25
Old Upper Envelope Equation (Black Dash Line)
$K_b = 3.53(r_c/w)^{-0.50}$
New Upper Envelope Curve (Pink)
$K_b = 3.3928(r_c/w)^{-0.3208}$
$R^2 = 1$
New Best Fit on All Data (Green)
$y = 2.5099(r_c/w)^{-0.3667}$
$R^2 = 0.6249$
Old Best Fit on All Data (Black Solid Line)
$K_b = 2.6281(r_c/w)^{-0.4691}$
$R^2 = 0.8004$
$K_b = \tau_b / \tau_o$
Centerline Radius of Curvature, r_c / Top Width, w

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Special Considerations

- Freeboard
  - Distance from WSE to top of channel wall or containment
  - Provides factor of safety against
    - Changes in hydrology
    - Urbanization
    - Sedimentation
    - Variations in channel/overbank roughness

Special Considerations

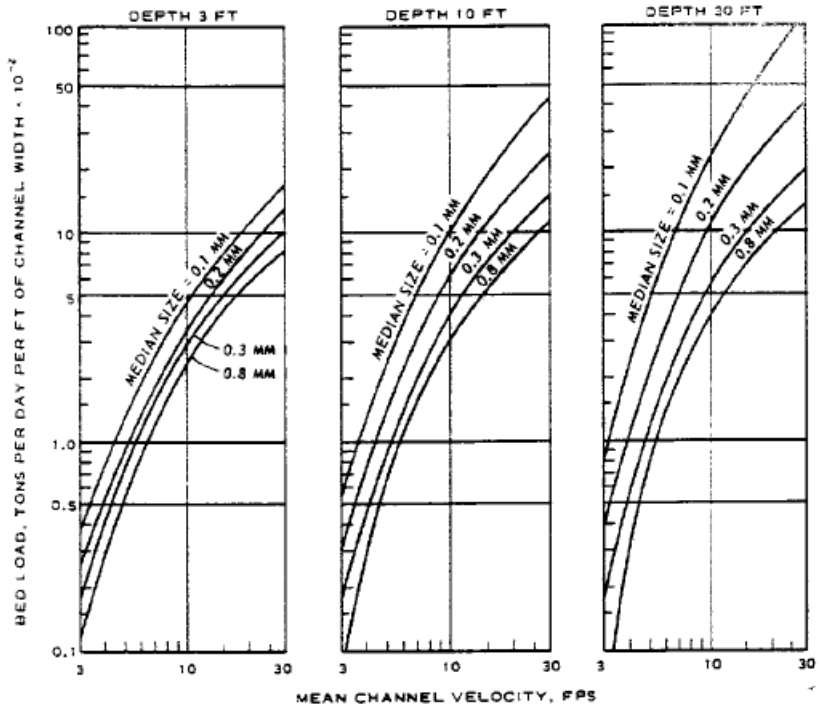
- Freeboard
  - Identify local areas where WSE may be indeterminate
    - Bridge piers
    - Hydraulic jumps
    - Transitions
    - Drop structures

Freeboard

- Amount
  - Concrete lined channels
    - Rectangular sections – 2 ft
    - Trapezoidal sections – 2.5 ft
  - Riprap channels
    - 2.5 ft
  - Earthen channels
    - 3 ft
- Consequence of damage!

Special Considerations

- Sediment Transport
  - EM-1601 provides limited information and is generally out of date
  - However..
    - Plate 27



BASIC DATA SOURCES

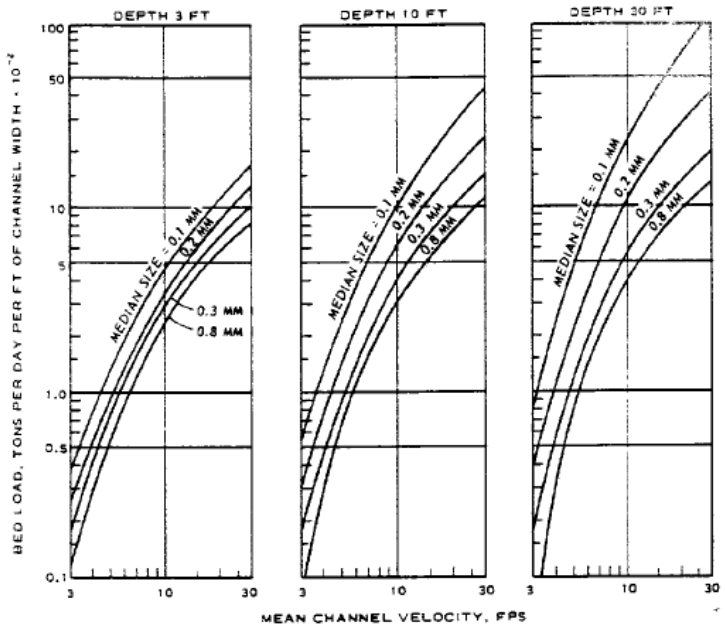
FIELD DATA		FLUME DATA	
RIVER	LOCATION	INVESTIGATOR	DATE OF TESTS
MIDDLE LOOP[a]	DUNNING, NEBRASKA	GILBERT	1914
NIOBRARA[a]	CODY, NEBRASKA	BARTON AND LIN	1955
ELKHORN	WATERLOO, NEBRASKA	SIMONS, ET AL.	1961
LOWER COLORADO	---	BROOKS	1958
PIGEON ROOST CK.	NORTHERN MISSISSIPPI		
MISSISSIPPI	ST. LOUIS, MO.		
CEDAR	NEBRASKA		
LITTLE BLUE	NEBRASKA		
NORTH LOOP	NEBRASKA		
SOUTH LOOP	NEBRASKA		
RIO GRANDE	NEW MEXICO		
RIO PUERCO	NEW MEXICO		

NOTE: SOURCES OF PUBLISHED FLUME AND FIELD DATA ARE GIVEN IN COLBY (1964A); SOME FIELD DATA HAVE NOT BEEN PUBLISHED. CURVES ARE FOR WATER TEMPERATURE OF 60 F AND NO SUSPENDED FINE SEDIMENT LOAD AND ARE EXTRAPOLATED FROM A PLOT IN THE REFERENCE.

[a] TOTAL LOAD MEASURED FOR THESE TWO STREAMS; SUSPENDED LOAD MEASURED AND BED LOAD COMPUTED FOR OTHER FIELD DATA.

Plate 27

Plate 27



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CEDAR	NEBRASKA		
LITTLE BLUE	NEBRASKA		
NORTH LOOP	NEBRASKA		
SOUTH LOOP	NEBRASKA		
RIO GRANDE	NEW MEXICO		
RIO PUERCO	NEW MEXICO		

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[a] TOTAL LOAD MEASURED FOR THESE TWO STREAMS; SUSPENDED LOAD MEASURED AND BED LOAD COMPUTED FOR OTHER FIELD DATA.

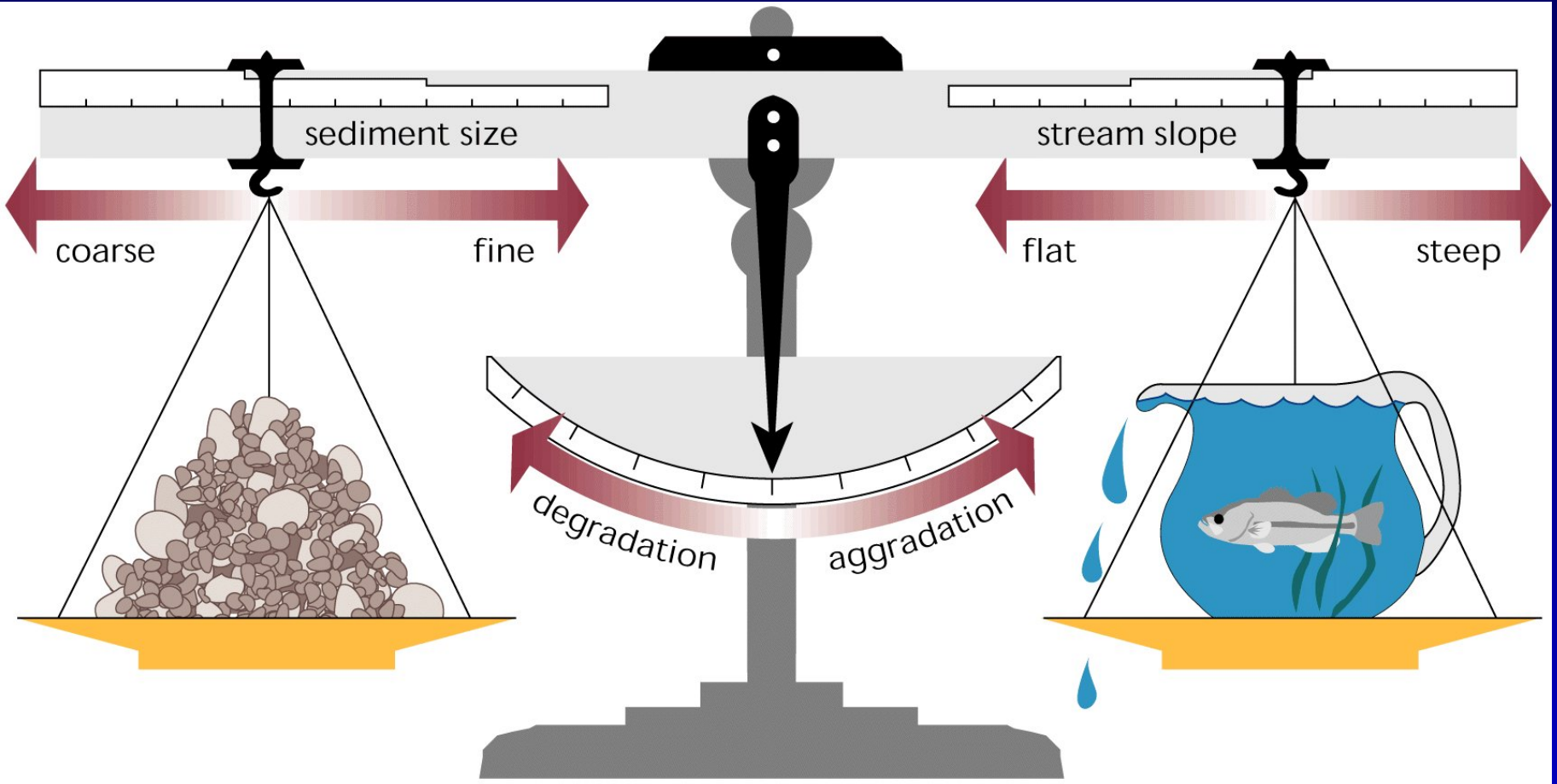
- Estimate relative effects of channel characteristics on bed-load movement
- Estimate equilibrium sediment discharge
- Estimate scour and/or deposition
- Estimate size of a detention basin

Open Channel Hydraulic Theory

- Physical Hydraulic Elements
- Hydraulic Design Aspects
- Flow Through Bridges
- Transitions
- Flow in Curved Channels
- Special Considerations
- Stable Channels

Stable Channels

- Lanes balance



$$Q_s \cdot D_{50} \propto Q_w \cdot S$$

From Rosgen (1996), from Lane, Proceedings, 1955.
Published with the permission of American Society of Civil Engineers.

Fig. 1.13 – Factors affecting channel degradation and aggradation: Concept of "Stream Balance.".
In Stream Corridor Restoration: Principles, Processes, and Practices, 10/98.
Interagency Stream Restoration Working Group (15 Federal Agencies of the US).

Stable Channels

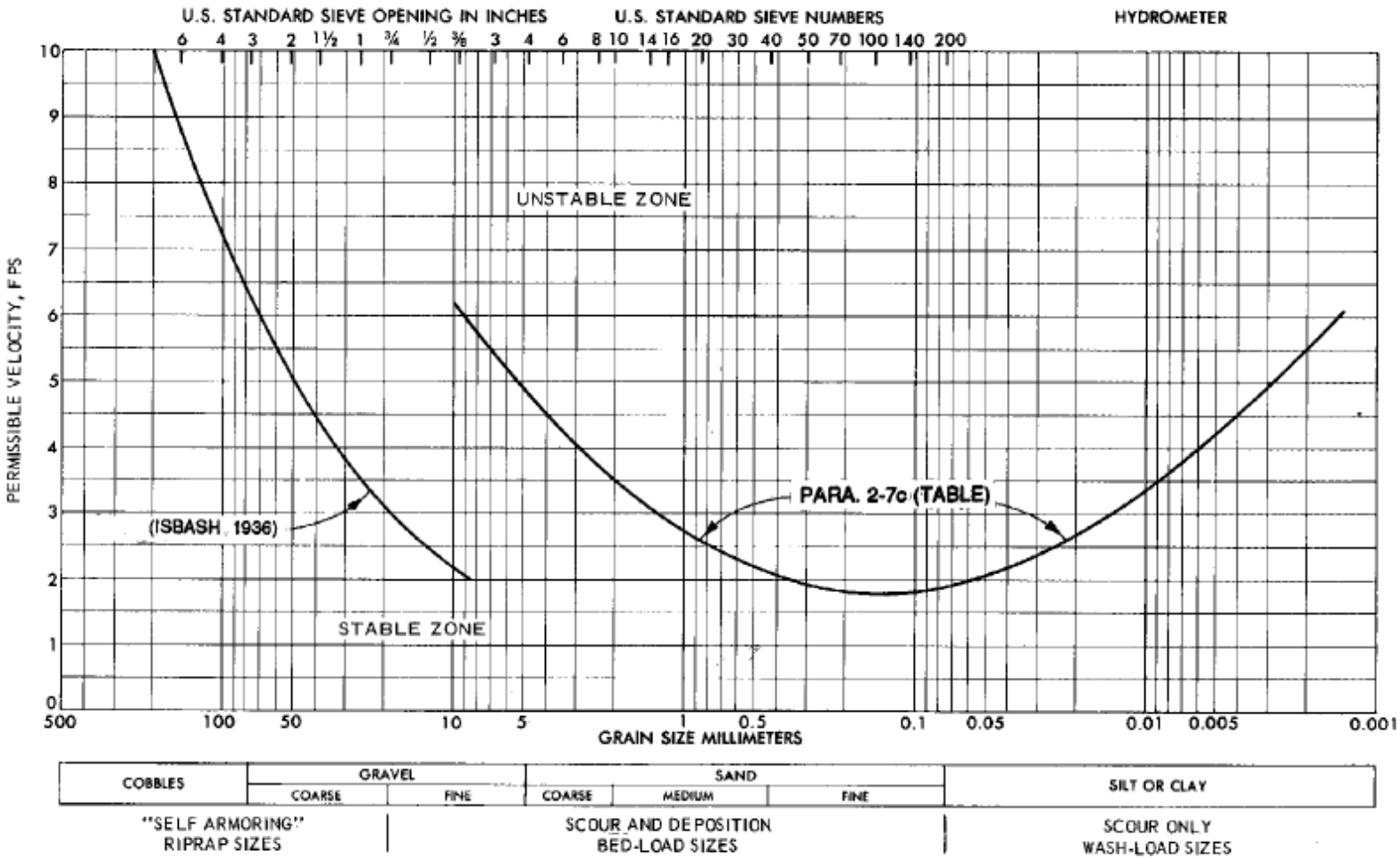
- Lanes balance
- Scour of channel bed and banks are typically main concerns
- Revetments designed through analysis of tractive force:
  - Limiting shear stress
  - Limiting velocity

Limiting Velocity

Table 2-5
Suggested Maximum Permissible Mean Channel Velocities

Channel Material	Mean Channel Velocity, fps
Fine Sand	2.0
Coarse Sand	4.0
Fine Gravel[1]	6.0
Earth	
Sandy Silt	2.0
Silt Clay	3.5
Clay	6.0
Grass-lined Earth (slopes less than 5%)[2]	
Bermuda Grass	
Sandy Silt	6.0
Silt Clay	8.0
Kentucky Blue Grass	
Sandy Silt	5.0
Silt Clay	7.0
Poor Rock (usually sedimentary)	10.0
Soft Sandstone	8.0
Soft Shale	3.5
Good Rock (usually igneous or hard metamorphic)	20.0

Limiting Velocity



NONCOHESIVE SEDIMENT GRADATION AND PERMISSIBLE VELOCITY

Plate 28

CECW-EH-D Engineer Manual 1110-2-1601	Department of the Army U.S. Army Corps of Engineers Washington, DC 20314-1000	EM 1110-2-1601 1 July 1991/ 30 June 1994
	Engineering and Design HYDRAULIC DESIGN OF FLOOD CONTROL CHANNELS	

Chapter 3

Riprap Protection