Experiments

When conducting experiments, it is almost always a good idea to take replicate measurements at the same conditions. The exact number will depend on the nature of the experiment, but you will usually want at least three replicates.

Data taken in real systems will yield different results each time, even at the same operating conditions.

How do we deal with these differences in the data?

By making use of appropriate **statistics**.

Useful statistics in uncertainty analysis

For repeated measurements, a useful statistical measure is the <u>standard deviation</u>, it shows how much variation or "dispersion" exists from the average (mean, or expected value).

The <u>mean</u> of the data values, μ, is given by

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \text{ , where N is the number of data points}$$

The <u>standard deviation</u>, σ, is then

$$\sigma = s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2} \quad \text{('corrected' sample standard deviation)}$$

However, sometimes the standard deviation is define as follows:

$$\sigma_N = s_N = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \quad \text{('uncorrected' sample standard deviation)}$$

Exercise

The following data points were taken on a system.

For either temperature or pressure, compute the mean and standard deviation of the replicate data points.

	Temperature (C)	Pressure (kPa)
replicate 1	27	211
replicate 2	25	203
replicate 3	31	180
replicate 4	22	192
mean, μ	?	?
standard deviation, σ	?	?

Exercise: Solution

|	Temperature (C)	Pressure (kPa)
replicate 1	27	211
replicate 2	25	203
replicate 3	31	180
replicate 4	22	192
mean, μ	26.25	196.5
standard deviation, σ	3.77	13.5

For the temperature:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{4}(27 + 25 + 31 + 22) = 26.25 \text{ }^\circ C$$

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2}$$

$$= \sqrt{\frac{1}{4-1}\left[(27-26.25)^2 + (25-26.25)^2 + (31-26.25)^2 + (22-26.25)^2\right]}$$

$$= 3.77$$

**Uncertainties on plots**

Error bars are a common way of representing uncertainty <u>in repeated</u> <u>measurements</u>. The <u>point</u> on the plot is usually the mean value of the data and the length of the <u>bar</u> from this point is typically the standard deviation in the data (though it can represent the standard error of the mean).

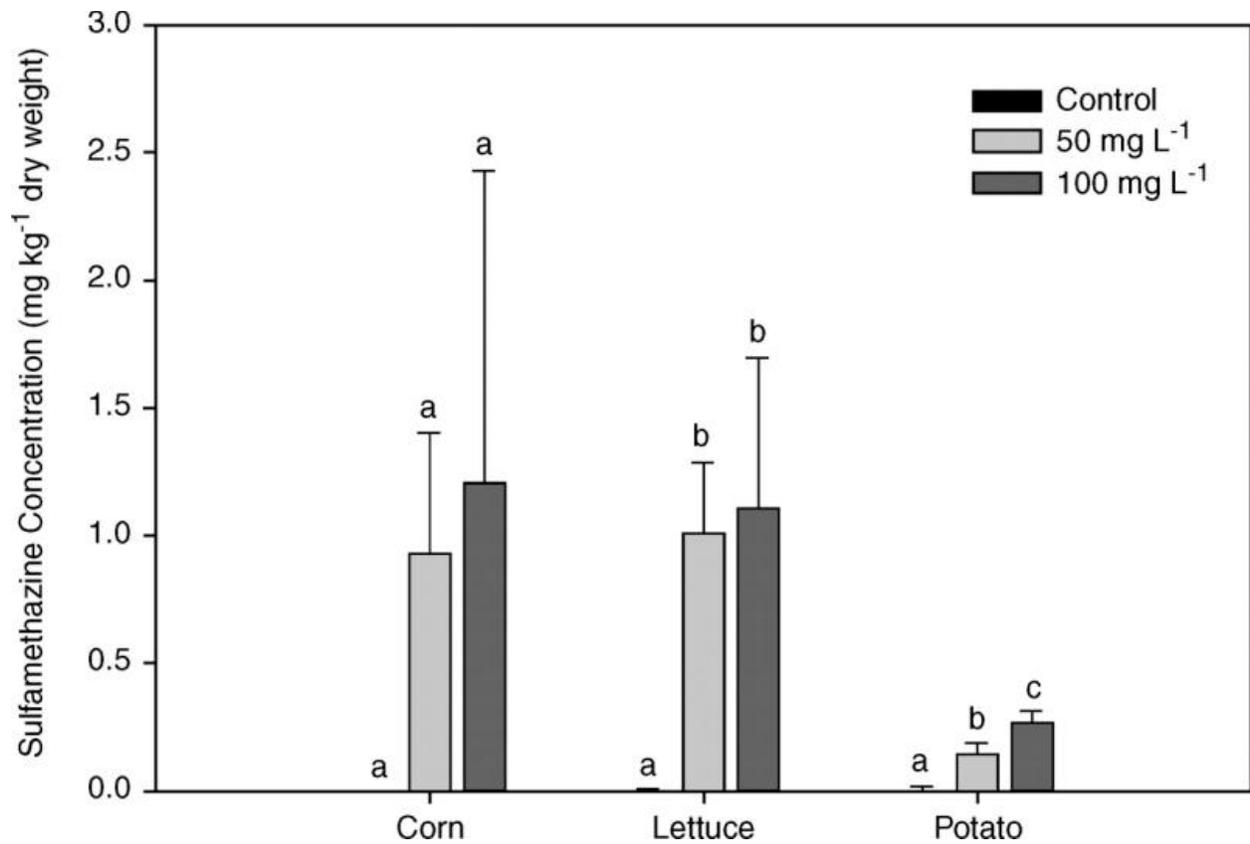In general, you will always want to show error bars in your plots of measured data.
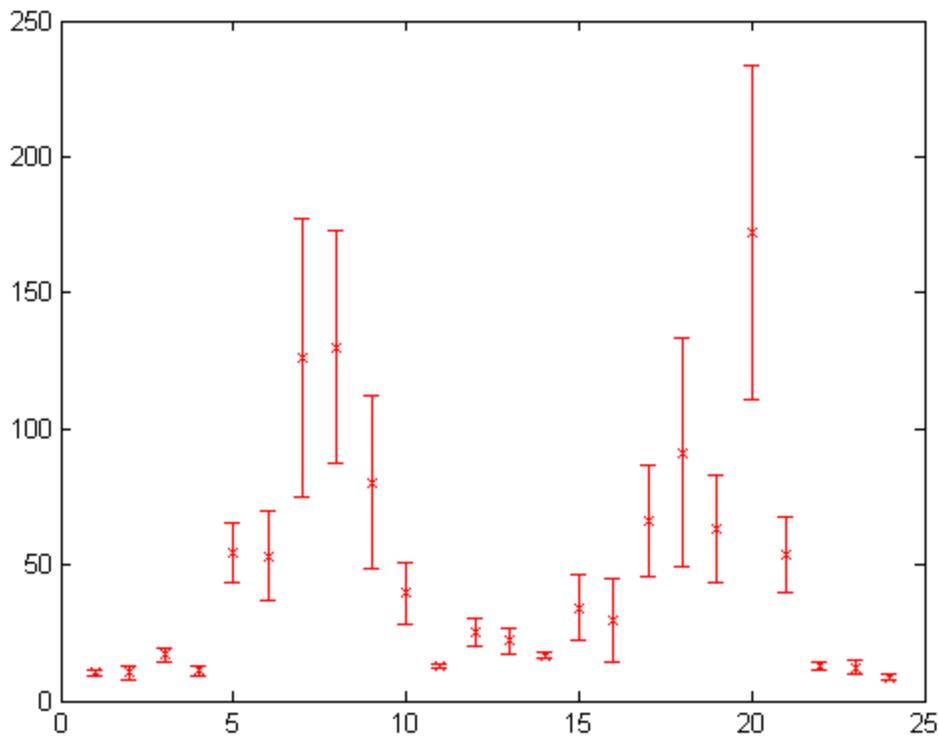
Here are some examples of plots with error bars from scientific papers:

The Effect of Fruit and Vegetable Intake on Risk for Coronary Heart Disease (Ann Intern Med. 19 June 2001;134(12):1106-1114)
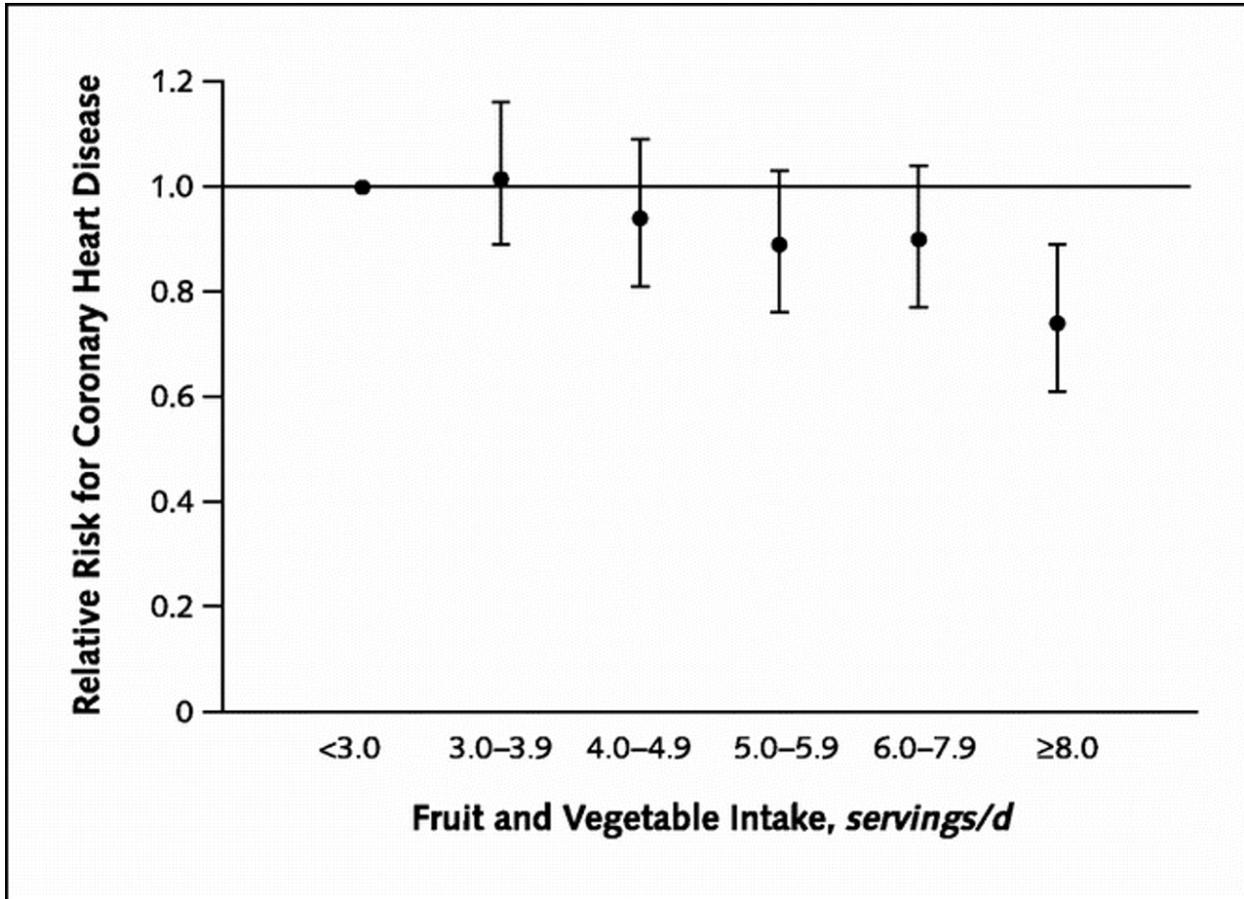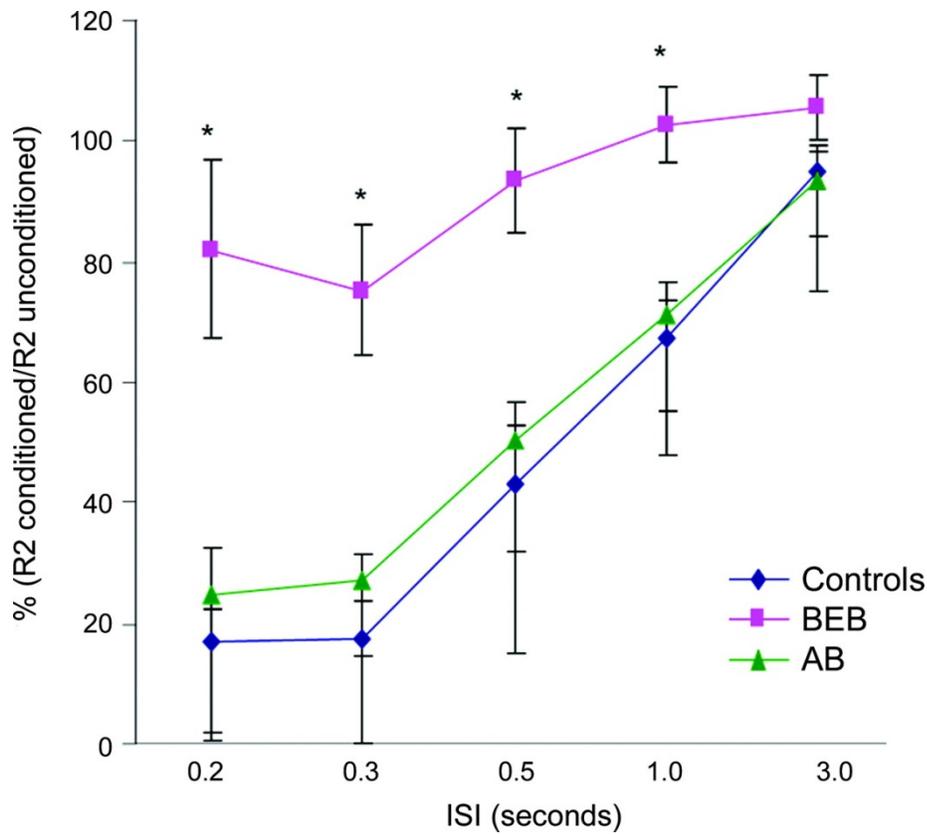

What conclusions can you draw from this plot?

Animal manure is applied to agricultural land as a means to provide crop nutrients. However, animal manure often contains antibiotics as a result of extensive therapeutic and sub-therapeutic use in livestock production.

What conclusions can you draw from this plot?

Without even knowing the specifics of the study, what conclusions can be drawn from this plot?

How significant are the differences between the controls, and the BEB and AB treatments?

Propagation of uncertainties

When you combine (add, subtract, multiple, divide) measurements with uncertainties, the uncertainties 'propagate'.

The uncertainty of the 'combined' or calculated quantity will always be larger than that of any of the individual uncertainties.

<u>Rule for the adding or subtracting quantities with uncertainties:</u>

$$q = x + \cdots + z - (u + \cdots + w)$$

$$\delta q = \sqrt{(\delta x)^2 + \cdots + (\delta z)^2 + (\delta u)^2 + \cdots + (\delta w)^2}$$

example:

$$q = (3.2 \pm 0.1) - (5.21 \pm 0.03) - (17.2 \pm 0.8)$$

$$q = (3.2 - 5.21 - 17.2) \pm ?$$

$$q = \text{-}19.21 \pm ?$$

$$\delta q = \sqrt{(0.1)^2 + (0.03)^2 + (0.8)^2} = 0.8086$$

Therefore, the answer, using appropriate significant figures, is

$$q = \text{-}19.2 \pm 0.8$$

<u>Rule for the multiplying or dividing quantities with uncertainties:</u>

$$q = \frac{x \times \cdots \times z}{u \times \cdots \times w}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}$$

example:

$q = (3.2 \pm 0.1) \times (5.21 \pm 0.03)/(17.2 \pm 0.8)$

$q = 3.2 \times 5.21 / 17.2 \pm ?$

$q = 0.9693 \pm ?$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{0.1}{3.2}\right)^2 + \left(\frac{0.03}{5.21}\right)^2 + \left(\frac{0.8}{17.2}\right)^2} = 0.0563$$

$$\frac{\delta q}{|0.9693|} = 0.0563$$
$$\delta q = 0.0546$$

Therefore, the answer, using appropriate significant figures, is

$q = 0.97 \pm 0.05$

Other rules

Other rules for powers, exponents, special functions, etc. of numbers with uncertainties are contained in most books on uncertainty analysis.

Software used for error analysis: Demo

It is essential that you know the mathematics behind uncertainty propagation before using software for these calculations.

There is a large list of software packages for uncertainty analysis on Wikipedia:

https://en.wikipedia.org/wiki/List_of_uncertainty_propagation_software

In addition, other examples are as follows:

- MATLAB: 'Propagation of Uncertainty' package
  https://www.mathworks.com/matlabcentral/fileexchange/17901-propagation-of-uncertainty

- python: 'uncertainties' module
  http://packages.python.org/uncertainties/

- Abacus
  http://www.eecs.berkeley.edu/~megens/abacus/abacus_description.html