Definitions and terminology

True value: Since the true value cannot be absolutely determined, in practice an accepted reference value is used. The accepted reference value is usually established by repeatedly measuring some NIST or ISO traceable reference standard. This value is not the reference value that is found published in a reference book. Such reference values are not “right” answers; they are measurements that have errors associated with them as well and may not be totally representative of the specific sample being measured

Accuracy is the closeness of agreement between a measured value and the true value.

Error is the difference between a measurement and the true value of the measurand (the quantity being measured).

Precision is the closeness of agreement between independent measurements of a quantity under the same conditions. It is a measure of how well a measurement can be made without reference to a theoretical or true value.

Uncertainty is the component of a reported value that characterizes the range of values within which the true value is asserted to lie. An uncertainty estimate should address error from all possible effects (both systematic and random) and, therefore, usually is the most appropriate means of expressing the accuracy of results.

Error/Uncertainty Analysis

What is an error or uncertainty?

In science, the word *error* does not carry the usual connotations of the terms *mistake* or *blunder.* Error in a scientific measurement means the inevitable uncertainty that attends all measurements.

As such, errors are not mistakes; you cannot eliminate them by being very careful. The best you can hope to do is to ensure that errors are as small as reasonably possible and have reliable estimate of how large they are.

No measurement is perfect

- The uncertainty (or error) is an estimate of a range likely to include the true value

Uncertainty in data leads to uncertainty in calculated results

- Uncertainty never decreases with calculations, only with better measurements

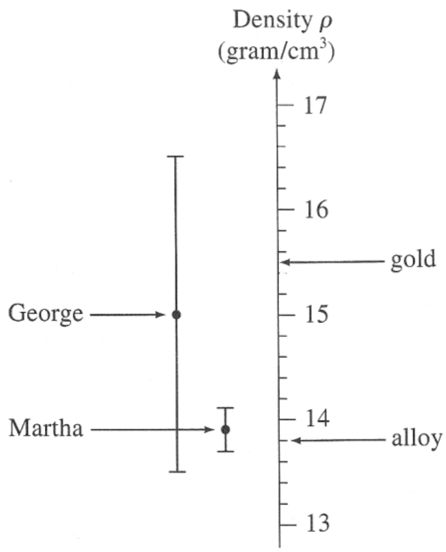
Reporting uncertainty is essential

- Knowing the uncertainty is critical to decision-making
- Knowing the uncertainty is the engineer's responsibility

Examples of the importance of understanding uncertainty

Determination of a physical property

Martha and George were assigned by the museum curator to determine if an artifact is made of gold or some alloy.

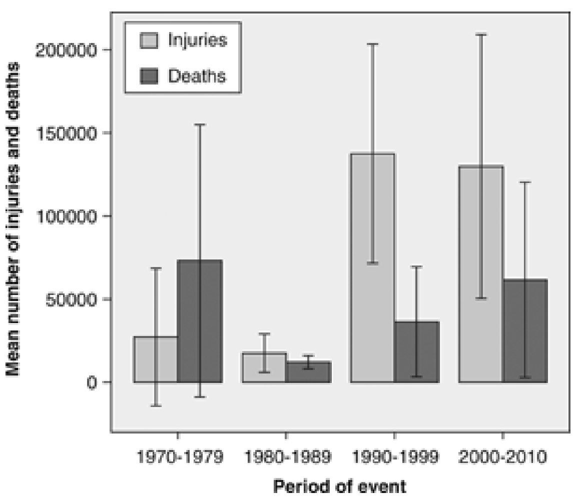


What is the artifact made of?

Can we draw a conclusion from George's measurements?

What about Martha's

Data comparing the incidence of injuries and death by decade



Can we draw meaningful conclusions when the uncertainties are so large?

Types of errors

1. Random errors: Errors inherent in apparatus.

   A random error makes the measured value both smaller and larger than the true value.

2. Systematic errors: Errors due to "incorrect" use of equipment or poor experimental design.

   A systematic error makes the measured value always smaller or larger than the true value, but not both.

   Examples:

   - Leaking gas syringes.
   - Calibration errors in pH meters.
   - Calibration of a balance
   - Changes in external influences such as temperature and atmospheric pressure affect the measurement of gas volumes, etc.
   - Personal errors such as reading scales incorrectly.
   - Unaccounted heat loss.
   - Liquids evaporating.
   - Spattering of chemicals

Reading scales

The reading error in a measurement indicates how accurately the scale can be read.

<u>analog scales</u>

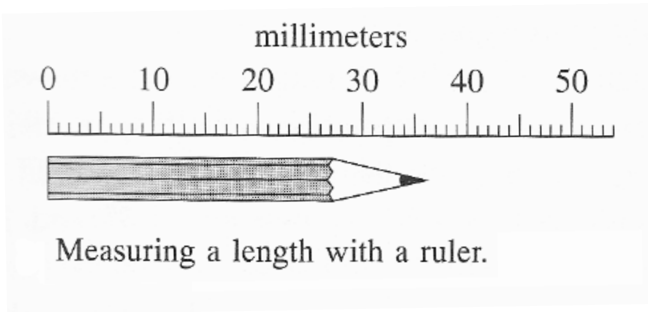
In analog readouts the reading error is usually taken as plus or minus half the smallest division on the scale, but can be one fifth of the smallest division, depending on how accurately you think you can read the scale.

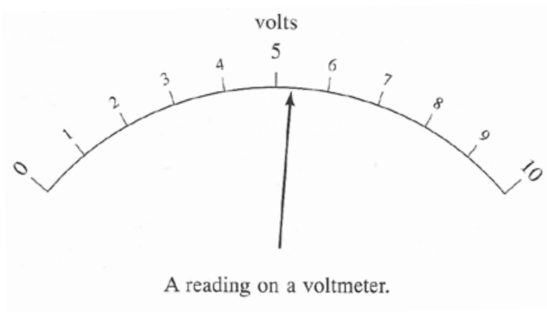
<u>digital scales</u>

In digital readouts the reading error is taken as plus or minus one digit on the last readout number.

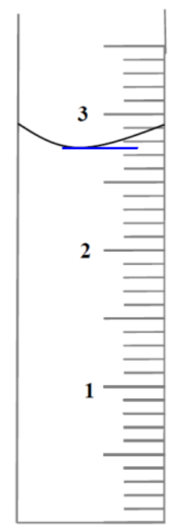
By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of plus or minus 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit - it could be a “2”; then again, it could be a “1” or a “3”. A mass of 13.20 g indicates an absolute uncertainty of plus or minus 0.01 g.

Examples





Assume that this graduated cylinder has units of ml →



Assume that this bathroom scale has units of lb:

What is your “best estimate” of the values above?

What is the uncertainty in each case?

Values from textbooks, handbooks, etc.

When we read a value in a textbook or other resource and the uncertainty is not explicitly stated, how should we estimate it?

Examples:

Textbook problem: Assume that the viscosity of the solution is 0.281×10^{-3} Pa-s and the volumetric flowrate is 10.3 cm^3/s.

Tabulated values:

Fluid	**Density** (g/cm^3)
water	0.99820
gasoline	0.66-0.69
ethyl alcohol	0.791
turpentine	0.8
glycerin	1.260
mercury	13.55

One common approach is to assume a value of ±1 in the smallest place value.

Examples:

Textbook problem: Assume that the viscosity of the solution is $(0.281 \pm 0.001) \times 10^{-3}$ Pa-s and the volumetric flowrate is 10.3 ± 0.1 cm^3/s.

Tabulated values:

Fluid	**Density** (g/cm^3)
water	0.99820 ± 0.00001
gasoline	0.66-0.69 ± 0.01
ethyl alcohol	0.791 ± 0.001
turpentine	0.8 ± 0.1
glycerin	1.260 ± 0.001
mercury	13.55 ± 0.01

Significant figures

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability.

The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures.

Significant figures are critical when reporting scientific data because they give the reader an idea of how well you could actually measure/report your data.

Exercise: Significant figures

How many significant figures are present in the following numbers?

Number
48,923
3.967
900.06
0.0004
8.1000
501.040
3,000,000
10.0

Exercise: Significant figures

How many significant figures are present in the following numbers?

Number	# Significant Figures
48,923	5
3.967	4
900.06	5
0.0004 (= 4 E-4)	1
8.1000	5
501.040	6
3,000,000 (= 3 E+6)	1
10.0 (= 1.00 E+1)	3

Reporting uncertainties

Proper specification of a quantity

Three elements must be present:

(i) 'best estimate' of the value, (ii) uncertainty in the value, (iii) and appropriate units

The standard form for reporting a measurement of a physical quantity x is

best estimate ± uncertainty units

(measured value of x) = $x_{best} \pm \delta x$ units,

where

x_{best} = (best estimate for x)

and

δx = (uncertainty or error in the measurement).

This statement expresses our confidence that the correct value of x probably lies in (or close to) the range from $x_{best} - \delta x$ to $x_{best} + \delta x$.

Rule for Stating Uncertainties

Experimental uncertainties should almost always be rounded to one significant figure.

Examples

(measured g) = 9.82 ± 0.02385 m/s^2.

(measured g) = 9.82 ± 0.02 m/s^2

measured speed = 6051.78 ± 30 m/s

measured speed = 6050 ± 30 m/s.

Rule for Stating Answers

The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.

For example, the answer 92.81 with an uncertainty of 0.3 should be rounded as

92.8 ± 0.3 .

If its uncertainty is 3, then the same answer should be rounded as

93 ± 3,

and if the uncertainty is 30, then the answer should be

90 ± 30.

Other best practices:

- Keep ‘extra’ figures during calculations, but use appropriate significant figures and uncertainties when reporting the result.

- Always write leading zeros to avoid errors and confusion:
  - write 0.543 instead of .543
  - write 0.613×10^{-9} instead of $.613 \times 10^{-9}$

Poor examples from the project reports of CBE students (Juniors)

Optical density and pH data

Time	OD	PH
0	0.042	7.871
2.4	0.056333	7.965333
5.05	0.070333	7.987
22.38	0.313	8.174667
25.16	0.378333	8.204667
47.26	0.960667	8.580333
52.24	1.13	8.663667
74.77	2.143333	9.908333
78.07	2.44	10.41667
100.12	3.933333	10.95333
123.566	5.673333	11.161
144.776	6.446667	10.81067
168.709	7.93	10.87267
191.175	8.39	10.50233
216.66	10.74	10.447
236.84	13.63333	10.684
265.606	16.2	10.34933
292.166	20.3	9.956

Drug concentrations in humans (fabricated data)

Formulation Np-098

Gender	Age	Total Dose	Time (hrs)					
(Male: 1 Female: -1)	(Years)	(mg)	0	1	2	3	4	5
1	32	700	0	15.44599	15.29663	12.52321	8.566103	4.886877
1	23	700	0	13.96295	15.57242	12.75647	8.039142	4.620582
1	18	700	0	14.63669	16.96609	13.1393	7.422146	4.451436
1	30	700	0	15.52564	17.72326	12.07007	7.605365	4.524576
1	19	700	0	15.31056	17.97465	11.40607	7.201673	4.295098
-1	31	600	0	17.48077	15.12329	11.72981	7.344404	5.201865
-1	33	600	0	17.48408	15.10815	12.71568	6.714871	5.194024
-1	33	600	0	16.31947	14.43027	12.44672	6.645299	5.476588
-1	32	600	0	16.63305	15.67287	12.92792	6.508079	5.224937

Fractional uncertainties

If x is measured in the standard form $x_{best} \pm \delta x$, the fractional uncertainty in x is as follows:

$$\text{fractional uncertainty} = \frac{\delta x}{|x_{best}|}$$

In most serious measurements, the uncertainty δx is much smaller than the measured value, x_{best} .

Because the fractional uncertainty $\frac{\delta x}{|x_{best}|}$ is therefore usually a small number, multiplying it by 100 and quoting it as the percentage uncertainty is often convenient. For example, the measurement

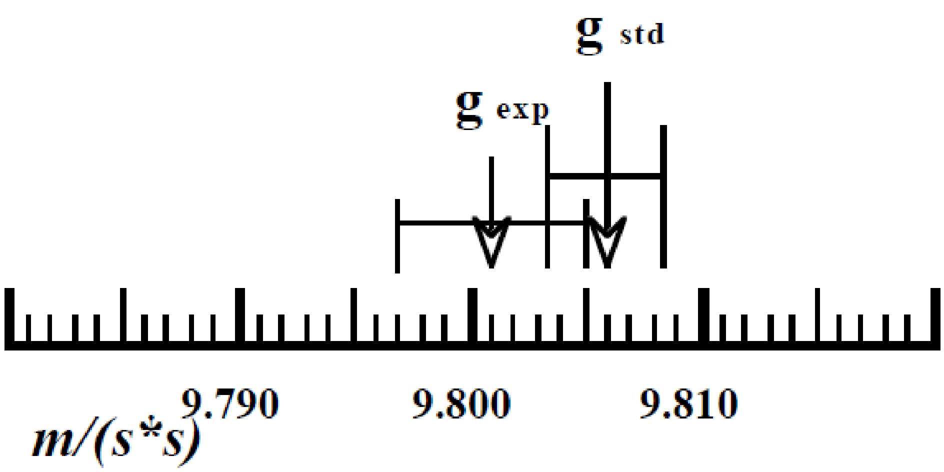
length = 50 ± 1 cm

has a fractional uncertainty of 0.02 and a percentage uncertainty of 2%. Thus, the result above could be given as

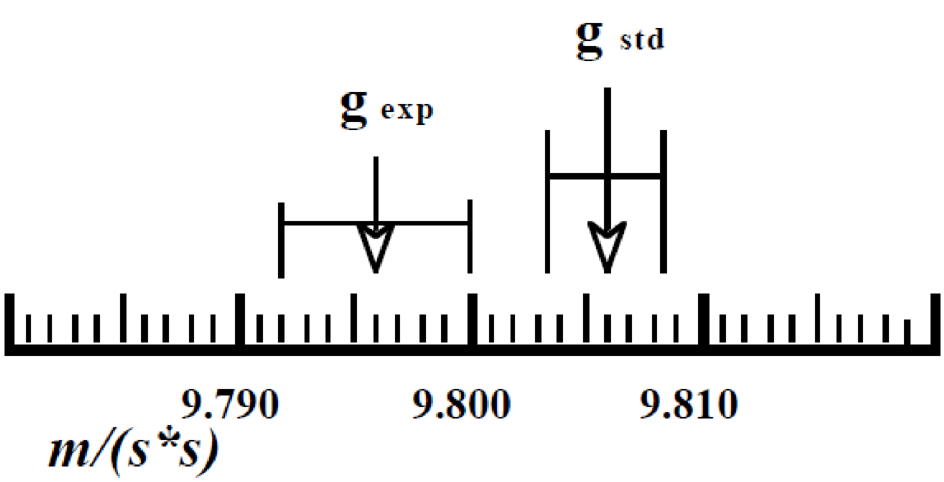
length = 50 cm ± 2%

Discrepancies

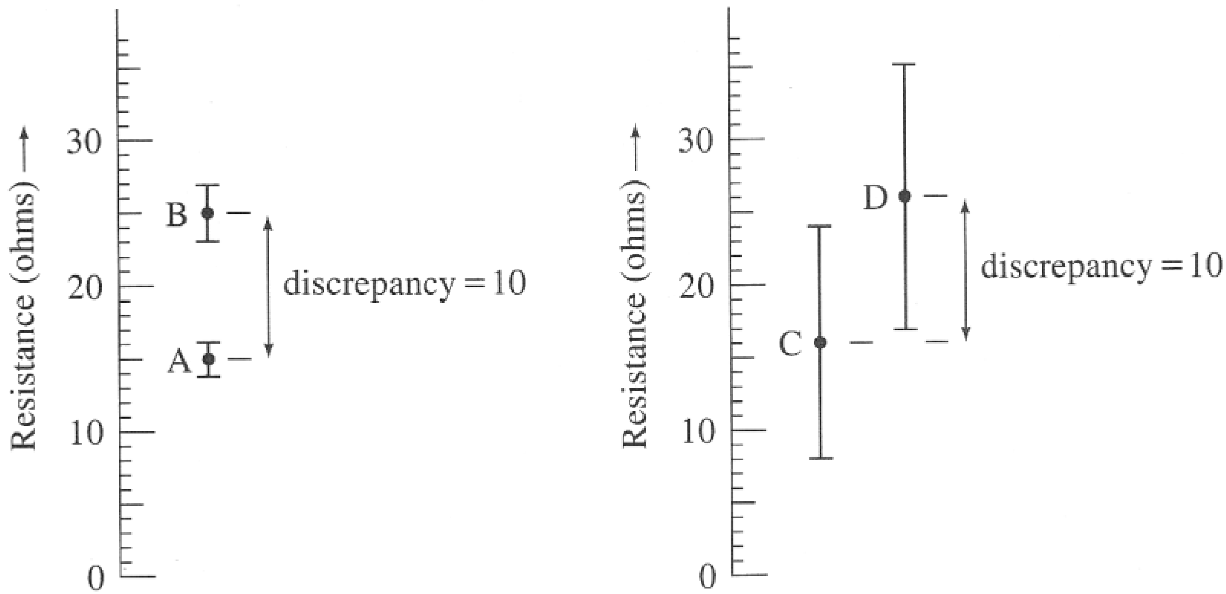
How do you judge if two measurements are significantly different?



a: two values in experimental agreement



b: two discrepant values


Resistance (ohms)
30
20
10
0
B
A
discrepancy = 10
C
D
discrepancy = 10

iClicker questions

1. When expressing a quantity, we should include what three elements?

   (a) value, precision, accuracy

   (b) value, range, precision

   (c) range, units, accuracy

   (d) best estimate, uncertainty, units

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2. Experimental uncertainties should almost always be rounded to how many significant figures.

   (a) zero

   (b) one

   (c) five

   (b) the same number as other quantities in the analysis

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3. You have a computed a mean value of density for a material of 31.276 kg/m^3 and have estimated the uncertainty to be 0.352 kg/m^3. What is the proper way to express the value of the measured density?

   (a) 31.276 $kg/m^3 \pm 0.352$

   (b) $31.28 \pm 0.35\ kg/m^3$

   (c) $31.3 \pm 0.4\ kg/m^3$

   (d) 31.276 ± 0.352

Chemical and Biological Engineering II (CBE 102)

Exercise: Error Analysis (I)

1. Rewrite the following results in their clearest forms with suitable numbers of significant figures:
   a. measured height = 5.03 ± 0.04329 m
   b. measured time = 1.5432 ± 1 s
   c. measured charge = $-3.21 \times 10^{-19} \pm 2.67 \times 10^{-20}$ C
   d. measured wavelength = 0.000,000,563 ± 0.000,000,07 m
   e. measured momentum = $3.267 \times 10^{3} \pm 42$ g-cm/s

2. My calculator gives the answer x = 1.1234, but I know that x has a fractional uncertainty of 2%. Restate my answer in the standard form, $x \pm \delta x$ properly rounded. How many significant figures does that answer really have?

3. Two students measure the length of the same rod and report the results 135 ± 3 mm and 137 ± 3 mm. Draw an illustration to represent these two measurements. What is the discrepancy between the two measurements, and is it significant?

Exercise: Solution

1.

a. 5.03 ± 0.04 m

b. There is a strong case for retaining an extra digit: 1.5 ± 1 s

c. $(-3.2 \pm 0.3) \times 10^{-19}$ C

d. $(5.6 \pm 0.7) \times 10^{-7}$ m

b. $(3.27 \pm 0.04) \times 10^{3}$ grams-cm/s

2.

1.12 ± 0.02, which has three significant figures.

3.

Discrepancy = 2 mm, which is not significant.