Celerity and Amplification of Supercritical Surface Waves

Pierre Y. Julien[1]; Noah Friesen[2]; Jennifer G. Duan[3]; and Richard Eykholt[4]

Abstract: The amplification of supercritical waves in steep channels is examined analytically using a one-dimensional dynamic solution of the Saint-Venant equations. Existing methods were modified to describe the amplification of surface waves over a normalized channel length rather than over a single wavelength. The results are strikingly different, and a generalized graph shows that short waves amplify the most over a fixed channel length. The maximum amplification parameter over a normalized channel length is 0.53 when $\mathsf{F}=3.44$. Applications to the flood drainage channel F1 in Las Vegas indicate that the amplitude of waves shorter than 100 m would increase by 65% over a channel length of 543 m. These theoretical results await field verification. Supercritical waves could be dampened by increasing channel roughness to reduce the Froude number below 1.5.

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Introduction

The hydraulic characteristics of unsteady flow in open channels have been analyzed extensively (Ferrick 2005; Field et al. 1998; Onizuka and Odai 1998; Wu et al. 1999; Odai et al. 2006; Ridolfi et al. 2006). Most studies are theoretical and very few relate to experimental or field flow conditions. Laboratory research on the formation of roll waves by Julien and Hartley (1986) showed experimental measurements on the amplification properties of surface waves in laminar sheet flow. Short surface waves amplified and resulted in roll waves in these laboratory experiments.

In turbulent flows, the Saint-Venant equations (Graf 1998; Julien 2002) are usually simplified and solved analytically to obtain kinematic, diffusive, quasi-steady dynamic, and full dynamic wave approximations (Ponce and Simons 1977; Tsai and Yen 2001; Yen and Tsai 2001; Tsai 2003). These simplified wave models are valid for specific flow conditions, typically subcritical flow (Mishra and Seth 1996; Tsai 2003; Chung and Kang 2006). Additional research has focused on the effects of downstream backwater (Chung et al. 1993; Tsai 2005), unsteady flow-rating curves (Schmidt and Yen 2001; Perumal et al. 2004), irrigation canals (Ponce et al. 1999), and the propagation of dam-break floods (Ponce et al. 2003).

[1]Professor, Dept. of Civil and Environmental Engineering, Colorado State Univ., Fort Collins, CO 80523 (corresponding author). E-mail: pierre@engr.colostate.edu

[2]Graduate Student, Dept. of Civil and Environmental Engineering, Colorado State Univ., Fort Collins, CO 80523.

[3]Assistant Professor, Dept. of Civil Engineering and Engineering Mechanics, Univ. of Arizona, Tucson, AZ 85755. E-mail: gduan@email.arizona.edu

[4]Associate Professor, Dept. of Physics, Colorado State Univ., Fort Collins, CO 80523. E-mail: eykholt@lamar.colostate.edu

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Simplified wave models present advantages over the dynamic wave model due to lower computational requirements and less detailed input data (Singh et al. 1998; Tsai 2003). Numerical schemes have been developed to solve the Saint-Venant equations (Mishra and Seth 1996; Moussa and Bocquillon 1996). These numerical models can produce quick and accurate results. There is still interest in searching analytical solutions, however, both as a means to evaluate the accuracy of the numerical models and to define asymptotic conditions to describe simplified unsteady flow dynamics (Ferrick and Goodman 1998; Lai et al. 2002).

There is a need to further investigate unsteady supercritical flow, specifically for waves that are significantly longer than the normal flow depth. This would improve the analysis of surface wave and flood wave propagation in steep channels. For instance, the Clark County Regional Flood Control District (CCRFCD 1999) operates flood control channels in the Las Vegas Valley. Designed and built on steep alluvial fans at slopes up to 3.5%, these channels convey supercritical flow at Froude numbers typically between 2–4, at their design flow discharge (Duan and Chen 2003). The hydrograph of a typical urban flood in the Las Vegas area shows a very short duration and a high peak. Better understanding of the hydraulic properties of supercritical flows is needed to design channels with sufficient freeboard to ensure the safe passage of flood surges.

This study analyzes the celerity and amplification of supercritical surface waves in steep channels. The full dynamic wave model is employed in the present study because of its general applicability to various flow scenarios in rectangular channels. The main objective is to determine the propagation of supercritical surface waves over a finite channel length rather than over a single wavelength. The main contribution is that the results of surface wave amplification over a fixed channel length may have practical implications in the design of supercritical conveyance channels. The flood drainage channels in the Las Vegas area also provide an opportunity to look at a real-world application. The F1 channel serves as an example to demonstrate the applicability of the dynamic wave model, and to provide guidelines for improved design of lined flood drainage channels.

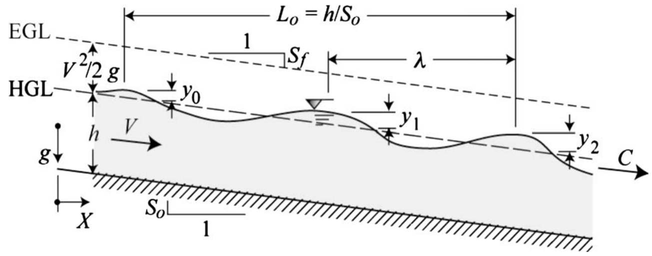

Fig. 1. Definition sketch for surface waves

Wave Amplification as a Function of Wave Number

The analysis of the amplification of surface waves is governed by the one-dimensional Saint-Venant equations (Graf 1998). These equations have been solved analytically using the small perturbation method. The approach used for this analysis is based on the recent theoretical developments by Tsai and Yen (2004). A modification thereof is proposed to present the wave amplification results over a fixed channel length in comparison with wave amplification results over a single wavelength.

General Solution to Wave Amplification as a Function of Wave Number

In such derivations the basic hydraulic parameters, as sketched in Fig. 1, include the channel bed slope S_0, the normal depth h (m), the base flow depth h_b (m), the depth-averaged flow velocity V (m/s), and the wave celerity c (m/s). The main characteristics of the surface wave include the wavelength λ (m), the period τ (s), and the wave amplitude y (m). A normalized channel length $2\pi L_0 = 2\pi h/S_0$ (m) is defined, while x (m) is the downstream distance along the channel. Results are typically presented in dimensionless form including the dimensionless celerity $c^* = c/V$, the dimensionless baseflow depth $h_b^* = h_b/h$, the dimensionless channel location $x^* = x/L_0$, the dimensionless time $t^* = tV/L_0$, the dimensionless period $\tau^* = \tau V/L_0$, the Froude number $\mathsf{F} = V/(gh)^{0.5}$, and the dimensionless wave number $\sigma^* = 2\pi L_0/\lambda$. The wave number is thus inversely proportional to wavelength, such that short waves have large wave numbers. The wave amplification parameter δ^* represents the natural logarithm of the change in wave amplitude over one period, or one wavelength, and is defined as $\delta^* = \ln\{y(x, t+\tau)/y(x,t)\}$. Values of $\delta^* > 0$ describe wave amplification and $\delta^* < 0$ denote wave attenuation.

Recent studies (Tsai and Yen 2001, 2004; Tsai 2003, 2005) have analyzed wave propagation in accelerating and decelerating flows, as well as backwater areas in reservoirs. In these studies, waves in channels having M1 and M2 profiles were examined, and the dynamic wave model was also compared with other simplified wave models (e.g., inertia and gravity waves). Previous studies were essentially limited to subcritical flows at Froude numbers less than 0.7. These equations are valid for supercritical flows as well as subcritical flows. The dimensionless wave celerity and amplification parameter are respectively given here without derivation from Eqs. (47) and (48) of Tsai and Yen (2004) as

$$c^* = [h_b^*]^{-1} + \frac{1}{\sigma^* \mathsf{F}^2}(A^2+B^2)^{1/4}\sin\left[\frac{\theta + 2\pi(j-1)}{2}\right] \quad (1)$$

$$\delta^* = 2\pi\left[\frac{\mathsf{F}^2[h_b^*]^{-2}\dfrac{dh_b^*}{dx^*} - [h_b^*]^{-1-\alpha} + (A^2+B^2)^{1/4}\cos\dfrac{\theta+2\pi(j-1)}{2}}{\left|(A^2+B^2)^{1/4}\sin\dfrac{\theta+2\pi(j-1)}{2} + \sigma^*\mathsf{F}^2[h_b^*]^{-1}\right|}\right] \quad (2)$$

where $j=1$ for a primary wave or $j=2$ for a secondary wave, and

$$A = h_b^{*-2-2\alpha} + \mathsf{F}^2[-\sigma^{*2}h_b^* - \alpha h_b^{*-3-\alpha}(dh_b^*/dx^*)]$$

$$B = \mathsf{F}^2[\sigma^*(dh_b^*/dx^*) - \alpha\sigma^* h_b^{*-2-\alpha}]$$

$$\theta = \cos^{-1}[A/(A^2+B^2)^{1/2}]$$

In this article, the base flow depth and normal depth are identical $h_b^*=1$, and only the primary wave is considered $j=1$.

Wave Amplification over a Single Wavelength for $h_b^*=1$

The solution of interest for field applications considers a base flow depth equal to the normal depth ($h_b^*=1$) without a gradient in base flow depth ($dh_b^*/dx^*=0$). Simulations will include subcritical and supercritical flows at Froude numbers ranging from 0.1–10. Flows with a Froude number between 2 and 4 are of particular interest with regard to field applications to the flood conveyance channels in the Las Vegas area. In the foregoing analysis, only the primary wave ($j=1$) is of practical interest and resistance to flow is described by Manning equation ($\alpha=4/3$). Accordingly, Eqs. (1) and (2) are reduced to the following:

$$c^* = 1 + [1/\sigma^*\mathsf{F}^2](A^2+B^2)^{1/4}\sin(\theta/2) \quad (3)$$

$$\delta^* = 2\pi\{[-1] + (A^2+B^2)^{1/4}\cos(\theta/2)/ [(A^2+B^2)^{1/4}\sin(\theta/2) + \sigma^*\mathsf{F}^2]\} \quad (4)$$

where $A = 1-[\sigma^{*2}\mathsf{F}^2]$; $B = -[4/3]\sigma^*\mathsf{F}^2$; and $\theta = \cos^{-1}[A/(A^2+B^2)^{1/2}]$.

Wave Amplification over a Fixed Channel Length

The application of Eqs. (3) and (4) to field conditions is problematic because results are expressed per single wavelength while conveyance channels are designed with a fixed channel length. Specifically, δ^* is a logarithmic amplitude increment/decrement that will occur as the wave travels over one wavelength. The actual amplification or attenuation that a wave will experience within a fixed channel length does not depend solely on δ^*. For instance, a wave with a large amplification parameter that also has a very long wavelength may amplify less in an absolute sense than a shorter wave with a smaller amplification parameter. There is thus a need to analyze wave amplification over a fixed channel length.

From Fig. 2, the amplification of shorter wavelengths over the same fixed distance depends on the number of wavelengths such that $m\delta^* = \ln[y_{t+m\tau}^*/y_t^*]$, where m=number of periods that have passed since time t. Effectively, the wave number σ^* is a measure of the number of wavelengths within a fixed channel length equal to $2\pi L_0 = 2\pi h/S_0$. The product $\sigma^*\delta^*$ is thus a measure of the total amplification/attenuation over a normalized length of $2\pi L_0$. A convenient way to express the wave amplification over a fixed channel length is simply obtained from

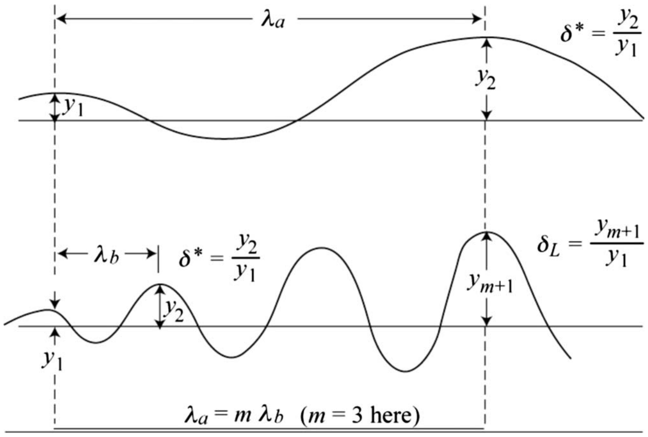

Fig. 2. Amplification over a fixed channel length for different wavelengths

$$\delta_L = \sigma^* \delta^* = \ln(y_d^*/y_u^*) \tag{5}$$

where y_d^*=downstream wave height and y_u^*=upstream wave height. The parameter δ_L determines the level of amplification ($\delta_L > 0$) or attenuation ($\delta_L < 0$) of surface waves.

Wave Amplification Results

Calculation results from Eqs. (3) and (4) with Froude numbers varying from 0.1–10 are plotted for a wide range of wave numbers on Fig. 3. Fig. 3(a) shows that the dimensionless celerity increases with wave number at low Froude numbers, as expected from Tsai (2005). However, the dimensionless celerity decreases with wave number when the Froude number is greater than 1.5. When the Froude number reaches $\mathsf{F}=1.5$, the dimensionless celerity remains a constant at 1.67 for all wavelengths. This is in agreement with the Kleitz-Seddon law for long waves with Manning resistance equation. It is also observed that the celerity of short waves decreases with the Froude number. Indeed, for very short waves, the dimensionless celerity asymptotically approaches $c^* = 1 + 1/\mathsf{F}$, which is obtained as the limit of Eq. (3) as σ^* approaches ∞. As a limit, the dimensionless celerity of short waves thus approaches unity as the Froude number becomes infinitely large.

Figs. 3(b and c), respectively, show the amplification parameters over a single wavelength and over a normalized channel length $2\pi L_0$. Fig. 3(b) shows that surface waves with $\mathsf{F} < 1.5$ always attenuate. When the Froude number equals 1.5, the amplification parameter remains 0 for any wave number or wavelength. The amplification of surface waves is a concern in the design of supercritical flow channels. As the Froude number increases above 1.5, the amplification increases, and the peak of the wave amplification over a single wavelength moves toward smaller wave numbers, thus longer wavelengths. The results in Fig. 3(b) imply that a relatively narrow range of wavelengths could be defined for maximum wave amplification at a given Froude number. Fig. 3(b) also indicates that the wave amplification over single wavelengths becomes negligible for short waves (high wave numbers).

Strikingly different results are shown on Fig. 3(c) when considering the wave amplification parameter δ_L over a normalized channel length $2\pi L_0$. The plot of δ_L illustrates that when growth is examined over a fixed channel length, short waves that were previously noted to have a negligible amplification over a single wavelength will have a greater amplification than long waves. This is perhaps the most important finding in this article. Conversely, for long waves (low wave numbers) that showed considerable growth in Fig. 3(b), the amplification over a normalized channel length will remain close to 0. In the design of supercritical flow channels, the wave amplification results shown in Fig. 3(c) should be used. Accordingly, the amplification of supercritical waves from Fig. 3(c) gradually increases with the wave number σ^* and asymptotically reach a constant value for short waves (large wave numbers). This asymptotic value of δ_L can be obtained from the limit of Eq. (4) as $\sigma^* -> \infty$ which yields

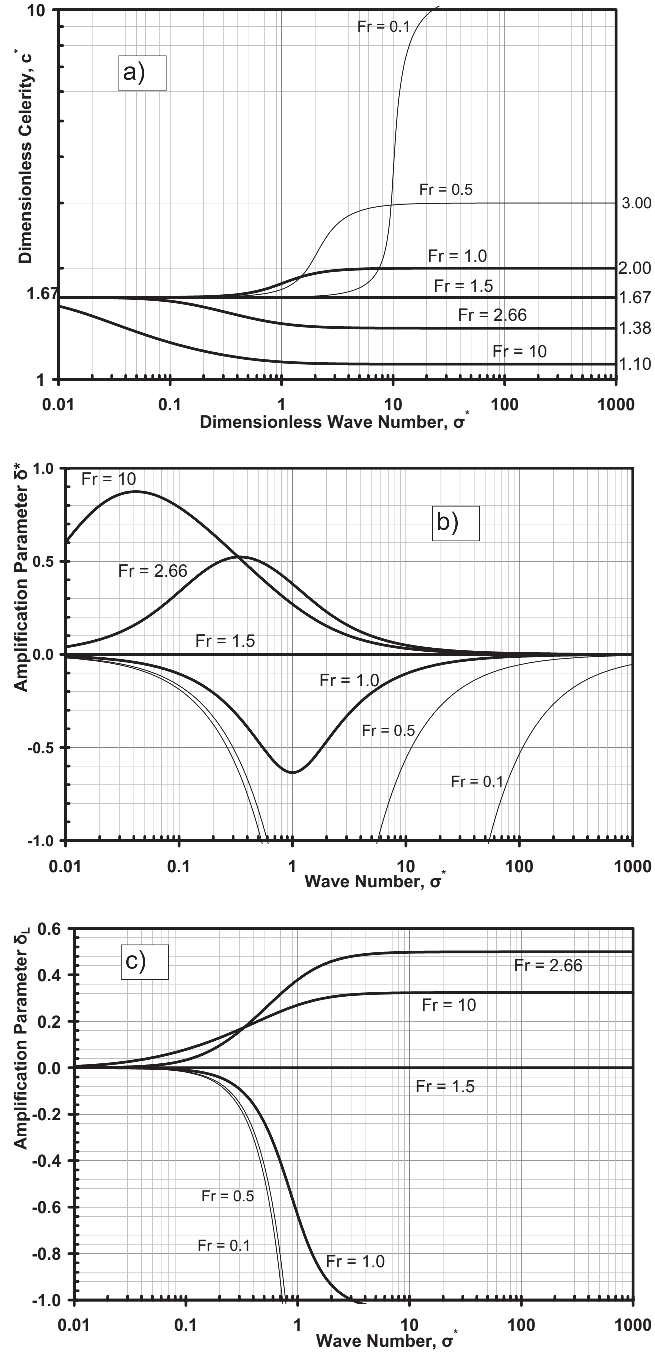

Fig. 3. (a) Dimensionless celerity c^*; (b) amplification parameter over a single wavelength δ^*; and (c) amplification parameter $*_L$ over a normalized channel length as a function of wave number σ^* and Froude number F

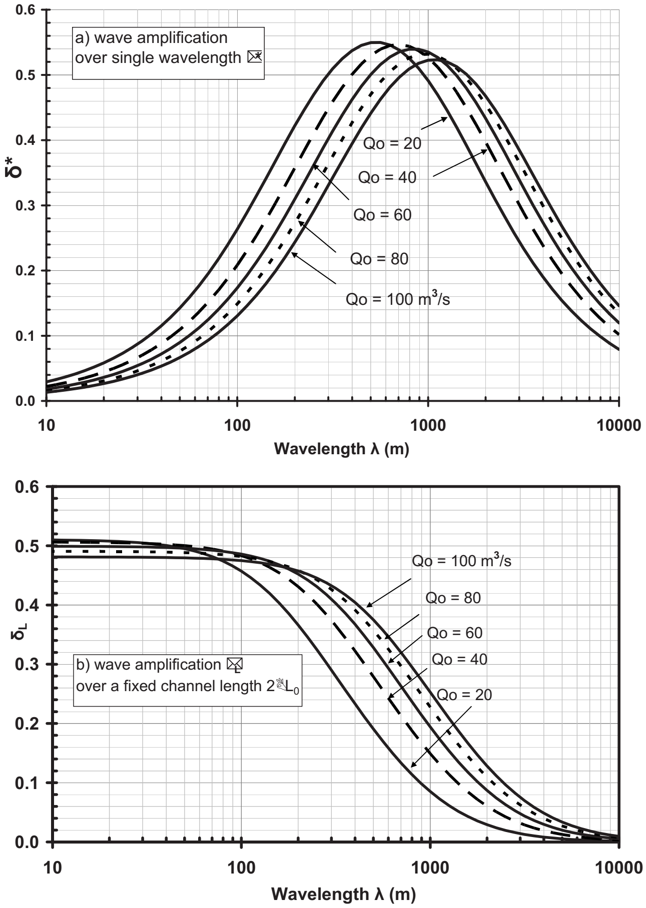

Fig. 4. Dimensionless amplification parameter (a) over a single wavelength δ^*; (b) over a fixed channel length δ_L for different wavelengths λ and flow discharges Q_0 in m^3/s for the F1 channel

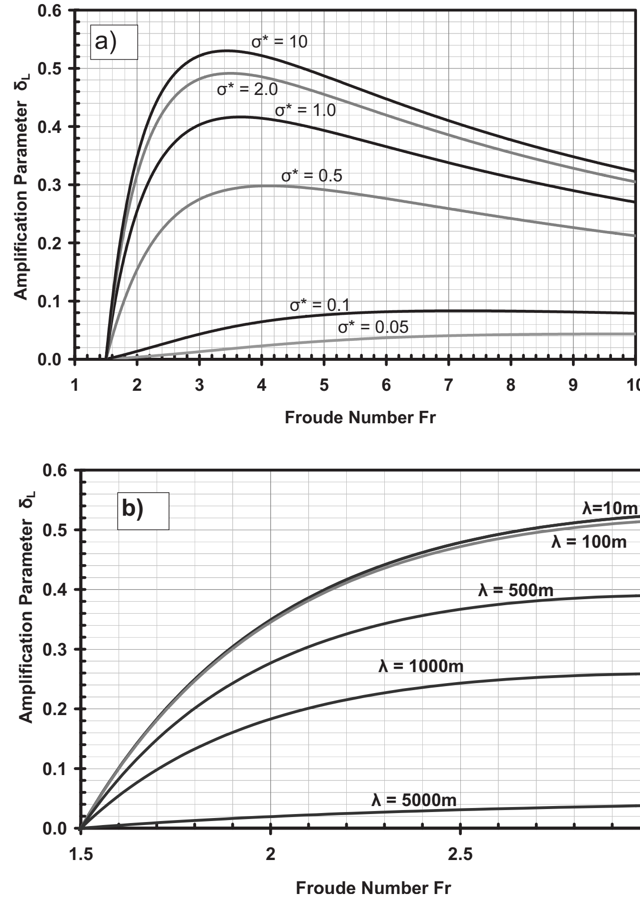

Fig. 5. Wave amplification parameter δ_L over a fixed channel length versus Froude number F (a) as a function of wave number σ^*; (b) at different wavelengths λ for the F1 channel

$$\delta_L = 2\pi[-1 + (2/3)\mathsf{F}]/[\mathsf{F} + \mathsf{F}^2] \qquad (6)$$

As expected, the numerator indicates that there is no amplification when $\mathsf{F}=1.5$ and surface waves attenuate ($\delta_L<0$) when $\mathsf{F}<1.5$. On the other hand, waves amplify when $\mathsf{F}>1.5$ and the amplification reaches a maximum value at $\mathsf{F}=3.44$ obtained from the first derivative of Eq. (6). This maximum wave amplification parameter is $\delta_{L\,\max}=0.53$.

Field Applications for the F1 Flood Channel in Las Vegas

The F1 flood channel is a concrete-lined open channel designed for conveying a 100-year design flood in the Las Vegas Valley (Duan and Chen 2003). The cross section is rectangular with a bottom width of 4.0 m and a downstream slope of 0.025. The Manning roughness coefficient n is 0.014 and the design discharge is 93.4 m^3/s. This corresponds to a normal flow depth of 2.0 m and normal flow velocity of 11.3 m/s, and a Froude number $\mathsf{F}=2.55$. The equations for calculating the dimensionless amplification parameters were employed with flow discharge varying from 20 to 100 m^3/s to encompass a range of discharges around the design flood discharge. Base flow was assumed uniform and equal to the normal depth, thus $h_b^*=1$, and the Froude number for all flow rates was between 2.5 and 2.8 with an average of 2.66. A large range of wavelengths from 1–10,000 m was investigated by Friesen (2007).

In Fig. 4(a), the amplification parameter over a single wavelength reaches a maximum value for wavelengths of about 800 m. The amplification parameter slightly shifts toward longer wavelengths at higher discharges. This diagram indicates that longer waves tend to amplify more over a single wavelength than shorter wavelengths. When considering wave amplification over the normalized channel length $2\pi L_0=2\pi h/S_0=543$ m, however, the results in Fig. 4(b) are strikingly different. Indeed, all wavelengths shorter than about 100 m will have the maximum amplification over the reach length. The maximum amplification is about 0.5, which corresponds to $e^{0.5}=1.64$ or a 64% increase in wave amplitude over this 543-m reach length. The amplification parameter over the fixed channel length is not very sensitive to flow discharge.

The F1 channel is designed at a Froude number $\mathsf{F}=2.55$ which is relatively close to the flow condition ($\mathsf{F}=3.44$) for the maximum amplification of surface waves. For a channel designed at a flow depth of 2 m plus freeboard (1 m), waves of small amplitude could overtop the channel at the design discharge. In practice, the possibility of increasing channel roughness in order to lower the Froude number below 1.5 could be considered. Fig. 5 shows the amplification over the normalized channel length as a function of the Froude number for a range of wavelengths. This figure definitely shows that the amplification of supercritical surface waves would be significantly reduced as the Froude number approaches 1.5. Decreasing the Froude number to a value of 1.5, where no amplification occurs, would simply require an increase in channel roughness from $n=0.014$ to 0.022.

These results of wave celerity and amplification in supercritical flows are the best that can be analytically derived at this time. Experimental verification of these theoretical results is awaited. Field and laboratory measurements will either confirm the results or lead to substantial improvements of the underlying theory. Meanwhile, field applications of these theoretical developments should be used with those limitations in mind.

Summary and Conclusions

The hydraulic properties of surface waves are studied for a broad range of Froude numbers from 0.1 to 10. This analysis specifically focuses on the celerity and amplification of supercritical surface waves propagating in steep open channels. The small perturbation method of the Saint-Venant equations is modified to find analytical solutions for wave growth over fixed channel lengths. The results presented here are specifically focused on supercritical flows in open channels where Manning's resistance equation is applicable. The main conclusions are the following:

1. Surface waves attenuate at Froude numbers less than 1.5 and amplify when the Froude number exceeds 1.5. Over a fixed channel length, short wavelengths always amplify more than long wavelengths. The celerity of short wavelengths reduces to approximately $c=(1+1/\mathsf{F})V$.
2. When considering the amplification over a single wavelength, very long and very short waves will have an amplification parameter near 0. The modification to describe surface wave amplification over a fixed channel length yields completely different results. As shown in Fig. 3(c), short waves amplify the most over a fixed channel length. The maximum amplification parameter over a normalized channel length $2\pi L_0=2\pi h/S_0$ is 0.53. This is obtained when $\mathsf{F}=3.44$ as shown from Eq. (6).
3. In terms of applicability to the F1 channel in Las Vegas, all wavelengths shorter than 100 m would have a 65% increase in wave amplitude over a normalized channel length of 543 m. The possibility of increasing resistance to flow to reduce the Froude number below 1.5 should be considered. These theoretical results await verification with field measurements.

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Notation

The following symbols are used in this technical note:

A = dummy variable used in Eqs. (1)–(4);
B = dummy variable used in Eqs. (1)–(4);
c = surface wave celerity (L/T);
$c^*=c/V$ = dimensionless wave celerity from the ratio of celerity to mean velocity;
$\mathsf{F}=V/\sqrt{gh}$ = Froude number;
g = gravitational acceleration (L/T^2);
h = flow depth (L);
h_b = base flow depth (L);
$h_b^*=h_b/h$ = dimensionless base flow depth, $h_b^*=1$ in this paper;
j = indicator of primary or secondary waves, $j=1$ in this paper;
$L_0=h/S_0$ = normalized channel length;
m = number of waves within a reach length;
n = Manning roughness coefficient;
S_0 = channel bed slope;
t = time (T);
$t^*=tV/L_0$ = dimensionless time;
V = depth-averaged flow velocity (L/T);
x = downstream distance along the channel (L);
$x^*=x/L_o$ = dimensionless location in the channel;
y = wave amplitude (L);
α = friction equation coefficient, $\alpha=4/3$ for Manning equation;
θ = dummy variable used in Eqs. (1)–(4);
λ = wavelength (L);
$\sigma^*=2\pi L_0/\lambda$ = dimensionless wave number;
τ = wave period (T);
$\tau^*=\tau V/L_0$ = dimensionless wave period;
δ_L = wave amplification/attenuation parameter over a normalized channel length $2\pi L_0$; and
δ^* = wave amplification parameter (+) or attenuation parameter (−) over a single wavelength.

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