NSF Workshop on Control Co-Design (CCD)

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Outline

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What should control researchers know about design research?

→ Optimization Model-based CCD

- Many control co-design (CCD) approaches leverage model-based CCD
- Often this is further structured as *optimization model-based CCD*
  - This comes with the value, goal, or objective measure(s)
  - We also must consider what is possible — the constraints

$$\begin{aligned}
\text{changing:}\quad & \text{plant decisions, control decisions} && \text{(1a)}\\
\text{(maximize or) minimize:}\quad & \text{goal} && \text{(1b)}\\
\text{subject to:}\quad & \text{what is possible} && \text{(1c)}
\end{aligned}$$

- Let's add some more structure to the CCD problem:

$$\begin{aligned}
\text{changing:}\quad & \text{plant decisions } \boldsymbol{x}_p, \text{ control decisions } \boldsymbol{x}_c && \text{(2a)}\\
\text{(maximize or) minimize:}\quad & \text{goal } J(\boldsymbol{x}_p, \boldsymbol{x}_c) && \text{(2b)}\\
\text{subject to:}\quad & \text{physical design-only constraints } \boldsymbol{g}_p(\boldsymbol{x}_p) && \text{(2c)}\\
& \text{control design-only constraints } \boldsymbol{g}_c(\boldsymbol{x}_c) && \text{(2d)}\\
& \text{coupled system constraints } \boldsymbol{g}_s(\boldsymbol{x}_p, \boldsymbol{x}_c) && \text{(2e)}
\end{aligned}$$

→ Design Coupling and Synergy Mechanisms

- We are here because of the purple coupled parts — they enable the investigation of design coupling and synergy mechanisms
- *Design coupling* — How design decisions in one domain influence the ideal design decisions in other domains
  - For example, plant decisions might impact controller gains, or control decisions modify the states that force the plant decisions to change
  - Is it strong or significant? Is it captured by the optimization model?
- *Synergy mechanism* — A specific underlying design mechanism that facilitates overall system performance improvements when two or more design elements are varied synergistically[1]
  - In wind energy, CCD enables the synergistic reduction in tower size with better-controlled maintenance of the optimal tip speed ratio, structural deflections, and stress[2]
- Certain simplified system dynamics (such as steady-state or pseudostatic models) or static analysis that neglects dynamic effects altogether don't readily support these ideas[3]

[1] Allison, Herber, and Deshmukh 2015 [2] Deshmukh and Allison 2015 [3] Allison and Herber 2014

→ What Should Our Goals Be? (Or What Have Our Goals Been?)

- What determines the system's value that we are driving at through a CCD perspective?
- Are we looking to understand trade-offs (multi-objective perspectives)?
- Sometimes the result of these questions leads to separable "design"-focused goals J_p and "control"-focused goals J_c
- In other CCD application spaces, such a distinction might be unnatural or unnecessary
  - In energy systems, this might be the levelized cost of energy (LCOE)[1]
  - Other areas are cost-driven (minimize cost within prescribed specifications)
- Still, limited or simplified consideration of the dynamics and controls occurs
  - For example, designing a counterbalanced robotic manipulator as a proxy for minimizing energy consumption[2]
- Overall we might consider appropriately "balanced" CCD approaches
  - Ones that identify the key system-level goals without undue influence of *either* area

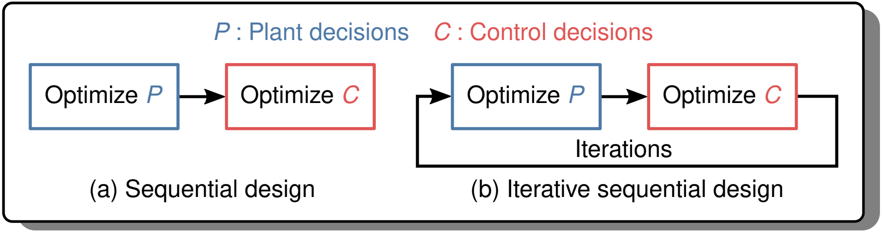
[1] Sundarrajan, Lee, et al. 2021 [2] Allison and Herber 2014

Consider the Limits

- A common perspective in the design community is understanding feasible system solutions through limits
  - Inequality constraints in the optimization context
- These might be simple bounds ($x \leq a$) or more complicated constraints on our independent decisions or derived quantities (e.g., outputs, states, and control signals)
  - Examples include cost, mass, geometric dimensions, deflection, stress, fatigue, packaging, temperatures, power, actuator limits, etc.
- Drivers are often failure theories, manufacturing limits, stakeholder preferences, or even engineering judgment

? A question then is what are effective CCD strategies assuming these concerns?
  - Many popular control paradigms don't directly handle such concerns
  - This has led some CCD researchers to explore methods with this specific situation in mind

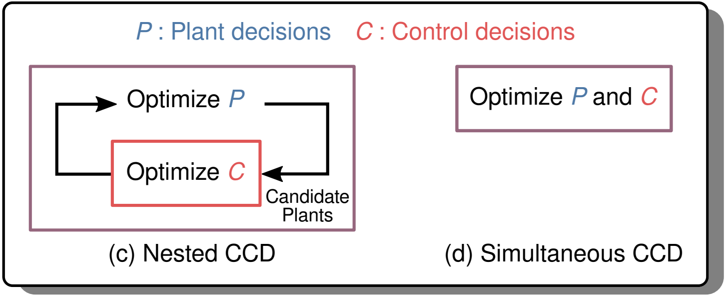
→ How Should We Solve It? — Sequential Perspective



- *Sequential design* — Determine the plant first, controller second ✖
- *Iterative sequential design* — Now we pass control design results back for plant redesign and iterate
  - What is communicated back? Might be a fixed controller and/or insights into changes related to the physical-design domain
  - This approach can suffer from slow convergence and well-posedness issues

? Can we do better?

→ How Should We Solve It? — Simultaneous and Nested CCD Strategies



- *Nested CCD* — Ask the question, if I made this physical system, what would the best controller be? This is the essence of the nested approach[1]
  - Embedded inner-loop optimization problem (control subproblem) within the outer loop
- *Simultaneous CCD* — Consider both at the same time in one problem
  - Could follow many paths toward the system-level optimum

[1] Herber and Allison 2018

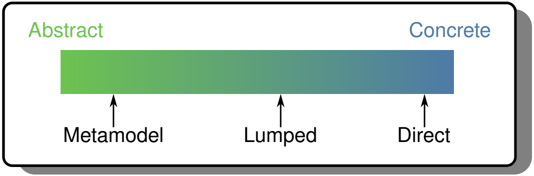
→ Simultaneous and Nested CCD Strategies — Which One?

1. **X** is faster and more scalable than **Y** — *It depends!*[1]
   - A poorly implemented **Y** is worse than a well implemented **X**
2. **X** is easier to implement than **Y** — *It depends!*
   - Sometimes it is easier to create one problem with simultaneous CCD
   - Sometimes it is easier to partition based on an existing control design technique for the inner-loop subproblem
3. **X** is more robust and accurate than **Y** — *It depends!*
   - Simultaneous CCD has more flexibility to explore since infeasibility is allowed while iterating/solving
   - Nested CCD can support hybrid approaches with focused exploration (often the physical design parameters) but might fail to converge if the inner loop does not always have a solution
4. **X** will result in the same solution than **Y** — *It depends!*
   - Many CCD problems do not readily support “nice” formulations
   - In certain CCD problems, concerns regarding local optima are valid

[1] Sundarrajan and Herber 2021

→ Modeling for Effective and Balanced CCD (1)

- Lots of ways to model and represent change to our physical systems or plants
- However, there is sometimes a disconnect between the more “controls-centric” plant modeling needs and the model concerns of physical system realization



- *Metamodel* — coefficients in a state-space model or transfer function
- *Lumped model* — physics-driven intermediate parameters
  - For example, the spring constant k in the $a_{2,3}$ coefficient k/m of the state-space model
- *Direct model* — independent decisions, more closely connected to manufacturing
  - Instead of k from before, we might consider the spring wire diameter directly

→ Modeling for Effective and Balanced CCD (2)

- More abstract representations might be considered plant requirements or targets, but the issue comes when there is a disconnect between this CCD result and what is physically possible, especially when plant-design constraints are ignored[1]
- This isn't to say there isn't value in more abstract CCD problems — we should consider the realizability of the outcome
- Linear vs. nonlinear models, low vs. high fidelity models — ensure that system performance assessment is sufficiently close to reality even if the primary (control) design methods are based on linear theory
  - Overly simplified plant models might not enable sufficient exploration and exploitation of design coupling

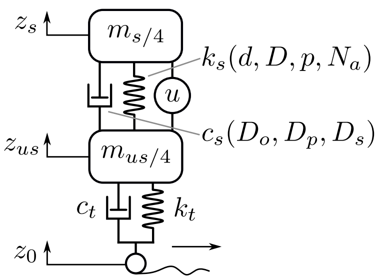
[1] Allison and Herber 2014

→ Active Suspension Case Study

The next 4 slides are from https://www.engr.colostate.edu/%7Edrherber/files/Sundarrajan2021a_presentation.pdf

- **Active vehicle suspension CCD problem in this work is the one from Allison, Guo, and Han 2014**
- The system consists of two masses: sprung mass $m_{s/4}$ and unsprung mass $m_{us/4}$
- The suspension is composed of a spring k_s and damper c_s, and a force actuator $u(t)$
- k_t and c_t are the spring damper constants of the tire, and $z_0(t)$ is the road input
- There are seven design geometric plant design variables associated with the spring and damper

Active Suspension CCD Problem



→ System Dynamics and Objective

- There are four states in the system $(z_s - z_{us}, \dot{z}_s, z_{us} - z_0, \dot{z}_{us})$
- The dynamics of the system are linear with respect to $(\boldsymbol{\xi}(t), u(t))$, and nonlinear with respect to $\boldsymbol{x}_p$:

$$\dot{\boldsymbol{\xi}}(t) = \boldsymbol{A}(\boldsymbol{x}_p)\boldsymbol{\xi}(t) + \boldsymbol{B}u(t) + \boldsymbol{E}\dot{z}_0(t) \tag{6a}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_t(\boldsymbol{x}_p)}{m_{us}/4} & \frac{-[c_s(\boldsymbol{x}_p)+c_t]}{m_{us}/4} & \frac{k_s(\boldsymbol{x}_p)}{m_{us}/4} & \frac{c_s(\boldsymbol{x}_p)}{m_{us}/4} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s(\boldsymbol{x}_p)}{m_s/4} & \frac{-k_s(\boldsymbol{x}_p)}{m_s/4} & \frac{-c_s(\boldsymbol{x}_p)}{m_s/4} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ \frac{-1}{m_{us}/4} \\ 0 \\ \frac{1}{m_s/4} \end{bmatrix}, \quad \boldsymbol{E} = \begin{bmatrix} -1 \\ \frac{c_t}{m_{us}/4} \\ 0 \\ 0 \end{bmatrix} \tag{6b-c}$$

- The objective function is a combination of quadratic penalties on handling $(z_{us} - z_0)$, passenger comfort $\ddot{z}_s$, and control effort u:

$$o = \int_{t_0}^{t_f} \left[w_1\xi_1^2 + w_2[\dot{\xi}_4(t, \boldsymbol{\xi}, u, \boldsymbol{x}_p)]^2 + w_3u^2 \right] dt \tag{7}$$

with $w_1 = 10^5$, $w_2 = 0.5$, and $w_3 = 10^{-5}$ from Ref. Allison, Guo, and Han 2014

- Two design load cases z_0 are simultaneously considered with a weighted sum: 1) ramp profile, 2) rough road profile

$$\min_{\boldsymbol{\xi}, u, \boldsymbol{x}_p} \; 10^{-2} o(\boldsymbol{\xi}_{\text{ramp}}, u_{\text{ramp}}, \boldsymbol{x}_p) + o(\boldsymbol{\xi}_{\text{rough}}, u_{\text{rough}}, \boldsymbol{x}_p) \tag{8}$$

→ Plant Design: Spring

- The spring physical design variables are the wire diameter d, helix diameter D, pitch p, and number of active coils N_a
- The main intermediate parameter is the spring constant $k_s(\boldsymbol{x}_p)$:

$$k_s = \frac{d^4 G}{8D^3 N_a \left[1 + \frac{d^2}{2D^2}\right]}$$

- There are six static constraints and four dynamic constraints

Spring Design

$$g_{o,1}(\boldsymbol{x}_p) = 4 - C \leq 0 \quad (10)$$

$$g_{o,2}(\boldsymbol{x}_p) = C - 12 \leq 0 \quad (11)$$

$$g_{o,3}(\boldsymbol{x}_p) = L_0 - 5.26D \leq 0 \quad (12)$$

$$g_{o,4}(\boldsymbol{x}_p) = L_0 - 0.40 \leq 0 \quad (13)$$

$$g_{o,5}(\boldsymbol{x}_p) = d + D - 0.25 \leq 0 \quad (14)$$

$$g_{o,6}(\boldsymbol{x}_p) = 1.2\tau(F_s) - S_{sy} \leq 0 \quad (16)$$

$$g_{i,1}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \max_t |\xi_3(t)| - L_0 + L_s + 0.02 + \delta_g \leq 0 \quad (17)$$

$$g_{i,2}(\boldsymbol{x}_p, \boldsymbol{\xi}) = 0.15 + 1 - \frac{L_0 - L_s}{\delta_g + 1.1\xi_3(t)} \leq 0 \quad (18)$$

$$g_{i,3}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \frac{1.2\tau(F_a)}{0.24S_{ut}} + \frac{\tau(F_m)}{S_{sy}} - 1 \leq 0 \quad (19)$$

$$g_{i,4}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \frac{1.2\tau(F_a)}{241 \times 10^6} - 1 \leq 0 \quad (20)$$

→ Plant Design: Damper

- The damper physical design variables are the valve diameter D_o, working piston diameter D_p, and damper stroke D_s
- The intermediate parameter is the damper constant $c_s(\boldsymbol{x}_p)$:

$$c_s = \frac{D_p^4}{8C_d C_2(D_o)D_o^2}\sqrt{\frac{\pi k_v \rho_1}{2}}$$

- There are three static and dynamic constraints

Damper Design

$$g_{o,7}(\boldsymbol{x}_p) = d - D + D_p + 0.022 \leq 0 \quad (22)$$

$$g_{o,8}(\boldsymbol{x}_p) = 2D_s - 0.394 \leq 0 \quad (23)$$

$$g_{o,9}(\boldsymbol{x}_p) = L_0 - L_s - D_s \leq 0 \quad (24)$$

$$g_{i,5}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \frac{4c_s(D_o)\max_t|\dot{\xi}_3(t)|}{\pi D_p^2} - 4.75 \times 10^6 \leq 0 \quad (25)$$

$$g_{i,6}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \max_t|\dot{\xi}_3(t)| - 5 \leq 0 \quad (26)$$

$$g_{i,7}(\boldsymbol{x}_p, \boldsymbol{\xi}) = \frac{4\pi D_o^2 c_s(D_o)\max_t|\dot{\xi}_3(t)|}{4k_v \pi D_p^2} - 0.03 \leq 0 \quad (27)$$

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Portrait of CCD research through now: physical-system design perspective

→ Introduction

- With concerns of . . .
  - Bidirectional coupling
  - General objective functions
  - Time-domain specifications
  - Inclusion of various limits
  - Comprehensive plant design representations, including independent design variables and nonlinear dynamics
  - Understanding system performance limits and optimal dynamic and control behaviors

. . . a certain direction of CCD research arose

→ One Thread of Historical CCD Development (1)

Early Integrated Design Methods

- 1980's–1990's: Control Structure Interaction (CSI) optimizing the structure and controller to minimize unwanted structural vibration modes[1]
- 1980's–present: Multidisciplinary Design Optimization (MDO) but developed around fundamentally static system models[2]

Initial CCD Research

A Breakthrough: Direct Optimal Control in CCD

CCD Method Maturation and Impact

Going Forward

[1] Crawley and Luis 1987; Manning 1991; S. S. Rao and Sunar 1994

[2] Sobieszczanski-Sobieski and Haftka 1997; Martins and Lambe 2013; Allison and Herber 2014

→ One Thread of Historical CCD Development (2)

Early Integrated Design Methods

Initial CCD Research

- Late 1990's/early 2000's: CCD theory and method development[1]
- Advances based on certain assumptions such as unidirectional design coupling and LQR/G control
- Cannot account for plant design in a comprehensive manner[2] (e.g., state-dependent failure modes)

***A Breakthrough*: Direct Optimal Control in CCD**

CCD Method Maturation and Impact

Going Forward

[1] Fathy et al. 2001; Reyer et al. 2001 [2] Allison and Herber 2014; Allison, Guo, and Han 2014; Herber and Allison 2018

→ One Thread of Historical CCD Development (3)

Early Integrated Design Methods

Initial CCD Research

***A Breakthrough*: Direct Optimal Control in CCD**

- 2011: First publication of CCD with direct transcription (DT) enabling comprehensive plant design while being generally efficient and scalable[1]
- 2017: Revised CCD theory for bi-directional problems[2]

CCD Method Maturation and Impact

Going Forward

[1] Allison, Guo, and Han 2014 [2] Herber and Allison 2018

→ One Thread of Historical CCD Development (4)

Early Integrated Design Methods

Initial CCD Research

***A Breakthrough*: Direct Optimal Control in CCD**

CCD Method Maturation and Impact

- Expanded applications, growing impact (new programs – NSF and ARPA-E)
- 2019: Labeled an engineering game changer[1]
- Deeper understanding of these methods and better implementations with solution time 100x less than initial efforts[2]
- Expansion beyond basic deterministic CCD with open-loop optimal control (e.g., distributed CCD, stochastic CCD, robust MPC, etc.)

Going Forward

[1] Garcia-Sanz 2019 [2] Sundarrajan and Herber 2021

One Thread of Historical CCD Development (5)

Early Integrated Design Methods

Initial CCD Research

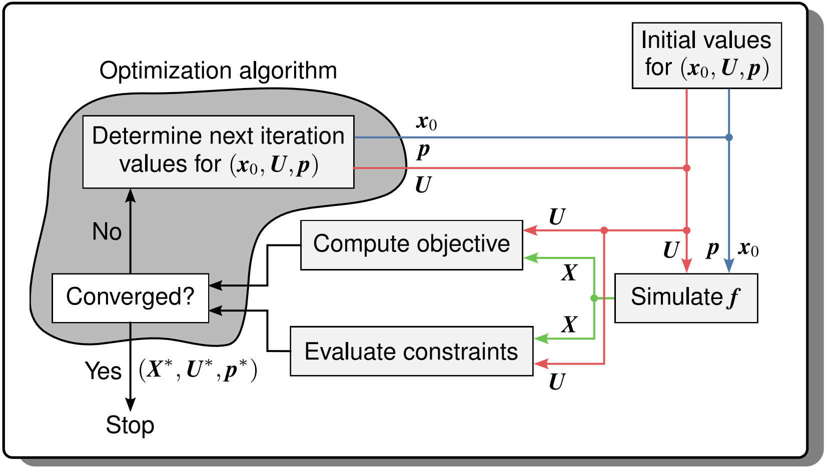
***A Breakthrough*: Direct Optimal Control in CCD**

CCD Method Maturation and Impact

Going Forward

- Incorporating detailed physical models (perhaps possible with surrogate modeling and machine learning)
- Account for uncertainty in the presence of design coupling
- Bridging the gap between the open-loop control insights and closed-loop control solutions
- Getting into the lab, physical experiments, and on actual products, especially when supporting higher-TRL development efforts

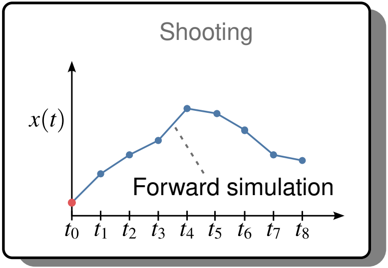
→ Simulation-based Method Block Diagram

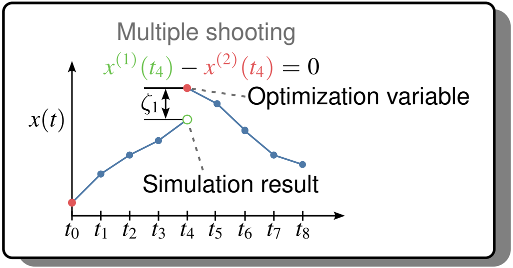


→ Multiple Shooting or Break Up the Long Simulation

- In the multiple shooting approach[1], we partition the time horizon into smaller time segments, and separate simulations are performed on each segment
- This results in a multiphase problem that requires continuity constraints, i.e., continuous states at each time segment:

$$\underbrace{\boldsymbol{x}^{(1)}(t_1)}_{\text{simulation result}} = \underbrace{\boldsymbol{x}^{(2)}(t_1)}_{\text{optimization variable}} \tag{3}$$

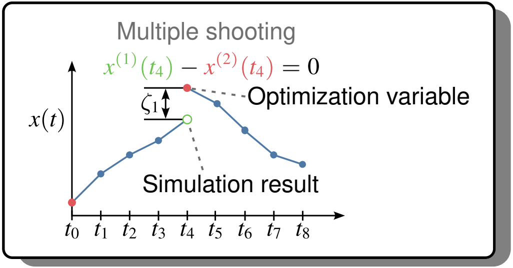


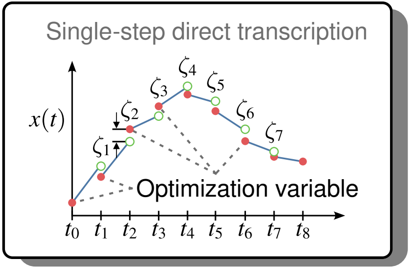


[1] Section 3.4 in *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*

→ Multiple Shooting vs. Direct Transcription

- ❓ What if we reduce the simulation (shooting) horizon to only two points?
- This idea is the essence of a single-step direct transcription (DT) (or time-marching or integral DT) method[1]





[1] See Chapters 2 & 3 in *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*, Chapters 8 & 10 in *Nonlinear Programming*, and Betts 1998; A. V. Rao 2010; Kelly 2017

→ Direct Transcription Comments

- Although it may be counterintuitive to create such a large problem with many more variables and constraints, it is in fact often better than the alternatives
  - For example, finite-horizon LQR is solved with matrix multiplications and an inverse
- There are many tools available to help construct and solve DT problems with a variety of different numerical methods (and some do support the inclusion of plant design variables)
  - You don't have to (and probably shouldn't) do this on our own
- DT is closely related to model predictive control (MPC); individual MPC problems are DT-like problems
  - An MPC strategy solves open-loop problems sequentially with feedback from what the system actually did under previous control actions
- Similar to linear-quadratic problems from classical control theory, linear-quadratic dynamic optimization problems can be efficiently solved as quadratic programs (QPs)
  - Used in some studies with LQDO-amenable CCD problems using the nested CCD strategy

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Thanks!