Overview of Uncertain Control Co-Design

NSF Workshop on Control Co-Design Research

Saeed Azad

Postdoctoral Fellow
saeed.azad@colostate.edu
Colorado State University, Department of Systems Engineering

May 4–6, 2023

①

Introduction

→ Introduction

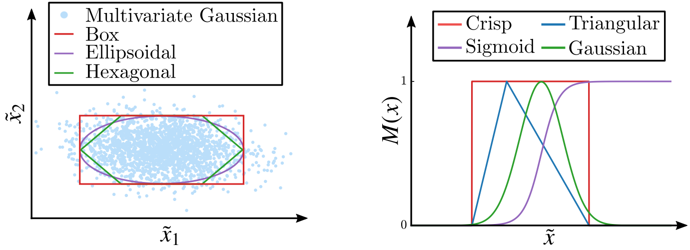
- Often some of the elements of CCD problem are **inherently uncertain** or **not entirely known**; we refer to these characteristics as **uncertainties**
- These uncertainties are related to the amount of **information** that is available to the designer at different stages
- The goal is to identify
  - ✓ ways for the mathematical **representation** of uncertainties,
  - ✓ approaches for their **integration** into CCD activity,
  - ✓ **solution strategies** for solving the resulting uncertain CCD (UCCD) problems

→ Representation of Uncertainties (continued)

Any uncertain variable may be represented in three ways[1]:

- **Stochastic**
- **Crisp**
- **Possibilistic**

Uncertainty Representations



[1] Beyer and Sendhoff 2007

②

Elements of UCCD Problem

→ Elements of UCCD Problem

- **Control trajectories** may be uncertain, for example due to electrical noise, actuator imprecision, etc.
  - ✓ Robust control used for a class of uncertain systems via disturbance observer-based control for a nonlinear MAGnetic LEViation (MAGLEV) suspension system[1]
- **State trajectories** may be uncertain due to uncertainties in $(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{d}})$, uncertain initial/final conditions, noise, mismodeled, and neglected dynamics, etc.
  - ✓ Robust adaptive fuzzy tracking controller developed to deal with parametric and unmodeled dynamics for a hypersonic flight vehicle[2]
- **Time-independent optimization variables** may be uncertain due to imperfect manufacturing processes, plant measurement errors, mass productions of plants, plant aging (model plant mismatch)
  - ✓ Robust design optimization used for the UCCD of a hybrid-electric vehicle powertrain with time-independent uncertainties[3]

[1] Yang et al. 2011 [2] X. Hu, Xu, and C. Hu 2018 [3] Azad and Alexander-Ramos 2021

→ Elements of UCCD Problem (continued)

- **Objective function** may be uncertain, but it can be added to the set of constraints through the addition of a new variable; this is known as the **epigraph** form
  - ✓ Epigraph representation demonstrated for an uncertain objective function[1]
- **Equality constraints** must be satisfied if they describe the laws of nature (i.e. Type I), otherwise then can be relaxed[2]
  - ✓ Equality constraints under uncertainties discussed in detail[3]
- **Risk** is characterized through some **measure** that maps the outcomes from an uncertain space into a quantity that can be easily interpreted; it captures the designer's attitude towards uncertainties through risk-averse, risk-neutral, or risk-taking approaches
  - ✓ Risk-averse approach implemented for optimal motion planning of a robot[4]
- **Inequality (path) constraints** can take different forms depending on the **availability of information** and the **risk attitude** of the designer

[1] Azad and Herber 2022a [2] Azad and Herber 2022b [3] Mattson and Messac 2003 [4] Nakka and Chung 2022

→ Elements of UCCD Problem (continued)

Inequality constraints $\mathbb{E}[\bar{g}_i(t, \tilde{\boldsymbol{u}}, \tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{d}})]$ can be modeled in different ways:

- **Nominal**: $g_i(t, \boldsymbol{u}_N, \boldsymbol{\xi}_N, \boldsymbol{p}_N, \boldsymbol{d}_N) \leq 0$
  - ✓ Nominal rough road profile used for CCD active suspension[1]
- **Expected value**: $g_{\mu,i}(t, \tilde{\boldsymbol{u}}, \tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{d}}) \leq 0$
  - ✓ Risk neutral expected value bidding model used for wind power[2]
- **Higher-order moments** (often with mean value): $\sqrt{\mathbb{E}[g_i(\cdot)^2] - g_{\mu,i}(\cdot)^2} = g_{i,\sigma}(\cdot) \leq \sigma_{a,i}$
  - ✓ Aircraft robust trajectory optimization performed using higher-order moments[3]

[1] Allison, Guo, and Han 2014 [2] AlAshery and Qiao 2018 [3] X. Li et al. 2014

→ Elements of UCCD Problem (continued)

- **Probabilistic chance-constrained**: $\mathbb{P}[g_i(t, \tilde{\boldsymbol{u}}, \tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{d}}) \geq 0] \leq \mathbb{P}_{f,i}$
  - ✓ Trajectory optimization of robotic spacecraft simulator[1]
- **Worst-case** is a conservative approach whose solution is feasible for all realizations within the uncertainty set $\mathcal{R}$: $\underset{(\cdot)\in\mathcal{R}}{\text{maximize}} \{g_i(t, \boldsymbol{u}, \boldsymbol{\xi}, \boldsymbol{p}, \boldsymbol{d})\} \leq 0$
  - ✓ Robust UCCD of an aircraft thermal management system[2]
- Other characterizations are possible including usage of expected utility theory, evidence theory, min-max regret, and possibilistic formulations

[1] Nakka and Chung 2022 [2] Nash, Pangborn, and Jain 2021

③

Solution Strategies

→ Solution Methods

- Uncertainty propagation (UP) methods are needed to solve UCCD problems with probabilistic representation of uncertainties
- In UCCD literature, UP methods are based on
  - **Sampling** methods, such as Monte Carlo Simulation[1]
  - **Local expansion** methods, such as first-order second moment (FOSM)[2]
  - **Functional expansion** methods such as generalized Polynomial Chaos (gPC)[3]
  - **Most-probable point** methods such as first-order reliability method (FORM)[4]
- The **worst-case robust** approach has been used when uncertainties in the UCCD problem are represented as crisp sets[5]

[1] Azad and Herber 2022a [2] Azad and Alexander-Ramos 2020a; Azad and Alexander-Ramos 2021 [3] Azad and Herber 2022a; Behtash and Alexander-Ramos 2021 [4] Azad and Alexander-Ramos 2020b; Cui, Allison, and Wang 2020; Cui, Allison, and Wang 2021 [5] Nash, Pangborn, and Jain 2021; Nash and Jain 2019

→ Some Potential Future Directions for UCCD Research

- **Comparing** different UCCD formulations and solution strategies to gain insights into questions regarding computational time, scalibility, and impact
- Identifying additional solution strategies, with an emphasis on **time-dependent**, **possibilistic**, and **hybrid** representation of uncertainties
- Discovering efficient methods for solving UCCD problems of **complex and large-scale systems** by
  - ✓ Improving **guidelines on selecting solution strategies** for different classes of problems
  - ✓ Investigating **decomposition methods** for more efficient UCCD problem structure

→ References

- M. K. AlAshery and W. Qiao (2018). "Risk management for optimal wind power bidding in an electricity market: A comparative study". *North American Power Symposium*. DOI: 10.1109/NAPS.2018.8600680
- J. T. Allison, T. Guo, and Z. Han (2014). "Co-design of an active suspension using simultaneous dynamic optimization". *J. Mech. Design* 136.8. doi: 10.1115/1.4027335. DOI: 10.1115/1.4027335
- J. T. Allison and D. R. Herber (2014). "Multidisciplinary design optimization of dynamic engineering systems". *AIAA J.* 52.4. doi: 10.2514/1.J052182
- L. Andrieu, G. Cohen, and F. Vázquez-Abad (2007). "Stochastic programming with probability Constraint". arXiv:0708.0281
- S. Azad and M. J. Alexander-Ramos (2021). "Robust combined design and control optimization of hybrid-electric vehicles using MDSDO". *IEEE Trans. Veh. Technol.* 70.5. DOI: 10.1109/TVT.2021.3071863
- S. Azad and M. J. Alexander-Ramos (2020a). "Robust MDSDO for co-design of stochastic dynamic systems". *J. Mech. Design* 142.1. doi: 10.1115/1.4044430

→ References (continued)

- S. Azad and D. R. Herber (2022a). "Investigations into Uncertain Control Co-design Implementations for Stochastic in Expectation and Worst-case Robust". *International Mechanical Engineering Congress and Exposition*. IMECE2022-95229. doi: 10.1115/IMECE2022-95229
- S. Azad and M. J. Alexander-Ramos (2020b). "A single-loop reliability-based MDSDO formulation for combined design and control optimization of stochastic dynamic systems". *J. Mech. Design* 143.2. doi: 10.1115/1.4047870
- S. Azad and D. R. Herber (2022b). "Control Co-Design Under Uncertainties: formulations". *International Design Engineering Technical Conferences*. DETC2022-89507. doi: 10.1115/DETC2022-89507
- M. Behtash and M. J. Alexander-Ramos (2021). "A reliability-based formulation for simulation-based control co-design using generalized polynomial chaos expansion". *Journal of Mechanical Design* 144.5. doi: 10.1115/1.4052906
- F. J. Bejarano, L. M. Fridman, and A. S. Poznyak (2009). "Output integral sliding mode for min-max optimization of multi-plant linear uncertain systems". *IEEE Trans. Automat. Contr.* 54.11. doi: 10.1109/TAC.2009.2031718

→ References (continued)

- A. Bemporad and M. Morari (1999). "Robust model predictive control: A survey". *Robustness in identification and control. Lecture Notes in Control and Information Science*. Springer. DOI: 10.1007/BFb0109870
- H.-G. Beyer and B. Sendhoff (2007). "Robust optimization–a comprehensive survey". *Comput. Method. Appl. M.* 196.33–34. doi: 10.1016/j.cma.2007.03.003
- R. A. Briggs (2019). "Normative Theories of Rational Choice: Expected Utility". *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Fall 2019. Metaphysics Research Lab, Stanford University
- A. E. Bryson and Y.-C. Ho (1975). *Applied Optimal Control*. doi: 10.1201/9781315137667. Taylor & Francis
- T. Cui, J. T. Allison, and P. Wang (2021). "Reliability-based control co-design of horizontal axis wind turbines". *Struct. Multidiscipl. Optim.* 64. doi: 10.1007/s00158-021-03046-3
- — (2020). "A comparative study of formulations and algorithms for reliability-based co-design problems". *J. Mech. Design* 142.3. doi: 10.1115/1.4045299
- D. R. Herber and J. T. Allison (2019). "Nested and simultaneous solution strategies for general combined plant and control design problems". *J. Mech. Design* 141.1. doi: 10.1115/1.4040705

→ References (continued)

- X. Hu, B. Xu, and C. Hu (2018). "Robust adaptive fuzzy control for HFV with parameter uncertainty and unmodeled dynamics". *IEEE Trans. Ind. Electron.* 65.11. DOI: 10.1109/TIE.2018.2815951
- Y. Kim et al. (2018). "Robust model predictive control with adjustable uncertainty sets". *IEEE Conference on Decision and Control*. doi: 10.1109/CDC.2018.8619158. DOI: 10.1109/CDC.2018.8619158
- X. Li et al. (2014). "Aircraft robust trajectory optimization using nonintrusive polynomial chaos". *J. Aircraft* 51.5. doi: 10.2514/1.C032474
- B. Liu (2002). "Toward fuzzy optimization without mathematical ambiguity". *Fuzzy Optim. Decis. Ma.* 1.1. doi: 10.1023/A:1013771608623
- R. Malak, B. Baxter, and C. Hsiao (2015). "A decision-based perspective on assessing system robustness". *Procedia Comput. Sci.* 44. doi: 10.1016/j.procs.2015.03.069. DOI: 10.1016/j.procs.2015.03.069
- C. Mattson and A. Messac (2003). "Handling equality constraints in robust design optimization". *AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*. DOI: 10.2514/6.2003-1780

→ References (continued)

- Z. K. Nagy and R. D. Braatz (2004). "Open-loop and closed-loop robust optimal control of batch processes using distributional and worst-case analysis". *J. Process Contr.* 14.4. DOI: 10.1016/j.jprocont.2003.07.004
- Y. K. Nakka and S.-J. Chung (2022). "Trajectory Optimization of Chance-Constrained Nonlinear Stochastic Systems for Motion Planning Under Uncertainty". *IEEE Trans. Robot.* DOI: 10.1109/TRO.2022.3197072
- A. L. Nash and N. Jain (2019). "Combined plant and control co-design for robust disturbance rejection in thermal-fluid systems". *IEEE. Trans. Control Syst. Technol.* 28.6. DOI: 10.1109/TCST.2019.2931493
- A. L. Nash, H. C. Pangborn, and N. Jain (2021). "Robust control co-design with receding-horizon MPC". *American Control Conference*. doi: 10.23919/ACC50511.2021.9483216
- P. N. Paraskevopoulos (2017). *Modern Control Engineering*. CRC Press
- S. Rahal and Z. Li (2021). "Norm induced polyhedral uncertainty sets for robust linear optimization". *Optim. Eng.* doi: 10.1007/s11081-021-09659-3
- A. Schöbel (2014). "Generalized light robustness and the trade-off between robustness and nominal quality". *Math. Methods Oper. Res.* 80.2. doi: 10.1007/s00186-014-0474-9. DOI: 10.1007/s00186-014-0474-9

→ References (continued)

- J. Yang et al. (2011). "Robust control of nonlinear MAGLEV suspension system with mismatched uncertainties via DOBC approach". *ISA Trans.* 50.3. doi: 10.1016/j.isatra.2011.01.006
- J. Yong (2020). "Stochastic Optimal Control—A Concise Introduction". *Math. Control. Relat. Fields.* doi: 10.3934/mcrf.2020027
- X. Zhang et al. (2017). "Robust optimal control with adjustable uncertainty sets". *Automatica* 75. doi: 10.1016/j.automatica.2016.09.016
- Y. Zhu (2009). "A fuzzy optimal control model". *J. Uncertain Syst.* 3.4

Questions?

Overview of Uncertain Control Co-Design
NSF Workshop on Control Co-Design Research

Saeed Azad
Colorado State University
saeed.azad@colostate.edu

④

Appendix A

→ Representation of Uncertainties

Each element in a UCCD problem belongs to one of four sets:

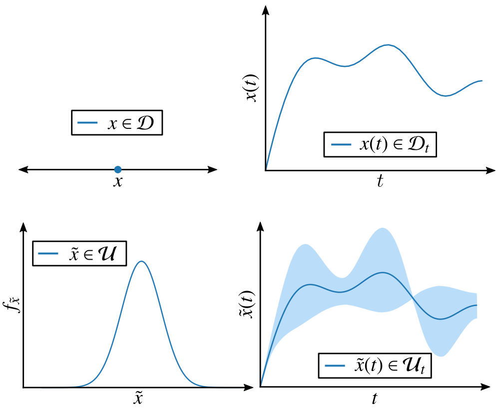
- Time-independent deterministic $\mathcal{D}$
- Time-dependent deterministic

$$\mathcal{D}_t := \{x(t) \mid t \in [t_0, t_f],\ x(t) \in \mathcal{D}\}$$

- Time-independent uncertain $\mathcal{U}$
- Time-dependent uncertain

$$\mathcal{U}_t := \{\tilde{x}(t) \mid t \in [t_0, t_f],\ \tilde{x}(t) \in \mathcal{U}\}$$

Deterministic and Uncertain Sets



→ Deterministic CCD

Here, we introduce the nominal continuous-time, deterministic, all-at-once (AAO), simultaneous, CCD problem[1]:

- $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ is open-loop control
- $\boldsymbol{\xi}(t) \in \mathbb{R}^{n_s}$ is state
- $\boldsymbol{p} \in \mathbb{R}^{n_p}$ is time-independent optimization variables:
  - $\boldsymbol{p}_p$ plant optimization variables
  - $\boldsymbol{p}_c$ is control gains
- $\boldsymbol{d} \in \mathbb{R}^{n_d}$ is problem data

Deterministic CCD

$$\begin{aligned}
\underset{\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p}}{\text{minimize:}} \quad & o = \int_{t_0}^{t_f} \ell(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d})\,\mathrm{d}t + m(\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) \\
\text{subject to:} \quad & \boldsymbol{g}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) \leq \boldsymbol{0} \\
& \boldsymbol{h}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) = \boldsymbol{0} \\
& \dot{\boldsymbol{\xi}} - \boldsymbol{f}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) = \boldsymbol{0} \\
\text{where:} \quad & \boldsymbol{\xi}(t_0) = \boldsymbol{\xi}_0,\ \boldsymbol{\xi}(t_f) = \boldsymbol{\xi}_f,\ \boldsymbol{u}(t) = \boldsymbol{u} \\
& \boldsymbol{\xi}(t) = \boldsymbol{\xi},\ \boldsymbol{d}(t) = \boldsymbol{d}
\end{aligned}$$

[1] Allison and Herber 2014; Herber and Allison 2019

→ A Generalized UCCD Formulation

Without any loss of generality, a generalized UCCD formulation can be defined in probability space[1]:

- $\tilde{\bullet}$ is a time-independent uncertain variable
- $\tilde{\bullet}(t)$ is a stochastic process
- $\bar{\bullet}(\cdot)$ is a function composition of $\bullet(\cdot)$, e.g.,
  - $\bar{o}(\cdot)$ is a function of the original objective function $o(\cdot)$
  - $\bar{\boldsymbol{g}}(\cdot)$ is a function of the original inequality constraint vector $\boldsymbol{g}(\cdot)$

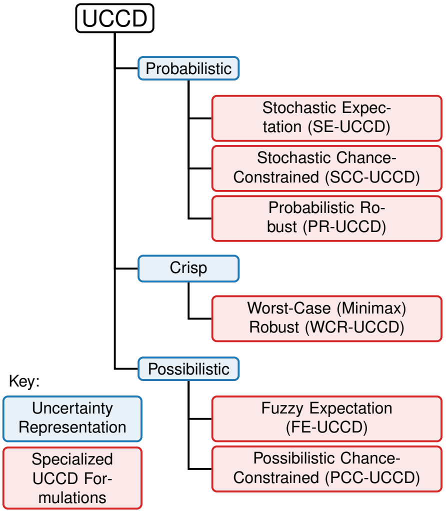
A Universal UCCD Formulation

$$\begin{aligned}
\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & \mathbb{E}\left[\bar{o}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] \\
\text{subject to:} \quad & \mathbb{E}\left[\bar{\boldsymbol{g}}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] \leq \boldsymbol{0} \\
& \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
& \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
\text{where:} \quad & \tilde{\boldsymbol{u}}(t) = \tilde{\boldsymbol{u}},\ \tilde{\boldsymbol{\xi}}(t) = \tilde{\boldsymbol{\xi}},\ \tilde{\boldsymbol{d}}(t) = \tilde{\boldsymbol{d}} \\
& \tilde{\bullet} \in \mathcal{V}_u,\ \tilde{\bullet}(t) \in \mathcal{T}_u(t)
\end{aligned}$$

[1] Azad and Herber 2022b

→ Specialized Formulations

Six specialized formulations can be derived from the generalized UCCD formulation on Slide 19



→ Stochastic Expectation and Stochastic Chance-Constrained

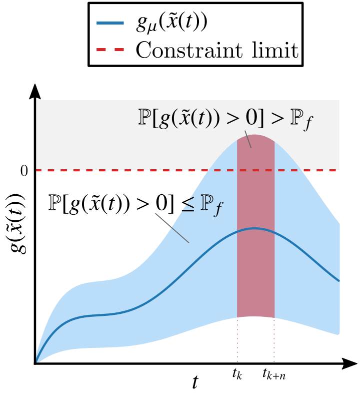
Stochastic in Expectation UCCD[1]

$$\begin{aligned} \underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \\ \text{subject to:} \quad & \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq \boldsymbol{0} \\ & (t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \in \mathcal{E} \end{aligned}$$

Stochastic Chance-Constrained UCCD[2]

$$\begin{aligned} \underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \\ \text{subject to:} \quad & \mathbb{P}[g_i(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) > 0] \leq \mathbb{P}_{f,i} \\ & i = 1,\ldots,n_g \\ & (t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \in \mathcal{E} \end{aligned}$$

Uncertain Probabilistic Constraint



[1] Andrieu, Cohen, and Vázquez-Abad 2007 [2] Azad and Alexander-Ramos 2020b

→ Probabilistic Robust UCCD

Probabilistic Robust UCCD[1]

$$\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad \alpha_w o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + (1-\alpha_w) o_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})$$

$$\text{subject to:} \quad \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + k_s \boldsymbol{g}_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \le \boldsymbol{0}$$

$$(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \in \mathcal{E}$$

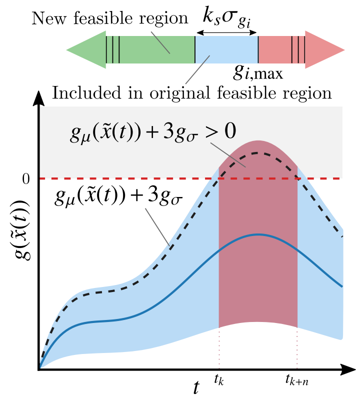
$$\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad \alpha_w o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + (1-\alpha_w) o_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})$$

$$\text{subject to:} \quad \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \le \boldsymbol{0}$$

$$\boldsymbol{g}_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) - \boldsymbol{\sigma}_a \le \boldsymbol{0}$$

$$(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \in \mathcal{E}$$

PR Constraint



[1] Nagy and Braatz 2004; Azad and Alexander-Ramos 2021; X. Li et al. 2014

→ Worst-Case Robust UCCD

- A solution is robust if it remains feasible for all uncertainty realizations **within the uncertainty set**
- Resembles a game between the optimizer and adversarial opponent[1]
- Uncertainties belong to their associated sets[2]

$$\mathcal{R}(\hat{\boldsymbol{q}}) = \{\mathcal{R}(\hat{\boldsymbol{p}}) \times \mathcal{R}(\hat{\boldsymbol{d}})\} \subseteq \mathcal{X}_{\text{crisp}}$$
$$\mathcal{R}_t(\hat{\boldsymbol{q}}) = \{\mathcal{R}(\hat{\boldsymbol{u}}) \times \mathcal{R}(\hat{\boldsymbol{\xi}}) \times \mathcal{R}(\hat{\boldsymbol{d}})\} \subseteq \mathcal{X}_{\text{crisp}}(t)$$

[1] Bryson and Ho 1975 [2] Rahal and Z. Li 2021

Worst-case Robust UCCD

$$\begin{aligned} \underset{\hat{\boldsymbol{u}},\hat{\boldsymbol{\xi}},\hat{\boldsymbol{p}}}{\text{minimize:}} \quad & v \\ \text{subject to:} \quad & \Phi_i(t,\hat{\boldsymbol{u}},\hat{\boldsymbol{\xi}},\hat{\boldsymbol{p}},\hat{\boldsymbol{d}}) \le 0 \ \text{ for } \ i = 1,\dots,n_g \\ & (t,\hat{\boldsymbol{u}},\hat{\boldsymbol{\xi}},\hat{\boldsymbol{p}},\hat{\boldsymbol{d}}) \in \mathcal{E} \\ & \boldsymbol{\psi}(\hat{\boldsymbol{u}},\hat{\boldsymbol{\xi}},\hat{\boldsymbol{p}},\hat{\boldsymbol{d}}) \le \boldsymbol{0} \end{aligned}$$

$\Phi_i(t,\hat{\boldsymbol{u}},\hat{\boldsymbol{\xi}},\hat{\boldsymbol{p}},\hat{\boldsymbol{d}})$:

$$\begin{aligned} \underset{\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}}{\text{maximize:}} \quad & g_i(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}) \\ \text{subject to:} \quad & (t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}) \in \mathcal{E} \\ & (\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{d}) \in \mathcal{R}_t(\hat{\boldsymbol{q}}_t) \\ & \boldsymbol{p} \in \mathcal{R}(\hat{\boldsymbol{q}}) \end{aligned}$$

→ Fuzzy Expected Value and Possibilistic Chance-Constrained

Fuzzy Expected Value UCCD[1]

$$\begin{aligned}\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}}\quad & \mathbb{E}[o(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})]\\ \text{subject to:}\quad & \mathbb{E}\left[\bar{\boldsymbol{g}}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right]\leq \boldsymbol{0}\\ & (t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\in\mathcal{E}\end{aligned}$$

Possibilistic Chance-Constrained[2]

$$\begin{aligned}\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}}\quad & \mathbb{E}[o(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})]\\ \text{subject to:}\quad & \mathbb{POS}[g_i(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})>0]\leq \mathbb{POS}_{f,i}\\ & (t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\in\mathcal{E}\end{aligned}$$

[1] Zhu 2009; Liu 2002 [2] Liu 2002

⑤

Appendix B

→ Discussion

Norm-Induced Sets

- Generally, norm-induced uncertainty sets are used in WCR formulations:

$$\mathcal{N} := \{\boldsymbol{q} \mid z(\hat{\boldsymbol{q}} - \boldsymbol{q}) \leq \boldsymbol{\eta}_q\}$$

- $z(\cdot)$ is a specified (norm) function that represents the geometry of the uncertainty set[1]
- The size of the uncertainty set $\boldsymbol{\eta}_q$ can be a modeling choice, so one can optimally leverage the uncertainty set's size, shape, and structure to obtain a meaningful solution for a given metric through **adjustable uncertainty sets**[2].
- To avoid the conservativeness of the WCR, other formulations such as the *min-max* regret have been developed[3]

Linking SCC and WCR Formulations

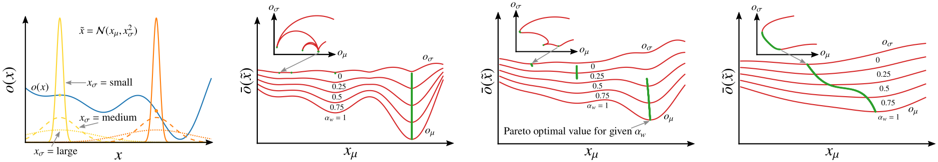
- Strict satisfaction of (infinitely many) hard constraints in WC is equivalent to the satisfaction of probabilistic constraints in SCC with an infinitesimally small failure probability

[1] Rahal and Z. Li 2021 [2] Zhang et al. 2017; Kim et al. 2018 [3] Schöbel 2014

→ Discussion (continued)

Robustness in the PR-UCCD Formulation

- PR-UCCD is not always a risk-averse formulation[1]



- Such limitations are addressed through concepts from normative decision theory such as representation theorems that result in a mathematical description of decision-maker's preferences through a utility function[2]

Insights From:

- Robust control theory[3], stochastic control theory[4], model predictive control[5] should be used when appropriate

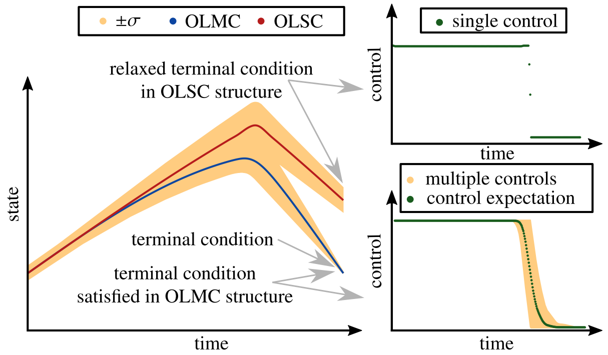
[1] Azad and Herber 2022b; Malak, Baxter, and Hsiao 2015 [2] Briggs 2019; Malak, Baxter, and Hsiao 2015
[3] Paraskevopoulos 2017 [4] Yong 2020 [5] Bemporad and Morari 1999

→ Discussion (continued)

Open-Loop Optimal Control Structure

- **Open-loop single-control** (OLSC) finds a single control command and is related to concepts from robust control theory and centralized control[1]
- **Open-loop multiple-control** (OLMC) elicits a range of optimal control responses based on the realization of uncertainties and is related to concepts from decentralized control

Open-Loop Optimal Control Structures



[1] Azad and Herber 2022a; Bejarano, Fridman, and Poznyak 2009