Control Co-Design Under Uncertainties: Formulations

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Introduction

→ Introduction

- Control co-design (CCD) refers to the **integrated** consideration of the **physical** and **control** system design
- With the growing complexity of dynamic systems, the need for effective control co-design strategies is ever present
- CCD is often studied in a deterministic manner[1], i.e., no uncertainties are present
- However, often some of the elements of CCD problem are **inherently uncertain** or **not entirely known**; we refer to these characteristics as **uncertainties**
- If we overlook the impact of such uncertainties in CCD, the solution may no longer be effective in realistic scenarios
- Uncertainty in CCD may stem from sources such as plant optimization variables, uncertain problem data, fidelity of dynamics, noise in the control channel, etc.
- These uncertainties transform CCD problem into an **uncertain CCD** or **UCCD** problem

[1] Herber and Allison 2019; Allison and Herber 2014

→ Motivations and Objectives

- Currently, **interpretations** associated with robust CCD, stochastic CCD, and other potential formulations are not clearly stated
- **Selection** of UCCD formulation is **not well-informed** according to the literature
- Current UCCD literature generally focuses on specific uncertainties and is **driven by particular solution techniques**[1]
- Different formulations, however, offer **different risk measures** and thus, their **formal representation** may inform application-focused UCCD
- This study aims to identify the **sources of uncertainties** and **formalize their inclusion** through a **universal UCCD** problem formulation
- From this universal UCCD formulation, **6 specialized UCCD formulations** can be derived

[1] Azad and Alexander-Ramos 2020b; Azad and Alexander-Ramos 2020a; Nash, Pangborn, and Jain 2021; Cui, Allison, and Wang 2020; Behtash and Alexander-Ramos 2021

→ Deterministic CCD

We begin by introducing the nominal continuous-time, deterministic, all-at-once (AAO), simultaneous, CCD problem[1]:

- $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ is open-loop control
- $\boldsymbol{\xi}(t) \in \mathbb{R}^{n_s}$ is state
- $\boldsymbol{p} \in \mathbb{R}^{n_p}$ is time-independent optimization variables:
  - $\boldsymbol{p}_p$ plant optimization variables
  - $\boldsymbol{p}_c$ is control gains
- $\boldsymbol{d} \in \mathbb{R}^{n_d}$ is problem data

Deterministic CCD

$$\begin{aligned}
\underset{\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p}}{\text{minimize:}} \quad & o = \int_{t_0}^{t_f} \ell(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d})\,\mathrm{d}t + m(\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) \\
\text{subject to:} \quad & \boldsymbol{g}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) \leq \boldsymbol{0} \\
& \boldsymbol{h}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) = \boldsymbol{0} \\
& \dot{\boldsymbol{\xi}} - \boldsymbol{f}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{\xi}_0,\boldsymbol{\xi}_f,\boldsymbol{d}) = \boldsymbol{0} \\
\text{where:} \quad & \boldsymbol{\xi}(t_0) = \boldsymbol{\xi}_0,\ \boldsymbol{\xi}(t_f) = \boldsymbol{\xi}_f,\ \boldsymbol{u}(t) = \boldsymbol{u} \\
& \boldsymbol{\xi}(t) = \boldsymbol{\xi},\ \boldsymbol{d}(t) = \boldsymbol{d}
\end{aligned}$$

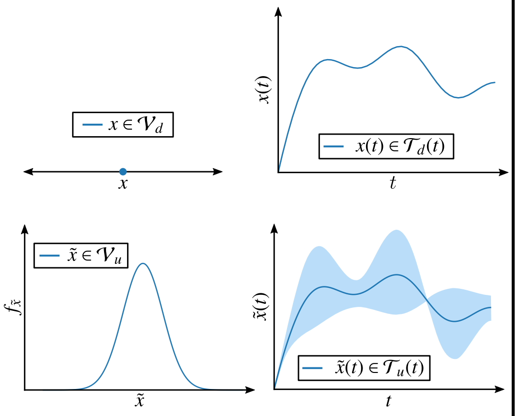
[1] Allison and Herber 2014; Herber and Allison 2019

→ Representation of Uncertainties

Each element in a UCCD problem belongs to one of four sets:

- Time-independent deterministic
  $\mathcal{V}_d := \{x \mid x \in \mathcal{V}_d\}$
- Time-dependent deterministic
  $\mathcal{T}_d(t) := \{x(t) \mid t \in [t_0, t_f],\ x(t) \in \mathcal{V}_d\}$
- Time-independent uncertain
  $\mathcal{V}_u := \{\tilde{x} \mid \tilde{x} \in \mathcal{V}_u\}$
- Time-dependent uncertain
  $\mathcal{T}_u(t) := \{\tilde{x}(t) \mid t \in [t_0, t_f],\ \tilde{x}(t) \in \mathcal{V}_u\}$

Deterministic and Uncertain Sets

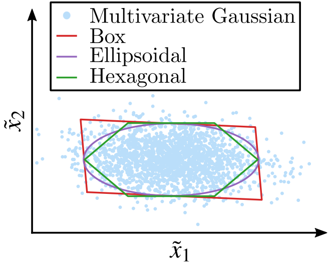


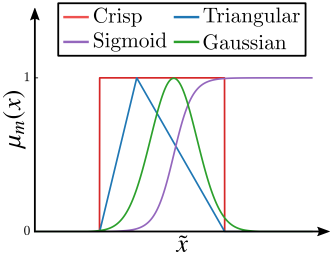
→ Representation of Uncertainties (continued)

Any uncertain variable belonging to $\mathcal{V}_u$ or $\mathcal{T}_u(t)$ may be represented in three ways[1]:

- **Stochastic**
- **Deterministic**
- **Possibilistic**

Uncertainty Representations





[1] Beyer and Sendhoff 2007

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Mathematical Foundations

→ A Universal UCCD Formulation

Without any loss of generality, a universal UCCD formulation can be defined in probability space because specialized forms of this formulation can be derived though the appropriate selection of the objective function and constraints

- $\tilde{\bullet}$ is a time-independent uncertain variable
- $\tilde{\bullet}(t)$ is a stochastic process
- $\bar{\bullet}(\cdot)$ is a function composition of $\bullet(\cdot)$, e.g.,
  - $\bar{o}(\cdot)$ is a function of the original objective function $o(\cdot)$
  - $\bar{\boldsymbol{g}}(\cdot)$ is a function of the original inequality constraint vector $\boldsymbol{g}(\cdot)$

A Universal UCCD Formulation

$$
\begin{aligned}
\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & \mathbb{E}\left[\bar{o}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] \\
\text{subject to:} \quad & \mathbb{E}\left[\bar{\boldsymbol{g}}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] \leq \boldsymbol{0} \\
& \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
& \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
\text{where:} \quad & \tilde{\boldsymbol{u}}(t) = \tilde{\boldsymbol{u}},\ \tilde{\boldsymbol{\xi}}(t) = \tilde{\boldsymbol{\xi}},\ \tilde{\boldsymbol{d}}(t) = \tilde{\boldsymbol{d}} \\
& \tilde{\bullet} \in \mathcal{V}_u,\ \tilde{\bullet}(t) \in \mathcal{T}_u(t)
\end{aligned}
$$

→ Considerations

- **Risk**[1]
  - Risk neutral
  - Risk averse
- **Decision space**
  - UCCD formulations must be balanced and non-biased
- **Objective function**[2]
  - No conceptual distinction between objective and inequality constraints
  - Epigraph representation
- **Equality constraints** [3]
  - Type I (must be satisfied)
  - Type II (Can't always be satisfied)

- **Inequality constraints**[4]
  - Worst-case

$$\mathbb{E}\left[\bar{g}_i(\cdot)\right] = \underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}}{\text{maximize:}} \quad g_i(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq 0$$

  - Expected-value

$$\mathbb{E}\left[\bar{g}_i(\cdot)\right] = g_{\mu,i}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq 0$$

  - Higher-order moments

$$\mathbb{E}\left[\bar{g}_i(\cdot)\right] = \sqrt{\mathbb{E}[g_i(\cdot)^2] - g_{\mu,i}(\cdot)^2} = g_{i,\sigma}(\cdot) \leq \sigma_{a,i}$$

  - Probabilistic chance-constrained

$$\mathbb{E}\left[\bar{g}_i(\cdot)\right] = \mathbb{E}\left[\mathbb{I}_E(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] = \mathbb{P}[E]$$

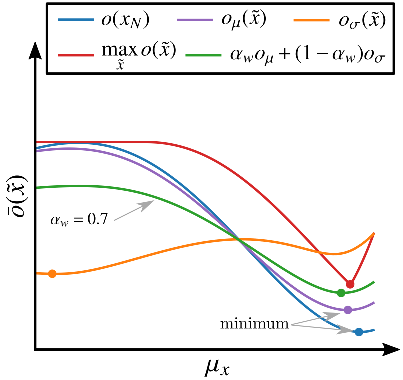
  - Possibilistic chance-constrained

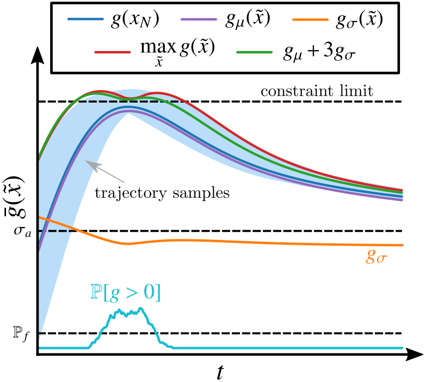
$$\mathbb{POS}[g_i(\cdot) \geq 0] \leq \mathbb{POS}_{f,i}$$

→ Considerations (Continued)

Illustrative representation of various objective and constraint descriptions:

Representation of Select Objectives & Constraints





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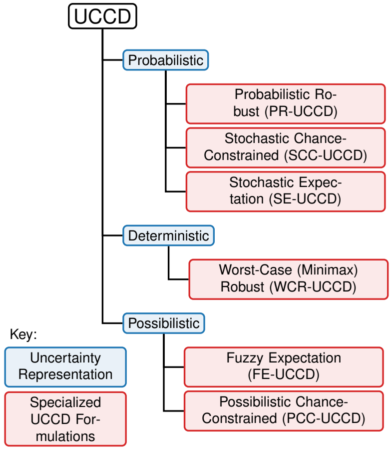
Specialized Formulations

→ Specialized Formulations

Six specialized formulations can be derived from the universal UCCD formulation on Slide 7

Specialized Formulations



→ Stochastic in Expectation and Stochastic Chance-Constrained

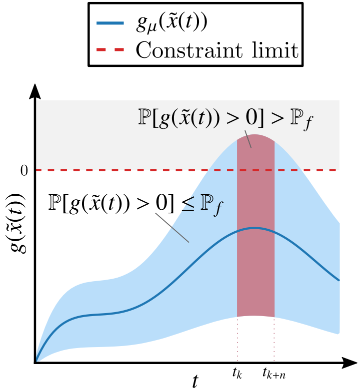
Stochastic in Expectation UCCD [1]

$$\begin{aligned}\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \\ \text{subject to:} \quad & \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq \boldsymbol{0} \\ & \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \qquad \text{(a.s)} \\ & \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \qquad \text{(a.s)}\end{aligned}$$

Stochastic Chance-Constrained UCCD [2]

$$\begin{aligned}\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \\ \text{subject to:} \quad & \mathbb{P}[g_i(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) > 0] \leq \mathbb{P}_{f,i} \\ & \quad i = 1,\ldots,n_g \\ & \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \qquad \text{(a.s)} \\ & \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \qquad \text{(a.s)}\end{aligned}$$

Uncertain Probabilistic Constraint



[1] Andrieu, Cohen, and Vázquez-Abad 2007 [2] Azad and Alexander-Ramos 2020b

→ Probabilistic Robust UCCD

Probabilistic Robust UCCD [1]

$$\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad \alpha_w o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + (1-\alpha_w) o_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})$$

$$\text{subject to:} \quad \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + k_s \boldsymbol{g}_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq \boldsymbol{0}$$

$$\boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}},\tilde{\boldsymbol{w}}) = \boldsymbol{0} \qquad \text{(a.s)}$$

$$\dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}},\tilde{\boldsymbol{w}}) = \boldsymbol{0} \qquad \text{(a.s)}$$

$$\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad \alpha_w o_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) + (1-\alpha_w) o_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})$$

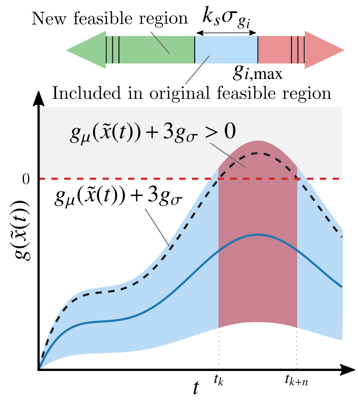
$$\text{subject to:} \quad \boldsymbol{g}_\mu(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq \boldsymbol{0}$$

$$\boldsymbol{g}_\sigma(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) - \boldsymbol{\sigma}_a \leq \boldsymbol{0}$$

$$\boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}},\tilde{\boldsymbol{w}}) = \boldsymbol{0} \qquad \text{(a.s)}$$

$$\dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}},\tilde{\boldsymbol{w}}) = \boldsymbol{0} \qquad \text{(a.s)}$$

Illustration



[1] Nagy and Braatz 2004; Azad and Alexander-Ramos 2021; X. Li et al. 2014

→ Worst-Case Robust UCCD

- A solution is robust if it remains feasible for all realizations of uncertainties **within the uncertainty set**
- This resembles a game between the optimizer and nature (adversarial opponent)[1]
- Results in a semi-infinite problem that can be replaced with the constraint maximization problem
- Uncertainties belong to their associated sets [2]

Worst-case Robust UCCD

$$\underset{\hat{\boldsymbol{u}},\hat{\boldsymbol{p}}}{\text{minimize:}} \quad v$$

$$\text{subject to:} \quad \Phi_i(t,\hat{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\hat{\boldsymbol{p}},\tilde{\boldsymbol{d}}) \leq 0$$

$$\text{for} \quad i = 1,\dots,n_g$$

$$\psi(\hat{\boldsymbol{u}},\hat{\boldsymbol{p}}) \leq \boldsymbol{0}$$

$$\underset{\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}}{\text{maximize:}} \quad g_i(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d})$$

$$\text{subject to:} \quad \boldsymbol{h}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}) = \boldsymbol{0}$$

$$\dot{\boldsymbol{\xi}}(t) - \boldsymbol{f}(t,\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{p},\boldsymbol{d}) = \boldsymbol{0}$$

$$\boldsymbol{u} \in \mathcal{R}_u(t), \boldsymbol{p} \in \mathcal{R}_p, \ \boldsymbol{d} \in \mathcal{R}_d(t)$$

[1] Bryson and Ho 1975 [2] Rahal and Z. Li 2021

→ Fuzzy Expected Value and Possibilistic Chance-Constrained

Fuzzy Expected Value UCCD[1]

$$\begin{aligned}
\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & \mathbb{E}[o(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})] \\
\text{subject to:} \quad & \mathbb{E}\left[\bar{\boldsymbol{g}}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})\right] \leq \boldsymbol{0} \\
& \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
& \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
\text{where:} \quad & \tilde{\boldsymbol{u}}(t) = \tilde{\boldsymbol{u}},\ \tilde{\boldsymbol{\xi}}(t) = \tilde{\boldsymbol{\xi}},\ \tilde{\boldsymbol{d}}(t) = \tilde{\boldsymbol{d}}
\end{aligned}$$

Possibilistic Chance-Constrained[2]

$$\begin{aligned}
\underset{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}}}{\text{minimize:}} \quad & \mathbb{E}[o(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}})] \\
\text{subject to:} \quad & \mathbb{POS}[g_i(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) > 0] \leq \mathbb{POS}_{f,i} \\
& \boldsymbol{h}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
& \dot{\tilde{\boldsymbol{\xi}}}(t) - \boldsymbol{f}(t,\tilde{\boldsymbol{u}},\tilde{\boldsymbol{\xi}},\tilde{\boldsymbol{p}},\tilde{\boldsymbol{d}}) = \boldsymbol{0} \\
\text{where:} \quad & \tilde{\boldsymbol{u}}(t) = \tilde{\boldsymbol{u}},\ \tilde{\boldsymbol{\xi}}(t) = \tilde{\boldsymbol{\xi}},\ \tilde{\boldsymbol{d}}(t) = \tilde{\boldsymbol{d}}
\end{aligned}$$

[1] Zhu 2009; Liu 2002 [2] Liu 2002

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Discussion

→ Discussion

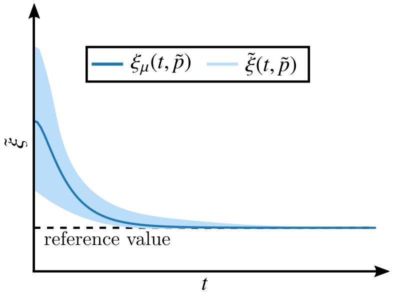
- **Linking stochastic and robust**
  - Assuming that the deterministic description of uncertainties is a just representation, the solution of the WCR-UCCD is equivalent in limit to the solution of SCC-UCCD with an infinitesimal probability of failure
- **Formulations from stochastic control theory**
  - Classical stochastic control focuses on idealized processes such as wiener and Poisson [1]
  - Methods from Iî̂to calculus are often used
- **Insights from robust control theory**
  - Various tools such as gain, phase, and disk margins, H_2, H_∞, and μ-synthesis have been developed to address uncertainty-related issues [2]
  - Regardless of control architecture, **uncertainties propagate in the dynamic system**

[1] Åström 1970; Yong 2020 [2] Paraskevopoulos 2017; Seiler, Packard, and Gahinet 2020

Discussion (Continued)

A notional infinite-horizon linear quadratic regulator (LQR) (which is an optimal controller for its associated cost function) reduces uncertainty in the system response over time to the reference value, assuming stability under the uncertainties.

LQR Example



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Questions?

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