A GAME THEORETIC APPROACH TO A RETAIL ELECTRICITY MARKET WITH A HIGH PENETRATION OF SMALL AND MID-SIZE RENEWABLE SUPPLIERS

BY

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A THESIS
SUBMITTED TO THE FACULTY OF

ALFRED UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

ALFRED, NEW YORK
APRIL, 2016

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ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor Dr. Ehsan Ghotbi for his valuable and constructive guidance during the development of this research work. His continuous support, motivation and patience helped me throughout this project. My grateful thanks are extended to the faculty and staff members of the Inamori School of Engineering, particularly Dean Edwards for her considerable support during my Master's study. I would also like to acknowledge my thesis committee members Dr. Wallace Leigh and Dr. Jalal Baghdadchi for their encouragement and helpful critiques of this research work. Lastly, I would like to thank my parents for their continuous support throughout my education.

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ABSTRACT

Game theory has provided a practical tool to model players' strategic behavior in electricity markets. In this work, a retail electricity market with a high penetration of renewable resources is modeled. Using game theory, the hourly clearing electricity prices, as well as the optimum behavior of market participants are obtained. In this model, which is inspired by the "Energy Internet" concept, consumers play an active role in managing their load demands. This highly dynamic model allows us to analyze consumers' reaction to price fluctuations. Spot pricing, which is employed here to model the electricity market, can make consumers react to the high electricity prices. This is particularly important in the demand side management, where consumers should modify their demand through financial incentives. Two types of active players are considered in this electricity market: small electricity suppliers and consumers. Electricity grid acts as an Independent system operator and is considered to be a complementary unit to compensate for the deficiency of power from small suppliers. The problem is formulated mathematically and a design of experiment (DOE) approach is employed to find the rational reaction set (RRS) of market participants. The intersection of these sets provides the Nash Equilibrium.

INTRODUCTION

The United States has one of the major energy markets in the world. As in 2014, end users in the U.S. consumed approximately 3863 billion kilowatt hours of energy in the form of electricity.[1] This huge amount of energy, which powers industry, transportation, residential and commercial units, requires an effective management of the critical components of the electricity market. Although the electricity markets have undergone massive changes in the last few decades, implementing an efficient market structure which addresses the major challenges in the energy market has not been an easy task. The non-storable nature of the electricity in large-scale has particularly distinguished the electricity market from other market structures. With the introduction of Plug-in-Electric vehicles to the market, the demand for electricity is expected to rise to unprecedented levels. Environmental concerns have made utilities shift their generation from fossil fuels to clean and sustainable resources. However, the intermittency of the renewable resources has posed new challenges to electricity providers. Implementing a reliable market framework that despite protecting the consumers from paying high rates, introduces competition at both retail and wholesale levels and is also consistent with environmental regulations, is at the core of every energy market. This study covers the challenges at the intersection of the foreseeable future technologies, namely smart grids and renewable integration, and the concept of game theory from the economic point of view. In the last few years, many scholars have applied game theory to wholesale electricity markets. However there is an urgent need to analyze the market in the presence of small suppliers in the context of a highly dynamic market framework. To the best of the author’s knowledge, this problem has not been previously studied in this context and with this game design. It is valuable to have a brief look at the history of the electricity markets in the U.S. and the challenges that they have been through since their founding in 1879.

A. History of electricity markets

Natural monopoly implies that efficiency in production would be higher when a single firm supplies the entire market.[2] Being considered a natural monopoly, electricity market was started as an unregulated industry. However, it has been subject to multiple periods of municipal, state and federal regulations since then. The main purpose of these regulations has been expressed to protect the consumers from paying unfair rates. Nevertheless, there's a controversy regarding the level to which these regulations have been effective in implementing a market structure that protects the consumers.

The electric utility industry has been subject to various levels of regulation since its founding in 1879. Initially, utilities built their own generating units and charged the customers for a bill that included all the costs associated to generation, transmission and distribution. Vertically integrated companies charged the consumers at monopoly prices. In microeconomics, economies of scale arise when the cost per unit decreases as output increases, because the fixed costs are spread out over more units of output. The development of the vertically integrated utilities in the market was primarily inspired by the economies of scale in the generation sector, as well as the complexities of coordinating the generation with transmission, which was deemed to be inseparable at the time. Economies of scale in generation convinced many that generation is also a natural monopoly.[3] On the other side, electric consumers were able to buy electricity from just one supplier and therefore, retail access was limited.

In 1899, a form of weak regulation called municipal franchise, was imposed on the utilities in the form of long-term contracts, which allowed firms to operate in particular cities and implement the required infrastructure in public areas like streets.[4] Although municipal franchises offered price ceilings, it was not until 1900 that a series of stricter regulations were imposed on utilities. The new regulation gave the commission the power to terminate a franchise by buying the assets of the utility to establish entry control. It allowed city authorities to unilaterally dictate the electricity rates after the franchise contract was expired. These regulations were basically developed to protect consumers against monopoly prices which were induced by the conventional wisdom that electricity is a natural monopoly and without regulation, prices will be based on the profit maximizing behavior.[5]

Shifting from municipal to the state regulations brought about a lot of controversy regarding the motives behind this major change. The main two conflicting theories are known as "public interest" and "positive" theories.[6] The first theory states that the competition during the municipal regulation was distorted by the municipal government's decisions in granting franchises, granting an excessive number of franchises or protecting a particular company from competition. As a result, consumers were exploited and state regulation was a necessity to protect consumers by enforcing a uniform regulation throughout the jurisdiction. The positive theory holds that the municipal regulations were successful in withholding utilities from exercising monopoly power and the introduction of the state regulations was primarily inspired by protecting utilities, rather than consumers, from competition. Jarrell's empirical results[7] based on the year that different states underwent the state regulation proved to be contradictory with the proposition of the public interest theory. He further noticed that the shift to the state regulation in states that joined the regulation prior to 1917 resulted in 25 percent increase in average price and 40 percent increase in average profit. This evidence supports the hypothesis that state regulations served to protect private utilities against the vigorous competition which was introduced through municipal regulations.

In 1935, the United States congress passed a law to facilitate the regulation of electric utilities at the state level. The Public Utility Holding Company Act (PUHCA) enforced state regulation by requiring the utility companies to incorporate in the same state where they are operating.[8] Under these regulations, the firms owned the monopoly of the market and prices were set based on the "fair rate of return". Averch et al. proposed that considering the fair rate of return is greater than the capital cost and lower than the monopoly price, then the firm tends to substitute the capital and operate at an output where cost is not minimized.[9]

From 1940s to 1960s, electric industry witnessed tremendous growth and success. During this period, utilities even voluntarily offered reduced prices and demand was growing. However, 1970s was the beginning of major challenges in the electric industry. The clean air act of 1970 authorized the development of regulations to limit emissions.[10] Three years later, the members of Organization of Arab Petroleum Export Countries (OAPEC) proclaimed an oil embargo, which resulted in dramatically increased oil prices.

This increase in the cost of fuels to operate power plants was reflected in retail power prices and utilities started to demand higher electricity prices from the state commission. As a response, the U.S. congress acted to reduce the country's dependency on foreign oil by passing the 1978 Public Utility Regulatory Policy Act (PURPA).[11] This law was designed to promote and encourage renewable resources and efficient generation technologies, such as natural gas-fired cogeneration. Utilities constructed power plants which relied on local fuels such as coal and Uranium. PURPA also required utilities to buy power from independent companies that could produce power for less than what it would have cost for the utility to generate the power, called "avoided cost",[12] and therefore promoted competition. PURPA signified the introduction of "competitive entry into the capital-intensive power generation business".[13] The Energy Policy Act of 1992 (EPACT) even promoted the competition more by removing the roadblocks in the way of independent generators. In particular, it created a competitive framework for the wholesale electricity market[14] by (i) creating a new class of exempt wholesale generators (EWG) and (ii) expanding the authority of the Federal Energy Regulatory Commission (FERC) to initiate open transmission access.[15] It was also aimed at increasing the energy conservation and efficiency at buildings' and utilities' level[14].

With the Three Mile Island accident and its consequences on the market, utilities and regulators started to realize that energy conservation and demand side management (DSM) could be less costly than constructing a new power plant.[16] Required by EPACT, states conducted Integrated Resource Planning (IRP) and evaluated the impact of purchased power contracts on the local distribution company.[17] In the upcoming years, the electricity industry underwent major regulatory reforms with the aim of introducing competition in various utility functions, which is broadly known as "electricity restructuring".[18]

By 1990s, the most prominent initiative was in California, where the California Power Exchange (PX) was established as a regional spot market for buyers and sellers to trade electricity. California's ISO, which started its operation in 1998, was designed to operate the state's power transmission grid and to provide open access to all qualified users.[19] California's electricity prices were among the highest in nation. Introduction of competition into the market was deemed to reduce these high costs, however, independent power producers and power brokers established a de facto oligopoly, which resulted in still high prices.[20] In the first two years of the operation of this market structure, the wholesale prices of electricity declined and average rates fluctuated between $20 and $50 per megawatt hour.[21] Figure 1 demonstrates price variations in the wholesale electricity market in California.[20] Order 888 by FERC required that utilities open their transmission to independent producers. This order was intended to introduce competition at the wholesale level. However, at 2000, the electricity sector went through unprecedented challenges. The prices broke the $100 per MWh and remained at extraordinarily high rates till the spring of 2001.

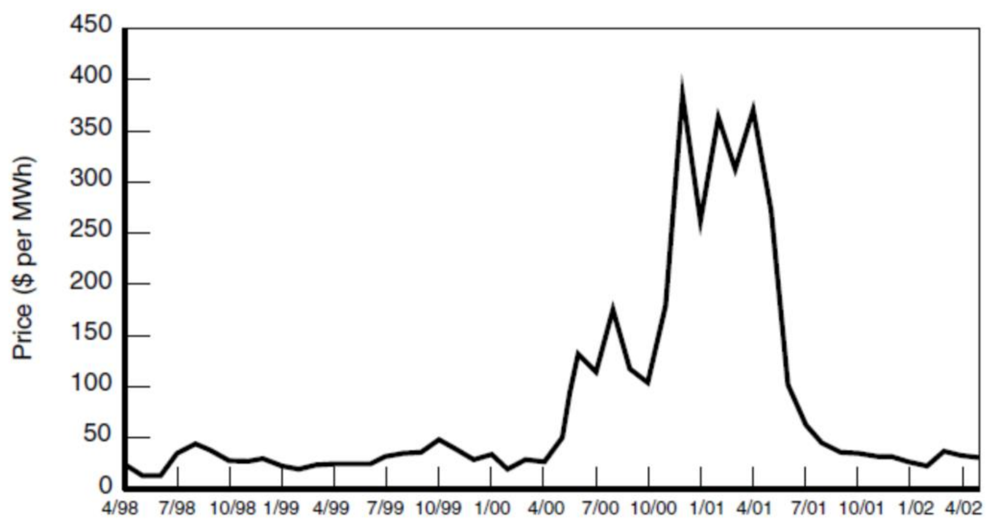

Figure 1. Average wholesale electricity prices in California 1998-2002[21].

While there's still a lot of controversy regarding the roots of the California's energy crisis, Weare mentions the following significant factors as the main reasons leading to this crisis[21]:

1- During the early years of the wholesale market in California, the generation capacity was ample, however, evidence demonstrate a tight electricity market in 2000 and the following years which seems to be an antecedent to the energy crisis.

2- California’s reliance on the imports from other states was influenced by the higher demand in those markets. This phenomenon is called the bottleneck effect.
3- Market manipulation and price gouging by the firms, which was a result of the oligopolistic nature of California’s market. This feature of the market allowed firms to increase the prices above the competitive levels.
4- As regulators were moving away from the traditional cost-based models, utilities abandoned their traditional role of constructing capacity.
5- The market failed to send strong signals to demonstrate a need for further investments.

Facing the energy crisis, several actions have been taken by California and federal government authorities to mitigate the impacts of the energy crisis.[22] Lessons from the California’s energy crisis hindered the restructuring of the electricity market in other states. Overwhelmed by the skyrocketing prices, electricity shortages and bankrupt utilities, the institutions established by the 1996 reform were dismantled. Policymakers were facing the daunting task of restructuring the market from scratch. However, in the struggle of the restructuring the market, a movement to reinstate elements of competition, particularly in the wholesale market was inevitable.[23]

In the last decade, the urgent need for a more efficient and reliable electricity market was triggered by the environmental concerns, as well as the harrowing experiences of the California’s energy crisis and northeastern major blackouts. The need for energy conservation and demand side management has called for a dynamic market structure in which the consumers play an active role in the electricity market. In addition, the integration of renewable resources requires a more flexible market structure. These challenges gave birth to the introduction of smart grids as the future of the electricity markets.

B. Smart Grids

The massive power blackout in the northeastern US and Canada in 2003 signified the role of system reliability. Moving towards a more competitive electricity market in both wholesale and retail markets, a more complicated and intelligent electricity network seems to be a necessity to address the challenges in the electricity market. Currently, the electrical grid suffers from a number of problems, among which, the following are the most important ones[24]:

- The average age of power plants is 35 years, thus the electricity grid is old.
- More than half of the electricity is generated through coal, thus it's dirty.
- The delivery efficiency of electricity is 35% and therefore, it's inefficient.
- Based on the major northeast blackouts, the grid is vulnerable.

As a result, "Smart Grid, which is an electricity network that can intelligently integrate the actions of all users connected to it, including generators, consumers and those that do both, was introduced in order to efficiently deliver sustainable, economic and secure electricity supplies".[25] The need for a Smart Grid basically stems from the environmental concerns, which entail providing a safe network to encourage electricity consumers to play an active role in the supply chain. This active involvement by the end users in the electricity market is not only limited to managing consumption in response to the price signals from the utility grid. Smart grids can accommodate the integration of a multitude number of small and mid-size renewable suppliers, particularly at the residential sector. This quality of the smart grids increases the generation capacity, and if properly regulated, can result in lower electricity rates for local end users. In addition, according to the anticipations of the high penetration of Plug-in Electric Vehicles (PEVs) in the U.S., these small suppliers can also act as storage devices for the utility grid at the times of need. Smart Power Management system, which is a main feature of smart grids, enables monitoring, analyzing, controlling and communication for power generation, transmission and distribution at every level of the system.[26]

Apart from the smart management system, smart grid entails a number of features that guarantee the efficiency of the network, among which smart infrastructure and smart protection systems are the most notable. Smart infrastructure system is the necessary energy, information and communication infrastructure underlying the network that enables the two-way flow of electricity and information.[27] Smart protection system provides advanced reliability analysis, failure protection and security and privacy protection.[27] Transition to the Smart Grid, while driven by market forces, entails selfcoordination, self-awareness, self-healing and self-reconfiguration[28] and that's what distinguishes smart grids from the traditionally designed markets.

Smart Grid will ensure bi-directional flow of electricity between supply and demand sources. In addition, active network management technologies are necessary for utilities to efficiently integrate distributed generation (DG), including residential micro generation.[25] Considering all the benefits of implementing an intelligent network, the American recovery and Reinvestment Act of 2009 (ARRA) was designed to provide additional stimulus to accelerate the smart grid transition which will eventually result in improvements in the following areas:[29]

- Reliability – the reliability of grid will be improved by several factors including less disturbances and blackouts.
- Economics – by reducing the electricity rates and creating new jobs and enhancing the U.S. gross domestic product.
- Efficiency – by reducing the costs associated with generation, transmission and distribution of the electricity.
- Environmental – by reducing the amount of emissions by increasing the integration of renewable resources.
- Security – by reducing the probability of manmade attacks and natural disasters.
- Safety – by reducing the consequences of any grid-related events.

The benefits of the smart grid at the societal level provided by a reliable network with a decreased probability of blackouts and downward pricing, is facilitated by the active participation of end-users at the supply chain. The "Plug-and-play" interface accommodates all the generation and storage options available to the grid. Smart grid's ability to integrate renewable resources leads to a broader generation portfolio, as well as improved carbon mitigation.[30] The intermittent nature of renewable resources, particularly wind and solar, requires a tool to diminish the consequences on the grid. Decision & control, as well as demand response programs are among the most common tools that have been employed to address this issue.[31] Facilitated by the smart grid, the interactions among end-users and utilities can be channeled towards the optimal usage of energy based on environmental concerns, price preferences and system's technical issues.[32]

Overcoming various technical challenges in design, control and implementation of smart grids requires the adoption of advanced technologies and methodologies. Maximizing the profit or reducing the costs is the main goal of all market participants. Since in modeling the market behavior, the decisions of all agents in the market will influence their competitors' choice, it is necessary to employ a tool which takes into consideration the strategic interactions among these players. In this respect, game theory is expected to provide a key analytical tool in the design and optimization of the market.

C. Game Theory

According to psychology of choice, which is based on human rationality, even in the simplest problems human judgement is susceptive to factors such as problem formulation, personal characteristics and habits of the decision maker.[33] These factors lead to sub-optimal decisions. While many design theories have focused on reducing the effects of these sub-optimal choices, game theory, as a branch applied mathematics provides promising solutions to address the situations with conflicting interests.

The concepts within game theory can be traced back to the works of Cournot, Bertrand and Von Stackeleberg.[34] Although the field was born with work of John Von Neumann and Oskar Morgenstern, the theory thrived extensively in 1950s by John Nash. The broad scope of game theory in economics, political and social sciences signifies the significance of this methodology. Game theory is basically concerned with games of strategy, as opposed pure games of chance. In other words, it is concerned with games whose outcome does not depend on chance alone, but on the decisions that players make during the course of play[35]. Game theory can be defined as the study of situations of conflict and cooperation between intelligent and rational decision makers and it provides analytical techniques for situations in which two or more individuals make decisions that will influence one another's welfare.[36]

In order to introduce the concept of strategy, Blackwell and Girshick[37] provided an example in which you are to play a white piece in a single game of chess, however, you are unable to be presents at the occasion. Yet, you can have a representative at the game. In order to make sure that the representative will carry out your instructions exactly, you need to create a set of instructions for every possible circumstance that might arise during the game. Thus, a strategy for White, must specify the first move and, for each possible reply by Black, a corresponding next move should be available, till the game ends. Therefore, a strategy of a single player in a given game can be defined as a complete behavioral plan which specifies the player's behavior that is his decisions for all possible circumstances that may arise during the game[35].

Every game consists of a number of entities and characteristics. The participants in the game are competitors or players. One of the underlying assumptions for any game-based design is the rationality of these players. In engineering design, rationality is defined as making decisions consistently in pursuit of one's own objectives.[36] Each player in the game has a set of available actions or strategies, and a set of payoffs corresponding to these strategies. The payoff for a given player depends not only on the chosen strategy by that player, but also on the strategies taken by the other players present in the game. Payoff, also called utility, reflects the desirability of an outcome to the player. Since the objective functions of the players are often conflicting, game theory provides a reliable tool to model the interactions among players.

There are various classifications for games. Games can be classified according to the number of players and the payoff. If the players make payments only to each other, this is said to be zero-sum.[38] In a zero-sum game, one person's gain equals another's loss. In another classification, a game is finite when every player has a limited number of moves. Generally, games can be classified into three major categories, based on the types of interactions that the players are allowed to carry on: Cooperative game, non-cooperative game (Nash), and extensive games. Non-cooperative game theory studies the strategic interactions among self-interested players. Therefore, the players don't have any prior knowledge about their competitors' strategic decisions and thus, they'll make assumptions regarding the strategies selected by other players.[39] On the contrary, in a cooperative game, players form a coalition and thus, players are aware of the strategies taken by other players. While extensive games can be considered non-cooperative, their main distinguishing feature is the fact that players make decisions sequentially.

Nash equilibrium is a strategy for each player where the player cannot improve upon unilaterally. In Nash equilibrium players have no incentive for changing their strategy, assuming that none of the other players is going to change theirs[40].

To better comprehend the prominence and applications of game theory, it is helpful to provide an example. The very well-known prisoner's dilemma, was first formed by Albert W. Tucker at a seminar at Stanford University.[41] It is a strategic game among two players. The players are two prisoners who are being interrogated in separate rooms. Note that in this game, there's no order among the players, as they act simultaneously and with no prior knowledge of the other player's action. This feature makes the symmetry possible in the game. The game will stay the same if the players are exchanged.

In this game of strategy, each player has two strategies, either to cooperate, or to defect. R is the "reward" payoff that players receive for cooperating. P is the "punishment" that they receive if both defect. T is the "temptation" that each player receives as the sole defector and S is the "sucker" payoff that each player receives as the sole cooperator. The payoff values satisfy the following chain of inequalities:

$$T > R > P > S$$

Figure 2 demonstrates the possible resulting payoffs in the game. For the sake of clarity, the payoffs associated with player 2 are expressed in bold.

		Prisoner 1	
		Cooperate	Defect
Prisoner 2	Cooperate	(**R**,R)	(**S**, T)
	Defect	(**T**, S)	(**P**,P)

Figure 2. Prisoner's dilemma possible payoffs.

Now, suppose that player 1 cooperates. Then prisoner 2 will get R by cooperation and will get T for deflecting. Thus, it is better to defect. Assuming that player 1 defects, player 2 gets S for cooperating and P for defecting and again it is better to defect. The symmetrical nature of this game guarantees the same results for the other player. Therefore, we can conclude that the dominant strategy for both players is to defect. Rational players will defect and get P, while two irrational players can cooperate and get a better result of R. Accordingly, (D, D) is the only Nash equilibrium in this game.[42] It is also worth noting that in the context of game theory, all players are assumed to be rational.

Having acquired a general knowledge of strategic games, we can now study the application of game theory to the multi-objective problems in cooperative and non-cooperative problems. Multi-objective problems can be modeled in a game theoretic context in which, each player is associated with an objective function and seeks to optimize its individual objective function.[39]

1. Non-Cooperative game theory in design

In a non-cooperative game, each player has control over a set of variables and strives to optimize his individual objective function, regardless of its impact on other players. If the players obtain an equilibrium solution, it is called the Nash Equilibrium, which is the most common concept of non-cooperative game. Finding the Nash equilibrium is a challenging task, especially when the number of players is more than two. One of the methods that has been proposed to find the Nash equilibrium is Nikaido-Isoda function.[43] Rational reaction set with DOE-RSM[44] (design of experiment - response surface method), proposed by Lewis and Mistree, has been used extensively by authors to investigate the Nash solution for non-cooperative games. For the sake of simplicity, this study employs DOE-RSM method to find the solution for the market model. Also, monotonicity analysis[45] is among the well-known methodologies to compute the Nash equilibrium. While in some design problems, closed form expressions of Nash equilibrium can be obtained, in general, numerical techniques are required to find the solution.[39]

2. Cooperative game theory in design

In a cooperative game, all a number of players form a coalition and therefore, are aware of the strategies chosen by other players. They cooperate to find a Pareto-optimal solution. It is worthy to note that cooperation can result in an improved payoff of a non-cooperative game. Various techniques are introduced to find the cooperation solution. Some of these methods are based on obtaining the Pareto optimal frontier set, while other employ bargaining functions to maximize the difference of the objective function from the worst value that they can get in the game.[39]

D. A game approach to a retail electricity market

Increasing the level of the competition, a worldwide trend in the evolution of electricity markets, has made game theory a notably popular approach to find the market equilibrium. In this study, we model a retail electricity market with a high penetration of renewable resources. Using game theory, the clearing electricity prices, as well as the optimum behavior of the market participants are obtained. In this model, which is inspired by the "Energy Internet" concept, consumers play an active role in managing their load demands. This highly dynamic model allows us to analyze consumers' reaction to price fluctuations. Spot pricing, which is employed here to model the electricity market, can make consumers react to the high electricity prices. This is particularly important in the demand side management, where consumers should modify their demand through financial incentives. Two types of active players are considered in this electricity market, small electricity suppliers and consumers. Electricity grid, while present in the market, only takes the responsibility to compensate for the deficiency of power from small and mid-size suppliers. The problem is formulated mathematically, subject to a number of local and global constraints to find the Nash equilibrium.

Environmental and economic issues have provided great incentives for traditionally regulated electricity markets to move forwards to a more liberated market. The deregulated framework of electricity market, which has been proposed by numerous scholars, calls for an increase in the level of competition. This popular trend has resulted in various market structures in different parts of the world which can be classified into (i) Power pools or centralized markets and (ii) Bilateral contact model or decentralized markets.[46] Due to the great diversity of the market structures, several methods have been introduced to analyze and optimize different aspects of deregulated electricity markets. The level of competition in these models is distinctively salient, since it establishes the degree to which the market is deregulated. These models vary significantly in the level of competition[47] from the most uncompetitive situation, Stackelberg,[48]to the most competitive model, Bertrand Competition.[49] Moving towards a highly competitive electricity market, it is necessary to implement safeguards and supportive market conditions.[3] The oligopolistic aspect of the deregulated electricity market makes suppliers act strategically in order to maximize their profit. These strategies require a game theoretic approach: traditional non-cooperative game theory, evolution game or agent-based modeling.[50]

In the last decade, the urgent need for a more efficient and reliable electricity market, which was triggered by environmental concern and estimations of high penetration of Plug-in Electric Vehicles (PEVs) into the market, has necessitated an intelligent construct within the electricity market. Smart grids can efficiently and effectively address the current concerns in the deregulated electricity markets. The future of electricity markets is highly reliant on renewable energies. The high penetration of renewable resources, particularly wind and solar into the grid enables small and mid-size generating units.[51] Renewable integration, coupled with volatile electricity prices,[52]signify the importance of energy management in smart grids. The high penetration of small and mid-size distributed generators in the residential sector alleviates the concerns regarding high load demand and sustainability issues. These agents supply their own electricity demand and sell the rest to consumers, who can manage their load demand in response to the spot market prices. The financial incentive that is taken into consideration in consumers' objective function provides enough motivation for consumers to respond to the volatile electricity prices. The widespread integration of renewable resources at the residential level, calls for an intelligent distribution system to tackle control and storage challenges. The ability of Distributed Generators (DGs) and Distributed Energy Storage Devices (DESDs) to plug and play, which is a notable feature of smart grids, enables energy sources and storage devices to be connected with the distribution grid, anywhere and anytime.[53]

The Future Renewable Electric Energy Delivery and management Systems Center (FREEDM) introduced a framework for future power distribution of DGs and DESDs through the concept of "Energy Internet".[54] This model was inspired by the decentralization process in computer industry in 1980s, where all individual users became a part of the infrastructure in the restructured industry. Huang et al.[54] proposed the "Energy Internet" model into the electricity market. The active role of consumers, which is facilitated by utilizing Distributed Energy resources or Storage devices, is a central quality of this model. The proposed model entails three important features: plug and play interface, an information router and an open standard operating system. The first feature, plug and play, enables all individual users to connect to the utility grid anytime, anywhere. Thus, all agents are easily recognized as soon as they are connected and necessary information regarding loads, storage and generation is collected and managed by the energy router. Finally, the open-standard operating system coordinates system management with other energy routers. This vision, allows for a large number of DG and DESD to participate in the electricity market and communicate information within the network. Since in this model, consumers not only purchase electricity, but also sell their excess generation to the grid, a bidirectional metering interface is required.[55] Information Communication technologies have facilitated the two way communication between different entities.[56] The resulting smart metering interface assists distribution operators to collect the required information about usage and generation pattern in order to increase the quality of their services.[57] Later, this data, which indicate consumers' reaction to price fluctuations, can be employed for a demand side management analysis.

Searching for the market equilibrium, a state where all market participants have made the most optimum decision is an objective for market participants.[58] The market equilibrium enables all players to make an optimal decision, based on their competitors' choice. The multitude number of suppliers and consumers in this market structure calls for a game theoretic approach to find the clearing electricity market prices.

This work takes a pragmatic stand point towards the implementation of the smart grid with a high penetration of the renewable resources as small and mid-size suppliers. While scholarly efforts in the last few years have generally failed to recognize the significance of the role of the consumers as the potential small suppliers, the model proposed here, liberates the consumers from the traditional boundaries by enabling them to communicate in a bi-directional networking framework, which is facilitated by the smart grid technologies. This communication enables them to interact with the Regional Transmission organizations (RTO) or Independent System Operators (ISO) to be able to optimize their market behavior. This paper employs game theory to analyze and optimize the market participant’s behavior.

MATERIALS AND METHODS

E. Market Framework

Efficient implementation of a restructured market is a challenging task. The harrowing experience of California's 2000 major black outs, which was a result of market manipulation by suppliers, hampered the deregulation in California's electricity market.[21] Liberalization of the traditionally regulated electricity markets, gave rise to the growth of Independent System Operators (ISO), which are mainly responsible for scheduling, dispatching power, reliability and ancillary services.[59] In the proposed Energy Internet market, small suppliers, which are composed of generating and storage units, such as wind turbines, solar systems, diesel generators and Distributed Energy Storage Devices (DESDs), can communicate and exchange information with other market participants through an internet-like structure. This communication is necessary for maximizing their profit. In this market structure, electricity suppliers become independent of the conventional electricity suppliers.[60] This model capacitates the large integration of renewable resources, which is a necessary factor in moving towards a more sustainable design for the future of the electricity market.

Maximizing the profit is a goal of market participants in every market. Since the decisions of all agents in the market will influence their competitors' choice, it is necessary to employ a tool which takes into consideration the strategic interactions among all these players. Game theory provides a suitable approach to model the interactions among these players.[34] The active participation of small distributed generators in the market results in a highly complex and dynamic market structure. The level of complexity and competition in the market play a determining role in selecting a game approach that best fits the market structure. This work applies game theory to the proposed electricity market. Considering that the distribution power operations of utility companies have been the central focus of literature in the last few years, increasing consumers' role in a deregulated market calls for an urgent attention to address distributed energy generation and storage issues.

In this market structure, utility grid no longer holds the monopoly of the whole market. In fact, it is considered a complementary unit which compensates for the deficiency of power for other suppliers. It makes profit by selling electricity, as well as providing ancillary services to consumers and small and mid-size suppliers. In this research, the role of utility grid as an active player is excluded from the game and it is not considered a separate player. The focus is on the interactions among a large number of suppliers and consumers, however, the significant role of the utility grid is evident when a small supplier makes a strategic decision to buy electricity from the grid instead of switching on a diesel generator with a high financial and environmental cost.

While in many market structures consumers have an insignificant impact on the framework of the market, this model enables them to control and manage their load demand in response to the price fluctuations, which leads to the minimization of costs. A non-cooperative game is employed among consumers and suppliers to find the Nash equilibrium, which is the main objective of this research. Firstly, the interactions among the consumers are modeled through a non-cooperative game. Since consumers act as separate entities, a non-cooperative game can properly model their behavior. On the contrary, suppliers' strategic behavior seems to be more efficient and reliable when all suppliers cooperate with each other to supply the total load demand. This collaboration results in a more stable market. Therefore, a cooperative game among suppliers is taken into consideration. Finally, a non-cooperative game among consumers and suppliers is taken into consideration to find the Nash equilibrium. The rational reaction set of market participants can be obtained using a Design of Experiment approach. These rational reaction sets are used to model the interactions among the consumers and the suppliers to find the Nash equilibrium. This Equilibrium entails the clearing electricity price at each hour.

In order to model the market, a series of assumptions are taken into consideration. This research employs the same assumptions as the Cournot model, in which all units produce a homogenous product. Cournot model is an economic model in which the firms compete on the amount of the output they will produce. Static in nature, each firm's strategy is the quantity of the output for a homogenous good. Other considerations include:

- The market price is influenced by the total supply and is fixed for all units.
- Supplier's output affects the market price.

- The number of suppliers is constant.
- Suppliers act simultaneously and compete in output quantity.
- All players are rational. They can manage their own generating units and dispatchable loads to maximize their profit.

In a traditionally regulated market, the price of electricity is set by regulators. However, in a retail competitive market, consumers' freedom in choosing from a wide range of suppliers with various services will eventually result in a market with lower prices. Moreover, companies cannot exercise market power to manipulate the price of electricity. The electricity cost function is based on the well-known Cournot model, which is widely used to approximate the competition in the electricity market.[61] In particular, the electricity price is considered to be a function of the aggregate electricity output of all suppliers. Since suppliers are infinitesimal, they have no effect on the market price.[62]

F. Problem Formulation

The mathematical model of the problem is formulated here. Two types of players are taken into consideration:

(i) Small and mid-size electricity suppliers that sell their excess power generation.
(ii) Consumers

Utility grid operates on a large scale and can provide electricity for the whole market. However, due to environmental and cost issues, small and mid-size renewable DGs have a higher priority in supplying the demand. In case of the power shortage from DGs, utility grid will act as a complementary unit. This approach enhances the market's efficiency by reducing the costs associated with coal or other costly generators. At the same time, it can secure the load demand by the least possible cost. The objective function for each entity is presented and the local and global constraints are taken into consideration to model the problem.

1. Objective functions

The objective function of the i^{th} supplier is defined as the summations of the differences between revenue and cost over 24 hours in an hourly interval. Suppliers seek to increase their profit according to the following equation:

$$\text{Max } F_i = \sum_{t=1}^{t=24} (R_{i,t} - C_{i,t}) \tag{1}$$

where $R_{i,t}$ and $C_{i,t}$ are the revenue and cost functions, respectively of the i^{th} player at t^{th} hour.

The electricity price is a function of the aggregate demand and in this market is characterized by an inverse-demand function for each hour, with a negative slope. This function is based on the basic notion of supply and demand. Two different groups, electricity suppliers and consumers are taken into consideration to determine the price of electricity at each time interval. This price is the intersection of the supply and demand curves and is called the market equilibrium. The law of demand states that for the same quality of goods, when the price goes up, the demand will fall. This explains the downward sloping of the demand curve in Figure 3. The supply curve demonstrates the relationship between the electricity price and the quantity that they can offer. Therefore, as the price increases, the quantity of the goods supplied will also go up. Equation (2) indicates the electricity price function employed in this work.

$$\lambda_t(Pd) = -\alpha \times Pd_t + \beta \tag{2}$$

where Pd_t is the total load demand of the consumers at t^{th} hour. λ_t is the hourly electricity price in \$/Kwh and α and β are load demand coefficients. The retail electricity price is assumed to be identical for the whole residential distribution system.

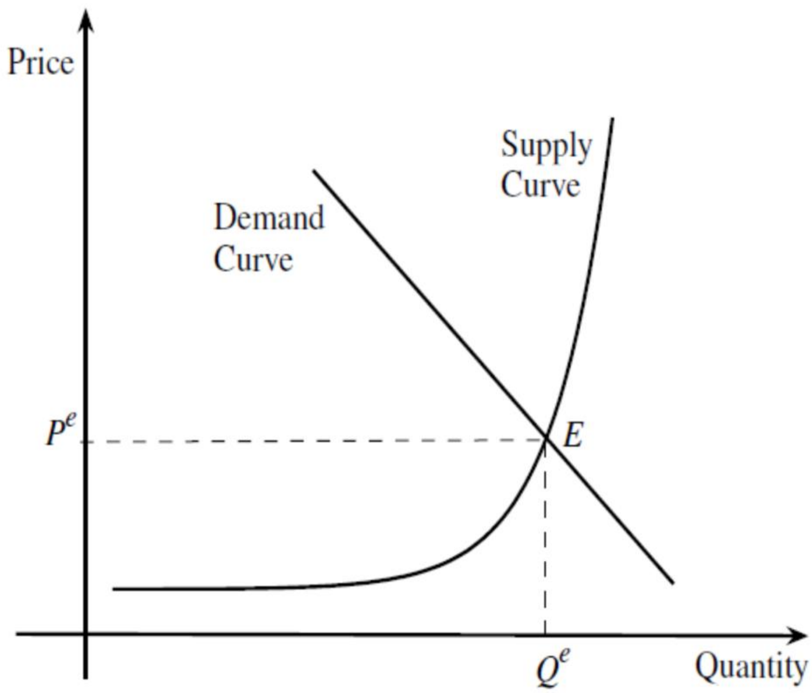

Figure 3. Supply and demand curves and market equilibrium price.

We assume that supplier units are composed of wind turbine, solar panels, diesel generators and storage devices. The electricity demand of each individual supplier is excluded from these calculations, as they are considered to be capable of supplying their own demand at all time. Therefore, the revenue function can be defined as:

$$R_{i,t} = \lambda_t(Pd) \times [Pw_{i,t} + Ps_{i,t} + Pdg_{i,t} + Pdesd_{i,t}] \quad (3)$$

where $Pw_{i,t}$ is the wind power output, $Ps_{i,t}$ the solar power output, $Pdg_{i,t}$ the diesel generator power output and $Pdesd_{i,t}$ the power from the distributed energy storage devices of the i^{th} supplier at t^{th} hour in Kw. It is worthy to note that $Pdesd_{i,t}$ can be positive or negative based on the charging status of each energy storage unit.

In this research, the cost of electricity is primarily a function of operating costs.[63] The cost function of player i can be expressed as:

$$C_{i,t} = Cw(Pw_{i,t}) + Cs(Ps_{i,t}) + Cdg(Pdg_{i,t}) + Cdesd(Pdesd_{i,t}) \quad (4)$$

where $C_{i,t}$ is the total cost function of player i at t^{th} hour. Cw and Cs present the costs associated with installation, maintenance and generation of power through wind turbine and solar systems, respectively. In long terms, the generation costs of renewable resources as well as the capital costs can be considered negligible, thus $Cw = 0$ and $Cs = 0$. Also, this study does not include the costs associated with the deterioration of storage devices, therefore $Cdesd = 0$. The cost function associated with diesel generators is approximated as a quadratic function:

$$Cdg_{i,t} = a_i Pdg_i^2(t) + b_i Pdg_i(t) + c_i \quad (5)$$

where a_i, b_i, and c_i are diesel generator's coefficients of cost function. The fuel consumption of a diesel generator is a function of its capacity, as well as the load it is operating at. The exact values for these coefficients are available for DGs with a high power rating.[64] In this research, diesel generators are included in the generation only for emergency backup. These generators are not efficient to operate at low rates and therefore, it is more reasonable for suppliers to avoid switching them on. An estimate of how much fuel a generator consumes is approximated as a quadratic function.[65]

According to the defined revenue and cost functions, equation (1) can be written as:

$$\text{Max } F_i = \sum_{t=1}^{t=24} (R_{i,t} - C_{i,t})$$
$$= \sum_{t=1}^{t=24} \lambda_t(Pd) \times [Pw_{i,t} + Ps_{i,t} + Pdg_{i,t} + Pdesd_{i,t}] - [a_i Pdg_i^2(t) + b_i Pdg_i(t) + c_i] \quad (6)$$

For the electricity consumers, the objective function is to minimize the cost by managing their own dispatchable load. For player i at t^{th} hour, the objective function can be expressed as:

$$\text{Min } F_i = \sum_{t=1}^{t=24} \lambda_t(Pd) \times Pd_{i,t} \qquad (7)$$

where $Pd_{i,t}$ is the load demand of the i^{th} player at t^{th} hour. In this type of energy cell, each player has the ability to manage and control its hourly load demand, subject to a local constraint.

As mentioned before, the main role of the utility grid in this model is to secure the critical load. In addition, suppliers can sell electricity to the gird at any time. However, a local constraint regarding the amount of tradable electricity of each unit is imposed on suppliers. The limited flexibility of the utility grid in this market structure makes it unable to exercise market power. An entity that is unable to exercise market power is known as price taker[66] and in this work, small suppliers are considered to be price takers, as they have no influence on the final price of the electricity. Here, the utility grid doesn't act as an active player; it doesn't have the monopoly of the electricity market. It is considered as an infinite source of energy with the capability to compensate for power deficiencies in the market. This feature can be considered as the result of buying the extra power generation from suppliers.

2. Constraints

Each energy cell is subject to a number of local and global constraints. These constraints either demonstrate the technical issues in renewable or diesel generation, or they are necessary assumptions in modeling the problem more accurately.

a. Local constraints

Wind generation is subject to the following constraint. Every wind turbine operates between a maximum and minimum level of generation. For the sake of simplicity, the technical details of wind power production are not taken into consideration.

$$Pw_{i,min} \leq Pw_{i,t} \leq Pw_{i,max} \quad (8)$$

where $Pw_{i,t}$ is the power output of wind turbine for the i^{th} player at t^{th} hour. $Pw_{i,min}$ and $Pw_{i,max}$ are the minimum and maximum power output of the wind turbine, respectively.

$$Ps_{i,t} \leq Ps_{i,max} \quad (9)$$

where $Ps_{i,t}$ corresponds to the power generation through solar energy for the i^{th} player at t^{th} hour. $Ps_{i,max}$ is the maximum power output of the solar system. Wind and solar power generation is modeled in SAM (System Advisor Model), which is developed by National Renewable Energy Laboratory.[67]

Technical limitations of diesel generators must be taken into consideration when modeling this problem. The power output of any generator must not exceed its rating. Moreover, for a reliable operation, generators' output must not drop below a certain value. Therefore, DGs, when active, must satisfy the following constraint:

$$Pdg_{i,min} \leq Pdg_{i,t} \leq Pdg_{i,max} \quad (10)$$

where $Pdg_{i,t}$ is the power output of the diesel generator of player i at the t^{th} hour. $Pdg_{i,min}$ and $Pdg_{i,max}$ are respectively the minimum and maximum power outputs of the diesel generator. In other words, the desired power output must be greater than a minimum level. High costs of operating a diesel generator, coupled with technical issues, restrict the suppliers from switching on the generator. In this model, we assume that the capacity of the diesel generators for the first and second suppliers, are 20 and 30 kw respectively. Every storage device is subject to the following constraints:

$$\mathrm{Pdesd}_{i,min} \leq \mathrm{Pdesd}_{i,t} \leq \mathrm{Pdesd}_{i,max} \quad (11)$$

where $\mathrm{Pdesd}_{i,t}$ is the power output of the energy storage unit of the i^{th} player at t^{th} hour. $\mathrm{Pdesd}_{i,min}$ and $\mathrm{Pdesd}_{i,max}$ are respectively the minimum and maximum power output of the storage unit. Also, the battery's State-of-Charge (SOC) imposes some constraints on any storage unit. SOC is defined as the energy stored at the moment divided by the maximum energy that can be stored.[68] A basic principle about state–of-charge must be taken into consideration. Units with higher SOC release more power when discharging, while units with lower SOC absorb more power when charging.[69] The statement of charge of each energy storage device must be within the safe range. To avoid any SOC imbalance, which can result in over charge/over discharge of a battery,[70] the following constraint must be satisfied:

$$\mathrm{SOC}_{i,min} \leq \mathrm{SOC}_{i,t} \leq \mathrm{SOC}_{i,max} \quad (12)$$

where $\mathrm{SOC}_{i,min}$ is the minimum battery storage state-of-charge and $\mathrm{SOC}_{i,max}$ is the maximum battery storage state-of-charge. In this paper, the values for $\mathrm{SOC}_{i,min}$ and $\mathrm{SOC}_{i,max}$ are considered to be 0.2 and 0.8, respectively. If the SOC of a storage unit goes beyond the safe range, the energy storage unit will switch to a stand-by mode.[60] The battery state –of-charge for each hour is calculated through the following equation:

$$\mathrm{SOC}_i(t+1) = \mathrm{SOC}_i(t) - \mathrm{Pdesd}_i(t)\frac{\Delta t}{\mathrm{Edesd}_i} \quad (13)$$

where Edesd_i is the battery capacity in Kwh. Δt refers to the time interval, which is considered to be 1 hour. Depending on charging or discharging status, Pdesd_i might be positive or negative.

Constraints (14) ensures the availability of a certain amount of electricity stored in DESD at the beginning of the next day.

$$\mathrm{SOC_{i,end}} \le \mathrm{SOC_{i,24}} \tag{14}$$

Although this market structure allows suppliers to buy or sell electricity from the grid, every entity is subject to the following constraint when attempt to sell electricity:

$$\left|\mathrm{Pg_{i,t}}\right| \le 0.1\,(\mathrm{Pw_{i,t}} + \mathrm{Ps_{i,t}} + \mathrm{Pdg_{i,t}} + \mathrm{Pdesd_{i,t}}) \tag{15}$$

where $\mathrm{Pg_{i,t}}$ is the power sold to the utility grid by the $\mathrm{i^{th}}$ player at the $\mathrm{t^{th}}$ hour. If $\mathrm{Pg_{i,t}}$ is positive, it implies buying electricity from the grid and if it's negative, it implies selling electricity to the grid.

All suppliers are subject to the aforementioned constraints. Since consumers have the ability to manage and control their dispatchable load at any given hour, the following constraint must be satisfied. This constraint indicates that despite the flexibility of consumers in managing their load demand, the variation in their demand is subject to a minimum and maximum. These high and low ends are derived from the basic load demand of each consumer and it fluctuates according to time.

$$\sigma_1 \mathrm{Pb_i(t)} \le \mathrm{Pd_i(t)} \le \sigma_2 \mathrm{Pb_i(t)} \tag{16}$$

where σ_1 and σ_2 are the minimum and maximum percentage of the manageable load, respectively and $Pb_i(t)$ is the basic load demand of the $\mathrm{i^{th}}$ player at $\mathrm{t^{th}}$ hour.

b. Global constraint

According to the concept of conservation of energy, the amount of generated power is equal to the consumed power. Due to small and mid-size capacity of the suppliers in this paper, the amount of power loss is considered to be negligible. This balance can be expressed as:

$$\sum_{i\in N}[Pw_i(t) + Ps_i(t) + Pdg_i(t) + Pdesd_i(t) + Pg_i(t) = \sum_{i\in N} Pd_i(t) \qquad (17)$$

Since the electricity cannot be stored, it is necessary to maintain that the generated electricity must be equal to the consumption of electricity. This equality constraint must be satisfied for the whole market model for any given hour.

In any market simulation, the behavior of market participants is of major significance and their strategic interactions must be taken into consideration. The structure of the market defines the extent to which market participants can influence market prices by their own behavior. Assuming that all players are acting rationally, power suppliers and consumers choose strategies to gain the maximum payoff. Power suppliers maximize their profit by decreasing the costs associated with power generation. Consumers try to minimize their objective function by managing their load demand as a reaction to high electricity prices. Game theory provides a tool to model this context. This work considers a case in which suppliers communicate and share information with each other to form a coalition. This coalition is expected to improve market efficiency and stability. No cooperation is considered among consumers since they are separate entities. After that, a DOE method is employed to find the Nash equilibrium for the game between suppliers and consumers.

Nash equilibrium provides the best possible strategy for any player, given the strategies of other players. In Nash equilibrium, there is no incentive for players to unilaterally deviate from their current strategy.[71] All players are assumed to be acting reasonably in order to increase their pay off functions. A game consists of the following three elements: a set of players, a set of actions and a payoff function available to each player. An action profiles is a list of actions available to each entity. A payoff function represents players' preferences over action profiles.

Considering the action profile a_i of every player i in a strategic game, a^* is the Nash equilibrium if a^* is at least as good for player i as the action profile (a_i, a^*_{-i}); where every other player j chooses a^*_j, while plyer i chooses a_i.[72] Thus:

$$U_i(a^*) \geq U_i(a_i, a^*_{-i}) \text{for every action } a_i \text{ of player i} \quad (18)$$

This means that if all players choose their equilibria profiles, no action profile generates a more preferable outcome for player i than the Nash equilibrium.

In order to find the Nash equilibrium, the rational reaction set (RRS) for each type of players should be obtained. The intersection of these sets provides the Nash Equilibrium. One approach to estimate the RRS would be sensitivity based approach.[73] The other approach would be a Design of Experiment (DOE). Although sensitivity based approach is more accurate than DOE [74], a DOE approach was employed due to simplicity. DOE techniques enable designers to scrutinize simultaneously the effects of many different factors that could influence the final output. Factorial experimentation is a method in DOE, in which the effects of each factor and combination of factors are estimated [75]. The following figures demonstrate a two and three factor design of experiment. Each point represents a unique combination of factors.

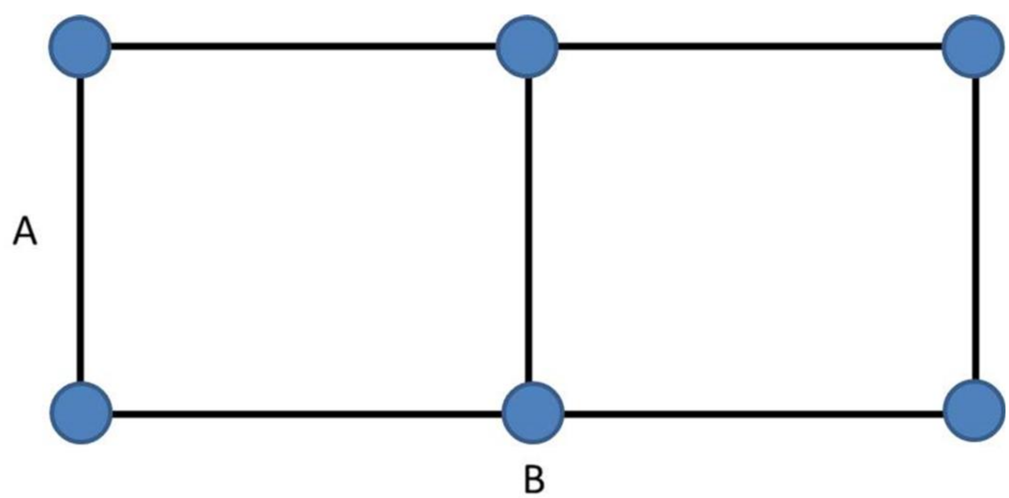

Figure 4. Two levels of factor A and three levels of factor B.

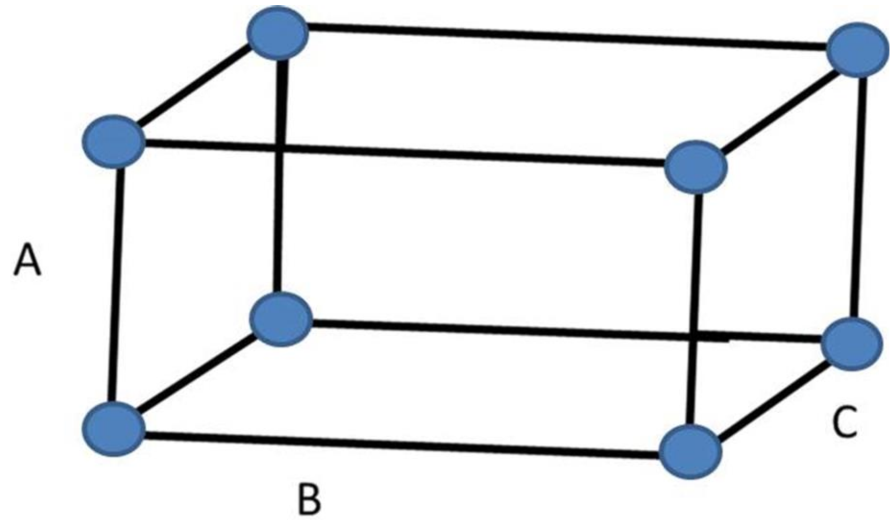

Figure 5. Three Factor DOE, two levels of each factor.

In this research, a factorial design of experiment is employed to find the sensitivity of each generating unit to the variations in the total demand. In addition, the same approach is employed to find the rational reaction set of each consumer in response to variations in the load demand from the other consumer. The results will help us to find the Nash equilibrium among consumers.

All simulations were performed on an Intel(R) Core(TM) i5-3470 CPU @ 3.20 GHz with 8.00 GB of installed memory in a 64-bit Operating system and on MATLAB R2014a software. All supplier units are considered to own a single wind turbine, solar panels and DGs. However, each of these components has a different capacity for each supplier. The strategic interactions among market participants are simulated with two consumers and two small scale electricity suppliers. Supplier one is considered to have an 11Kw wind turbine. Supplier two operates with a 5 Kw wind turbine. Both suppliers are equipped with solar systems, however, different features of these systems result in various solar generations by each individual energy cell. The MATLAB code to solve the suppliers' and consumers' problem is presented in the following.

RESULTS AND DISCUSSION

In this section, the hourly clearing prices of electricity, as well as the optimal behavior for all players is presented. In Table I diesel generators' cost function coefficients, as well as the load demand curve coefficients for the electricity price function are presented.

Table I. Cost and Price Coefficients.

	A	B	C	α	β
Units	$/Kw³h	$/Kw²h	$/Kwh	$/Kw²h	$/Kwh
Supplier 1	-0.0067	0.3333	0	0.001	0.24
Supplier 2	-0.0085	0.4972	0	0.001	0.24

The entire problem can be separated into three sections. First, the cooperative game among suppliers is modeled. Applying DOE to the game among suppliers, the rational reaction set of each player can be approximated as a function of the total load demand. In this context, the primary factor is considered to be Pd. Thus, 20 levels of values for Pd, each composed of data for 24 hours, are integrated into the suppliers' problem. These values must satisfy constraint 16. In other words, the values for which are used in the DOE, are compatible with the constraint associated with the dispatchable load. In the next step, we run the problem for each set of Pd and the optimized values for $\mathrm{Pw_i}$, $\mathrm{Ps_i}$, $\mathrm{Pdg_i}$, $\mathrm{Pdesd_i}$ and $\mathrm{Pg_i}$ are achieved. Finally, a linear equation as a function of Pd was obtained for each hour through regression for each of the generating units. For instance, equations (19) to (23) show a RRS of supplier one, as a function of total load demand at 12th hour. The RRS for the entire 24 hours for each supplier have been obtained.

$$\mathrm{Pw_{1,12}} = 0.219519 \times \mathrm{Pd} + 1.394027 \qquad (19)$$

$$Ps_{1,12} = 0.189506 \times Pd + 0.72126 \quad (20)$$

$$Pdg_{1,12} = 0 \quad (21)$$

$$Pdesd_{1,12} = 0.146153 \times Pd - 2.11172 \quad (22)$$

$$Pg_{1,12} = -0.05552 \times Pd - 0.00036 \quad (23)$$

On the other hand, finding Nash equilibrium among consumers also requires a factorial design method. In this context, every player's load demand was divided into 20 levels and used when solving the other players' problems. Every player solves its own problem for every level of the other players load demand. Finally using regression, each consumer's hourly load demand could be modeled as a linear equation as a function of the other players' load demand during the same time interval. Finding the intersection of the hourly linear equations provides us with the Nash equilibrium among consumers. Equation (24) shows the RRS of consumer 1 at 9th hour. It is a sample RRS to show how a RRS looks like for consumers at a specified hour.

$$Pd_{1,9} = 0.739359 \times Pd_{2,9} - 0.9095 \quad (24)$$

Finally, as the third part of the problem, optimum demand values of consumers were substituted into the suppliers' equations to acquire Nash equilibrium. No need to mention that, at Nash equilibrium, no player can obtain a higher payoff function through changing its own strategy unilaterally.

Figure 6 represents the generated power by supplier 1 through wind, solar, diesel generator, DESD and the power to/from the gird. It is worthy to note that the negative values for Pg_i refer to the electricity sold to the grid. When Pg_i is positive, the utility grid assists the suppliers in securing the demand from consumers. This particularly happens at times when the renewable resources are not available and the high costs of switching on a diesel generator motivate the suppliers to secure the power through the utility grid. For instance, at 5^{th} hour, supplier 1 sells approximately 1.2 Kwh to the grid.

At the same time interval, this player generates approximately 7 Kwh. The variations in power generation in Figure 6 result from two major factors. First, the natural uncertainty which is inherent in the renewable resources, for instance, the wind speed is not necessarily similar among two different time slots. The second factor that has a considerable impact on the power generation in different hours is the electricity price at that hour, which is a function of the electricity demand from the consumer's side. According to the Eq. (17), generation has to meet the demand at each time slot. For instance, if the generation is at its maximum at 7 p.m., the electricity price has to be at its minimum at 7 p.m. This results from the market behavior of the consumers which is demonstrated in Figure 7.

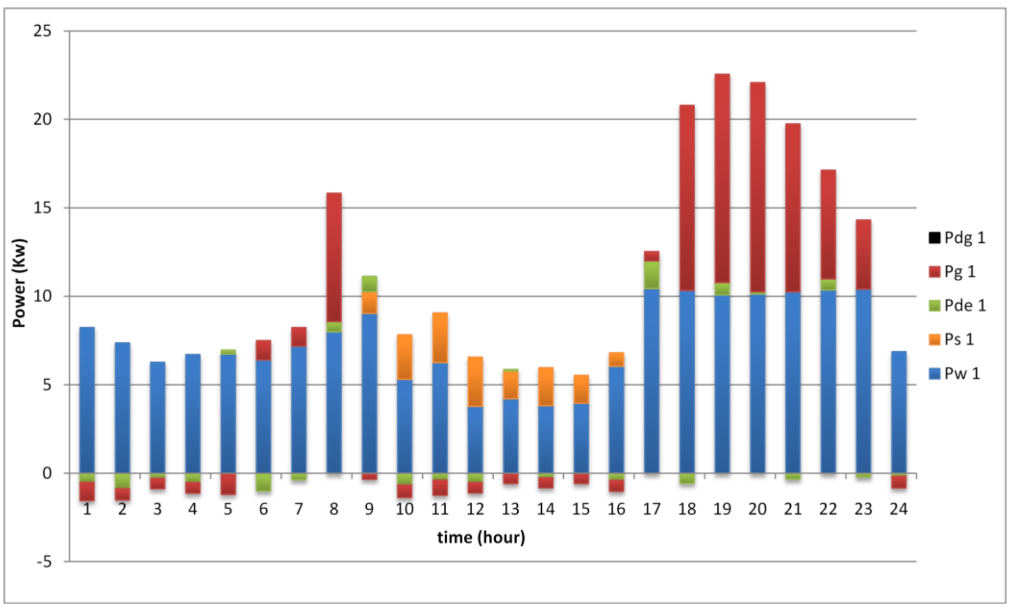

Figure 6. Provided power by player 1 through generation or grid.

Figure 7 shows the difference in consumer one's behavior before and after the optimization. Empirical evidence demonstrate that demand increases in response to a short-term price increase,[76] mainly because the level of convenience provided by electricity is so ingrained in consumers' lifestyle that they probably don't reduce their level of comfort to cut the electricity bill.[77] According to Figure 7, consumer 1 has increased its load demand at peak hours. This results from the electricity price function that is employed in this paper. The inverse proportionality of the electricity price to the aggregated load demand, makes consumers increase their load demands to decrease the electricity price.

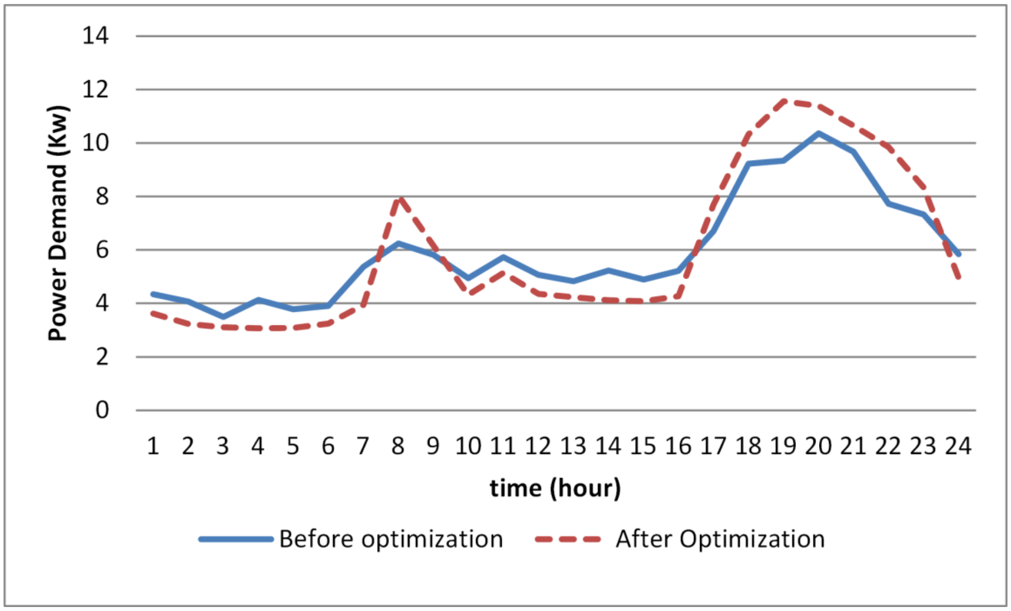

Figure 7. Difference in consumer one's behavior before and after the optimization.

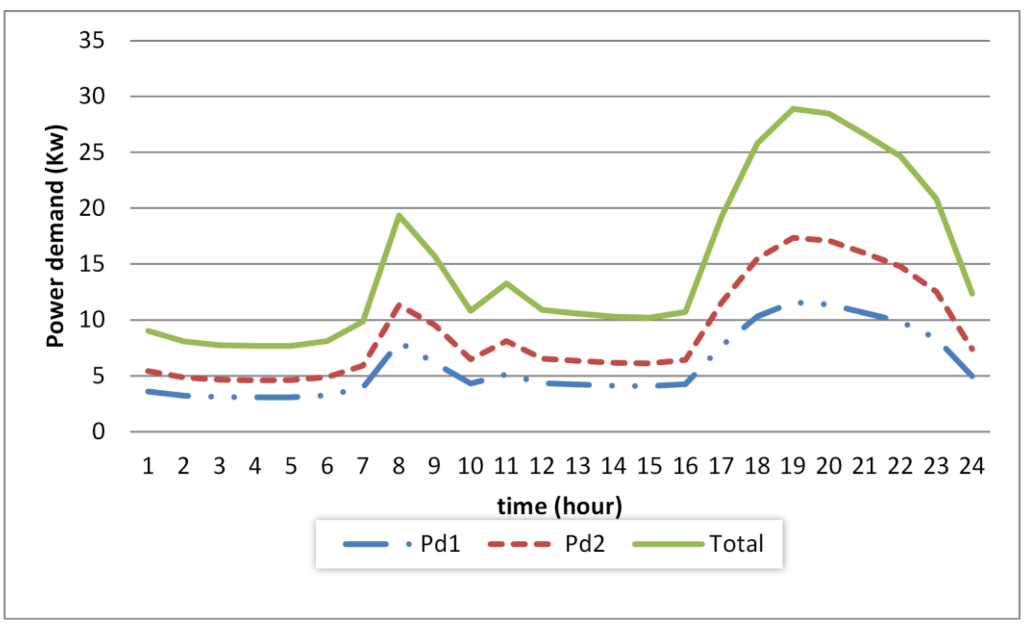

Figure 8. Consumers load demand.

The total load demand from the consumers' side is presented in Figure 8. Accordingly, it is apparent that the consumers have a tendency to increase their load demand at peak hour in an attempt to decrease the price of electricity. This pricing function does not incorporate the desirable incentives for consumers to cut their demand at peak hours and thus, is not suitable for demand side management techniques. The amount of generated power by each player and the aggregate power is presented in Figure 9. Given the equilibrium solution, the clearing price of the proposed restructured electricity market could be found for each hour and is shown in

Table II.

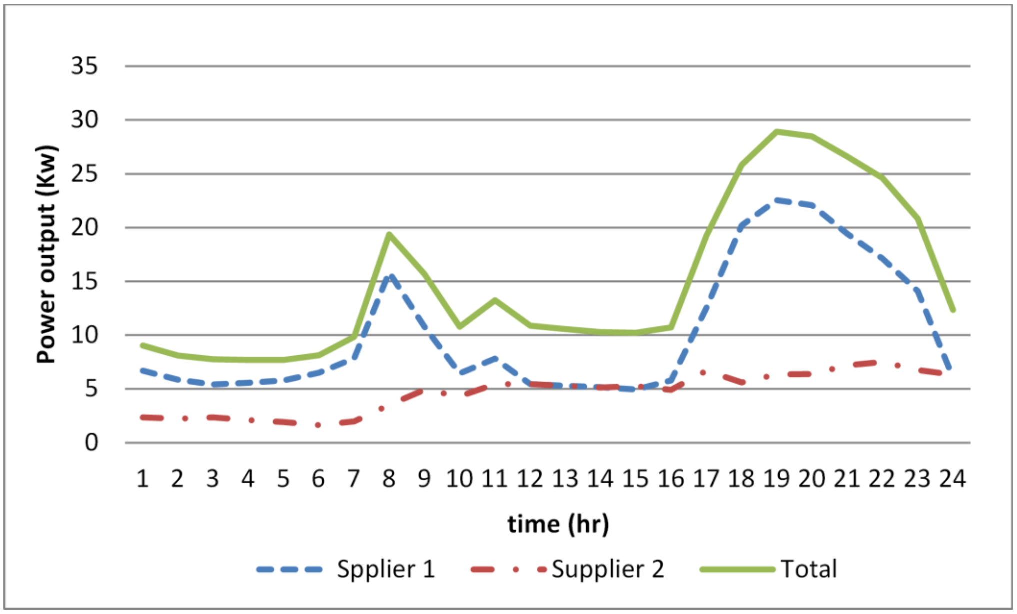

Figure 9. Power generation by each player.

Table II. Hourly Clearing Electricity Prices.

t	Price ($/Kwh)	T	Price ($/Kwh)	T	Price ($/Kwh)	t	Price ($/Kwh)
1	0.230963	7	0.230153	13	0.229435	19	0.211093
2	0.231907	8	0.220645	14	0.229715	20	0.211530
3	0.232246	9	0.224304	15	0.229793	21	0.213386
4	0.232319	10	0.229209	16	0.229290	22	0.215374
5	0.232301	11	0.226743	17	0.220764	23	0.219154
6	0.231877	12	0.229112	18	0.214184	24	0.227644

The retail electricity price is inversely proportional to the load demand. Therefore, the electricity price drops when the load demand increases. That's why at $t = 19$, the price of electricity is the lowest compared to the other hours. This behavior is also expressed in Figure 10.

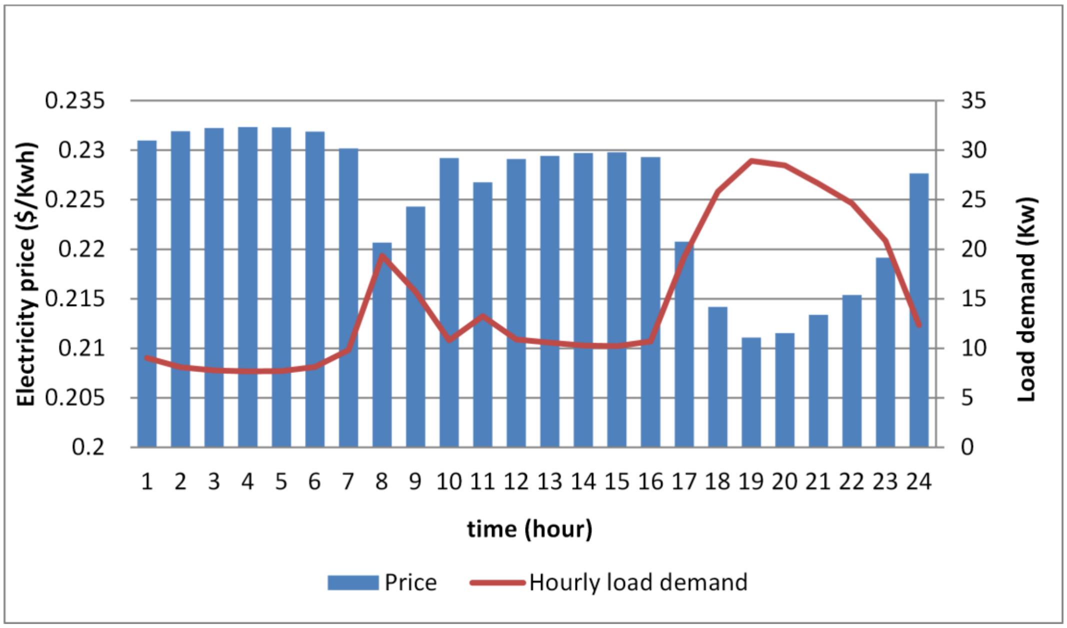

Figure 10. Electricity price and the total load demand.

Due to the high cost function of diesel generators, at Nash equilibrium, every generating entity prefers to switch off the diesel generator. Therefore, a portion of electricity needs to be secured by the electricity grid. The rest of the load demand is generated through wind and solar energies or it might be secured through one of the storage units. Figure 11 shows the reliance of each supplier on the electricity grid. There is a high load demand by consumers between hours 17 to 24. Due to the limitations of renewable resources in these hours, especially solar energy, both suppliers have the most dependency on the electricity grid at the aforementioned time.

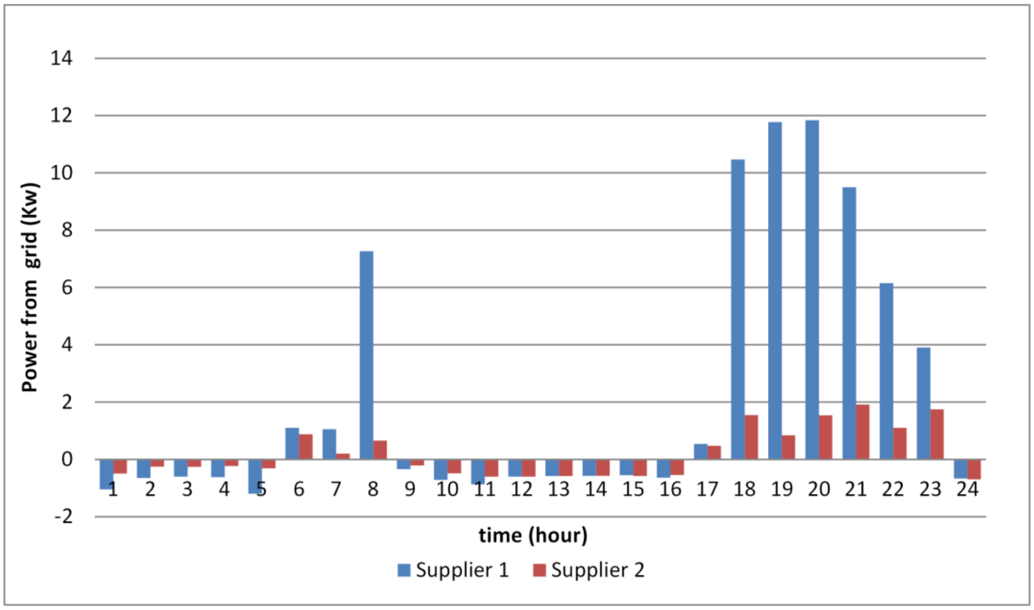

Figure 11. Suppliers' dependency on the utility grid.

Total renewable generation of suppliers is presented in Figure 12. Due to the differences in wind turbine generator features, supplier 1 has a greater share in generating power through renewable resources.

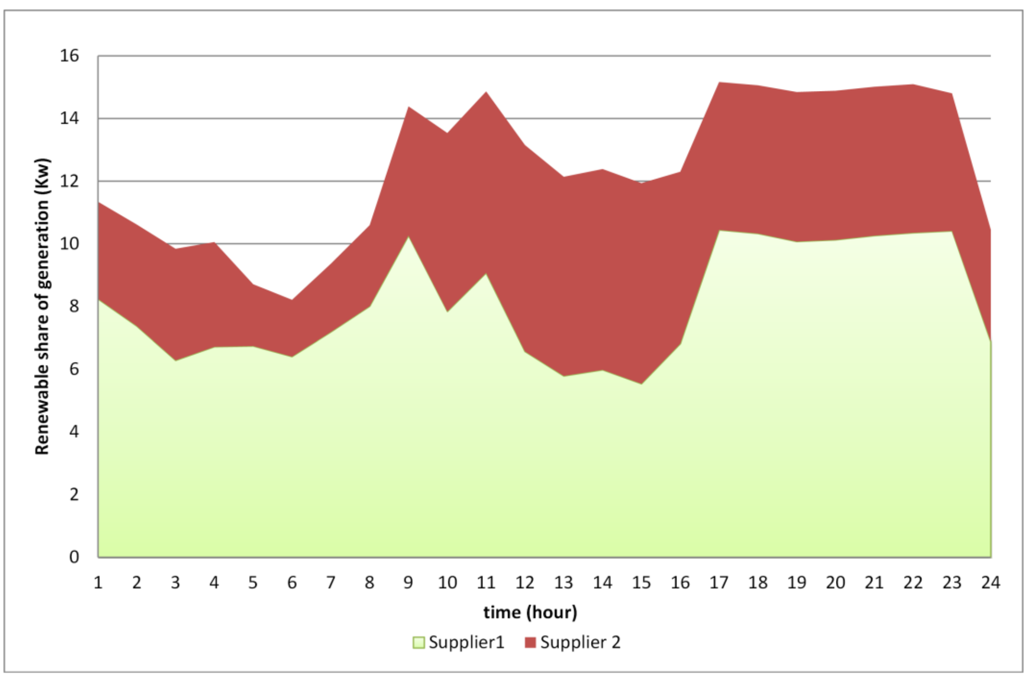

Figure 12. Renewable share of power generation by suppliers.

SUMMARY AND CONCLUSIONS

We have described a game based model for the restructuring electricity market, which is distinguished from other game-based approaches[78,79] in electricity market. The active participation of consumers in electricity generation in addition to their role in demand side load management is of crucial importance in this model. The current developments in electricity markets, particularly in smart grids, have necessitated a more significant role for consumers. In most cases, the influence of consumers on the market framework and structure is negligible. However, the current developments in zero energy buildings,[80]-[82] coupled with the increasing level of competition in the market with the goal of reducing the electricity prices for consumers ought to raise consumers' involvement within the market. Employing distributed generator units which operate on renewable resources in residential sector facilitates this process. A game theoretic approach provides a tool to analyze and optimize market participants' behavior.

The outcome of this simulation demonstrates the role and significance of pricing structures in managing the demand side load. While the inverse relationship of demand and price can precisely model the market behavior of various items, it cannot be applied to electricity markets. Basically, in any electricity market, regulators and market architectures seek for solutions to alleviate the variations in electricity consumption. In other words, a flat rate of consumption is more preferable to the one with peak and valleys. The electricity price function employed in this research makes consumers increase their load demand at peak hours, which is not a desirable output.

The market equilibrium is highly reliant on the mathematical model of electricity price function. Therefore, employing a pricing strategy which accurately reflects the complexities of the electricity market is of crucial importance. Pricing in a deregulated electricity market is a well-studied problem. With the currents efforts in reducing the peak load demand, it is necessary to recognize that a huge section of consumers, particularly in the residential sector, are capable of shifting and managing their load demand, if properly inspired by incentives. It is necessary to apply various electricity pricing models, particularly efficient personal pricing into the proposed electricity market. It assists in examining the volatility of market prices and the consequences on both suppliers and consumers. Once the economics of a smart grid with a high penetration of small suppliers is resolved, the procedures in this paper can be employed to optimize the behavior of the market participants.

FUTURE WORK

The main concern in modeling the market behavior arises from the electricity price function. Therefore, in the future work, it is necessary to comprehend thoroughly, the role and significance of this function. However, any slight variation in the electricity price function would change the whole market framework drastically and is some cases it eliminates the factor which makes the game possible among consumers and suppliers. In our ongoing research, we have tried to anticipate all the challenges in changing the electricity price function. At the same time, we have focused on modeling the market according to the reality of electricity markets in the U.S. The market model is basically inspired by the concept of micro-grid. In this model, the reliance of small suppliers on the utility grid is reduced to non-peak hours. As a result, the small and mid-size suppliers present in the market might need to switch on their diesel generators at peak hours in order to be able to secure the demand.

One important assumption in modeling the market is the fact that both consumers and suppliers are price-takers. In other words, electricity prices are imposed on the micro-grid by the utility grid, which also plays the role of an Independent System Operator (ISO). In other words, both consumers and suppliers have no say in the final price of the electricity and can only optimize their objective function according to the imposed prices. Consumers try to manage their own load by shifting a particular percentage of their hourly load demand. Due to the variations in the electricity price function and imposed prices on the market, consumers have no interaction with each other and thus, their behavior cannot be modeled as a game. Instead, a simple optimization problem is employed to optimize each customer's objective function.

After careful consideration of various pricing structures, Time of Use (TOE) pricing structure is considered to model the market. These hourly prices reflect the variations in the wholesale electricity market. Consumers receive the price signals one day ahead and they can manage their load demand accordingly. The required data for this type of pricing is extracted from the American Illinois website.[83] Table III shows the hourly electricity prices for a random day in winter.

Table III. TOU Electricity Prices.

T	Price ($/Kwh)	t	Price ($/Kwh)	t	Price ($/Kwh)	T	Price ($/Kwh)
1	0.042	7	0.055	13	0.046	19	0.052
2	0.042	8	0.051	14	0.045	20	0.05
3	0.042	9	0.049	15	0.045	21	0.048
4	0.043	10	0.048	16	0.045	22	0.046
5	0.044	11	0.047	17	0.048	23	0.045
6	0.048	12	0.046	18	0.056	24	0.043

Although there's no game among consumers, the suppliers are still in a game to acquire a larger share of the market, particularly in hours with a higher price for electricity. While suppliers can sell their extra power to the grid, the market is designed in way such that there's more incentives for suppliers to sell their power within the market. This feature of the market signifies the importance of storage units. This market feature results from the difference in the electricity price in the micro-grid and the utility grid.

In order to find the Nash equilibrium among suppliers, optimum sensitivity approach can be employed. As the Taylor's series expansion suggests, derivatives of a function can be used to approximate the function. This prominent feature of the derivatives results in the development of the sensitivity analysis. This function entails the necessary conditions of Kuhn-Tucker.

Below, a typical optimization problem subject to a number of equality and inequality constraints is introduced.

$$\text{Min } f(x, y) \tag{25}$$

Subject to:

$$h_i(x, y) = 0 \qquad i = 1,2, \ldots, p \tag{26}$$

$$g_j(x, y) \leq 0 \qquad j = 1,2, \ldots m \tag{27}$$

where $x \in R^n$ is the design variable vector and p is the given problem parameter. $f(x, y)$ is the objective function and $h_i(x, y)$ and $g_j(x, y)$ are equality and inequality constraints. p and m are the number of equality and inequality constraints, respectively. The Lagrangian function can be defined as:

$$L = f + \sum_{i}^{p} r_i h_i + \sum_{j}^{m} \lambda_j g_j \quad (28)$$

where r_i and λ_j are Lagrange multipliers. Assuming that $x^* \in R^n$, it is then known that x^*satisfies the Kuhn-Tucker conditions.[84]

$$\frac{dL}{dx}(x^*, y) = 0 \quad (29)$$

$$h_i(x^*, y) = 0 \qquad i = 1,2, \ldots, p \quad (30)$$

$$g_j(x^*, y) \leq 0 \qquad j = 1,2, \ldots, m \quad (31)$$

$$\lambda_j g_j(x^*, y) = 0 \qquad j = 1,2, \ldots, m \quad (32)$$

$$\lambda_j \geq 0 \qquad j = 1,2, \ldots, m \quad (33)$$

Note that the inequality constraints can be classified into active and inactive constraints. For active constraints:

$$g_j(x^*, y) = 0 \qquad j \in \overline{m} \quad (34)$$

$$\lambda_j \geq 0 \qquad j \in \overline{m} \quad (35)$$

For inactive constraints:

$$g_i(x^*, y) < 0 \qquad j \notin \overline{m} \quad (36)$$

$$\lambda_j = 0 \qquad j \notin \bar{m} \tag{37}$$

where $\bar{m}$ is the collection of active constraints. Equation (28) can be written as:

$$L = f + \sum_{i}^{p} r_i h_i + \sum_{j\in \bar{m}} \lambda_j g_j \tag{38}$$

The Kuhn-Tucker necessary conditions show that the optimal solution x^* and the Lagrange multipliers are subject to change, with variations in y. Thus, the optimal solutions are functions of problem parameter y and their derivatives with respect to y, are called optimum sensitivity derivatives.[85] When the problem parameter changes by a small amount, we assume that equations (29-30) and equations (34 to (38) remain unchanged. Differentiating these equations with respect to y will lead to:

$$\frac{\partial^2 L}{\partial x^2}\frac{dx^*}{dy} + \sum_{i}^{p} \frac{\partial h_i}{\partial x}\frac{dr_i}{dy} + \sum_{j\in\bar{m}} \frac{\partial g_j}{\partial x}\frac{d\lambda_j}{dy} + \frac{\partial^2 L}{\partial x\,\partial y} = 0 \tag{39}$$

$$[\frac{\partial h_i}{\partial x}]^T \frac{dx^*}{dy} + \frac{\partial h_i}{\partial y} = 0 \qquad i= 1,2, \ldots,p \tag{40}$$

$$[\frac{\partial g_j}{\partial x}]^T \frac{dx^*}{dy} + \frac{\partial g_j}{\partial y} = 0 \qquad j \in \bar{m} \tag{41}$$

where the Hessian of L can be written as:

$$\frac{\partial^2 L}{\partial x^2} = \frac{\partial^2 f}{\partial x^2} + \sum_{i}^{p} r_i \frac{\partial^2 h_i}{\partial b^2} + \sum_{j\in\bar{m}} \lambda_j \frac{\partial^2 g_j}{\partial x^2} \tag{42}$$

Finally, the aforementioned equations can be put in a matrix form for the sake of simplicity.

$$\begin{matrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial h}{\partial x} & \frac{\partial g}{\partial x} \\ \left[\frac{\partial h}{\partial x}\right]^T & 0 & 0 \\ \left[\frac{\partial g}{\partial x}\right]^T & 0 & 0 \end{matrix} \quad \begin{matrix} \frac{db^*}{dx} \\ \frac{dr}{dy} \\ \frac{d\lambda}{dy} \end{matrix} = - \begin{matrix} \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial h}{\partial y} \\ \frac{\partial g}{\partial y} \end{matrix} \qquad (43)$$

This matrix can be employed to find the rational reaction set of the suppliers. Due to the multitude number of inequality constraints in our problem, this matrix would be of considerable size.

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APPENDIX

G. MATLAB code

1. MATLAB code for suppliers

```
clc
clear all
Theta= 0.001;
Beta=0.24;
Pwind1=[8.83495 6.001320193 7 6.182734696 6.73975 6.40149 7.1869 8.00596 9.0395 3.13168 7 5.0024 5.0096 5 5 7.5 10.4268 10.3289 10.0711 10.1254 10.2601 10.3513 10.3009 10.3829];
Pwind2=[3.09896 3.23719 2.2571494 3 1.96896 1.81121 2.17802 2.58839 3.2167 2.57708 2.68142 2.5 2.5 2.5 2.5 4.42446 4.72015 4.73015 4.77272 4.75965 4.75196 4.74351 4.37805 4.61476];
Psolar1=[0 0 0 0 0 0 0 0 1.2241 3.61882 3.5865 4 2.51 3 2 1.33088 0.0171051 0 0 0 0 0 0 0];
Psolar2=[0 0 0 0 0 0 0 0 0.906923 3.11935 3.10721 3.06 2 2.2688 2.81419 1.06094 0 0 0 0 0 0 0 0];
Pdg1 =[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pdg2 =[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pdesd1=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pdesd2=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pgrid1 =[-0.2944 0 0 0 1.6462 1.8278 3.5744 5.4652 1.6642 0 -0.9334 0 0 0 0 -0.5506 0.0014 6.7976 9.7427 9.3137 6.9377 3.5204 2.7404 -0.9811];
Pgrid2 =[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pdi=[9.03688 8.09304 7.754 7.68136 7.6988 8.1228 9.84744 15.10512 14.58316 11.7282739 14.65652 12.4289884 10.85050739 10.30396782 10.51989218 13.20291393 15.389 25.8156 28.9074 28.46976 26.6142 24.6264 20.5724564 14.85132];
x0=[Pwind1,Psolar1, Pdg1,Pdesd1,Pgrid1;Pwind2,Psolar2,Pdg2,Pdesd2,Pgrid2];
options = optimset('Display','iter','Algorithm','active-set','MaxIter', 1000,
'MaxFunEvals',1e6);
```

```
[x,xval] = fmincon(@(s)
cooperativegame(s,Pdi),x0,[],[],[],[],[],[],@Cooperativegameconstr,options);
```

Where the two functions cooperativegame(s,Pdi) and Cooperativegameconstr(s) are presented below in a respective order.

```
function [profit]= cooperativegame(s,Pdi)
Beta=0.24;
Theta= 0.001;
a=[-0.0067;-0.0085];
b=[0.3333;0.4972];
for i=1:2
   for t=1:24
       Pwind(i,t)=s(i,t);
       j=t+24;
       Psolar(i,t)=s(i,j);
       k=t+48;
       Pdg(i,t)=s(i,k);
       l=t+72;
       Pdesd(i,t)=s(i,l);
       m=t+96;
       Pgrid(i,t)=s(i,m);
   end
end
for t=1:24
   P(t)= ((-Theta*(Pdi(t))+Beta)* (Pwind(1,t)+Psolar(1,t)+ Pdg(1,t)+Pdesd(1,t))-
(a(1)*Pdg(1,t)^2 + b(1)*Pdg(1,t)))*((-Theta*(Pdi(t))+Beta)* (Pwind(2,t)+Psolar(2,t)+
Pdg(2,t)+Pdesd(2,t))-(a(2)*Pdg(2,t)^2 + b(2)*Pdg(2,t)));
end
profit=-sum(P);
end
```

```
function [c,ceq] = Cooperativegameconstr(s)
Pdi=[9.03688 8.09304 7.754 7.68136 7.6988 8.1228 9.84744 15.10512 14.58316 11.7282739 14.65652 12.4289884 10.85050739 10.30396782 10.51989218 13.20291393 15.389 25.8156 28.9074 28.46976 26.6142 24.6264 20.5724564 14.85132];
a=[-0.0067;-0.0085];
b=[0.3333;0.4972];
Deltat=1;
Edesd=2;
Theta= 0.001;
Beta=0.24;
Pdgmin=[5;7.5];
Pdgmax=[20;30];
Pdesdmin=3.84;
Pdesdmax=0.816;
SOCmin=0.2;
SOC1=[0.5;0.5];
SOCmax=0.8;
SOCend=0.5;
Pwindmax=[8.83495 9.06266 9.54897 9.23104 6.73975 6.40149 7.1869 8.00596 9.0395 7.98221 8.15781 9.4389 10.5132 10.6179 10.5894 10.3779 10.4268 10.3289 10.0711 10.1254 10.2601 10.3513 10.3009 10.3829; 3.09896 3.23719 3.55642 3.34128 1.96896 1.81121 2.17802 2.58839 3.2167 2.57708 2.68142 3.48534 4.4299 4.68902 4.75766 4.42446 4.72015 4.73015 4.77272 4.75965 4.75196 4.74351 4.37805 4.61476];
Psolarmax=[0 0 0 0 0 0 0 0 1.2241 3.61882 3.5865 6.51982 6.50493 4.26972 3.31097 1.33088 0.0171051 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0.906923 3.11935 3.10721 5.94713 5.93777 3.75698 2.85016 1.06094 0 0 0 0 0 0 0 0];
Pwind=zeros(2,24);
Psolar=zeros(2,24);
Pdg=zeros(2,24);
```

```
Pdesd=zeros(2,24);
Pgrid=zeros(2,24);
c=zeros(2,313);
ceq=zeros(1,24);
Lambda=zeros(1,24);
for i=1:2
    for t=1:24
        Pwind(i,t)=s(i,t);
        j=t+24;
        Psolar(i,t)=s(i,j);
        k=t+48;
        Pdg(i,t)=s(i,k);
        l=t+72;
        Pdesd(i,t)=s(i,l);
        m=t+96;
        Pgrid(i,t)=s(i,m);
    end
end
 %Wind Constraints************************************
 for i=1:2
    for t=1:24
        c(i,t)=Pwind(i,t)/Pwindmax(i,t)-1;
    end
end
for i=1:2
    for t=1:24
        c(i,t+24)=0-Pwind(i,t);
    end
end
  %DG Constraints****************************************
  for i=1:2
```

```
    for t=1:24
        c(i,t+48)=0-Pdg(i,t);
    end
 end
 for i=1:2
    for t=1:24
        if Pdg(i,t) <= 0.1
            c(i,t+72)=0;
            c(i,t+96)=0;
       else
             c(i,t+96)=(Pdg(i,t)/Pdgmax(i))-1;
        end
    end
 end
%Solar Constraints***********************************
 for i=1:2
   for t=1:24
       c(i,t+120)=0-Psolar(i,t);
   end
end
for i=1:2
   for t=1:24
       c(i,t+144)=Psolar(i,t)-Psolarmax(i,t);
   end
end
 %Desd Constraints[Pdesd can be negative]************
 for i=1:2
   for t=1:24
       if Pdesd(i,t)<=0  %charge
         c(i,t+168)= (abs(Pdesd(i,t))/Pdesdmax)-1;
       elseif Pdesd(i,t)>0 %decharge
```

```
        c(i,t+168)= (Pdesd(i,t)/Pdesdmin)-1 ;
      end
   end
end
for i=1:2
   for t=1:24
      c(i,t+192)= (SOCmin/SOC1(i,t))- 1;
      c(i,t+216)= (SOC1(i,t)/SOCmax)- 1;
      SOC1(i,t+1)=SOC1(i,t)- (Pdesd(i,t)*Deltat)/Edesd;
   end
end
for i=1:2
   c(i,241)= SOCend/SOC1(i,24)-1; %Only for the last SOC
end
for i=1:2
   for t=1:24
      if Pgrid(i,t)<0  %Selling to grid%
        c(i,t+265)=                                   abs(Pgrid(i,t))-0.1
*(Pwind(i,t)+Psolar(i,t)+Pdg(i,t)+Pdesd(i,t));%Selling to grid%
      end

   end
end
for i=1:2
   for t=1:24
      c(i,t+289)=(a(i)*Pdg(i,t)^2        +        b(i)*Pdg(i,t))-((-Theta*(Pdi(t))+Beta)*
(Pwind(i,t)+Psolar(i,t)+ Pdg(i,t)+Pdesd(i,t)));
   end
end

for t=1:24
```

```
    ceq(t)=Pdi(t)-
sum(Pwind(1,t)+Psolar(1,t)+Pdg(1,t)+Pdesd(1,t)+Pgrid(1,t)+Pwind(2,t)+Psolar(2,t)+Pdg(
2,t)+Pdesd(2,t)+Pgrid(2,t));
end
for t=1:24
    Lambda(t)= -Theta* Pdi(t)+Beta;
end
end
```

2. MATLAB code for consumers

```
clear all
Theta= 0.001;
Beta=0.24;
Pbase1=[4.5184 4.0465 3.8770 3.8407 3.8494 4.0614 4.9237 6.2938 6.0763 5.3954
5.6371 5.4453 5.2826 5.1423 5.1036 5.3665 6.4121 8.6052 9.6358 9.4899 8.8714 8.2088
6.9485 5.7120];
Pdi1=[4.3370 4.0717 3.4904 4.1355 3.7758 3.8999 5.3674 6.2442 5.8183 4.9471 5.7298
5.0675 4.8315 5.2251 4.8862 5.2183 6.7004 9.2219 9.3427 10.3663 9.6762 7.7262
7.3213 5.8440];
Pdi2=[5.727158107 5.700717045 5.067346565 6.029151241 6.235227004 5.743641928
6.650054279 8.587164928 10.2260405 6.901379203 9.624010253 8.658452852
7.894164231 6.646866664 6.598130996 7.495259587 10.71691127 12.84621982
15.44657593 15.14150585 13.186224 14.11884236 9.040825074 9.83402609];
x0=[Pdi1];
options = optimset('Display','iter','Algorithm','active-set','MaxIter', 1000,
'MaxFunEvals',1e6);
[x,xval] = fmincon(@(s)
Consumer1game(s,Pdi2),x0,[],[],[],[],[],[],@Consumer1constr,options);
```

Where the functions Consumer1game(s,Pdi2) and Consumer1constr(s) are presented below in the respective order.

```
function [profit]= Consumer1game(s,Pdi2)
Beta=0.24;
Theta= 0.001;
Pbase1=[4.5184  4.0465  3.8770  3.8407  3.8494  4.0614  4.9237  6.2938  6.0763  5.3954
5.6371 5.4453 5.2826 5.1423 5.1036 5.3665 6.4121 8.6052 9.6358 9.4899 8.8714 8.2088
6.9485 5.7120];
for t=1:24
   Pdi1(t)=s(t);
end
for t=1:24
   P(t)=(-Theta*(Pdi1(t)+Pdi2(t))+Beta)*(Pdi1(t));
end
profit=sum(P);
end

function [c,ceq] = Consumer1constr(s)
Theta= 0.001;
Beta=0.24;
Sigma1=0.8;
Sigma2=1.2;
c=zeros(2,48);
ceq=zeros(1,24);
for t=1:24
     Pdi1(t)= s(t);
end
%Pbase for 17.78 families
Pbase1=[4.5184  4.0465  3.8770  3.8407  3.8494  4.0614  4.9237  6.2938  6.0763  5.3954
5.6371 5.4453 5.2826 5.1423 5.1036 5.3665 6.4121 8.6052 9.6358 9.4899 8.8714 8.2088
6.9485 5.7120];
for t=1:24
```

```
        c(t)=Sigma1*Pbase1(t)/Pdi1(t)-1;
        c(t+24)=1-Sigma2*Pbase1(t)/Pdi1(t);
end
    ceq(1)=sum(Pdi1(1,:))/sum(Pbase1(1,:))-1;
end
```