Gamma and Related Functions

The complete gamma function is defined as,
\[ \Gamma(\nu) = \int_0^\infty t^{\nu-1} e^{-t} \, dt \]

The incomplete gamma function is defined as,
\[ \gamma(\nu, x) = \int_0^x t^{\nu-1} e^{-t} \, dt \]

The complementary gamma function required is defined as,
\[ \Gamma(\nu, x) = \int_x^\infty t^{\nu-1} e^{-t} \, dt \]

The Nash model question in your homework requires that you evaluate the following integral,
\[ \Gamma(n, x) = (n-1)! e^{-x} \sum_{k=1}^{n} \frac{x^{k-1}}{(k-1)!} = (n-1)! e^{-x} \left( 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^{a-1}}{(n-1)!} \right) \]

Therefore, because \( \Gamma(\nu) = \gamma(\nu, x) + \Gamma(\nu, x) \),
\[ \frac{\gamma(n, t/k)}{\Gamma(n)} = 1 - \frac{\Gamma(n, t/k)}{\Gamma(n)} \]

For integer values of \( \nu \) (which is the case for this homework), the complementary gamma function can be evaluated using the following expression,