Turbulent Velocity Profile

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Turbulent Flow Equations

\[\begin{align*}
\frac{\partial v_x}{\partial t} + v_y \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} &= g_x - \frac{1}{\rho_m} \frac{\partial p}{\partial x} + \nu_m \nabla^2 v_x - \left[ \frac{\partial v_x^2}{\partial x} + \frac{\partial v_y^2}{\partial y} + \frac{\partial v_z^2}{\partial z} \right] \\
\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} &= g_y - \frac{1}{\rho_m} \frac{\partial p}{\partial y} + \nu_m \nabla^2 v_y - \left[ \frac{\partial v_x v_y}{\partial x} + \frac{\partial v_y^2}{\partial y} + \frac{\partial v_z v_y}{\partial z} \right] \\
\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} &= g_z - \frac{1}{\rho_m} \frac{\partial p}{\partial z} + \nu_m \nabla^2 v_z - \left[ \frac{\partial v_x v_z}{\partial x} + \frac{\partial v_y v_z}{\partial y} + \frac{\partial v_z^2}{\partial z} \right]
\end{align*}\]
Incipient Motion
Open Channel

- **Pressure Distribution**
  \[ \int_0^h dp = \rho \int_z^h -g \, dz \]
  \[ p = \rho g (h - z) \]

- **Shear Stress**
  \[ \tau_{zx} = \rho g (h - z) S_f \]
  \[ \tau_a = \rho g h S_f \]

- **Shear Velocity**
  \[ u_s = \sqrt{\frac{\tau_a}{\rho}} = \sqrt{gh S_f} \]
Saint-Venant Equation

**Bed Slope**

\[ S_o = -\frac{\partial z_o}{\partial x} \]

**Free Surface Slope**

\[ S_w = -\frac{\partial (z_o + h)}{\partial x} = S_o - \frac{\partial h}{\partial x} \]

**Energy Slope**

\[ S_f = -\frac{\partial \left( z_o + h + \frac{V^2}{2g} \right)}{\partial x} = S_o - \frac{\partial h}{\partial x} - \frac{V \partial V}{g} \]

**Unsteady Flow Saint-Venant Equation**

\[ S_f = S_o - \frac{\partial h}{\partial x} - \frac{V \partial V}{g} - \frac{\partial V}{g \partial t} \]
Logarithmic Velocity Profile

\[ \frac{v_x}{u_*} = \frac{1}{\kappa} \ln \frac{z}{z_o} \]
Logarithmic Velocity Profile

\[ v_x \delta = \frac{11.6 u}{\delta} = 104 z_o \]

\[ \delta = \frac{11.6 v}{u_\ast} \]

\[ \delta = \begin{cases} 3 d_s < \delta & \text{Smooth} \\ \frac{d_s}{3} & \text{Transition} & 4 < \text{Re}_\ast < 70 \\ d_s > 6 \delta & \text{Rough} & \text{Re}_\ast > 70 \end{cases} \]

\[ v = \frac{u_\ast}{\kappa} \ln \left( \frac{9.05 \frac{z u_\ast}{v}}{\nu} \right) \]

\[ v = \frac{u_\ast}{\kappa} \ln \left( \frac{30.2 \frac{z \chi}{k_s}}{\nu} \right) \]

\[ v = \frac{u_\ast}{\kappa} \ln \left( \frac{30.2 \frac{z}{k_s}}{\nu} \right) \]
Depth Average Velocity

- **One Point Method**
  - Measured down from water surface at 60% of the total flow depth

- **Two Point Method**
  - Average the velocity at 20 and 80% of the total flow depth

- **Three Point Method**
  - Average of the one-point and two-point methods.

- **Surface Method**
  - Determine surface velocity using a float and multiply the velocity by a coefficient to determine the average velocity
Resistance to Flow

\[ V = 5 \left( \frac{h}{d_{50}} \right)^{1/6} \sqrt{g \cdot h \cdot S_f} \]

\[ \sqrt{\frac{8}{f}} = 5.75 \log \frac{2h}{d_{50}} \]

\[ n = 0.064 \cdot d_{50}^{1/6} \text{ with } d_{50} \text{ in m} \]

Manning-Strickler
Example – Rhine River

In 1998 a flood was observed on the Rhine River, in the Netherlands. The following is the data that was obtained on the Rhine on November 3rd.

Q = 9,464 cms
S = 13.12 cm/km
h = 9.9 m (from velocity profile)
W = 260 m
d_{90} = 12.190 mm
d_{50} = 1.182 mm

The velocity profile is given as follows:

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Velocity (m/s)</th>
<th>Depth (m)</th>
<th>Velocity (m/s)</th>
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Determine the shear, mean and fall velocities.
Example - Shear Velocity

\[
u_\ast = \sqrt{ghS_f} = \sqrt{\frac{9.81 m}{s^2} \times 9.9 m \times 0.0001312}
\]

\[
u_\ast = 0.113 \frac{m}{s}
\]
Example - Mean Velocity

- **Method A** – at 60%
  \[ h = 9.9m \]
  \[ z = h_{60\%} = 3.96m \]
  \[ v_{\text{mean}} = 1.86 \frac{m}{s} \]

- **Method B** – at 20% and 80%
  \[ h = 9.9m \]
  \[ z = h_{20\%} = 7.92m \]
  \[ z = h_{80\%} = 1.98m \]
  \[ v_{\text{mean}} = \frac{2.04 \frac{m}{s} + 1.59 \frac{m}{s}}{2} \]
  \[ v_{\text{mean}} = 1.82 \frac{m}{s} \]

- **Method C** – Average of Method A and Method B
  \[ v_{\text{mean}} = \frac{1.86 \frac{m}{s} + 1.82 \frac{m}{s}}{2} \]
  \[ v_{\text{mean}} = 1.84 \frac{m}{s} \]
Turbulent Velocity Profiles and Resistance to Flow

Problem #1 (100%)
Field measurements along a vertical profile of the Rhine River are shown below. The navigable channel width covers 260 m. Consider a rectangular section to determine the hydraulic radius. The bed material is typically $d_{10} = 0.4$ mm, $d_{50} = 1.3$ mm and $d_{90} = 10$ mm. The measured slope of the Energy Grade Line was 13.12 cm per km on Nov. 3. Show the velocity profile on linear scale, and also provide a semi-log plot with a fitted line to the data to graphically determine the value of kappa.

Determine the following parameters in SI:

a) flow depth
b) hydraulic radius
c) ratio of hydraulic radius to flow depth
d) shear stress in Pascals
e) shear velocity
f) von Kármán constant
g) mean flow velocity in m/s (3 points)
h) Froude number
i) Manning n
j) laminar sublayer thickness in mm

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Positio from axis</th>
<th>Depth m water</th>
<th>Concentration u mg/l</th>
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